On the stability of truncating functional equations of three-dimensional Yang-Mills theory

Markus Q. Huber

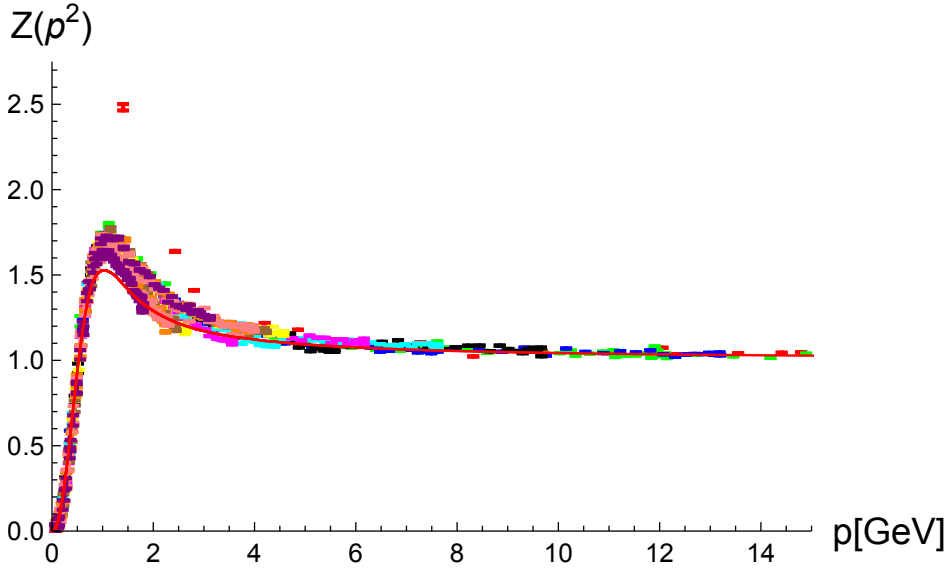
Institute of Physics, University of Graz

ACHT 2016 - Non-perturbative aspects of Quantum Field Theory, Čakovec, Croatia

Oct. 6, 2016

ACHT 2015

Preliminary results for $d = 3$ dimensional Yang-Mills theory:



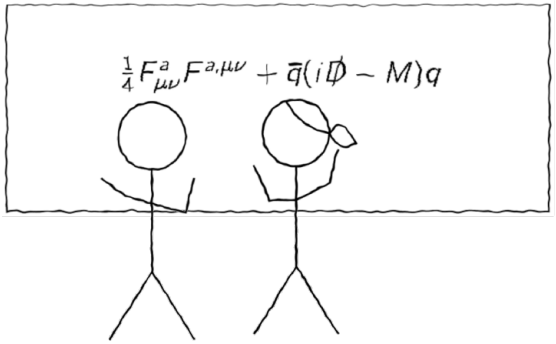
[MQH, ACHT2015]

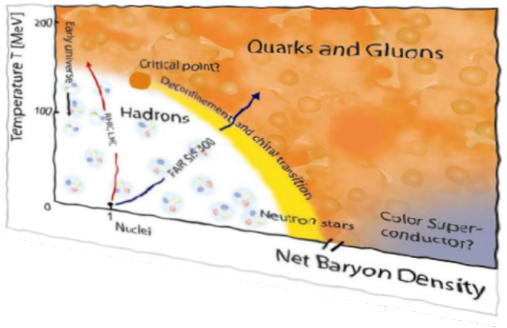
Today: Final results [MQH, PRD93, 1602.02038; MQH, 1606.02068].

Goal: QCD phase diagram

$$\frac{1}{4}F^a_{\mu\nu}F^{a,\mu\nu} + \bar{q}(i\not{D} - M)q$$

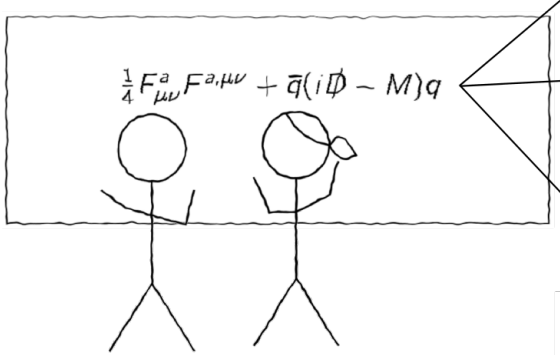
Goal: QCD phase diagram

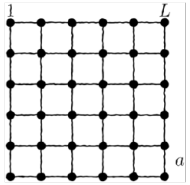


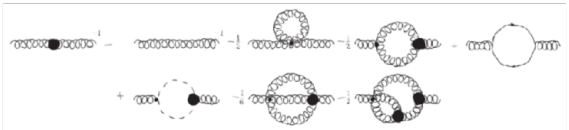


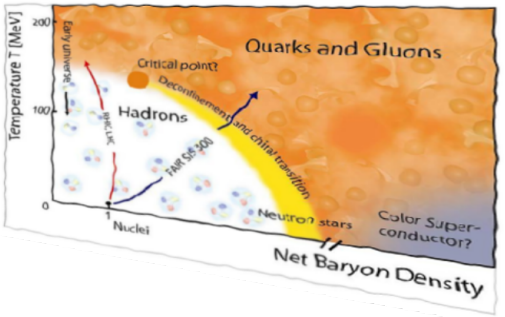
Goal: QCD phase diagram

$$\mathcal{L}_{\text{eff}} = \frac{1}{2}(\partial^\mu \pi)(\partial_\mu \pi) + \dots$$

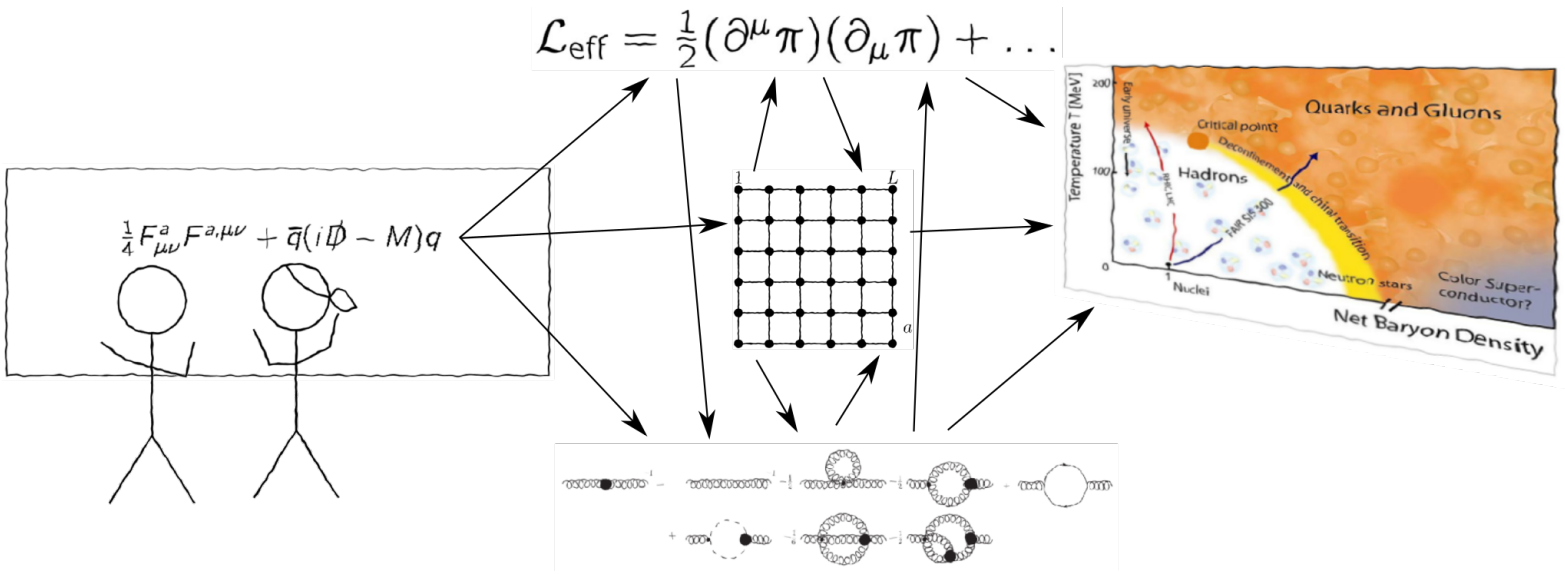




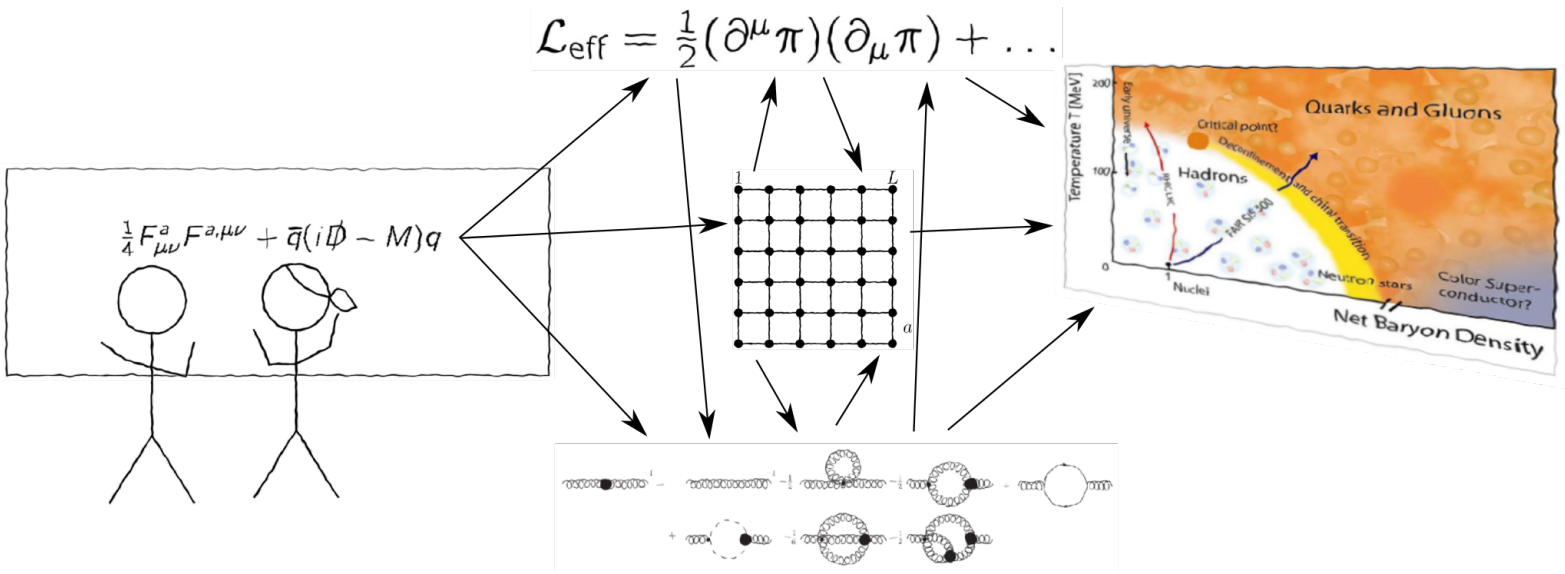




Goal: QCD phase diagram



Goal: QCD phase diagram

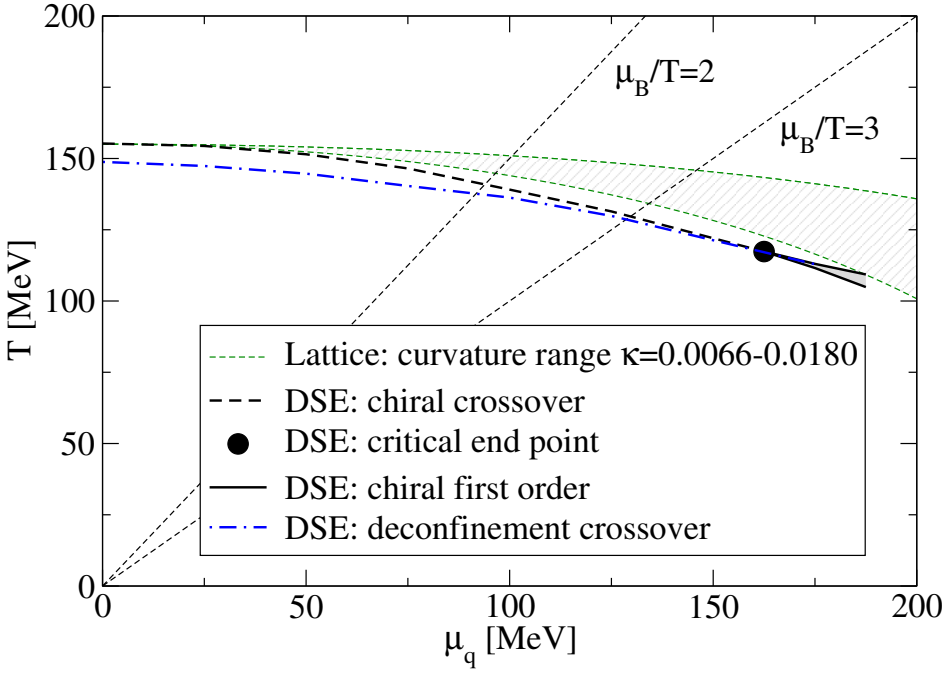


- Challenges for all methods at $\mu > T$, e.g.
  - Lattice QCD: complex action problem
  - Models: parameters
  - Functional methods: reliability of truncations

QCD phase diagram from functional equations

2+1 flavor QCD from DSEs

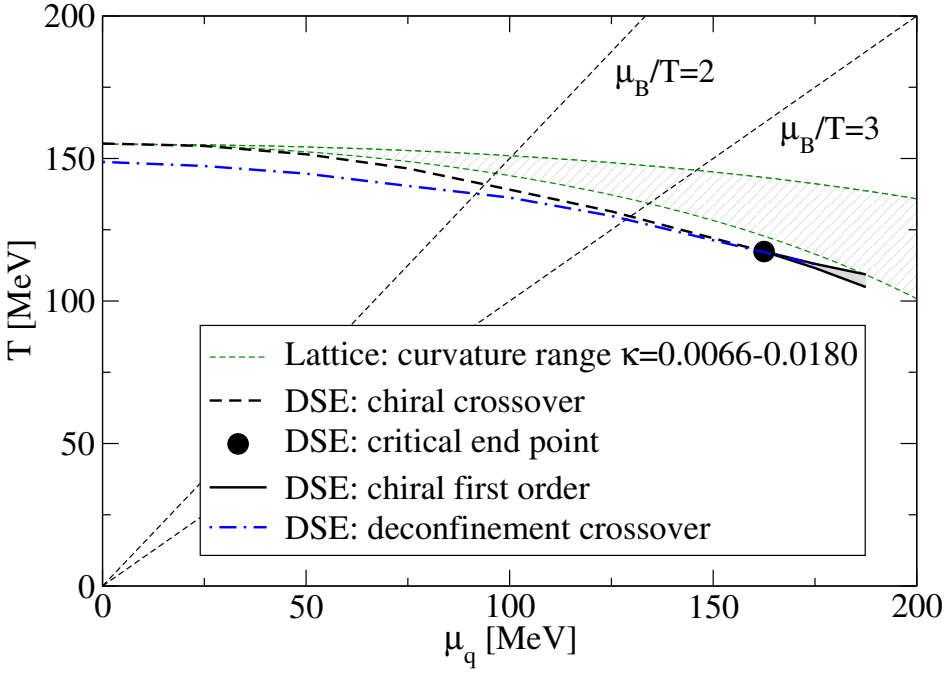
[Fischer, Lücker, Welzbacher '14]:



QCD phase diagram from functional equations

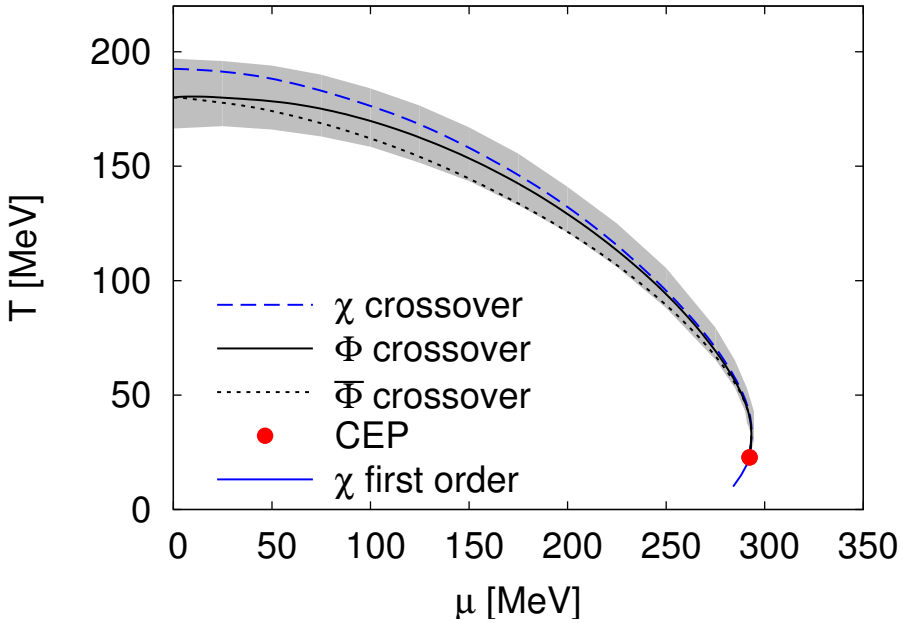
2+1 flavor QCD from DSEs

[Fischer, Lücker, Welzbacher '14]:



2 flavor QCD from PQM with FRG

[Herbst, Pawlowski, Schaefer '10]:



QCD phase diagram from functional equations

Input for DSEs (see also talk by Contant):

- model for quark-gluon vertex (parameters fixed at $\mu = 0$)
- fits for gluon and ghost propagators at $\mu = 0$ from the lattice

QCD phase diagram from functional equations

Input for DSEs (see also talk by Contant):

- model for quark-gluon vertex (parameters fixed at $\mu = 0$)
- fits for gluon and ghost propagators at $\mu = 0$ from the lattice

Possible improvements:

- fully dynamical propagators
- fully dynamical quark-gluon vertex

QCD phase diagram from functional equations

Input for DSEs (see also talk by Contant):

- model for quark-gluon vertex (parameters fixed at $\mu = 0$)
- fits for gluon and ghost propagators at $\mu = 0$ from the lattice

Possible improvements:

- fully dynamical propagators $\rightarrow$ require other vertices
- fully dynamical quark-gluon vertex $\rightarrow$ requires propagators & other vertices

QCD phase diagram from functional equations

Input for DSEs (see also talk by Contant):

- model for quark-gluon vertex (parameters fixed at $\mu = 0$)
- fits for gluon and ghost propagators at $\mu = 0$ from the lattice

Possible improvements:

- fully dynamical propagators $\rightarrow$ require other vertices
- fully dynamical quark-gluon vertex $\rightarrow$ requires propagators & other vertices

Ultimately, full control over Yang-Mills part required!

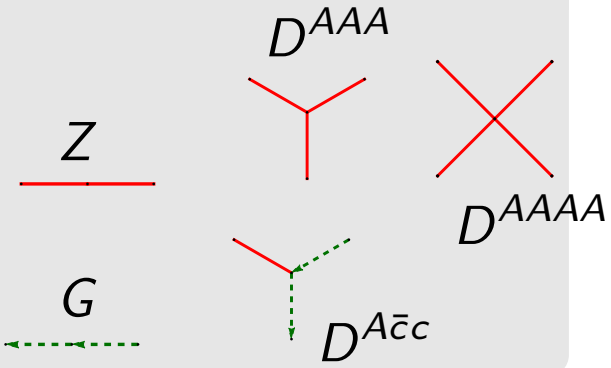
Landau gauge Yang-Mills theory

Gluonic sector of quantum chromodynamics: Yang-Mills theory

$$\mathcal{L} = \frac{1}{2} F^2 + \mathcal{L}_{gf} + \mathcal{L}_{gh}$$
$$F_{\mu\nu} = \partial_\mu \boldsymbol{A}_\nu - \partial_\nu \boldsymbol{A}_\mu + i\, g\, [\boldsymbol{A}_\mu, \boldsymbol{A}_\nu]$$

Landau gauge

- simplest one for functional equations
- $\partial_\mu \boldsymbol{A}_\mu = 0$: $\quad \mathcal{L}_{gf} = \frac{1}{2\xi} (\partial_\mu \boldsymbol{A}_\mu)^2, \quad \xi \to 0$
- requires ghost fields: $\quad \mathcal{L}_{gh} = \bar{\boldsymbol{c}} \left(-\Box + g\, \boldsymbol{A} \times \right) \boldsymbol{c}$



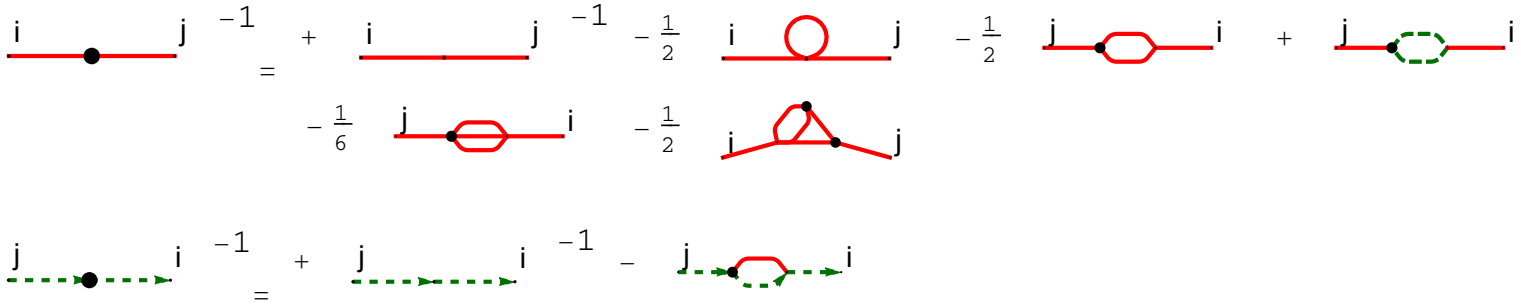
Truncation of Yang-Mills system

Neglect all non-primitively divergent Green functions.

Truncation of Yang-Mills system

Neglect all non-primitively divergent Green functions.

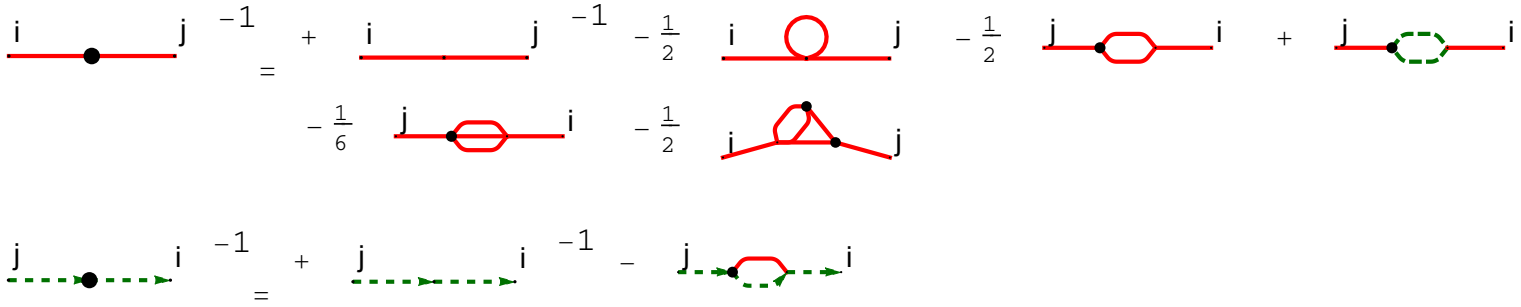
Full propagator equations (two-loop diagrams!):



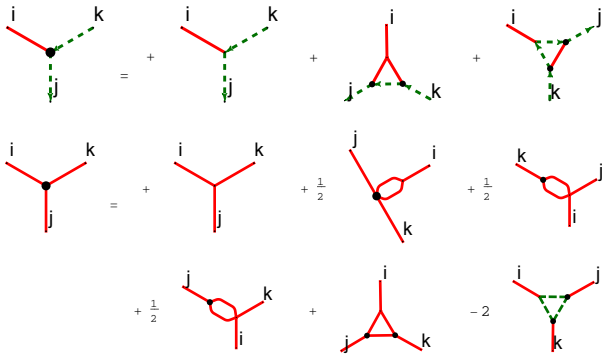
Truncation of Yang-Mills system

Neglect all non-primitively divergent Green functions.

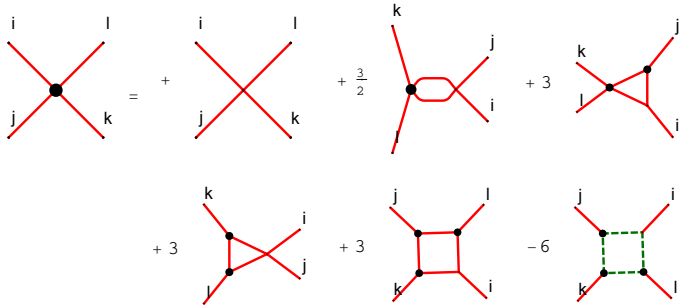
Full propagator equations (two-loop diagrams!):



Truncated three-point functions:



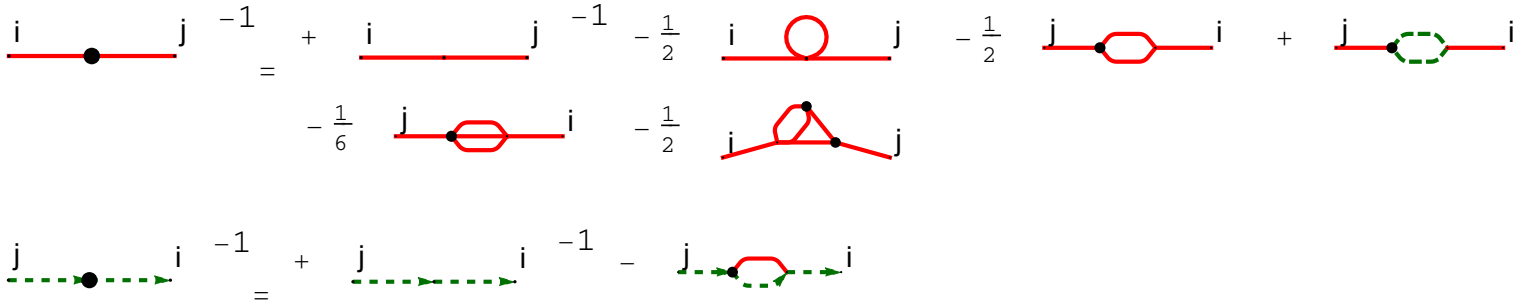
Truncated four-gluon vertex:



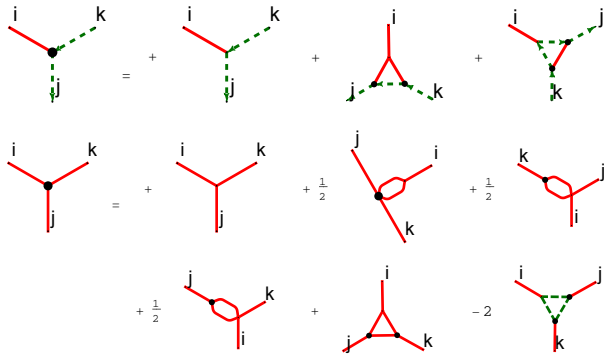
Truncation of Yang-Mills system

Neglect all non-primitively divergent Green functions.

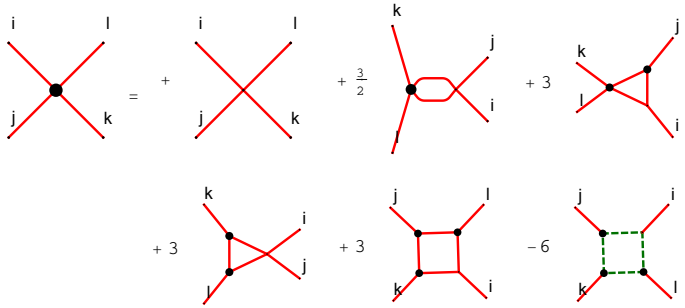
Full propagator equations (two-loop diagrams!):



Truncated three-point functions:



Truncated four-gluon vertex:



Technical questions: spurious divergences in gluon propagator, RG resummation

Yang-Mills theory in 3 dimensions: Why again?

Historically interesting because cheaper on the lattice $\rightarrow$ easier to reach the IR.

However: Numerically not cheaper for functional equations of 2- and 3-point functions.

Yang-Mills theory in 3 dimensions: Why again?

Historically interesting because cheaper on the lattice $\rightarrow$ easier to reach the IR.

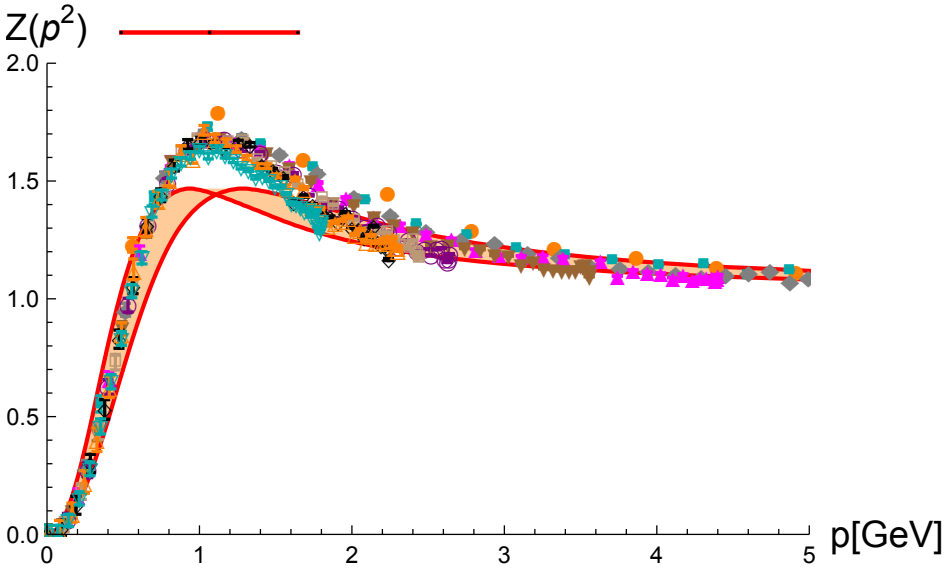
However: Numerically not cheaper for functional equations of 2- and 3-point functions.

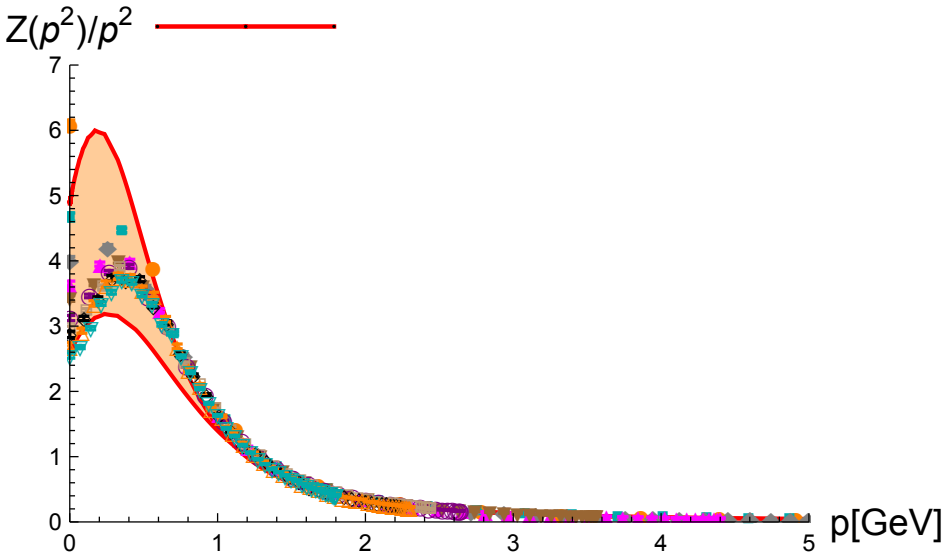
Advantages:

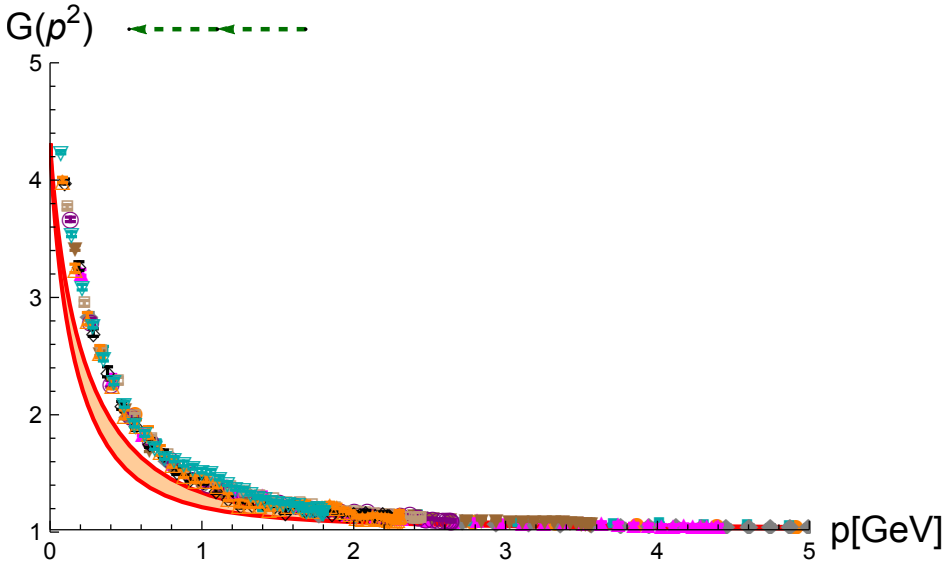
- UV finite: no renormalization, no anomalous running
- Spurious divergences easier to handle

$\Rightarrow$ Many complications from $d = 4$ absent. $\rightarrow$ Focus on truncation effects.

Results: Propagators







Bands from uncertainty in setting the physical scale.

[lattice: Maas '14]

Non-perturbative gauge fixing

Gribov copies: Gauge equivalent configurations that fulfill the Landau gauge condition $\partial A = 0$.

Up to here the minimal Landau gauge was shown for lattice data.

Non-perturbative gauge fixing

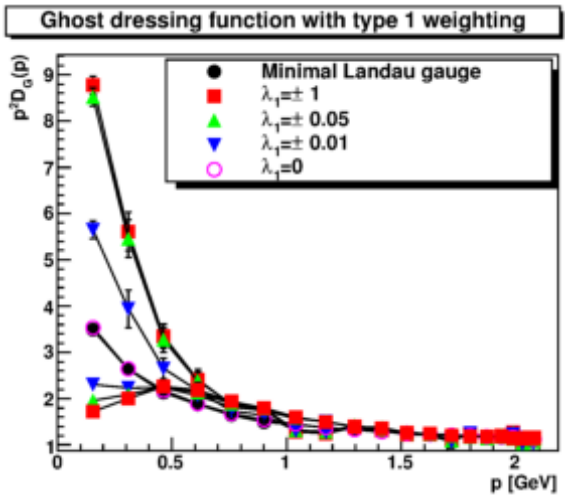
Gribov copies: Gauge equivalent configurations that fulfill the Landau gauge condition $\partial A = 0$.

Up to here the minimal Landau gauge was shown for lattice data.

Another possibility: Absolute Landau gauge (global minimum of gauge fixing functional)

$\rightarrow$ Different solutions on the lattice,

e.g. [Maas '09, '11; Cucchieri '97; Bogolubsky et al. '05; Sternbeck, Müller-Preussker '12].



[Maas '13]

NB: Different solutions also from functional equations [Boucaud et al. '08; Fischer, Maas, Pawlowski '08].

Non-perturbative gauge fixing

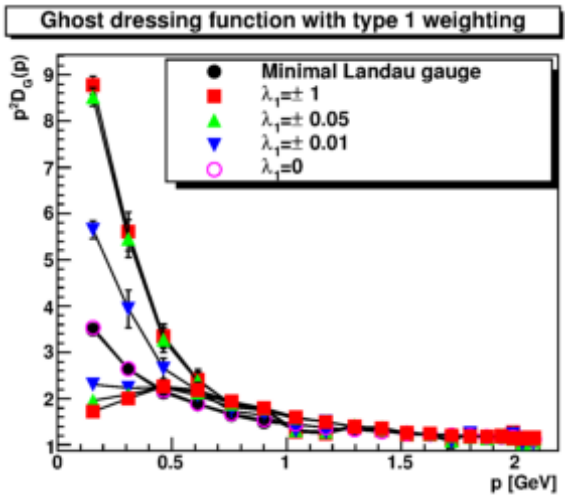
Gribov copies: Gauge equivalent configurations that fulfill the Landau gauge condition $\partial A = 0$.

Up to here the minimal Landau gauge was shown for lattice data.

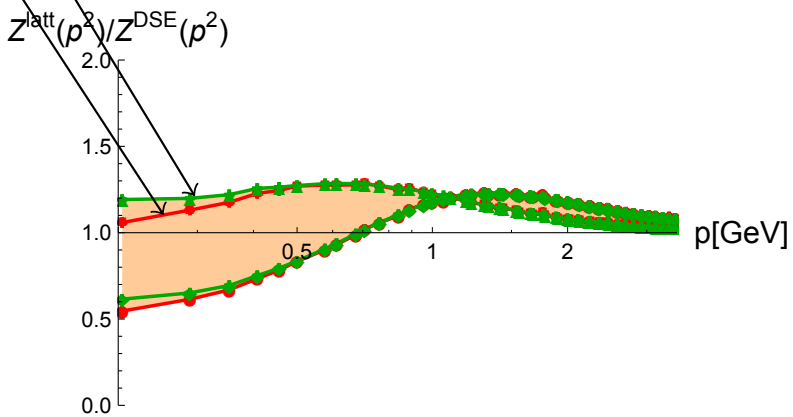
Another possibility: Absolute Landau gauge (global minimum of gauge fixing functional)

$\rightarrow$ Different solutions on the lattice,

e.g. [Maas '09, '11; Cucchieri '97; Bogolubsky et al. '05; Sternbeck, Müller-Preussker '12].



[Maas '13]



[lattice: Bornyakov, Mitrjushkin, Rogalyov '13]

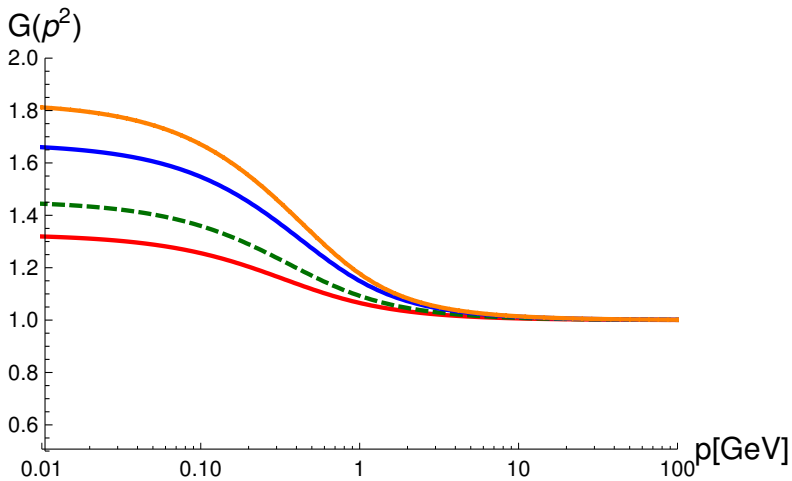
NB: Different solutions also from functional equations [Boucaud et al. '08; Fischer, Maas, Pawlowski '08].

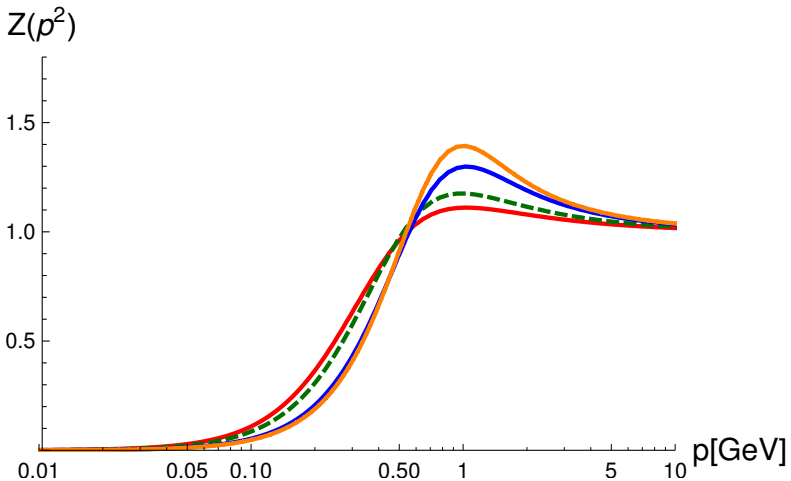
Family of solutions

Cf. FRG results: Bare mass parameter from modified STIs [Cyrol, Fister, Mitter, Pawlowski, Strodthoff '16].

DSEs: Enforce family of solutions by fixing the gluon propagator at $p^2 = 0$.

Simple toy system with bare vertices [MQH, 1606.02068]:



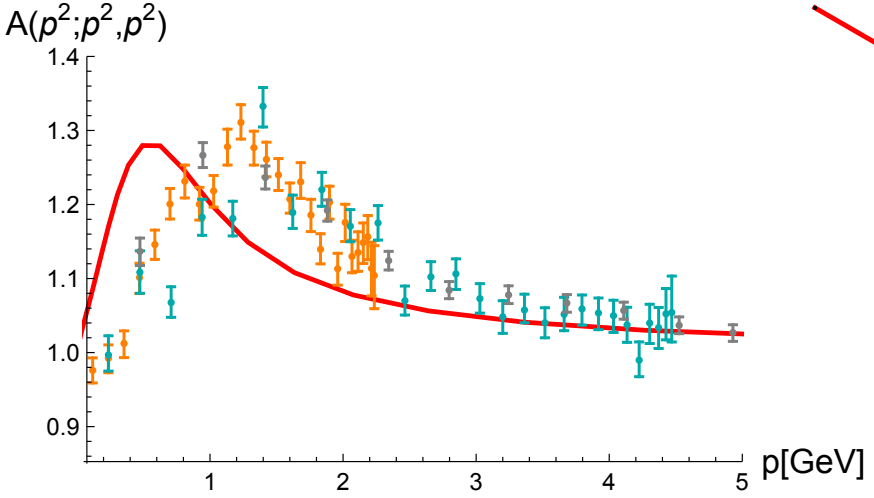


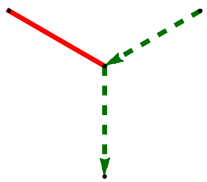
$\Rightarrow$ Possibility of family of solutions.

NB: Effect overestimated here since vertices are fixed.

Results: Three-point functions

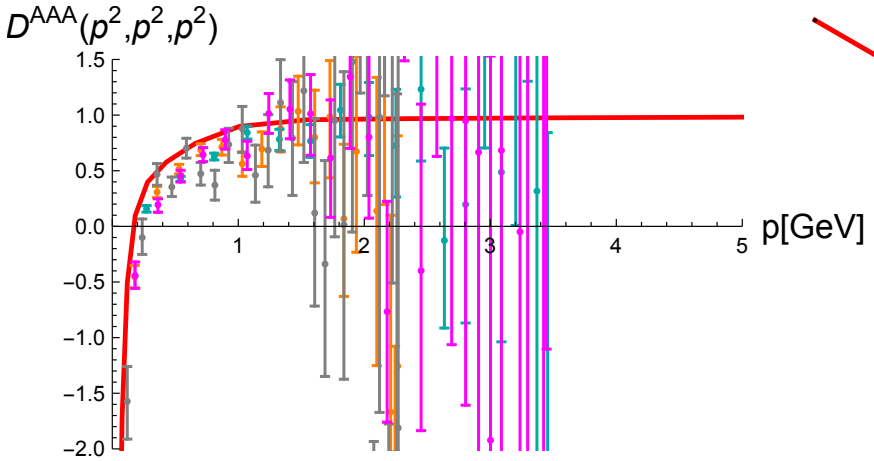
Dressings:

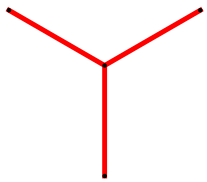




[lattice: Maas, unpublished]

- Maximum position shifted.
- Bump height ok.





[lattice: Cucchieri, Maas, Mendes '08]

- Good agreement with lattice data.
- Linear IR divergence.

Varying the four-gluon vertex

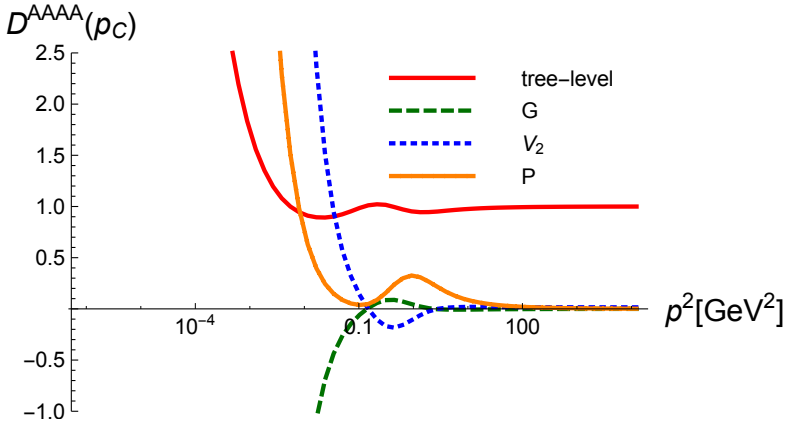
How stable is the truncation?

Varying the four-gluon vertex

How stable is the truncation?

Compare:

- full four-gluon vertex
- bare four-gluon vertex

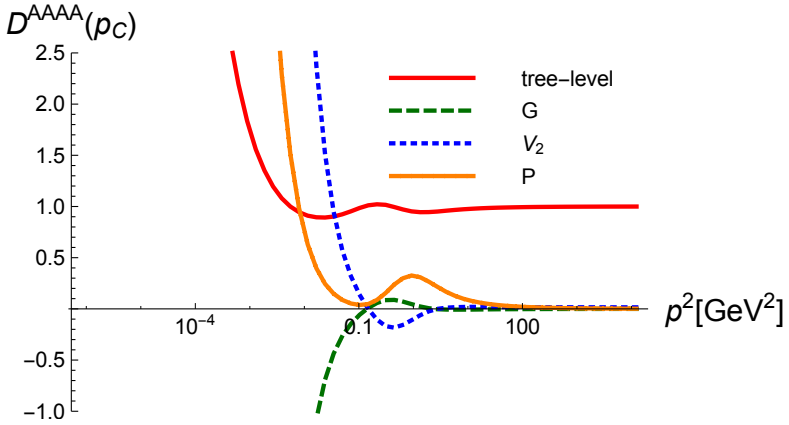


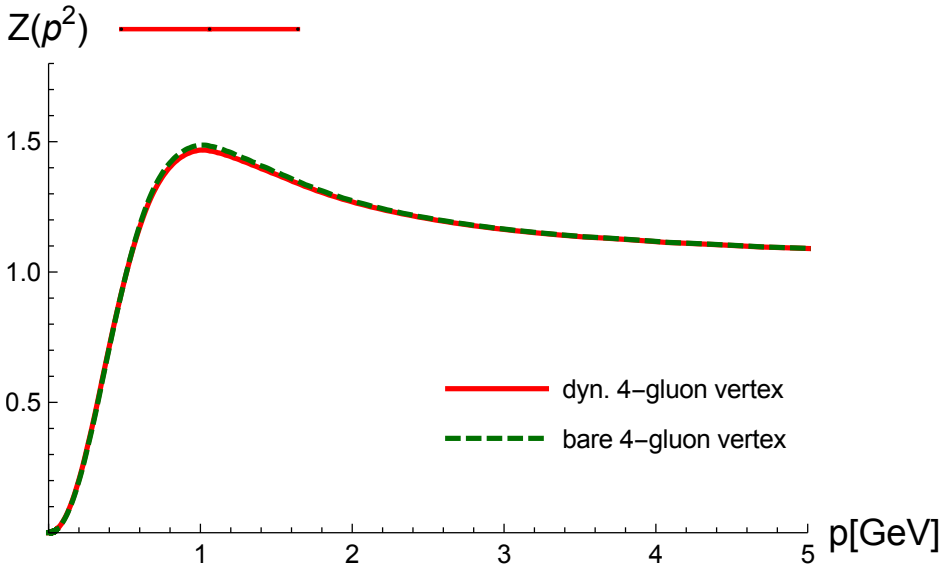
Varying the four-gluon vertex

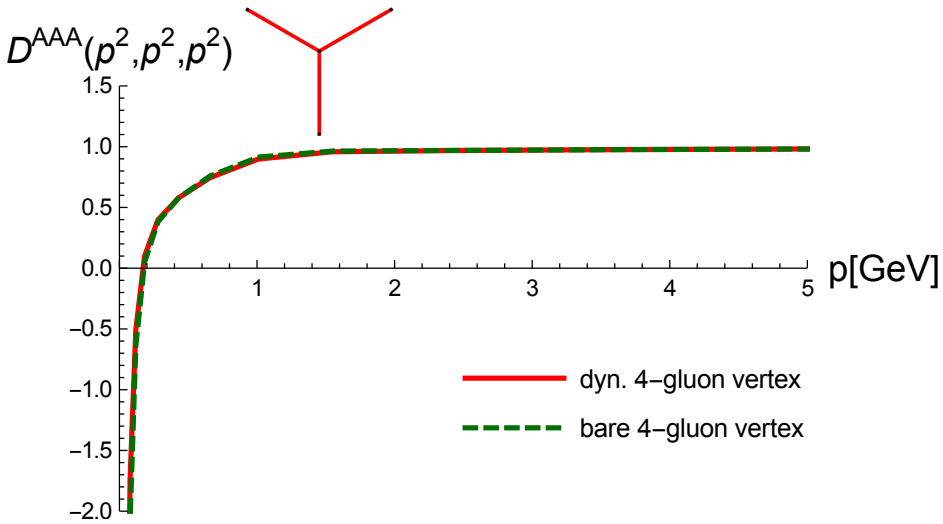
How stable is the truncation?

Compare:

- full four-gluon vertex
- bare four-gluon vertex







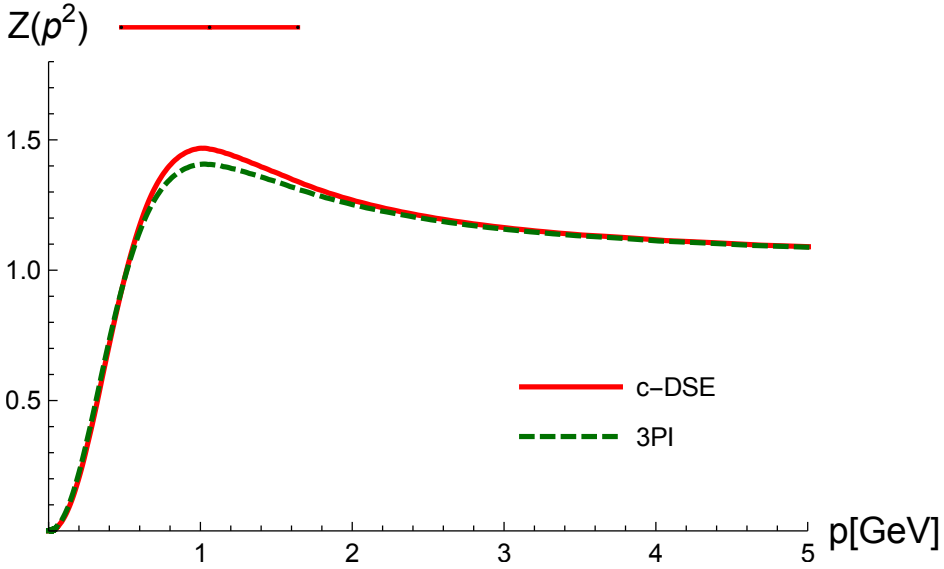
→ Very similar results.

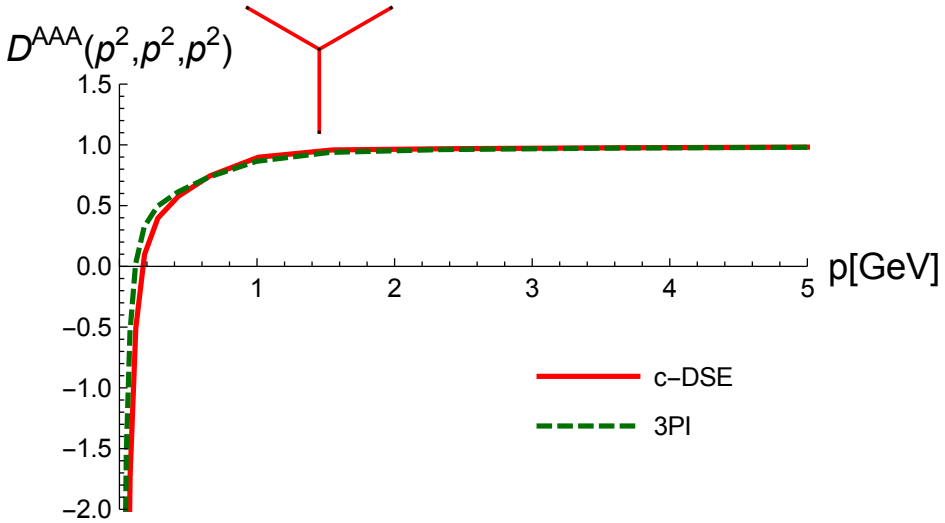
Solution from the 3PI effective action

Different set of functional equations:
equations of motion from 3PI effective action (at three-loop level)

Solution from the 3PI effective action

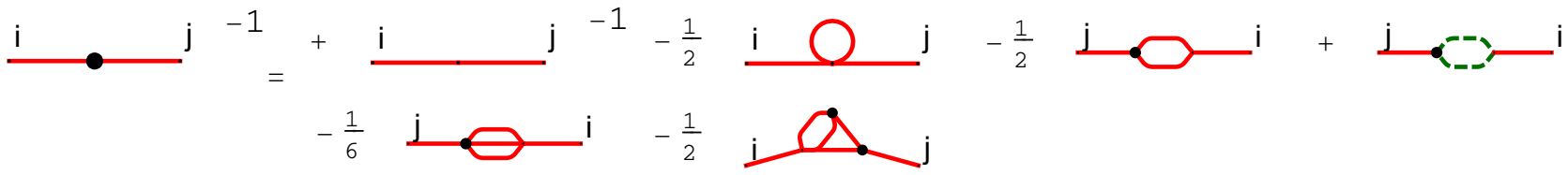
Different set of functional equations:
equations of motion from 3PI effective action (at three-loop level)

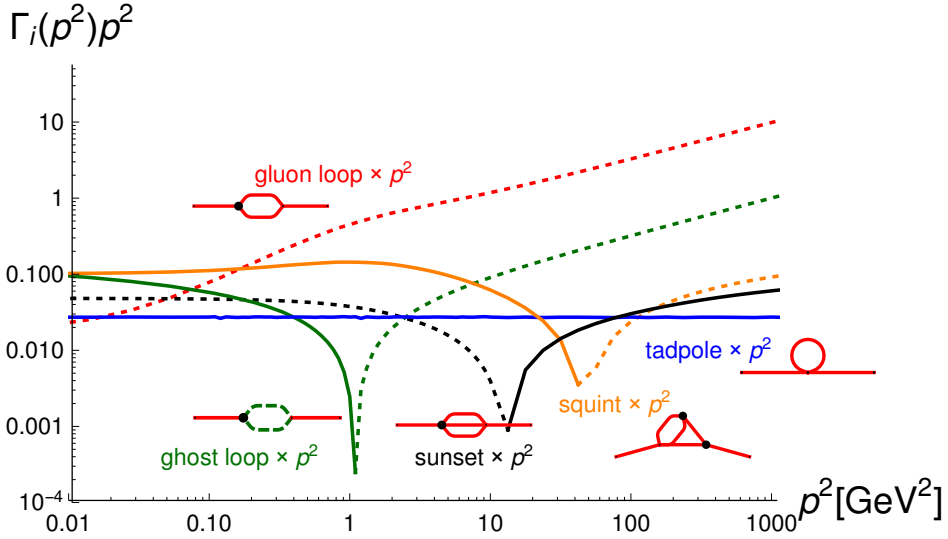




→ Very similar results.

Gluon propagator: Single diagrams

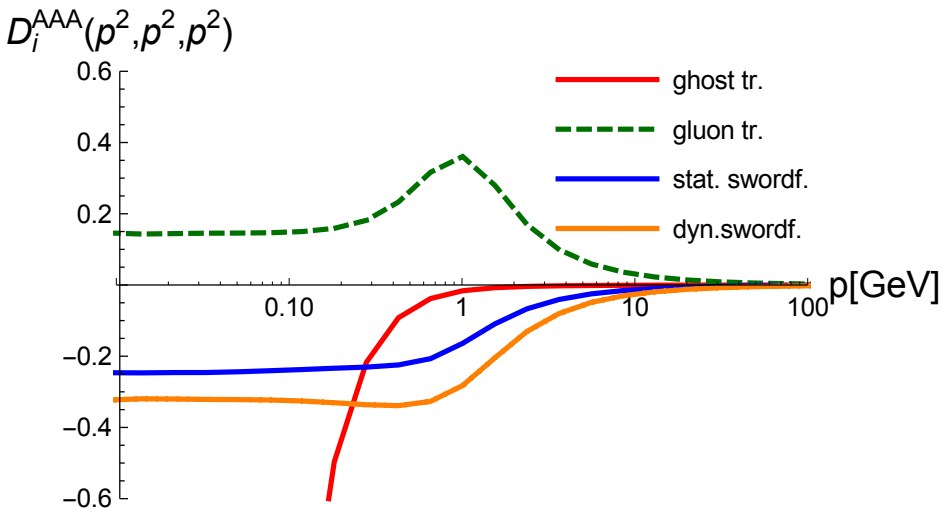




- Squint important in midmomentum regime.
- Sunset contribution small.

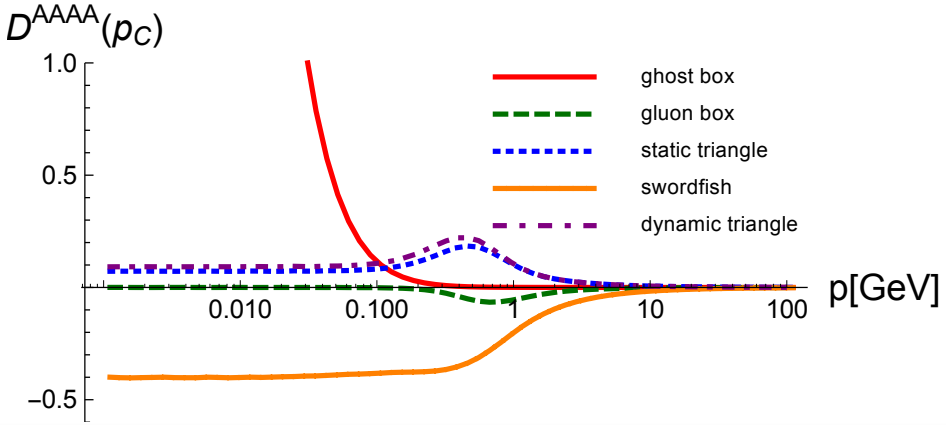
Cancellations in gluonic vertices

Three-gluon vertex:



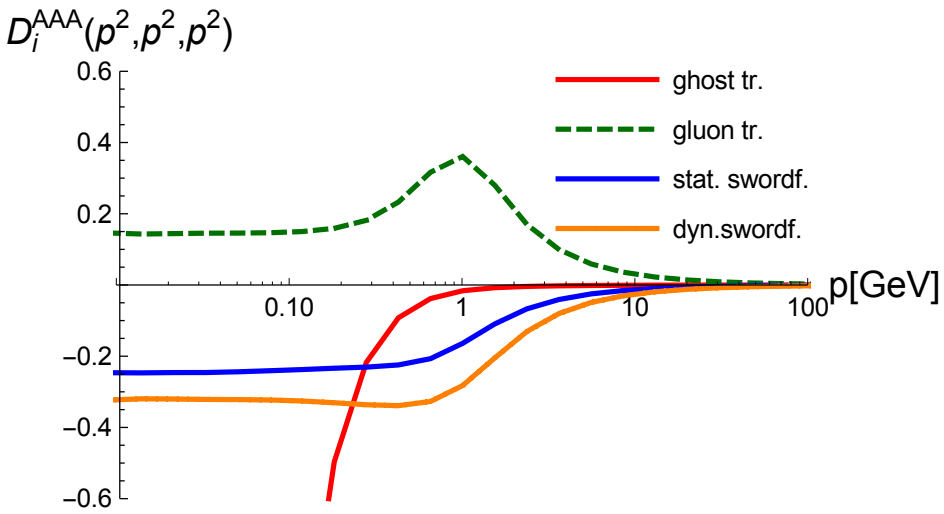
- Individual contributions large.
- Sum is small.

Four-gluon vertex:

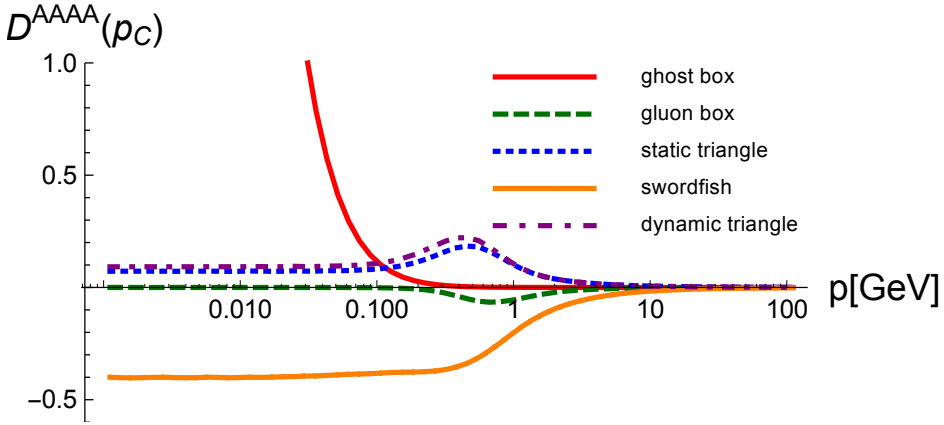


Cancellations in gluonic vertices

Three-gluon vertex:



Four-gluon vertex:



- Individual contributions large.
- Sum is small.

$\Downarrow$

Higher contributions:

- Higher vertices close to 'tree-level'? $\rightarrow$ Small.
- If pattern changes (higher vertices large): cancellations required.

Summary and conclusions

Test truncation effects in $d = 3$, where spurious divergences and RG resummation are understood:

- Used a self-contained truncation $\rightarrow$ no model parameters.
- Truncation stable under all tested variations:
  - comparison with 3PI
  - changing the four-gluon vertex
  - different DSEs for the ghost-gluon vertex
- Hierarchy of diagrams identified.
- Direct relation between different solutions in continuum and on the lattice to be understood.

Summary and conclusions

Test truncation effects in $d = 3$, where spurious divergences and RG resummation are understood:

- Used a self-contained truncation $\rightarrow$ no model parameters.
- Truncation stable under all tested variations:
  - comparison with 3PI
  - changing the four-gluon vertex
  - different DSEs for the ghost-gluon vertex
- Hierarchy of diagrams identified.
- Direct relation between different solutions in continuum and on the lattice to be understood.

Thank you for your attention.