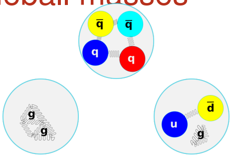
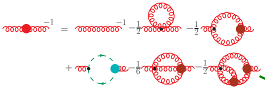
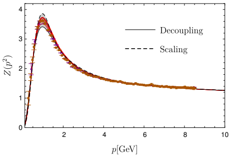
On the role of truncations of the 3PI effective action for glueball masses

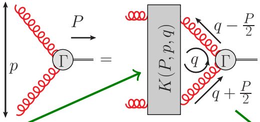
Markus Q. Huber

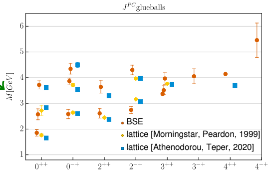
Institute of Theoretical Physics, Giessen University











QuantFunc

September 4, 2024

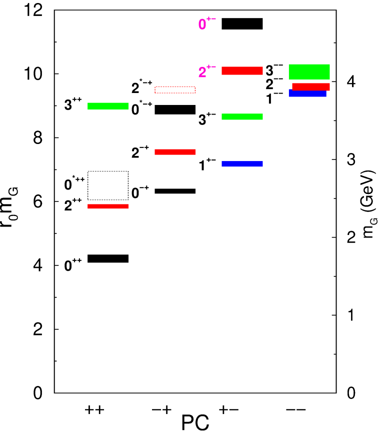
Valencia, Spain

JUSTUS-LIEBIG-UNIVERSITÄT GIESSEN

DFG Deutsche Forschungsgemeinschaft

Glueball results

Pure gauge theory glueballs known [Morningstar, Peardon, Phys. Rev. D60 (1999)].



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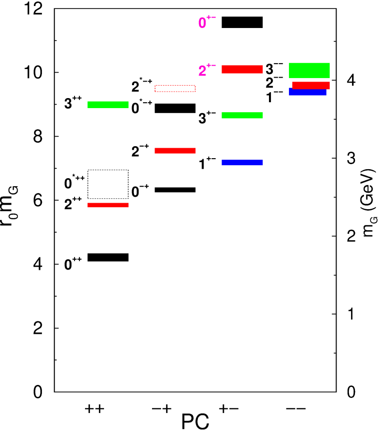
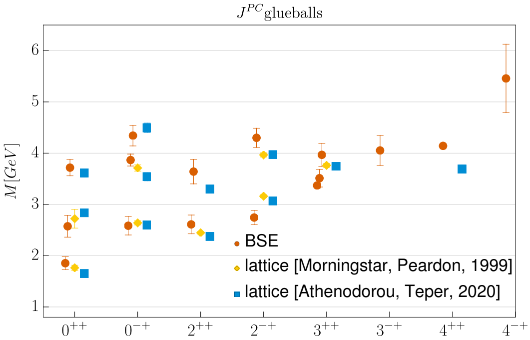


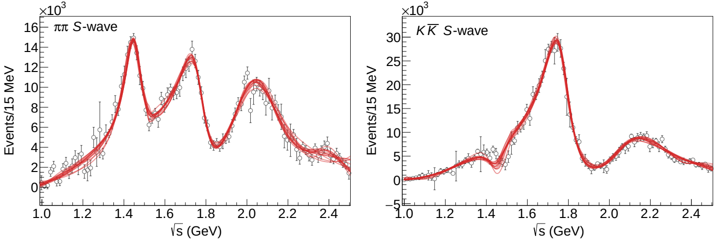


Lattice update: [Athenodorou, Teper, JHEP11 (2020)]

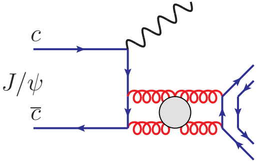
Functional results: [MQH, Fischer, Sanchis-Alepuz, Eur.Phys.J.C81 (2021)]

$\rightarrow$ Agreement with lattice results.

Glueballs from J/ψ decay



[JPAC Coll., Rodas et al., Eur.Phys.J.C 82 (2022)]



Scalar glueball candidate: Coupled-channel analyses of exp. data (BESIII)

- +add. data, largest overlap with $f_0(1770)$ [Sarantsev, Denisenko, Thoma, Klempt, Phys. Lett. B 816 (2021)]
- largest overlap with $f_0(1710)$ [JPAC Coll., Rodas et al., Eur.Phys.J.C 82 (2022)]

Pseudoscalar glueball candidate:

- $X(2370)$ [Ablikim et al. (BESIII), PRL132 (2024)]

Challenges/Reliability

Lattice:

- Statistical method $\rightarrow$ error bars
- Continuum extrapolation

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Yang-Mills:

Lattice:

- Statistical method $\rightarrow$ error bars
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Functional:

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QCD:

Lattice:

- Statistical method, but poor signal-to-noise ratio
- Continuum extrapolation
- Operator basis
- Mixing between operators
- Physical quark masses

Functional:

- Truncation errors
- Time-like momenta $\rightarrow$ Extrapolation errors
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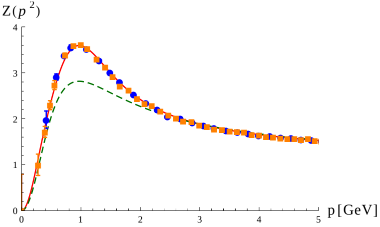
Input for bound state equations: 2013?

Bound state equations require propagators and vertices as input.

Propagators from 2013 [Sternbeck, hep-lat/0609016; MQH, von Smekal, JHEP 04 (2013)]:

Gluon dressing function:



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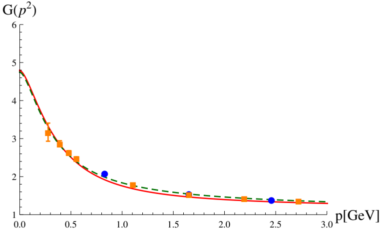

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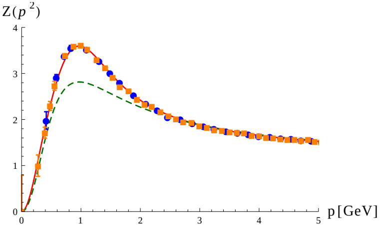
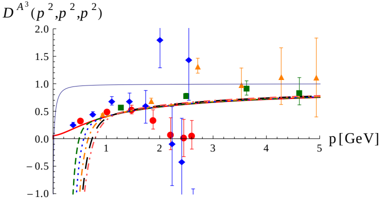


Three-gluon vertex [Cucchieri, Maas, Mendes, Phys. Rev. D 77 (2008); MQH, Phys. Rev. D 101 (2020)]:



Used a three-gluon vertex model tailored to reproduce lattice results.
$\rightarrow$ Insufficient for bound states.

Model based BSE calculations ($J = 0$)

- [Meyers, Swanson, Phys.Rev.D87 (2013)]
- [Sanchis-Alepuz, Fischer, Kellermann, von Smekal, Phys.Rev.D92, (2015)]
- [Souza et al., Eur.Phys.J.A56 (2020)]
- [Kaptari, Kämpfer, Few Body Syst.61 (2020)]

$\rightarrow$ Calculations possible, but no quantitative prediction yet.

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Better models? Alternative: Self-consistently calculated input.

Input for bound state equations: 2020?

[MQH, Phys. Rev. D 101 (2020)]

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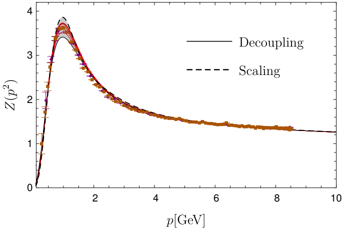

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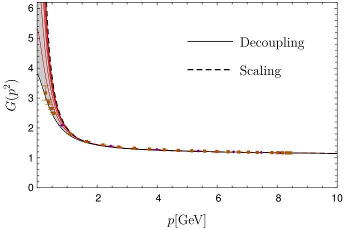

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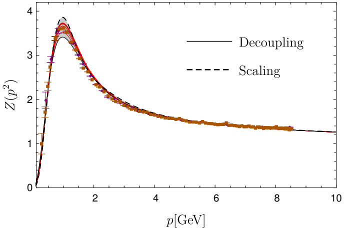

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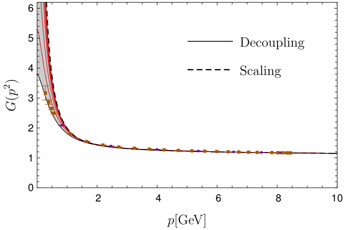

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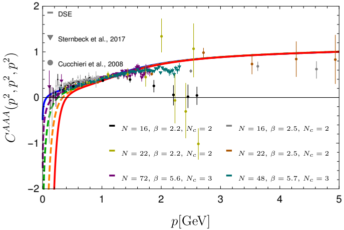

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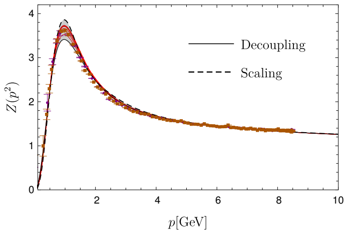

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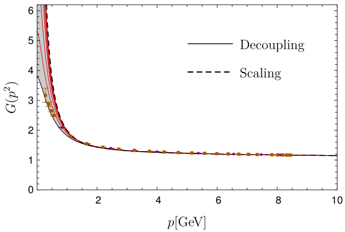

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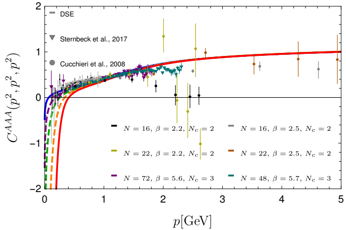

Still truncations in equations. . .

How far should we trust the results?

Overview

How to assess the quality of a truncation?

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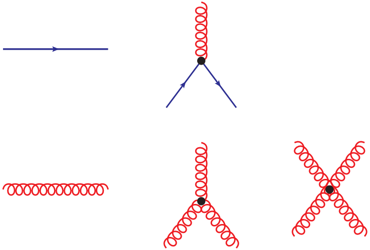
- 3PI effective action and its truncation
- Correlation functions: equations of motion, truncations, impact of extensions
- Bound state equations: kernels and their truncations, impact of extensions

Effective actions

QCD: non-Abelian gauge theory



Lagrangian density of QCD $\mathcal{L}_{\text{QCD}}$:

$\rightarrow$ 1PI effective action $\Gamma[\bar{q}, q, A, \bar{c}, c]$:

- Generating functional of correlation functions
- Equations of motion: DSEs

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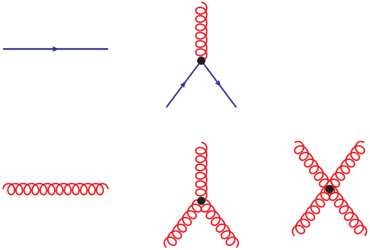

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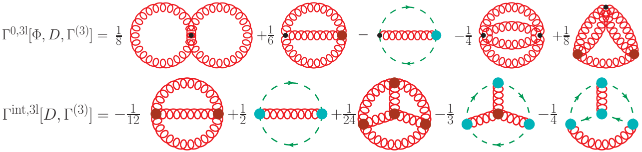
$\rightarrow$ 3PI effective action $\Gamma[\bar{q}, q, A, \bar{c}, c, D, \Gamma^{(3)}]$:

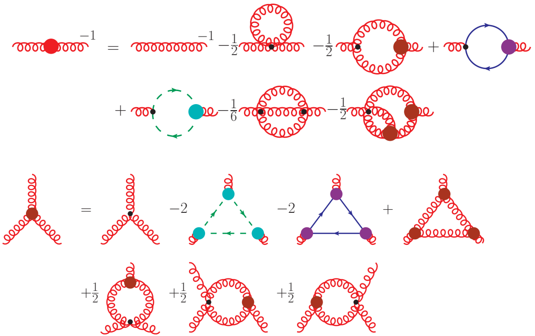
- Equations of motion from stationarity conditions
- Nonperturbative loop-expansion
  $\rightarrow$ three-loop truncation: $\Gamma^{3l} = \Gamma^{0,3l} + \Gamma^{\text{int},3l}$

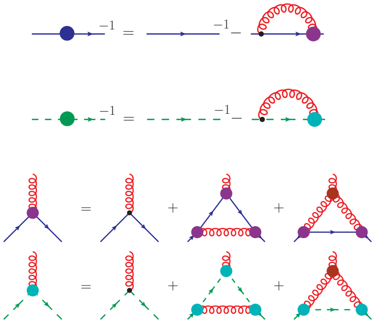


[Berges, Phys. Rev. D 70 (2004); Carrington, Gao, Phys. Rev. D 83 (2011)]

Equations of motion

Stationarity conditions: $\frac{\delta\Gamma}{\delta D} = 0$, $\frac{\delta\Gamma}{\delta\Gamma^{(3)}} = 0 \rightarrow$ Equations of motion (already truncated):

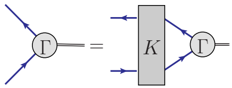




(Similar to DSEs, but still different.)

Bound state equations

Generic form:



Kernels can be obtained from the 3PI effective action [Fukuda, Prog. Theor. Phys 78 (1987); McKay, Munczek, Phys. Rev. D 40 (1989); Sanchis-Alepuz, Williams, J. Phys: Conf. Ser. 631 (2015)]:

$$K = -2\frac{\delta^2\Gamma^{3l}}{\delta D^2}$$

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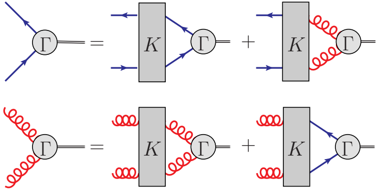

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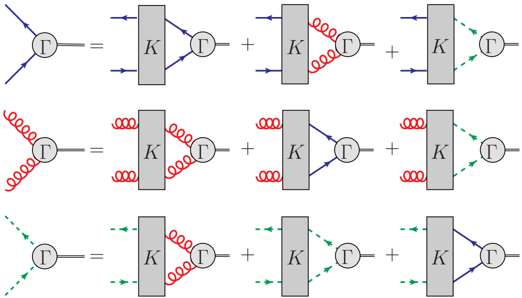

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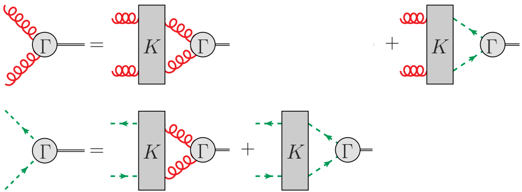
Bound state equations

Generic form:

Focus on pure glueballs.



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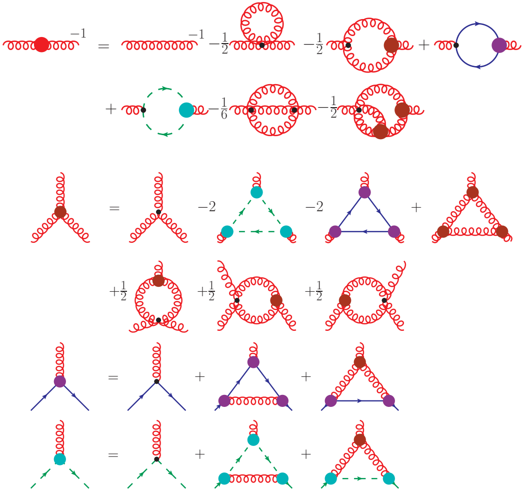
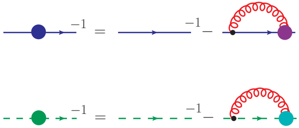
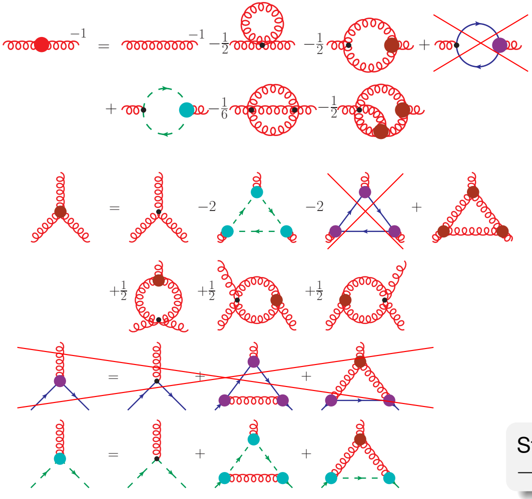
Equations of motion from 3PI–3-loop

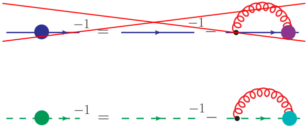




- Truncation at level of action → System of equations complete!
- → Self-contained: Only parameters are the strong coupling and the quark masses!
- Parts: propagators, ghost-gluon vertex, three-gluon vertex, four-gluon vertex
  - Individual calculations
  - Combination with lattice results
  - Complete setup

Equations of motion from 3PI–3-loop





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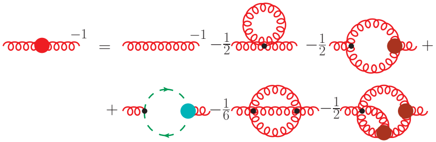
Start with pure gauge theory.
→ [MQH, Phys.Rev.D 101 (2020)]

Landau gauge propagators



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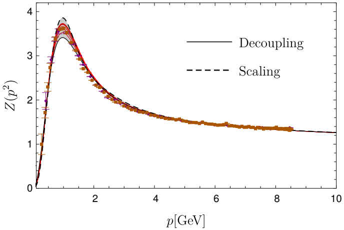

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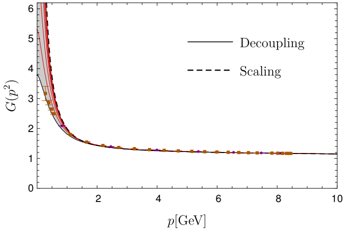
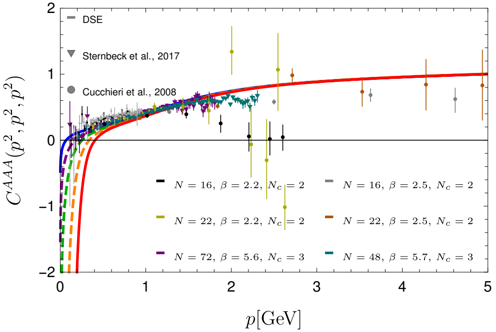


[Sternbeck, hep-lat/0609016; MQH, Phys. Rev. D 101 (2020)]

(Family of solutions [von Smekal, Alkofer, Hauck, PRL79 (1997); Aguilar, Binosi, Papavassiliou, Phys.Rev.D 78 (2008); Boucaud et al., JHEP06 (2008); Fischer, Maas, Pawlowski, Ann.Phys. 324 (2008); Alkofer, MQH, Schwenzer, Phys. Rev. D 81 (2010)])

Three-gluon vertex: Kinematics



- IR suppression with zero crossing

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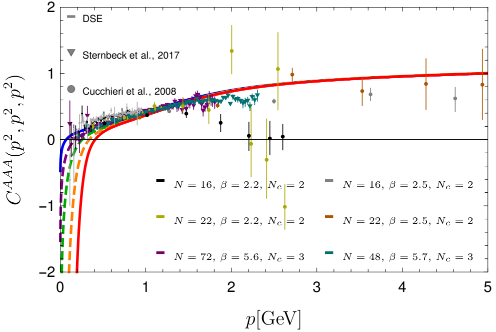
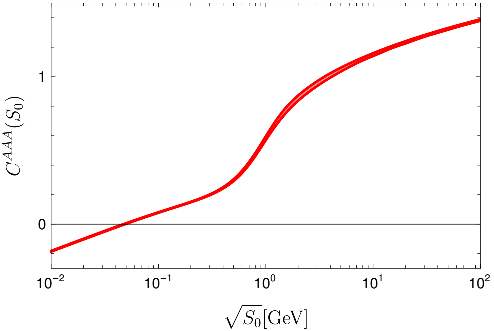




[Cucchieri, Maas, Mendes, Phys. Rev. D 77 (2008); Sternbeck et al., 1702.00612; MQH, Phys. Rev. D 101 (2020)]

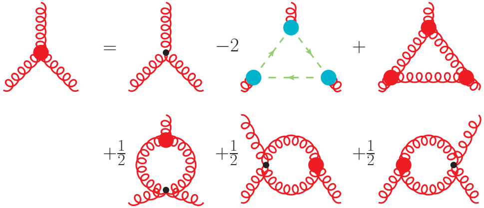
- IR suppression with zero crossing
- Simple kinematic dependence (singlet variable S_0 of S_3), “planar degeneracy“

First observation: [Eichmann, Williams, Alkofer, Vujinovic, Phys.Rev.D89 (2014)], but already in old data [Blum, Huber, Mitter, von Smekal, Phys.Rev.D89 (2014)]; lattice: [Pinto-Gómez et al., Phys.Lett.B838 (2023)]; [Aguilar et al., Eur.Phys.J.C83 (2023)]

$\rightarrow$ Talks by de Soto, Rodríguez-Quintero

Three-gluon vertex: Individual diagrams

Importance of individual diagrams?



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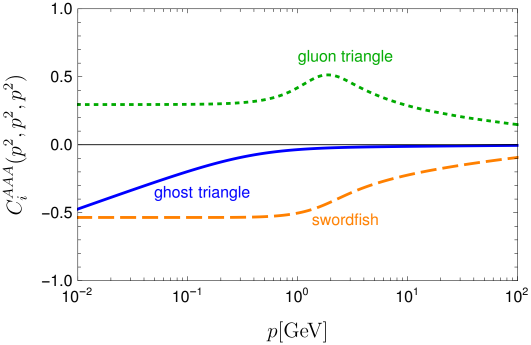
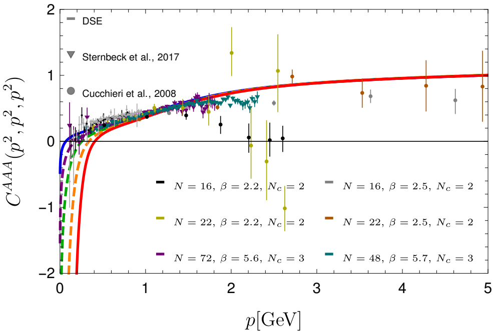




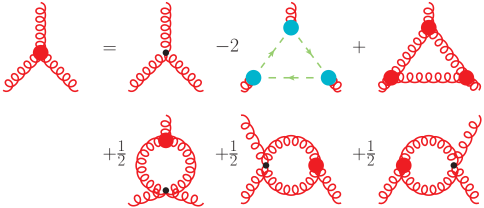
[MQH, Phys. Rev. D 101 (2020)]

$\rightarrow$ Cancellations between diagrams important.

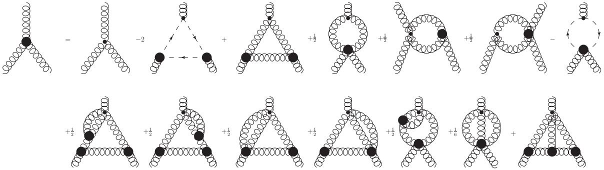
Three-gluon vertex: 1PI vs. 3PI

[MQH, Phys. Rev. D 101 (2020)]

3PI:



1PI:



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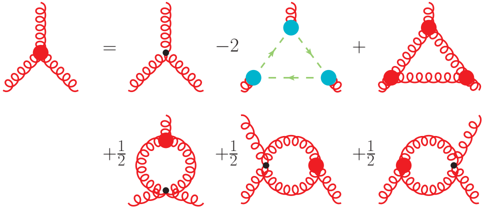
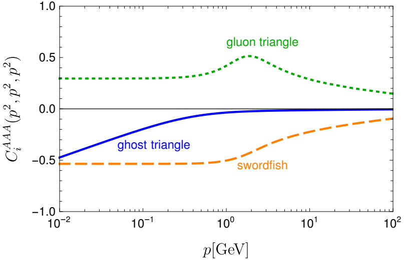
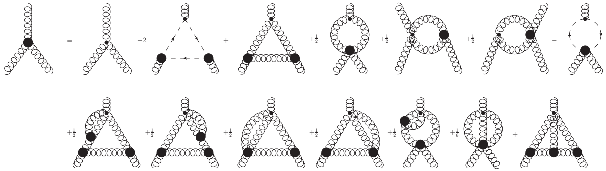
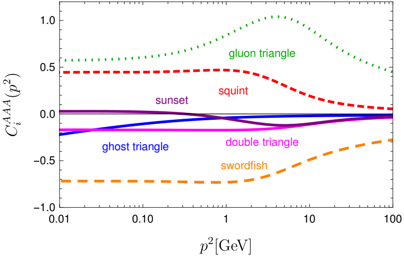




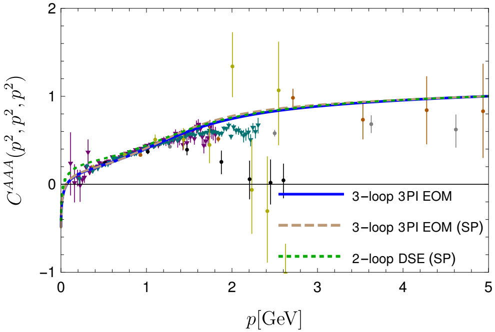
1PI:

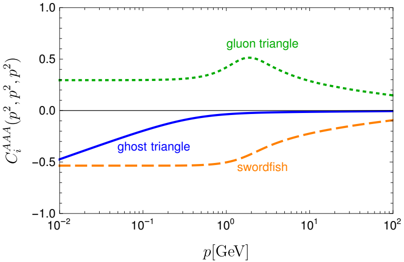


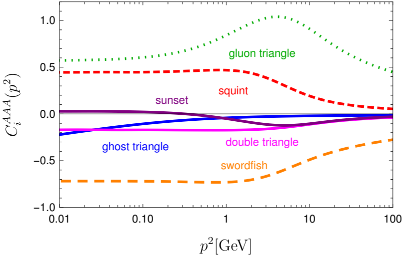


Three-gluon vertex: 1PI vs. 3PI

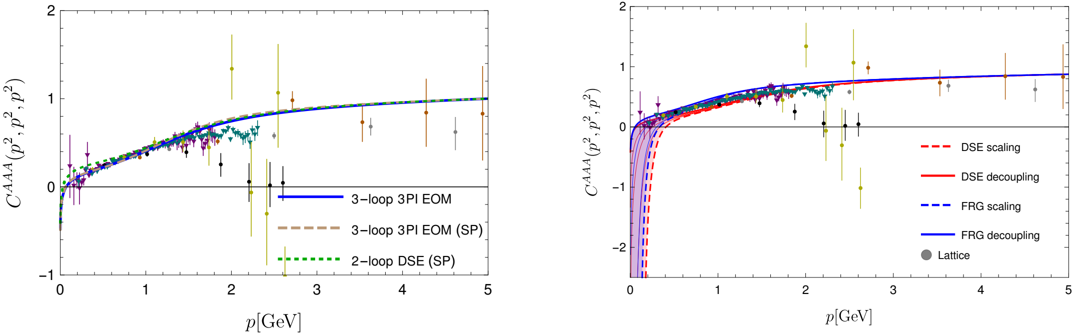
[MQH, Phys. Rev. D 101 (2020)]







Three-gluon vertex: More methods



[Cucchieri, Maas, Mendes, Phys.Rev.D77 (2008); Sternbeck et al., Proc.Sci. LATTICE2016 (2017); Cyrol et al., Phys.Rev.D 94 (2016); MQH, Phys.Rev.D101 (2020)]

$\rightarrow$ Agreement between lattice, FRG, DSE and 3PI.

Stability of the three-gluon vertex results

- Agreement with lattice and other functional results. ✓

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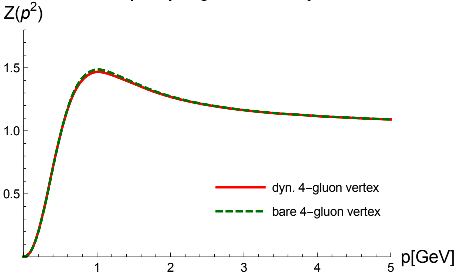

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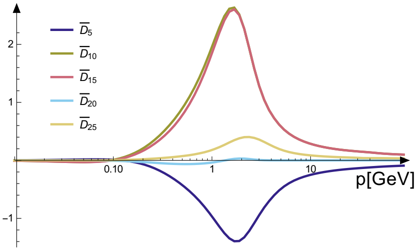
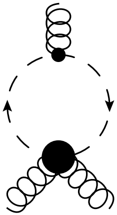
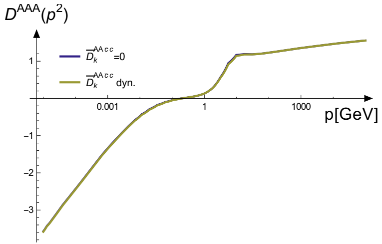
- Two-ghost-two-gluon vertex with 25 dressings [MQH, Eur. Phys.J.C77 (2017)]: ✓ (FRG: [Corell, SciPost Phys. 5 (2018)])







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A new benchmark quantity as new results become available.

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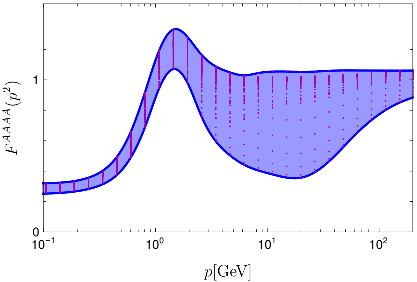
Four-gluon vertex from DSE using 3PI input: [MQH, Phys. Rev. D 101 (2020)]



Calculated for three kinematic variables (singlet and doublet of S_4).

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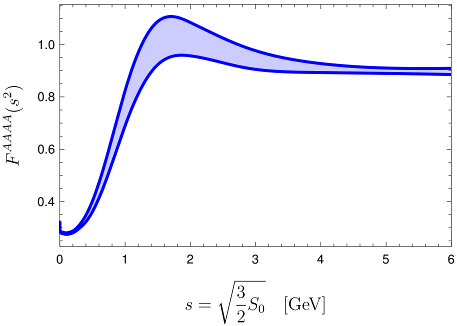

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Shown: Subset of one vanishing gluon momentum.

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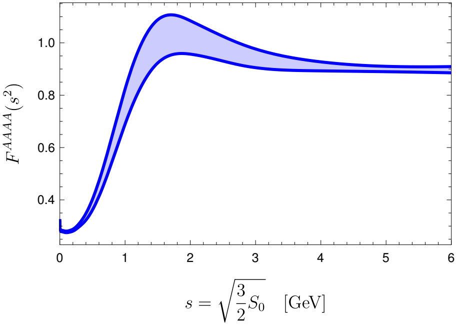

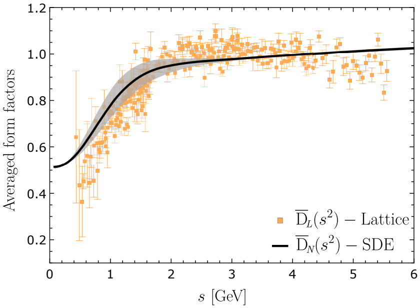

Shown: Subset of one vanishing gluon momentum.

Talks by Oliveira, de Soto, Santos [Colaco, Oliveira Silva,Phys.Rev.D 109 (2024); Aguilar et al., 2408.06135]

lattice, 4PI effective action $\rightarrow$ possibilities for checks between three methods

Possible approximations for bound state equations

Basic and complexity considerations:

Possible approximations for bound state equations

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Tensor basis:

Full? Relevant subset?

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- Pion has one dominant amplitude out of four.
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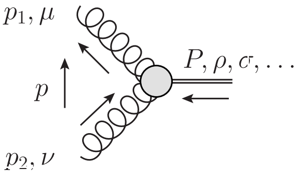
Input:

- Availability
- Kinematics
- Tensors

Tensor bases for glueballs

[MQH, Fischer, Sanchis-Alepuz, Eur.Phys.J.C81 (2021)]

$$\Gamma_{\mu\nu\rho\sigma\ldots}(p_1,p_2)=\sum \tau^i_{\mu\nu\rho\sigma\ldots}(p_1,p_2)h_i(p_1,p_2)$$



Increase in complexity:

- 2 gluon indices (transverse)
- J spin indices (symmetric, traceless, transverse to P)
- Linear independence (nontrivial for P $=-1$)

Numbers of tensors:

J	P $=+1$	P $=-1$
0	2	1
1	4	3
>2	5	4

Low number of tensors, but high-dimensional tensors!

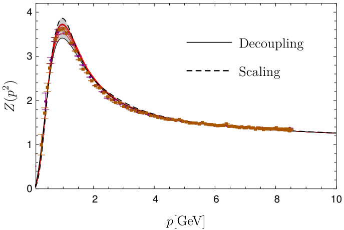
$\rightarrow$ Computational cost increases with J.

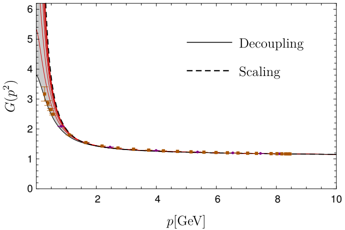
$\rightarrow$ Full tensor basis used.

Input

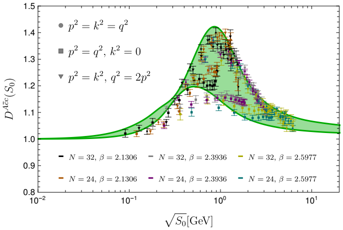
Propagators: Full

[MQH, Phys. Rev. D 101 (2020)]

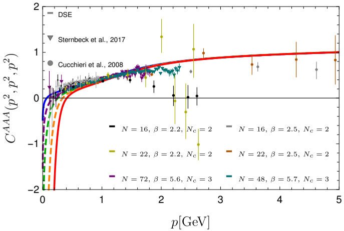




Ghost-gluon vertex: Full

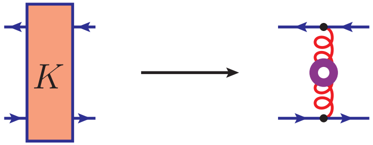


Three-gluon vertex: Leading tensor, full kinematics



Kernels from 3PI–3-loop

One-particle exchange kernels, e.g., ladder truncation (long-time work horse):



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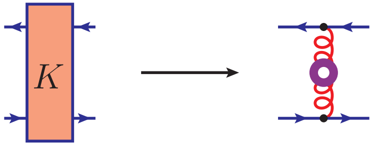
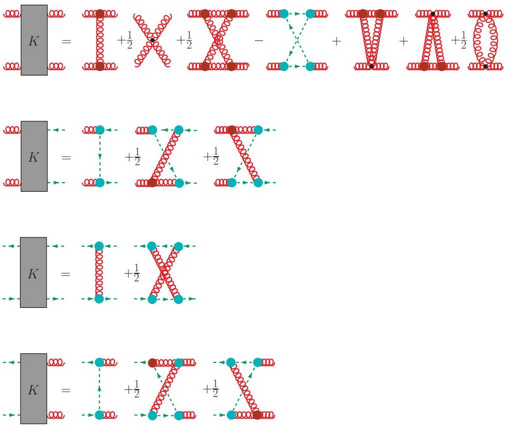
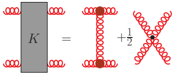
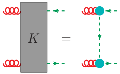


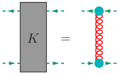


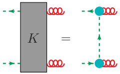
$\rightarrow$ Two-loop diagrams in BSEs!

Results from one-particle exchange kernels









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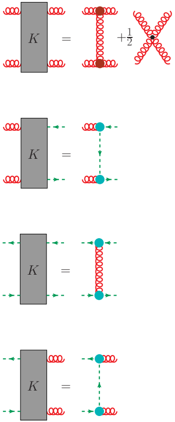

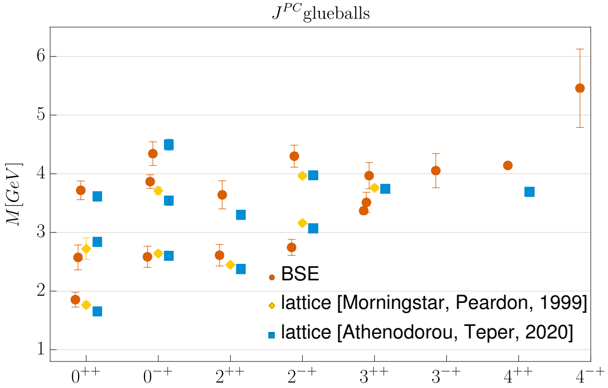

[MQH, Fischer, Sanchis-Alepuz, Eur.Phys.J.C81 (2021)]

Spectral reconstruction [Pawlowski et al., Phys.Rev.D 108 (2023)]:
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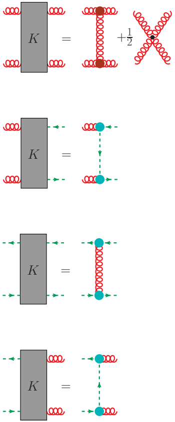

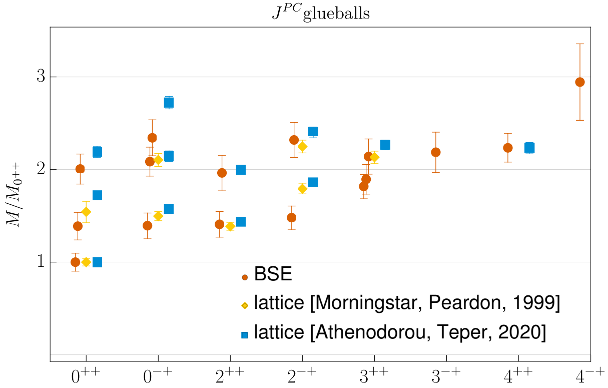

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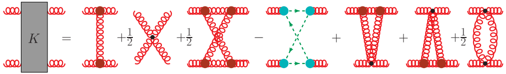
Beyond one-particle exchange

Gluon-gluon interactions leading $\rightarrow$ Test its full 3PI–3-loop kernel:



+ Exploit planar degeneracy of the three-gluon vertex.

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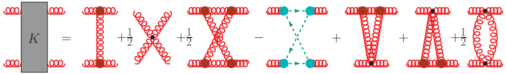

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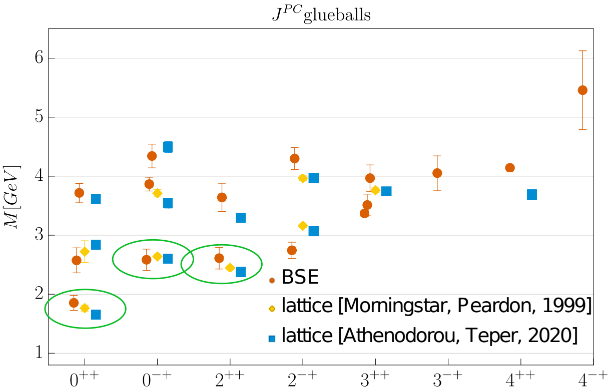

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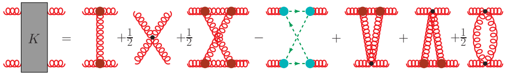

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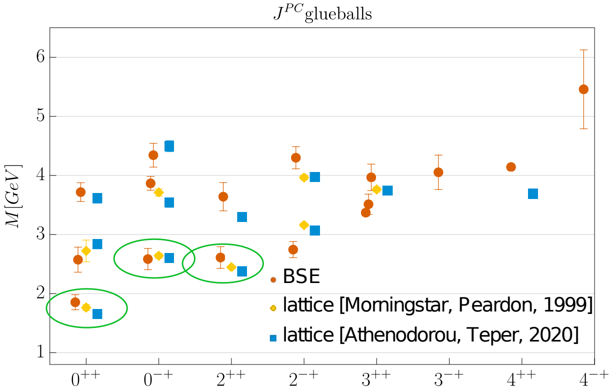

- 0^{-+}: none
  [MQH, Fischer, Sanchis-Alepuz, EPJ Web Conf. 258 (2022)]
- 0^{++}: $< 2\%$
  [MQH, Fischer, Sanchis-Alepuz, HADRON2021, Rev.Mex.Fis.Suppl. 3 (2022)]
- 2^{++}: none
  [MQH, Fischer, Sanchis-Alepuz, HADRON2023, Nuovo Cim.C 47 (2024)]

Summary

Glueballs from functional equations

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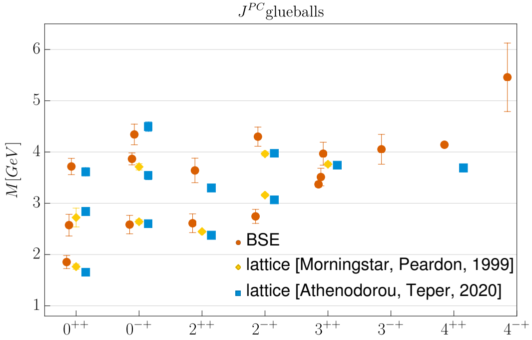

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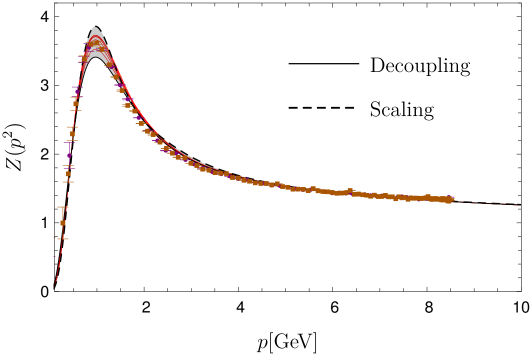

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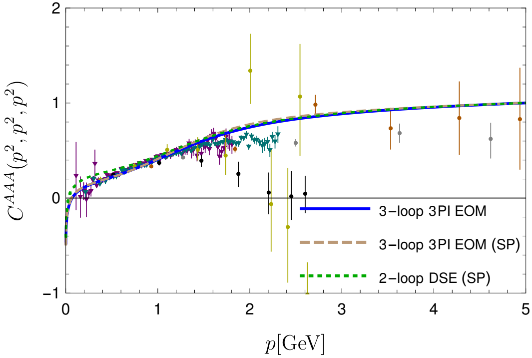

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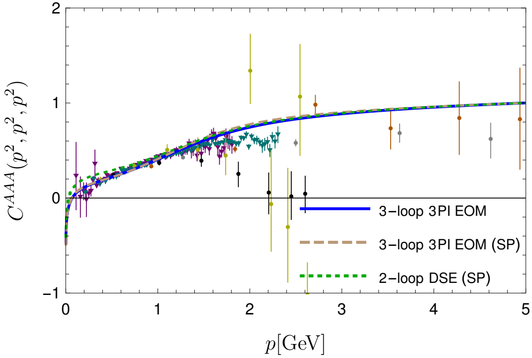

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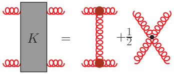
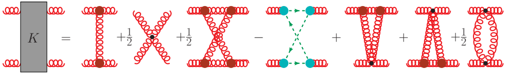


vs.



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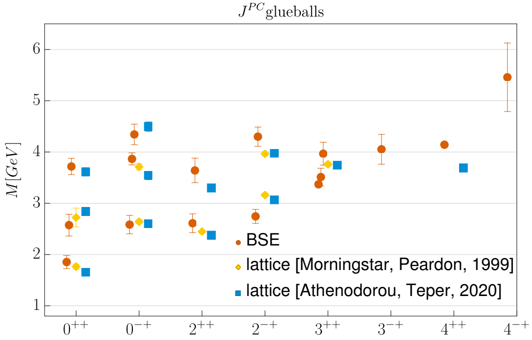
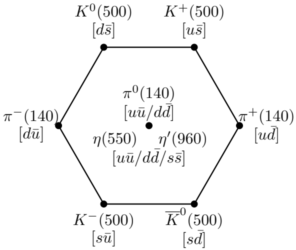


Thank you for your attention!

Scalar sector

$J^{PC} = 0^{++} \to q\bar{q}$ mesons, tetraquarks and glueballs

$m_u \sim m_d < m_s$



Pseudoscalar:

mass hierarchy ✓

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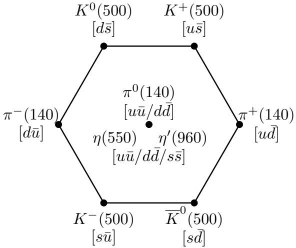

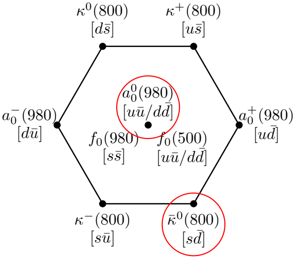

Pseudoscalar:
mass hierarchy ✓

Scalar:
Mass hierarchy?

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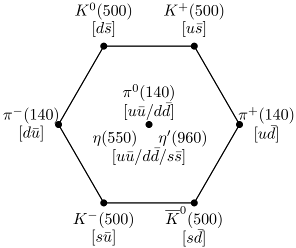

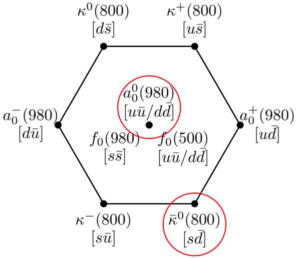

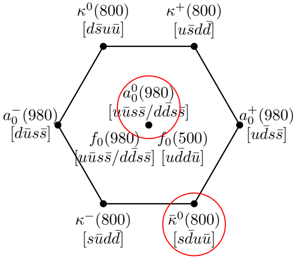

Functional review:
[Eichmann, Fischer, Santowsky, Wallbott, Few-Body Syst.61 (2020)]

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Inverted mass hierarchy if tetraquarks
[Jaffe, Phys. Rev. D 15 (1977)]

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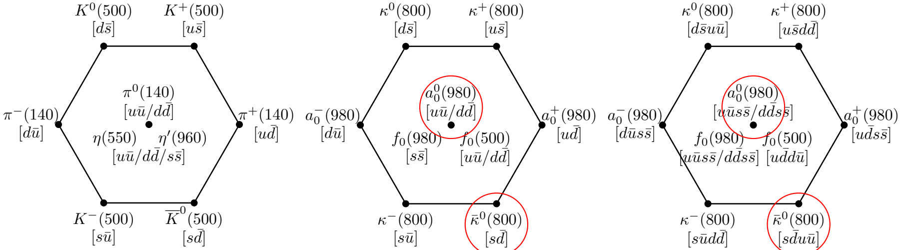

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Scalar glueballs: $Q = 0$, isoscalar

$f_0(500)$	$f_0(1370)$
$f_0(980)$	$f_0(1500)$
	$f_0(1710)$

← glueball candidates ($f_0(1370)$, $f_0(1500)$, $f_0(1710)$)

$J = 1$ glueballs

Landau-Yang theorem

Two-photon states cannot couple to $J^P = \mathbf{1}^{\pm}$ or $(2n+1)^-$
[Landau, Dokl.Akad.Nauk SSSR 60 (1948); Yang, Phys. Rev. 77 (1950)].
($\rightarrow$ Exclusion of $J = 1$ for Higgs because of $h \rightarrow \gamma\gamma$.)

Applicable to glueballs?

$\rightarrow$ Not in this framework, since gluons are not on-shell.

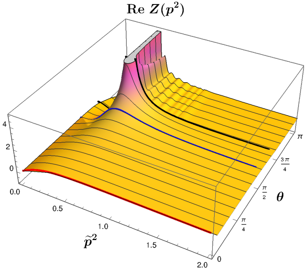
$\rightarrow$ Presence of $J = 1$ states is a dynamical question.

$J = 1$ not found here.

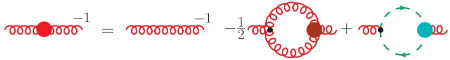
Correlation functions in the complex plane

Gluon

[Fischer, MQH, Phys.Rev.D 102 (2020)]

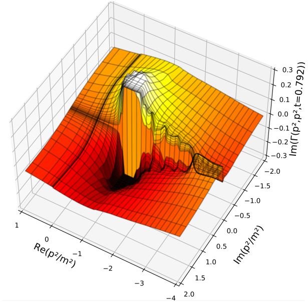


Simpler truncation:



Vertex (scalar toy theory)

[MQH, Kern, Alkofer, Phys.Rev.D107 (2023)]



- Numerically more demanding due to calculations close to cuts.
- Branch cut

Extrapolation of $\lambda(P^2)$

Extrapolation method

- Extrapolation to time-like P^2 using Schlessinger's continued fraction method (proven superior to default Padé approximants) [Schlessinger, Phys.Rev.167 (1968)]
- Average over extrapolations using subsets of points for error estimate

$$f(x) = \frac{f(x_1)}{1 + \frac{a_1(x-x_1)}{1+\frac{a_2(x-x_2)}{1+\frac{a_3(x-x_3)}{\cdots}}}}$$

Coefficients a_i can be determined such that $f(x)$ exact at x_i.

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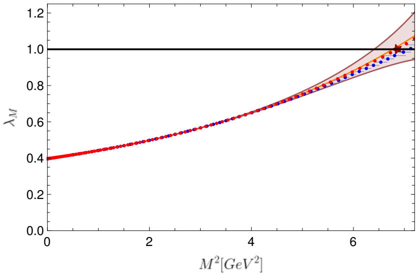
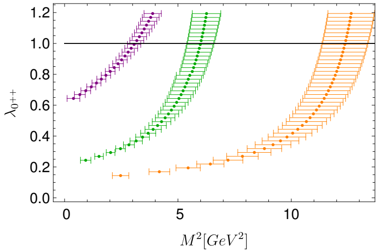
Test extrapolation for solvable system:
Heavy meson [MQH, Sanchis-Alepuz, Fischer, Eur.Phys.J.C 80 (2020)]

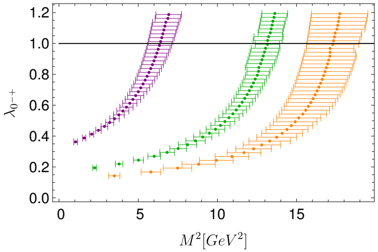


Extrapolation for glueball eigenvalue curves

0^{++}:



0^{-+}:



Several curves: ground state and excited states.

Gauge invariance

[MQH, Phys. Rev. D 101 (2020)]

Couplings can be extracted from each vertex.

- Slavnov-Taylor identities (gauge invariance): Agreement perturbatively (UV) necessary.
  [Cyrol et al., Phys.Rev.D 94 (2016)]
- Difficult to realize: Small deviations → Couplings cross and do not agree.
- Here: Vertex couplings agree down to GeV regime (IR can be different).