Infrared Behavior of Yang-Mills Green Functions in the Maximally Abelian Gauge

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SIC!QFT

A Possible Mechanism for Confinement

Dual superconductor scenario (electric $\leftrightarrow$ magnetic components)

Magnetic monopoles condense and the electric flux is squeezed into vortices ("strings").

Classical configurations of monopoles are constructed within the maximal Abelian subalgebra (Cartan subalgebra), i.e. its generators commutate:

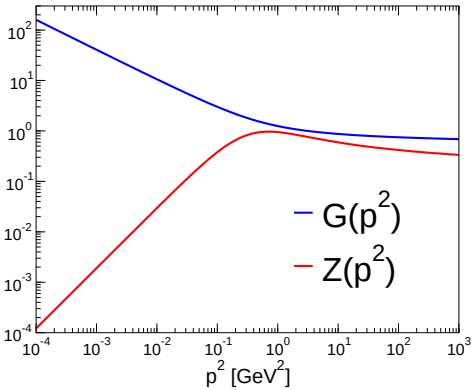
$$[T^i, T^j] = 0.$$

Hypothesis of Abelian Dominance

$\Rightarrow$ Abelian part should dominate in the infrared part of the theory (Ezawa, Iwazaki, 1981)

Use maximally Abelian gauge (MAG).

Landau gauge:



Fischer, Alkofer, Phys. Lett. B 536, 2002

- Gluon propagator vanishes at zero momentum
  $\Rightarrow$ gluon confinement.
- Ghost diverges.
- Gribov-Zwanziger, Kugo-Ojima scenarios
- Quark-gluon vertex
  $\Rightarrow$ quark confinement

Maximally Abelian Gauge:

- All propagators have finite values at zero momentum (extended Gribov-Zwanziger framework) [Capri et al., Phys. Rev. D77, 2008].
- Where does confinement come from?
- What about vertex functions?

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The Maximally Abelian Gauge (MAG)

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Split the gauge field in color space

$$A_\mu = A^i_\mu T^i + B^a_\mu T^a$$

diagonal generators T^i
off-diagonal generators T^a

In total: 4 fields

Fortunate: Diagonal ghosts decouple $\Rightarrow$ 3 fields (A, B, c)

"Additional" interactions

[overview e.g. Capri et al., Phys. Rev. D77, 2008]

- Gauge fixing: Minimize the off-diagonal gluon components

$$\Rightarrow D^{ab}_\mu B^b_\mu = 0.$$

- Renormalizability: quartic ghost interaction

Interactions in the MAG

Number depends on the gauge group:

	$SU(2)$	$SU(3)$	$SU(N>2)$	Landau gauge
components	3	8	N^2-1	N^2-1
diagonal components	1	2	$N-1$	$N-1$
three-point vertices	2	4	4	2
four-point vertices	5	7	7	1

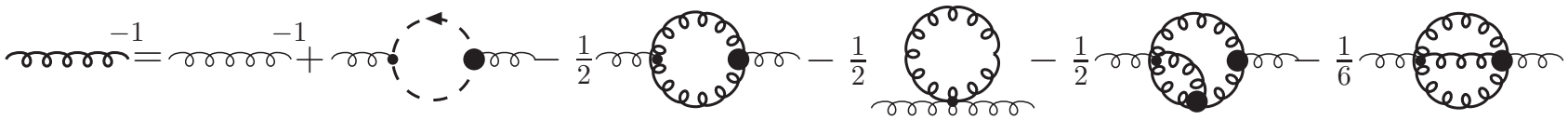
$\Rightarrow$ There are considerably more interactions in MAG than in Landau gauge!!!

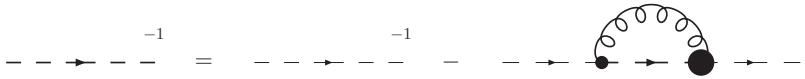
Is it a good idea to choose an approach whose intricacy depends on the number of interactions?
$\longrightarrow$ Having the right technique at hand, yes.

Dyson-Schwinger Equations (DSEs)

- Equations of motion of propagators and vertices
- Describe non-perturbatively the propagation of a particle, i.e. all interactions included.
- Compare perturbation theory: propagation of a particle at small coupling.
- DSEs at small coupling $\Rightarrow$ perturbation series

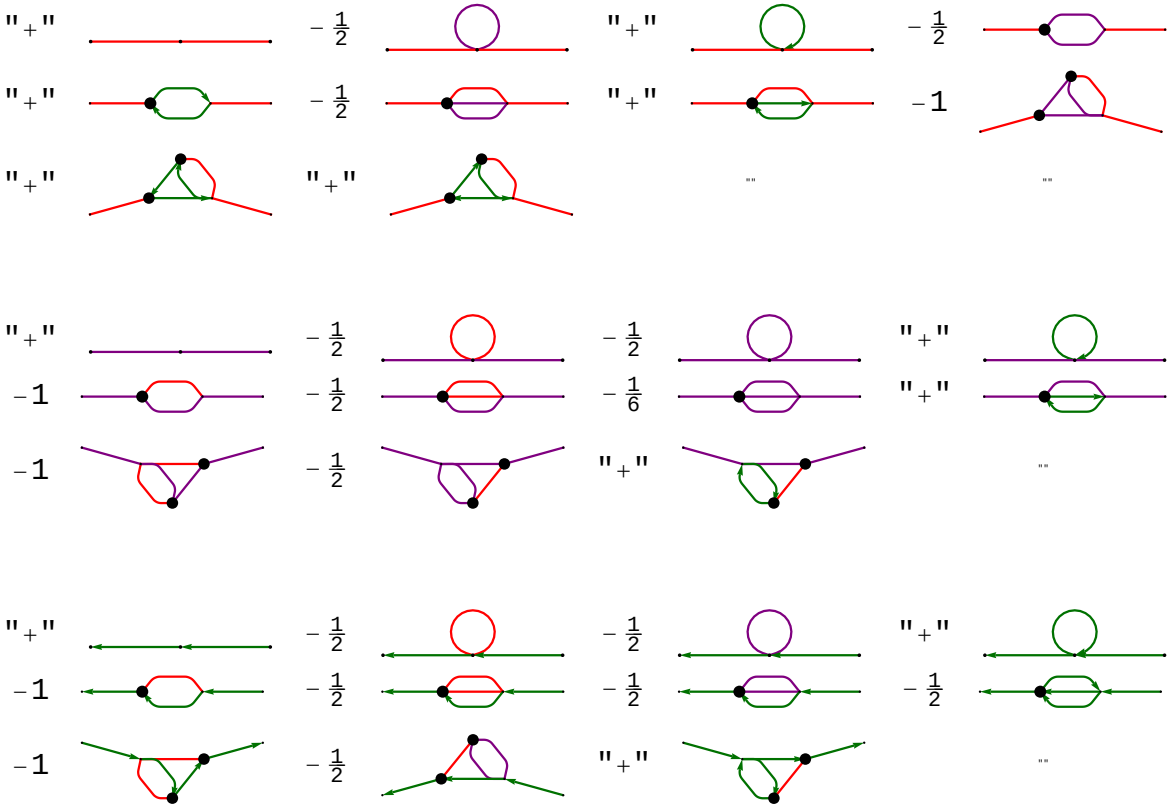
The more possibilities to interact, the more terms in the DSEs!
Compare Landau gauge (2 three-point, 1 four-point)





MAG DSEs for $SU(2)$

diagonal gluon:



off-diagonal gluon:

off-diagonal ghost:

[*DoDSE*: Alkofer, M.Q.H., Schwenzer, arXiv:0808.2939]

Infrared Analysis

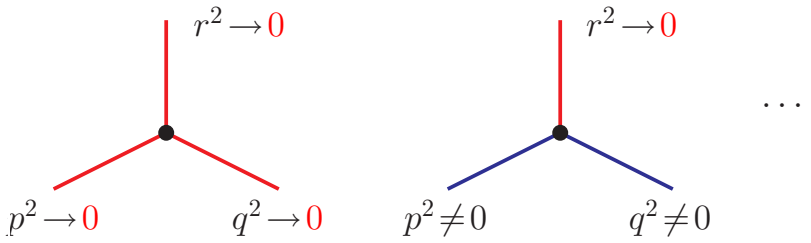
Investigation of the Infrared

- Dressing functions in the IR can be described by power laws, e.g. for ghost propagator

$$D_G = -\frac{G(p^2)}{p^2}, \qquad G(p^2) \overset{p^2 \to 0}{=} (p^2)^{\delta_{gh}}$$

- IR exponent: $\delta_{gh} > 1 \to$ propagator vanishes, $\delta_{gh} < 0 \to$ propagator IR enhanced.
- If external momenta small, integral dominated by small momenta.
- Upon integration loop momenta are transformed into external momenta.

1. Uniform limit, i.e. $p^2, q^2, r^2 \to 0$
2. Kinematic limit, i.e. p^2, q^2 const., $r^2 \to 0$ [Alkofer, M.Q.H., Schwenzer, arXiv:0801.2762]



Power Counting

DSEs are an infinitely big set of coupled equations → solvable?

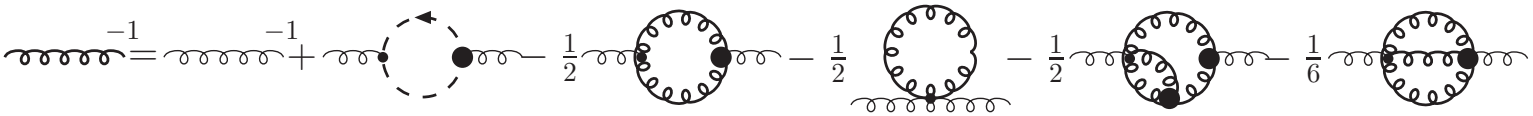
Ways to obtain information about the IR exponents

- Non-renormalization of the ghost-gluon vertex in Landau gauge [v. Smekal, Hauck, Alkofer, Ann. Phys. 267, 1998]
- Include the functional renormalization group equations (RGEs) [Fischer, Pawlowski, Phys. Rev. D 75, 2006].
- Assumptions about skeleton expansion [Alkofer, M.Q.H., Schwenzer, arXiv:0801.2762]

Constraints from DSEs

Still, at the end there is a big system of inequalities that has to be solved, e.g. gluon propagator in Landau gauge:

$$-\delta_g = \min(\underbrace{0}_{\text{bare prop.}}, \underbrace{2\delta_g + \delta 3g}_{\text{gh loop}}, \underbrace{2\delta_{gh} + \delta_{gg}}_{\text{gl loop}}, \underbrace{\delta_g}_{\text{tadpole}}, \underbrace{4\delta_g + 2\delta_{3g}}_{\text{squint}}, \underbrace{3\delta_g + \delta_{4g}}_{\text{sunset}})$$



min-function $\Rightarrow$ set of inequalities,
e.g. from sunset $-\delta_g \leq 3\delta_g + \delta_{4g}$.

IR Exponent for Arbitrary Diagram

System difficult to solve. Look for alternatives or shortcuts.

Arbitrary Diagram v

Numbers of vertices and propagators related $\Rightarrow$ possible to get a formula for the IR exponent.

Function of:

- propagator IR exponents δ_{X_i}
- number of external legs m^{X_i}
- number of vertices.

$$\delta_v = -\frac{1}{2}\sum_i m^{X_i}\delta_{X_i} + \\ + \sum_i (\#\text{ of dressed vertices})_i C_1^i + \sum_i (\#\text{ of bare vertices})_i C_2^i$$

Bounds for coefficients C_1^i and C_2^i:

$$C_1^i \geq 0, \qquad C_2^i \geq 0$$

$\Rightarrow$ Maximally IR divergent solution: $\delta_v = -\frac{1}{2}\sum_i m^{X_i}\delta_{X_i}$

MAG Constraints, $SU(2)$

Instead of large system of inequalities, we use constraints:

$$\delta_{BBBB} + 2\delta_B \geq 0, \qquad \delta_{cccc} + 2\delta_c \geq 0,$$
$$\delta_{AABB} + \delta_A + \delta_B \geq 0, \qquad \delta_{AAcc} + \delta_A + \delta_c \geq 0,$$
$$\delta_B \geq 0, \qquad \delta_c \geq 0,$$
$$\delta_A + \delta_B \geq 0, \qquad \delta_A + \delta_c \geq 0,$$

$$\delta_{BBcc} + \delta_B + \delta_c \geq 0, \qquad \delta_B + \delta_c \geq 0,$$
$$\delta_{ABB} + \frac{1}{2}\delta_A + \delta_B \geq 0, \qquad \frac{1}{2}\delta_A + \delta_B \geq 0,$$
$$\delta_{Acc} + \frac{1}{2}\delta_A + \delta_c \geq 0, \qquad \frac{1}{2}\delta_A + \delta_c \geq 0$$

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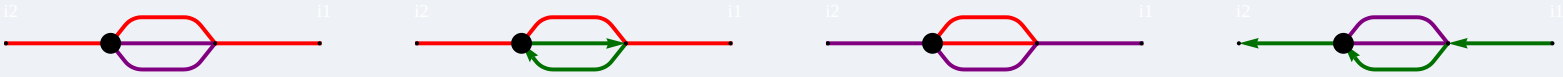
$$\boldsymbol{\delta_B \geq 0}, \qquad \boldsymbol{\delta_c \geq 0},$$

$$\delta_A + \delta_B \geq 0, \qquad \delta_A + \delta_c \geq 0,$$

Results for the MAG

Leading Diagrams in Propagator DSEs

All sunsets with 2 diagonal & 2 off-diagonal fields



possibly some squints

- IR enhanced diagonal gluon:

$$\delta_B = \delta_c =: \kappa \geq 0, \qquad \delta_A = -\kappa$$

- Supports Abelian dominance hypothesis.
- Vertices with 2 diagonal & 2 off-diagonal fields do NOT scale, i.e. $\delta_{AABB} = \delta_{AAcc} = 0$.
- Several solutions for vertex functions possible, e.g. $\delta_{ABB} = -\kappa$ or 0.

$SU(2)$ vs. $SU(3)$

Even more diagrams in $SU(3)$

Inequalities from additional terms in $SU(3)$

$$\delta_{BBB} + \frac{3}{2}\delta_B \geq 0, \qquad \frac{3}{2}\delta_B \geq 0,$$

$$\delta_{Bcc} + \frac{2}{\delta_B} + \delta_c \geq 0, \qquad \frac{1}{2}\delta_B + \delta_c \geq 0,$$

$$\delta_{ABcc} + \frac{1}{2}\delta_A + \frac{1}{2}\delta_B + \delta_c \geq 0, \qquad \frac{2}{\delta_A} + \frac{1}{2}\delta_B + \delta_c \geq 0,$$

$$\delta_{ABBB} + \frac{1}{2}\delta_A + \frac{3}{2}\delta_B \geq 0, \qquad \frac{1}{2}\delta_A + \frac{3}{2}\delta_B \geq 0$$

$SU(2)$ vs. $SU(3)$

Even more diagrams in $SU(3)$

Inequalities from additional terms in $SU(3)$

All inequalities less restrictive

$\Rightarrow$ **color $SU(3)$ has the same qualitative IR behavior as color $SU(2)$.**

Summary: General Results from Power Counting

- Maximally IR divergent solution depends on number of legs and propagator IR exponents:

$$\delta_v = -\frac{1}{2}\sum_i m^{x_i}\delta_{x_i}.$$

  - All diagrams scale equally in functional RGEs.
  - Leading diagrams in DSEs determined by their bare vertices.
- At least one vertex does not scale.
- Self-interacting fields (e.g. gluon in Landau gauge) have non-negative IR exponents $\Rightarrow$ no IR enhancement.

Summary: Results for the MAG

- $\delta_B = \delta_c =: \kappa \geq 0, \qquad \delta_A = -\kappa$
- Diagonal gluon $\boldsymbol{A}$ IR enhanced $\Rightarrow$ supports Abelian dominance hypothesis.
- Off-diagonal fields ($\boldsymbol{B}$, $\boldsymbol{c}$) are IR suppressed (compared to tree-level).
- Four-point functions with 2 diagonal and 2 off-diagonal fields do not scale $\Rightarrow$ 2-loop diagrams IR leading.
- For other vertices several solutions possible (for now...).

- No qualitative difference between $SU(2)$ and $SU(3)$.