From propagators to resonances in QCD

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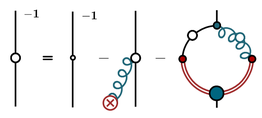
ABSTRACT:

We show results for propagators and vertices in Yang-Mills theory and QCD obtained from the 3PI effective action. Their use in calculating bound-states and resonances requires access to complex momenta, necessitating special care. We perform appropriate deformations of the integration contour to avoid cuts and poles and present corresponding results for bound states as well as first results for resonances using a simpler truncation of the functional equations.

3PI effective action

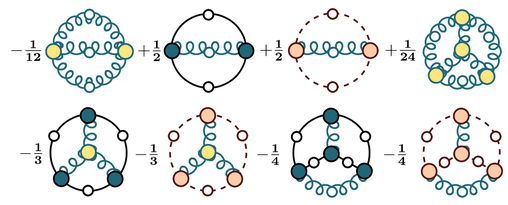
Dyson-Schwinger equations are the equations of motion from

- 1PI effective action: Legendre transformations with respect to **fields**
- 3PI effective action: further Legendre transformations with respect to **propagators** and **vertices**

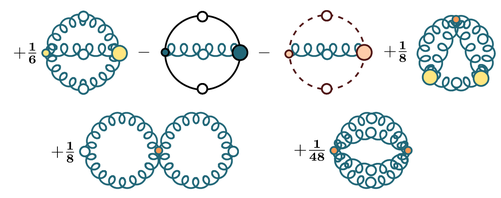


Example 1PI generating functional

1PI: Infinitely many equations with a finite number of terms, truncation necessary



Interacting part of the 3PI-3L effective action



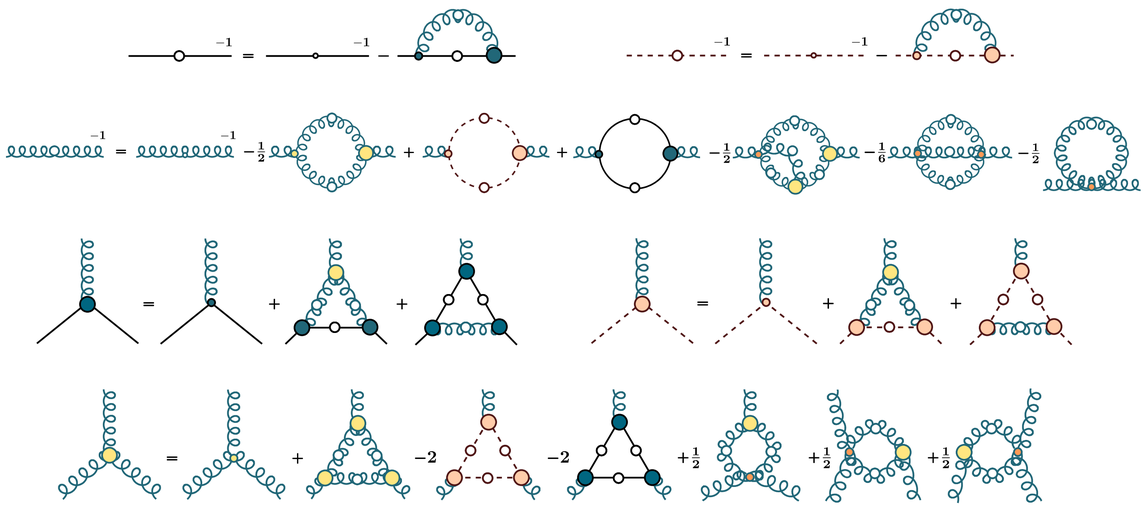
Non-interacting part of the 3PI-3L effective action

3PI: Finite number of equations with infinitely many terms, truncation via loop expansion

Equations of motion

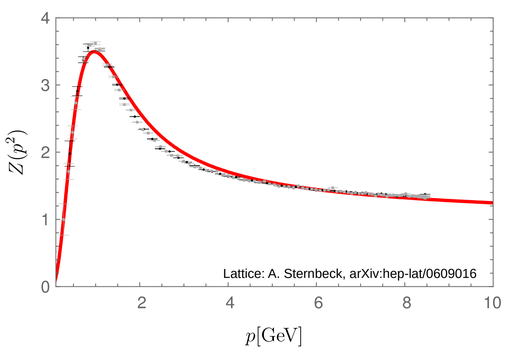
Solution of stationary conditions leads to the quantum equations of motion:

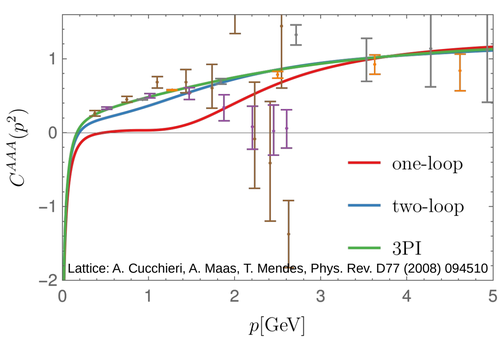
- Constitutes a numerical tool for the calculation of correlation functions.
- Can be formulated in both the vacuum and in the medium (finite temperature and finite density).
- Analytic as well as numerical analysis is possible.
- Provides access to space-like and time-like momentum regions.



Propagators and Vertices[1-5]

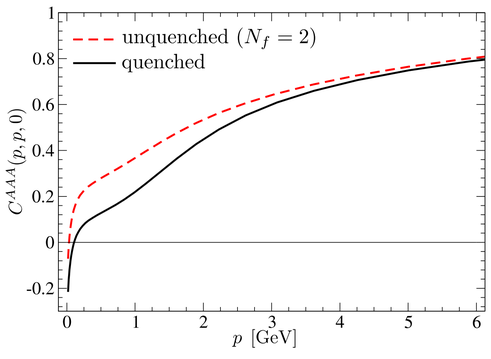
- Infinitely large system of coupled non-linear integral equations.
- Present truncation excludes only the sub-leading components of the three-gluon vertex.





Three-gluon vertex:

- Quantitative influence in all other equations.
- Suppression at small (infrared) momenta, together with a zero crossing.
- Similar results for three-loop 3PI system and a DSE truncation with two-loop diagrams.
- Effect of diagram with non-primitively divergent vertex in DSE negligible.
- Quark loops screen the gluon propagator: dynamical interplay leads to anti-screening in the three-gluon vertex.



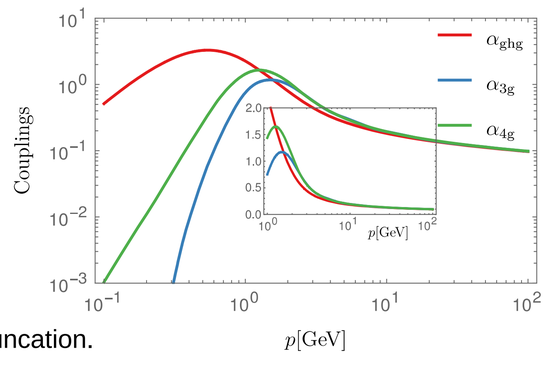
Running Couplings[3-6]

- A running coupling can be extracted for each primitively divergent vertex:

$$\alpha(p^2) = \alpha(\mu^2) Z(p^2)^{n_{gl}} G(p^2)^{n_{gh}} \Gamma(p^2)^{n_v}$$

- Couplings agree perturbatively due to their respective Slavnov-Taylor identities.

Provides a non-trivial check on the results of the truncation.



Time-like momenta

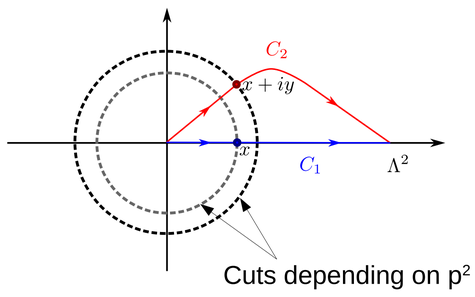
Integrand contains a pole with angular dependence.

$$I(p^2) = \int d^4q \frac{C(p,q)}{p^2 + q^2 + 2\,p\cdot q}$$

Angular integration creates a cut which has to be avoided by contour deformation.

$$I(p^2 = x) \propto \int_{C_1} dq^2 \int dz \sqrt{1-z^2} \frac{C(p^2,q^2,z)}{p^2+q^2+2\,p\,q\,z}$$

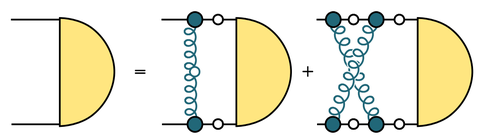
$$I(p^2 = x + i\,y) \propto \int_{C_2} dq^2 \int dz \sqrt{1-z^2} \frac{C(p^2,q^2,z)}{p^2+q^2+2\,p\,q\,z}$$



Path deformation avoiding cut structure in quark DSE

Details, e.g., in R. Alkofer, W. Detmold, C. S. Fischer, P. Maris, Phys. Rev D70 (2004) 014014; A. Windisch, M. Q. Huber, R. Alkofer, Acta Phys.Polon.Supp. 6 (2013) no.3, 887

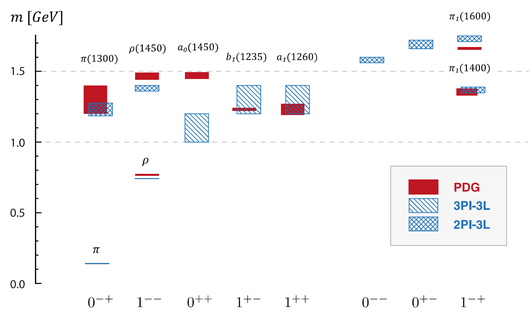
Bound-States[1,7]



Truncated BSE for the 3PI action at three-loop

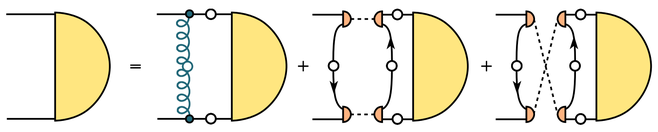
Repulsion in scalar and axial-vector channels when compared to rainbow-ladder

- Light scalar meson above 1 GeV
- Degenerate axial-vectors
- Correct vector—axial-vector splitting.



Light-meson spectrum with 2PI and 3PI actions at 3L

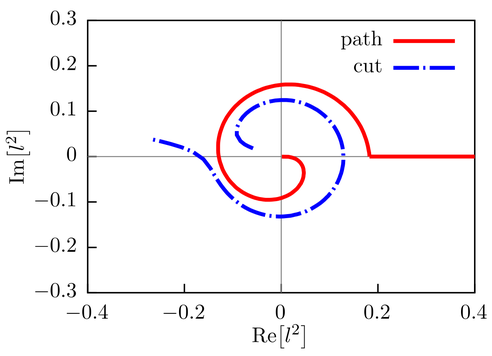
Resonances[8]



Dynamical resonance due to the inclusion of its two-body decay

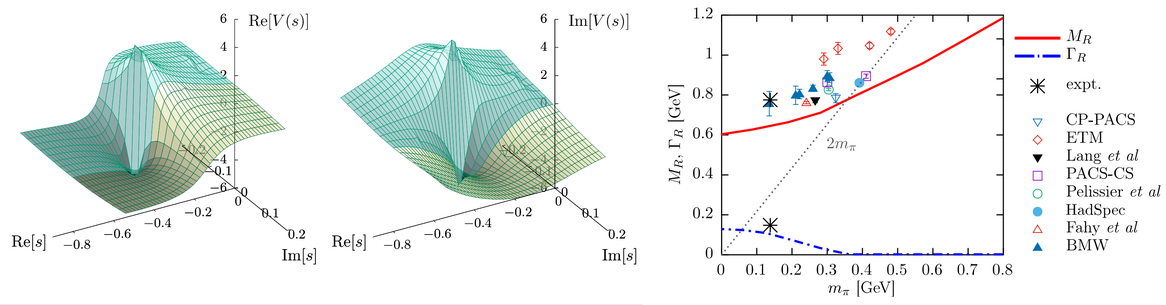
- Intermediate pion poles sweep out cuts in the radial integration. Correct integration path is **deformed**.

$$l^2_{\rm cut} = \left[-z\sqrt{t} + \sqrt{t\,(z^2-1) - m_\pi^2}\right]^2, \quad t = P^2/4$$



- Direct numerical evaluation is restricted to the first Riemann sheet.
- Analytic continuation to the 2nd Riemann sheet yields bound-state poles and access to decay width.

$\rho \to \pi\pi$



Analytic continuation of the BS vertex from 1st to 2nd Riemann sheet

Literature

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[2] M. Q. Huber, Eur. Phys. J. C77 (2017) 733
[3] A. Blum, M. Q. Huber, M. Mitter, L. von Smekal, Phys. Rev. D89 (2014) 061703
[4] G. Eichmann, R. Williams, R. Alkofer, M. Vujinovic, Phys. Rev. D89 (2014) 105014
[5] M. Q. Huber, unpublished
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