

# Infrared analysis of Yang-Mills theory in the Landau gauge and the maximally Abelian gauge

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# Contents of the talk

- Maximally Abelian gauge: Why do we need this complicated gauge, anyway? And what is its IR behavior?
- Landau gauge: Does (partly) solving the Gribov problem change the infrared behavior?
- Non-perturbative tool: Dyson-Schwinger equations; is there an easy way to derive them?

# Confinement of quarks and gluons

- **Confinement** is a long-range  $\leftrightarrow$  **IR phenomenon**: We do not see individual  $\sim$  infinitely separated quarks or gluons.
- One expects that the property of being **confined** is **encoded in the particles' propagators**.
- Different confinement criteria for the propagators:
  - Positivity violations: negative norm contributions  $\rightarrow$  not a particle of the physical state space
  - Kugo-Ojima: quartet mechanism, e. g. Gupta-Bleuler formalism in QED: timelike and longitudinal photon cancel each other.
  - Gribov-Zwanziger (Landau gauge, Coulomb gauge): IR suppression of the gluon propagator  $\rightarrow$  no long-distance propagation.

# Infrared regime of Yang-Mills theory in Landau gauge

## Scaling solution [Alkofer, Fischer, Maas, Pawłowski, von Smekal, ...]

- Dressing functions obey power laws.
- Qualitative IR solution of ALL correlation functions is known.
- Horizon condition  $\leftrightarrow$  IR enhanced ghost.
- Picture of confinement: IR vanishing gluon ( $\rightarrow$  gluon confinement) and IR enhanced ghost propagator ( $\rightarrow$  long-range force to confine quarks).
- Method easily transferable to some other gauges.

## Decoupling solution [Boucaud, Fischer, Papavassiliou, Pawłowski, Sorella, ...]

- Different renormalization of the ghost propagator  $\Rightarrow$  tree-level like.  
 $\leftrightarrow$  boundary condition for DSEs [Fischer et al., Ann. Phys. 324; Maas, 0907.5185]
- Seen in most lattice calculations [Cucchieri, Ilgenfritz, Mendes, Mueller-Preussker, Sternbeck, ...].
- Positivity violation  $\Rightarrow$  confined. Explanation of confinement?

# Hypothesis of Abelian dominance

## Dual superconductor picture of confinement (Mandelstam, 't Hooft)

- Picture a conventional superconductor, where the electric charges condense and force the magnetic flux into vortices.
- Change "electric" and "magnetic" components and you get a dual superconductor, where condensed magnetic monopoles squeeze the electric flux into flux tubes.
- QCD: No free chromoelectric charges. Are they confined by condensed magnetic monopoles?

Ezawa and Iwazaki [PRD 25 (1981)]: Magnetic monopoles live in Abelian part of the theory. → Abelian part dominates in the IR?

⇒ Hypothesis of Abelian dominance

# Lattice results on Abelian dominance

- String tension calculated from the Abelian part is almost the same as the one from the full theory. Even more, the string tension from the monopole part is almost the same, too.
- Suzuki et al. [PRD 80]: Without gauge fixing the string tension was extracted and agreed to 100%. Maybe MAG is a simple way to get monopoles?
- Available lattice results of MAG [Cucchieri, Mendes, Mihara, 2008]: all propagators massive, Abelian fields have lowest mass  
⇒ other fields decouple

# Definition of the maximally Abelian gauge

Look for dominance of Abelian part. What is the Abelian part?

Gauge field components:

$$A_\mu = \mathbf{A}_\mu^i \mathbf{T}^i + \mathbf{B}_\mu^a \mathbf{T}^a, \quad i = 1, \dots, N-1, \quad a = N, \dots, N^2-1$$

Abelian subalgebra:  $[\mathbf{T}^i, \mathbf{T}^j] = 0$ , can be written as diagonal matrices

$\Rightarrow$  Abelian  $\leftrightarrow$  diagonal fields  $\mathbf{A}$ ,

non-Abelian  $\leftrightarrow$  off-diagonal fields  $\mathbf{B}$ .

E.g.  $\mathbf{T}^1 = \frac{1}{2}\lambda^3$ ,  $\mathbf{T}^2 = \frac{1}{2}\lambda^8$  for  $SU(3)$ .

Which interactions are possible?

$$f^{ijk} = 0, \quad f^{ija} = 0, \quad f^{iab} \neq 0$$

$$SU(2): \quad f^{abc} = 0, \quad SU(N > 2): \quad f^{abc} \neq 0$$

$\Rightarrow$  2 off-diagonal and 1 diagonal field can interact; 3 off-diagonal fields can only interact in  $S(N > 2)$

# Gauge fixing condition

Stress role of diagonal fields  $\Rightarrow$  minimize norm of off-diagonal field  $B$ :

$$\|B_U\| = \int dx B_U^a B_U^a \rightarrow \text{minimize wrt. gauge transformations } U$$


$$D_\mu^{ab} B_\mu^b = (\delta_{ab} \partial_\mu - g f^{abi} A_\mu^i) B_\mu^b = 0 \quad \text{non-linear gauge fixing condition!}$$

Remaining symmetry of diagonal part:  $U(1)^{N-1}$

Fix gauge of diag. gluon field  $A$  by Landau gauge condition:  $\partial_\mu A_\mu = 0$   
 $\Rightarrow$  diagonal ghosts decouple (like in QED).



# Lagrangian for the MAG

 diagonal gluon

 off-diagonal gluon

 ghost



***ABB***



***Acc***



***AABB***



***AAcc***



***BBcc***



***BBBB***



***cccc***

$SU(N > 2)$

***BBB Bcc ABBB ABcc***

# Deriving Dyson-Schwinger equations

Integral of a total derivative vanishes:

$$\int [D\phi] \frac{\delta}{\delta\phi} e^{-S+J\Phi} = \int [D\phi] \left( J - \frac{\delta S}{\delta\phi} \right) e^{-S+J\Phi} = 0.$$

$\Rightarrow$  DSEs for **all Green functions** (full, connected, 1PI) by further differentiations.

Doing it by hand?

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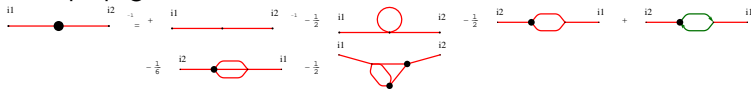
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Doing it by hand?

For example: Landau gauge, only 2 propagators (**AA**, **cc**), 3 interactions (**A<sub>cc</sub>**, **AAA**, **AAAA**)

# Landau Gauge: Propagators

Gluon propagator:

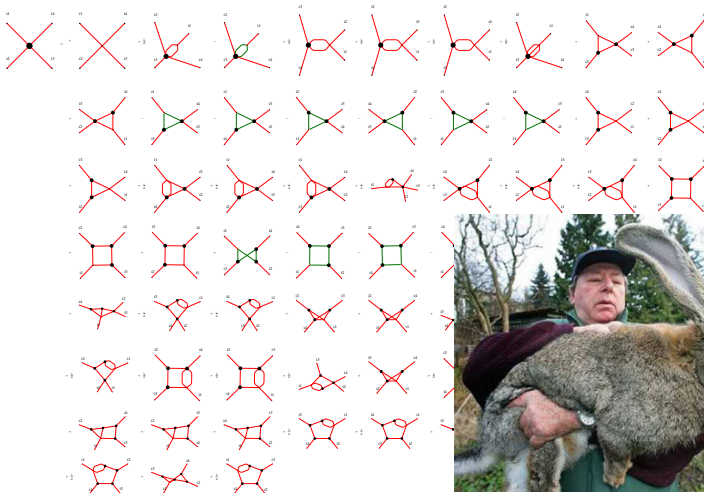


Ghost propagator:



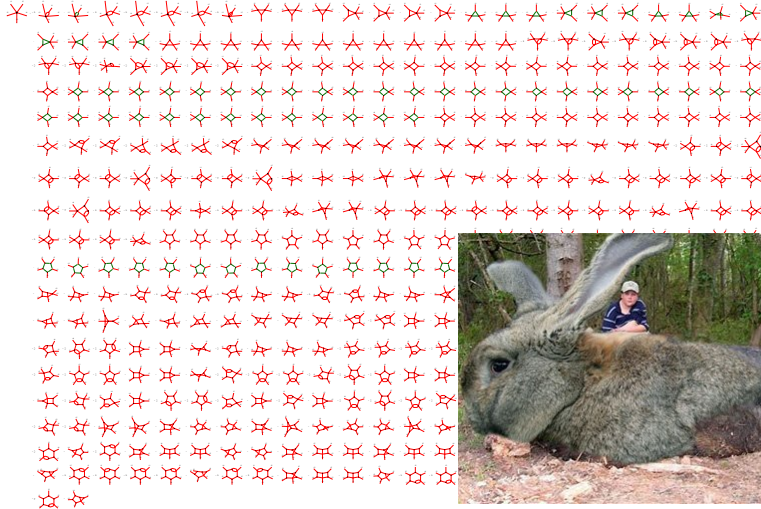
# Landau Gauge: Four-Gluon Vertex

66 terms



# Landau Gauge: Five-Gluon Vertex

434 terms



⇒ *DoDSE* [Alkofer, M.Q.H., Schwenzler, CPC 180 (2009)]

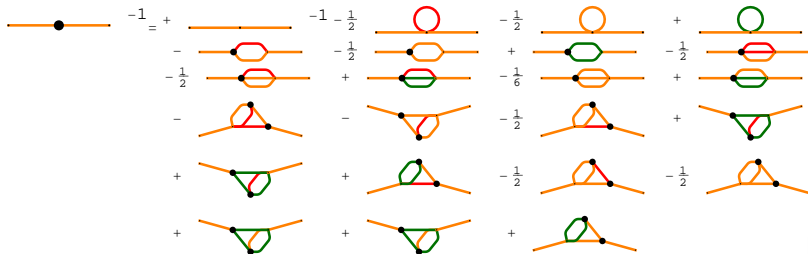
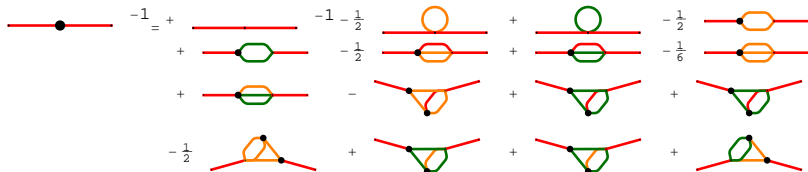
Given a structure of interactions, the *DSEs are derived symbolically* using *Mathematica*.

Example (Landau gauge):

- only input: interactions in Lagrangian (AA, AAA, AAAA, cc, Acc)
  - Which DSE do I want?
- 
- Step-by-step calculations possible.
  - Can handle mixed propagators (then there are really many diagrams  
→ Gribov-Zwanziger action).

Upgrade: *Symb2Alg* produces algebraic from the symbolic expressions.

# DSEs of the MAG





# Infrared power counting

## Generic propagator

$$T_{(\mu\nu)} \cdot \frac{D(p^2)}{p^2},$$

assume **power law** behavior at low  $p^2$

$$D^{IR}(p^2) = A \cdot (p^2)^\delta$$

**IR exponent**



- Vertices also assume power law behavior [Alkofer, Fischer, Llanes-Estrada, PLB 611 (2005) (skeleton expansion)].
- Limit of all momenta approaching zero simultaneously.
- Upon integration all momenta converted into powers of external momenta.  $\Rightarrow$  Counting of IR exponents

# System of inequalities

- IR exponent for every diagram
- lhs is dominated by at least one diagram on rhs and rhs cannot be more divergent than lhs.  $\rightarrow \delta_{lhs} \leq \delta_{rhs, any\ diagram}$ .
- Not knowing which diagram is leading on the rhs, we can write inequalities from all diagrams.



$$-\delta_{gl} \leq 2\delta_{gh} + \delta_{gg}, \quad -\delta_{gl} \leq 2\delta_{gl} + \delta_{3g}, \quad \dots$$

That's the basic idea.

Still, for a large system a lot of work.

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$$\text{wavy line with dot}^{-1} = \text{wavy line}^{-1} - \frac{1}{2} \text{wavy line with star} - \frac{1}{2} \text{wavy line with star} + \text{dashed circle} - \frac{1}{6} \text{wavy line with star} - \frac{1}{2} \text{wavy line with star}$$

$$-\delta_{gl} \leq 2\delta_{gh} + \delta_{gg}, \quad -\delta_{gl} \leq 2\delta_{gl} + \delta_{3g}, \quad \dots$$

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Still, for a large system a lot of work.

All inequalities relevant?

# Relevant inequalities

A closed form for all relevant inequalities can be derived from DSEs and RGEs.

2 types:

type		derived from	#
dressed vertices	$C_1 := \delta_{vertex} + \frac{1}{2} \sum_{\text{legs } j \text{ of vertex}} \delta_j \geq 0$	RGEs	infinite
prim. div. vertices	$C_2 := \frac{1}{2} \sum_{\text{legs } j \text{ of prim. div. vertex}} \delta_j \geq 0$	DSEs/RGEs	finite

Some inequalities are contained within others.

E. g. in MAG:  $\delta_B \geq 0$  and  $\delta_c \geq 0$  render  $\delta_B + \delta_c \geq 0$  useless.

NB: These inequalities explicitly show that the skeleton expansion used in previous studies is a consistent expansion. However, the [skeleton expansion is now obsolete](#).

# Infrared exponent for an arbitrary diagram

Having so many diagrams, isn't there a shorter way than writing all expressions down explicitly?

## Arbitrary Diagram $v$

Numbers of vertices and propagators related  $\Rightarrow$  possible to get a formula for the IR exponent by pure combinatorics in terms of:

- propagator IR exponents  $\delta_{\phi_i}$
- number of vertices
- number of external legs  $m^{\phi_i}$

$$\delta_v = \boxed{-\frac{1}{2} \sum_i m^i \delta_i} + \sum_i (\# \text{ of dressed vertices})_i C_1^i + \sum_i (\# \text{ of bare vertices})_i C_2^i$$

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## Arbitrary Diagram $\nu$

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lower bound on IRE

Only depends on the external legs  $\rightarrow$  equal for all diagrams in a DSE/RGE [M.Q.H., Schwenzler, Alkofer, arXiv:0904.1873].

[Similar formula for Landau gauge with slightly different arguments: Fischer, Pawłowski, arXiv:0903.2193]

# Scaling relations

## General analysis of propagator DSEs

At least one inequality from a prim. divergent vertex has to be saturated,

i. e.  $C_2^i = 0$  for at least one  $i$ .

Necessary condition for a scaling solution.

Related to bare vertices in DSEs: Fischer-Pawlowski consistency condition DSEs  $\leftrightarrow$  RGEs [Fischer, Pawlowski, PRD 75 (2007)].

$\Rightarrow$  One primitively divergent vertex is not IR enhanced.

This does not necessarily mean that it is bare:

- Dependence on momentum configuration.
- Consider different dressing functions: Vanishing or constant.

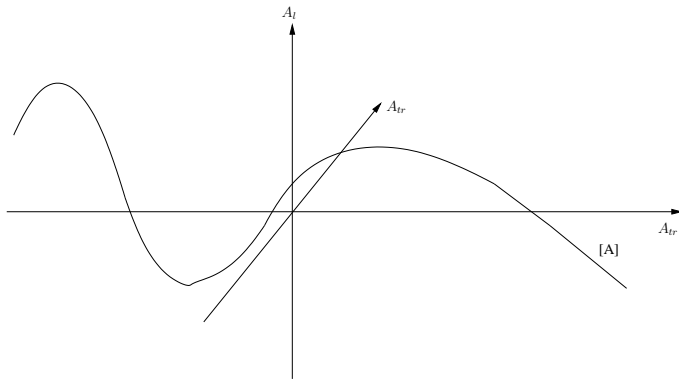
The non-enhancement of at least one primitively divergent vertex is now established for all scaling type solutions. [M.Q.H., Schwenzer, Alkofer, arXiv:0804.1873]

# IR scaling solution of the MAG

- The **Abelian fields are IR enhanced**.  $\rightarrow$  realization of Abelian dominance?
- Off-diagonal fields are IR suppressed.
- $SU(2)$  and  $SU(N > 2)$  have the same solution.
- Qualitative solutions for tower of all Green functions.
- Abelian configurations transformed to Landau gauge lie on Gribov horizon [Greensite, Olejnik, Zwanziger, PRD78].
- **Gribov region of MAG unbounded in diagonal direction** [Capri et al., PRD79].
- Two-loop diagrams are IR leading (sunset, squint).  $\rightarrow$  UV/IR preserving truncation?

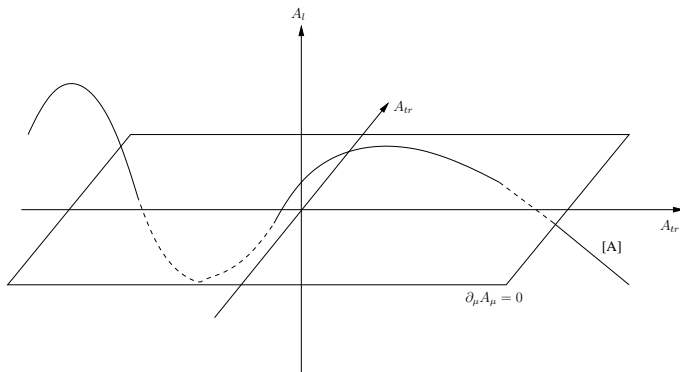


# Gauge orbits and Gribov copies



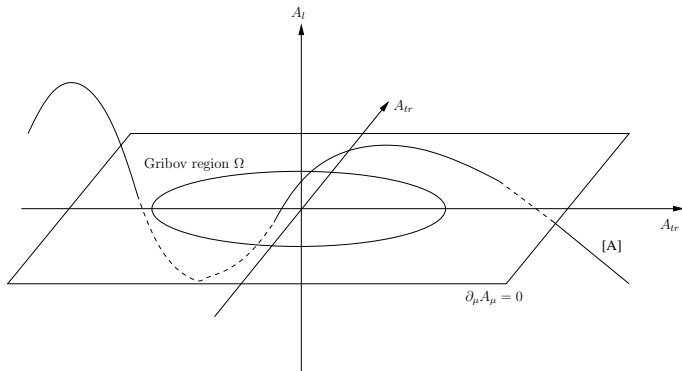
Gauge equivalent configurations (gauge orbit  $[A]$ )  $\Rightarrow$  integration in path integral is overcomplete.

# Gauge orbits and Gribov copies



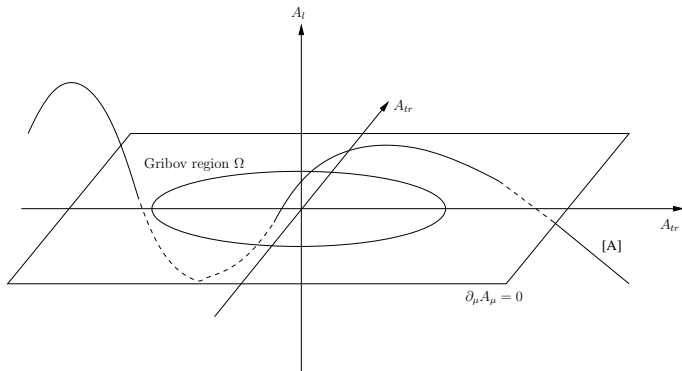
Faddeev and Popov: Restriction of integration to **single representative** of each gauge orbit possible? Gauge symmetry replaced by BRST symmetry!

# Gauge orbits and Gribov copies



Restriction to Gribov horizon: almost unique gauge fixing.

# Gauge orbits and Gribov copies



Restriction to Gribov horizon: almost unique gauge fixing.

Restriction to Gribov region is done via adding a non-local term to the Lagrangian.  $\rightarrow$  New parameter  $\gamma$ , determined by horizon condition.

# How do DSEs usually deal with this?

Integral of a total derivative vanishes:

$$\int [D\phi] \frac{\delta}{\delta\phi} e^{-S+J\Phi} = \int [D\phi] \left( J - \frac{\delta S}{\delta\phi} \right) e^{-S+J\Phi} = 0.$$

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$\Rightarrow$  DSEs for all Green functions (full, connected, 1PI) by further differentiations.

$$\int_{\Omega} [D\phi] \left( J - \frac{\delta S}{\delta\phi} \right) \delta(\partial \cdot A) \det(M) e^{-S_{YM}+J\Phi} = 0.$$

# Local renormalizable action

Non-local term can be localized with auxiliary fields

$(\bar{\varphi}_\mu^{ab}, \varphi_\mu^{ab}, \bar{\omega}_\mu^{ab}, \omega_\mu^{ab}) \rightarrow$  local Gribov-Zwanziger action:

$$\mathcal{L}_{GZ} = \bar{\varphi}_\mu^{ac} M^{ab} \varphi_\mu^{bc} - \bar{\omega}_\mu^{ac} M^{ab} \omega_\mu^{bc} + \gamma^2 g f^{abc} A_\mu^a (\varphi_\mu^{bc} - \bar{\varphi}_\mu^{bc}) - \gamma^4 d(N^2 - 1)$$

Horizon condition in local form:

$$\langle g f^{abc} A_\mu^a (\varphi_\mu^{bc} - \bar{\varphi}_\mu^{bc}) \rangle = 2\gamma^2 d(N^2 - 1),$$

- Restriction breaks BRST invariance.
- Mixing at the level of two-point functions, e. g.  $\langle A_\mu^a \varphi_\nu^{bc} \rangle$ .  
 $\Rightarrow$  (3x3)-matrix relation between propagators and two-point functions:

$$D^{\Phi\Phi} = (\Gamma^{\Phi\Phi})^{-1}, \quad \Phi \in \{A, \varphi, \bar{\varphi}\}$$



## More fields ...

Simplify to (2x2)-matrix relation by splitting into real and imaginary part  
[Zwanziger, 0904.2380]:

$$\varphi = \frac{1}{\sqrt{2}} (U + i V), \quad \bar{\varphi} = \frac{1}{\sqrt{2}} (U - i V).$$

$$\mathcal{L}'_{GZ} = \mathcal{L}_U + \mathcal{L}_V + \mathcal{L}_{UV} - \bar{\omega}_\mu^{ac} M^{ab} \omega_\mu^{bc},$$

$$\mathcal{L}_U = \frac{1}{2} U_\mu^{ac} M^{ab} U_\mu^{bc},$$

$$\mathcal{L}_V = \frac{1}{2} V_\mu^{ac} M^{ab} V_\mu^{bc} + i g \gamma^2 \sqrt{2} f^{abc} A_\mu^a V_\mu^{bc},$$

$$\mathcal{L}_{UV} = \frac{1}{2} i g f^{abc} U_\mu^{ad} V_\mu^{bd} \partial_\nu A_\nu^c \stackrel{LG}{=} 0,$$

Simplify even further:

$$c, \bar{c}, U, \omega, \bar{\omega} \longrightarrow \eta, \bar{\eta}$$

# Truncation of tensors

Propagators  $\langle A_\mu^a V_\nu^{bc} \rangle$  and  $\langle V_\mu^{ab} V_\nu^{cd} \rangle$  can have many tensors with different dressing functions,  
e. g. color space:  $f^{abc}$ ;  $\delta^{ab}\delta^{cd}$ ,  $\delta^{ac}\delta^{bd}$ ,  $\delta^{ad}\delta^{bc}$ ,  $f^{abe}f^{cde}$ ,  $f^{ace}f^{bde}$ .

Truncation: Take only tree-level tensors of two-point functions.

$$\Gamma^{\Phi\Phi} = \begin{pmatrix} \Gamma^{AA} & \Gamma^{AV} \\ \Gamma^{VA} & \Gamma^{VV} \end{pmatrix},$$

$$\Gamma_{\mu\nu}^{AA,ac} = \delta^{ac} p^2 c_A^\perp(p^2) P_{\mu\nu} + \delta^{ac} \frac{1}{\xi} c_A^\parallel(p^2) p_\mu p_\nu,$$

$$\Gamma_{\mu\nu}^{VV,abcd} = \delta^{ac}\delta^{bd} p^2 c_V(p^2) g_{\mu\nu},$$

$$\Gamma_{\mu\nu}^{AV,cab} = f^{cab} p^2 c_{AV}(p^2) g_{\mu\nu},$$

# Propagators of the GZ action

$$D_{cd}^{\eta\bar{\eta},ab} = (\Gamma_{cd}^{\eta\bar{\eta},ab})^{-1} = -\delta^{ab}\delta^{cd} \frac{c_{\eta}(p^2)}{p^2}$$

$D^{VV}$  has two tensors  $\rightarrow$  non-trivial truncation:

$$\begin{aligned} D_{\mu\nu}^{AA,ab} &= \delta^{ab} \frac{1}{p^2} P_{\mu\nu} \frac{c_V(p^2)}{c_A^{\perp}(p^2)c_V(p^2) + 2N c_{AV}^2(p^2)}, \\ D_{\mu\nu}^{VV,abcd} &= \frac{1}{p^2} \frac{1}{c_V(p^2)} \delta^{ac}\delta^{bd} g_{\mu\nu} - \\ &\quad - f^{abe}f^{cde} \frac{1}{p^2} P_{\mu\nu} \frac{2c_{AV}^2(p^2)}{c_A^{\perp}(p^2)c_V^2(p^2) + 2N c_{AV}^2(p^2)c_V(p^2)}, \\ D^{AV,abc} &= -i f^{abc} \frac{1}{p^2} P_{\mu\nu} \frac{\sqrt{2}c_{AV}(p^2)}{c_A^{\perp}(p^2)c_V(p^2) + 2N c_{AV}^2(p^2)} \end{aligned}$$

Appearance of the determinant  $c_A^{\perp}(p^2)c_V(p^2) + 2N c_{AV}^2(p^2)$

# The four possibilities

Which part of the determinant  $c_A^\perp(p^2)c_V(p^2) + 2N c_{AV}^2(p^2)$  dominates in the IR?

$$c_{ij}(p^2) = d_{ij} \cdot (p^2)^{\kappa_{ij}}$$

- I:  $c_{AV}^2 > c_{ACV} \leftrightarrow \kappa_A + \kappa_V > 2\kappa_{AV}$
- II:  $c_{ACV} > c_{AV}^2 \leftrightarrow 2\kappa_{AV} > \kappa_A + \kappa_V$
- III:  $c_{AV}^2 \sim c_{ACV} \leftrightarrow \kappa_A + \kappa_V = 2\kappa_{AV}$ , no cancellations
- IV:  $c_{AV}^2 \sim c_{ACV} \leftrightarrow \kappa_A + \kappa_V = 2\kappa_{AV}$ , cancellations

Cancellations: Leading contributions cancel and some less dominant term takes over.

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Cancellations: Leading contributions cancel and some less dominant term takes over.

Two solutions lead to inconsistencies.

## Case II: Recovery of standard Landau gauge solution

$$c_{ACV} > c_{AV}^2 \leftrightarrow 2\kappa_{AV} > \kappa_A + \kappa_V$$

- The  $VV$ -propagator becomes

$$\frac{1}{p^2} \frac{1}{c_V(p^2)} \delta^{ac} \delta^{bd} g_{\mu\nu}$$

$\Rightarrow$   $VV$ -propagator could be integrated out in the IR and the FP theory is recovered.

- All contributions containing an  $AV$ -propagator are suppressed.  $\Rightarrow$  DSEs reduce in the IR to the same system as in FP theory.
- Formula for IR exponent of arbitrary  $n$ -point functions is obtained.
- IR exponent of  $AV$ -propagator is not fixed by scaling relation; calculated numerically.

Several solutions: 0.0668776, 0.981386 and higher.

In the IR this is completely the same as FP theory!

## Case III: The "strict" scaling solution

All IR exponents are connected by the scaling relations ( $\kappa := \kappa_V = \kappa_\eta$ ):

$$\kappa_A + 2\kappa = \kappa + 2\kappa_{AV} = 0$$

$\Rightarrow$  Mixed propagator IR suppressed.

The determinant remains as it is.  $\Rightarrow$  Non-linear relations between the coefficients of the dressing functions.

# Summary Gribov-Zwanziger action

Explicitly restricted integration to Gribov region by using the Gribov-Zwanziger action.

Mixed propagators complicate the analysis. Two candidates remain:

- **Scaling relation** between FP ghost and gluon unaltered:  
 $\kappa_A + 2\kappa_c = 0$ .
- All solutions have the **same qualitative behavior**.
- Maybe mixed propagator contributions IR suppressed?  
→ Completely the same solution as for FP theory.
- Input for numerical solution of the equations.



# Summary maximally Abelian gauge

- Existence and form of scaling solutions can easily be obtained directly from the interactions.
- Fischer-Pawlowski consistency condition  $\leftrightarrow$  one vertex remains bare in the IR.
- Scaling solution may exist in MAG
  - Abelian gluon field is IR enhanced.  $\rightarrow$  Support of hypothesis of Abelian dominance.
  - Complete numerical solution required.
  - Two-loop terms are IR leading  $\leftrightarrow$  UV/IR preserving truncation?
  - Relation to chromomagnetic monopoles?

# IR Scaling solutions for other gauges

The analysis can be used also for other gauges. Beware: This corresponds to a naive application!

Linear covariant gauges	Ghost-antighost symmetric gauges
scaling solution only, if the longitudinal part of the gluon propagator gets dressed, but gauge fixing condition $\Rightarrow$ longitudinal part bare	quartic ghost interaction $\rightarrow \delta_{gh} \geq 0$ $\rightarrow$ with non-negative IREs only the trivial solution can be realized

This is valid for all possible dressings and agrees with the results from [Alkofer, Fischer, Reinhardt, v. Smekal, PRD 68 (2003)], where only certain dressings were considered.

$\Rightarrow$

- Either the existence of a scaling solution is something special (?) or
- a more refined analysis is needed in these cases.