The puzzle of confinement: putting together some pieces

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Nov. 10, 2011

Darmstadt, IKP Theorie-Seminar

Unterstützt von / Supported by
Alexander von Humboldt Stiftung/Foundation

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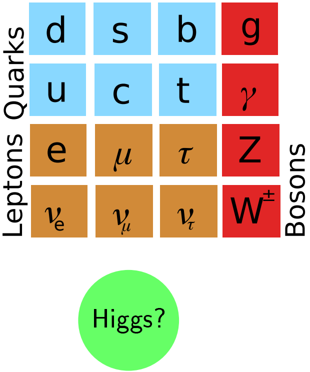
Our view of the world in terms of particles

The standard model:



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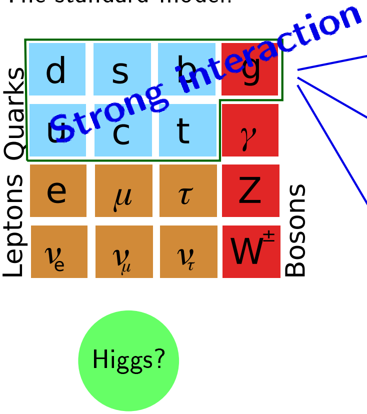
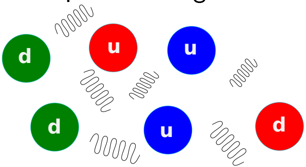
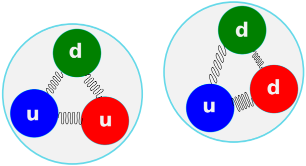


fundamental degrees of freedom:
quarks and gluons, # few
u, d quarks are light $\sim$ MeV



physical degrees of freedom:
hadrons, # many: p, n, $\pi^{0,\pm}$, $K^{0,\pm}$, ...,
hadrons are heavy $\gtrsim$ 140 MeV



Hadrons are bound states of quarks,
but they cannot be split into their building blocks.

Confinement

Inelastic scattering:

Atoms $\longrightarrow$ constituents

Hadrons $\longrightarrow$ more hadrons

"Absence of free quarks and gluons."

"Quarks and gluons are not part of the physical state space."

Quarks: no fractional charges have been found

We observe hadrons but
use (unobservable) quarks and gluons as elementary fields.

Several confinement scenarios/mechanisms

View the problem from different perspectives $\rightarrow$ not mutually exclusive but different aspects emphasized.

see e.g. [Alkofer, Greensite, JPG34 (2007)] for a short review

Confinement scenarios: a selection

Dual superconductor & monopoles

...

Abelian infrared dominance

Center vortices

Gribov-Zwanziger

Kugo-Ojima

Dual superconductor picture of confinement

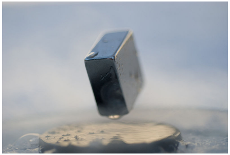
'tHooft 1976, Mandelstam 1976

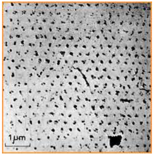
Conventional type-II superconductor:

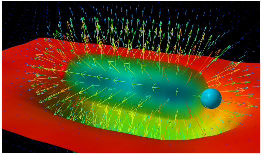
- magnetic flux squeezed into vortices
- condensation of Cooper pairs

Dual superconductor:

- electric flux squeezed into vortices
- condensation of magnetic monopoles







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Dual superconductor picture of confinement

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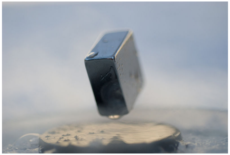
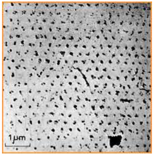
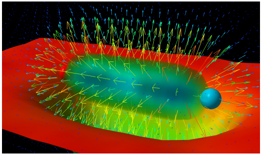
Dual superconductor:

- chromoelectric flux squeezed into vortices
- condensation of chromomagnetic monopoles







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Theories where confinement was proven:
compact $U(1)$, Georgi-Glashow model, deformed $\mathcal{N}=2$ SUSY Yang-Mills

$\rightarrow$ In all condensation of magnetic monopoles!

Abelian infrared dominance

Hypothesis of Abelian infrared dominance [Ezawa, Iwazaki, PRD25 (1981)]: based on dual superconductor picture

Chromomagnetic monopoles live in Abelian part of the theory.

Monopoles dominant for long-range phenomena.

Abelian part dominant in the infrared.

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What is the Abelian part?

$$[A^a_\mu, A^b_\nu] = 0\,?$$

Gauge actions

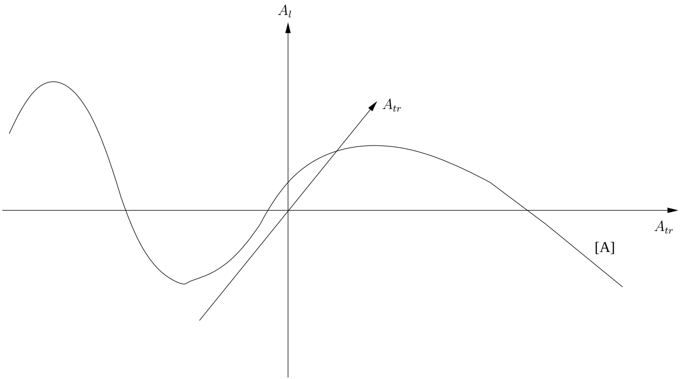
Action is invariant under gauge transformations.
$\rightarrow$ (Infinitely) Many equivalent configurations.
Space of field configurations $\mathcal{A}$ is not the physical space!

Gauge transformations connect physically equivalent configurations.
$\rightarrow$ Physical space is the quotient space over the gauge group G:

$$\mathcal{A}_{phys} = \mathcal{A}/G$$

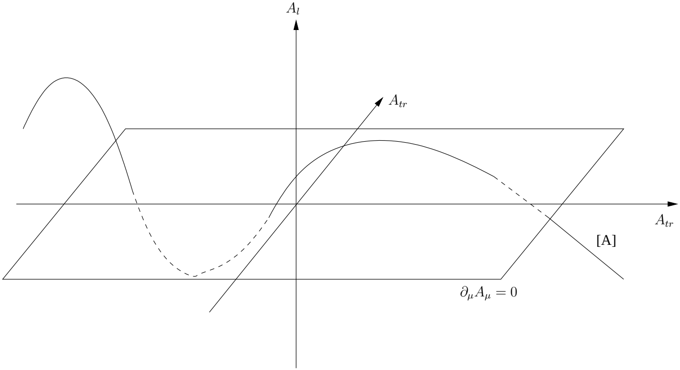
Realization: Fix a gauge.

Gauge orbits and Gribov copies



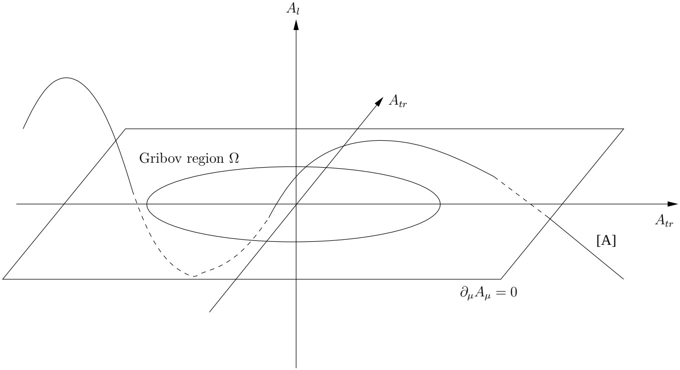
Gauge equivalent configurations (gauge orbit $[A]$)
$\Rightarrow$ integration in path integral is overcomplete.

Gauge orbits and Gribov copies



Faddeev and Popov: Restriction of integration to single representative of each gauge orbit possible?
Gauge symmetry replaced by BRST symmetry!

Gauge orbits and Gribov copies



Restriction to Gribov horizon: almost unique gauge fixing.
Can be implemented in action.

Gauge fixing

Restriction of integration, e.g. to a hypersurface $\Gamma : f[A] = 0$ (gauge fixing condition, e.g., $\partial A = 0$)

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Freedom to choose any gauge!

- Perturbation theory: Feynman gauge convenient, propagator simple $(g_{\mu\nu}/p^2)$
- Dyson-Schwinger equations: Landau gauge, simplification of calculations. Helped us learn the method.
- Deep inelastic scattering: light-cone gauge
- Dual superconductor scenario: Abelian gauges

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(All these gauges are non-perturbatively not complete:
There are still equivalent configurations on Γ. $\rightarrow$ Gribov problem
Physical space $\mathcal{A}_{phys}$ is topological not trivial!

Theorem of Singer 1978:
No unique gauge fixing with a continuous gauge fixing condition.)

Hypothesis of Abelian IR dominance

Choose a gauge suitable for investigating hypothesis of Abelian IR dominance.

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Landau gauge?

$\rightarrow$ no monopoles, meaning of Abelian component of the gauge field?

Need a gauge where meaning of Abelian component is clear.

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Need a gauge where meaning of Abelian component is clear.

$\Rightarrow$ maximally Abelian gauge

The maximally Abelian gauge I

Dual superconductor picture based on Abelian symmetry.

What is it for $SU(3)$?

Approach here:
Split the gauge field into diagonal and off-diagonal parts.

The maximally Abelian gauge II

Gauge field components:

$$A_\mu = A^i_\mu T^i + B^a_\mu T^a, \quad i = 1, \ldots, N-1, \quad a = N, \ldots, N^2-1$$

Abelian subalgebra: $[T^i, T^j] = 0$, can be written as diagonal matrices

Abelian $\leftrightarrow$ diagonal fields A,
non-Abelian $\leftrightarrow$ off-diagonal fields B.

E.g. $T^1 = \frac{1}{2}\lambda^3$, $T^2 = \frac{1}{2}\lambda^8$ for $SU(3)$.
Which interactions are possible ($[T^r, T^s] = i f^{rst} T^t$)?

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	$SU(2)$	$SU(N>2)$
f^{ijk}	0	0
f^{ija}	0	0
f^{iab}	✓	✓
f^{abc}	0	✓

$\rightarrow$ $SU(2)$ and $SU(3)$ different?

Gauge fixing condition

Stress role of diagonal fields $\Rightarrow$ minimize norm of off-diagonal field $\boldsymbol{B}$:

$$\|B_U\| = \int dx\, \boldsymbol{B}^a_U \boldsymbol{B}^a_U \rightarrow \text{minimize wrt. gauge transformations } U$$

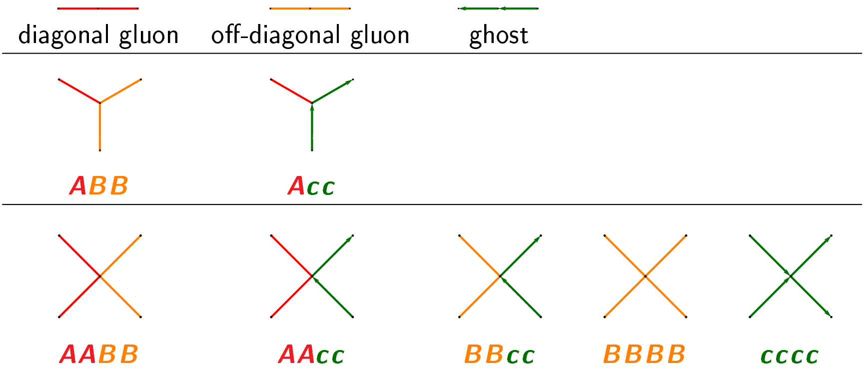
$$D^{ab}_\mu \boldsymbol{B}^b_\mu = (\delta_{ab}\partial_\mu - g\, f^{abi} \boldsymbol{A}^i_\mu)\boldsymbol{B}^b_\mu = 0$$ non-linear gauge fixing condition!

Gauge fixing introduces off-diagonal ghost fields.

Remaining symmetry of diagonal part: $U(1)^{N-1}$

Fix gauge of diag. gluon field $\boldsymbol{A}$ by Landau gauge condition: $\partial_\mu \boldsymbol{A}_\mu = 0$
$\Rightarrow$ diagonal ghosts decouple (like in QED).

Lagrangian for the MAG



$SU(N>2)$

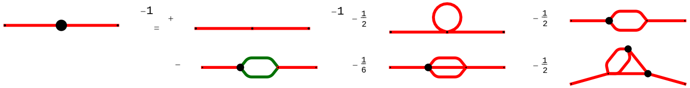
BBB Bcc $ABBB$ $ABcc$

Functional equations

Green functions:
Propagators and vertices $\leftrightarrow$ describe how fields propagate and interact.

Exact relations between Green functions given by, e.g.,
Dyson-Schwinger and functional renormalization group equations.

DSE for the gluon propagator (Landau gauge):

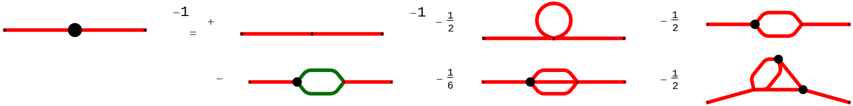


Valid **non-perturbatively**:
dressed propagators and vertices!

Infinite tower of coupled equations:
every eq. contains higher Green functions.

Landau Gauge: Propagators

Gluon propagator:



Ghost propagator:

Functional equations of the maximally Abelian gauge

Partial DSE of the diagonal propagator:

```
In[384]:= getAE[AA, {{A, i1, p, i, μ}, {A, i2, -p, j, ν}}, explicit → True]

Out[384]= {delta[i, j] (-(1 - 1/ξ) FVD[p, μ] FVD[p, ν] + MTD[μ, ν]) SPD[p],

 -1/2 g² delta[a$40620, c$40620] fd[a$40620, oDummy[130], i] fd[c$40620, oDummy[130], j] (2 MTD[μ, ν] MTD[μ$40624, ρ$40624] - (1 - 1/α) MTD[μ$40624

  ((α DB1[SPD[q1]] FVD[q1, μ$40624] FVD[q1, ρ$40624])/SPD[q1]² + (DB[SPD[q1]] (MTD[μ$40624, ρ$40624] - (FVD[q1, μ$40624] FVD[q1, ρ$40624])/SPD[q1]))/SPD[q1]),

 (2 g² Dc[SPD[q1]] delta[c$40620, a$40620] fd[a$40620, oDummy[131], i] fd[c$40620, oDummy[131], j] MTD[μ, ν])/SPD[q1],

 1/2 g² delta[a$40620, e$40620] delta[c$40620, g$40620] fd[a$40620, c$40620, i] fd[e$40620, g$40620, j] GABBD[{ν, τ$40624, β$40624}, {-p, -q1, p

  (FVD[-p + q1, ρ$40624] MTD[μ$40624, μ] + FVD[-p - 2 q1, μ] MTD[μ$40624, ρ$40624] + FVD[2 p + q1, μ$40624] MTD[ρ$40624, μ] + (-FVD[-p - q1, ρ$40624]

  ((α DB1[SPD[-p - q1]] FVD[-p - q1, β$40624] FVD[-p - q1, ρ$40624])/SPD[-p - q1]² + (DB[SPD[-p - q1]] (MTD[ρ$40624, β$40624] - (FVD[-p - q1, β$40624] FVD[-p - q1, ρ$40624])/SPD[-p - q1]))/SPD[-p - q1])

  ((α DB1[SPD[q1]] FVD[q1, μ$40624] FVD[q1, τ$40624])/SPD[q1]² + (DB[SPD[q1]] (MTD[μ$40624, τ$40624] - (FVD[q1, μ$40624] FVD[q1, τ$40624])/SPD[q1]))/SPD[q1]),

 (g² Dc[SPD[-p - q1]] Dc[SPD[-q1]] delta[c$40620, g$40620] delta[e$40620, a$40620] fd[a$40620, c$40620, i] fd[e$40620, g$40620, j] FVD[-p - 2 q1
  /(SPD[-p - q1] SPD[-q1])

  delta[b$40620, f$40620] delta[c$40620, h$40620] fd[b$40620, oDummy[132], i] fd[c$40620, oDummy[132], a$40620] GAABBD[{j, d$40620, f$40620, h

  (2 MTD[μ, μ$40624] MTD[ν$40624, ρ$40624] - (1 - 1/α) MTD[ν$40624, μ$40624] MTD[ρ$40624, μ] - (1 - 1/α) MTD[ν$40624, μ] MTD[ρ$40624, μ$40624])

  ((2 ξ DA1[SPD[q1]] FVD[q1, μ$40624] FVD[q1, σ$40624])/SPD[q1] + (DA[SPD[q1]] (MTD[μ$40624, σ$40624] - (FVD[q1, μ$40624] FVD[q1, σ$40624])/SPD[q1]))/SPD[q1])

  ((α DB1[SPD[-p - q1 - q2]] FVD[-p - q1 - q2, α$40624] FVD[-p - q1 - q2, ν$40624])/SPD[-p - q1 - q2]² + (DB[SPD[-p - q1 - q2]] (MTD[ν$40624, α$40624] - (FVD[-p - q1 - q2, α$40624] F
   /SPD[-p -
  /SPD[-p - q1 - q2]
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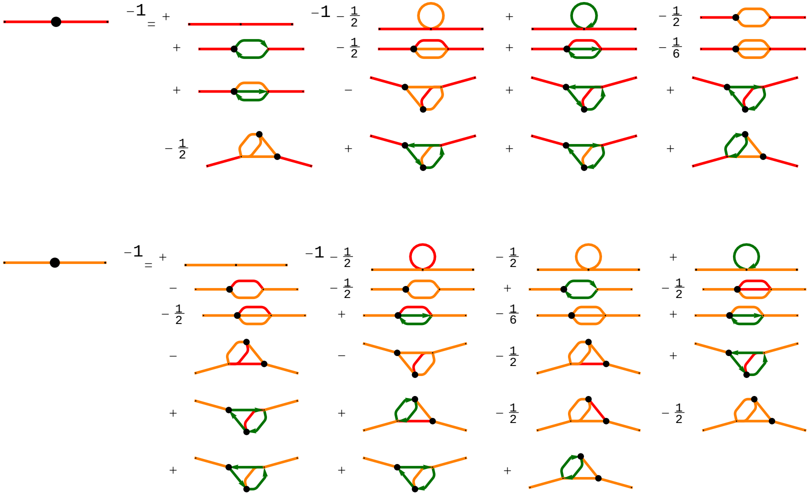
DSEs of the MAG

diagonal, off-diagonal, ghost



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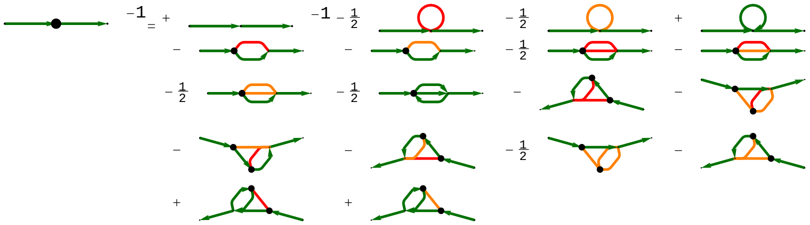

Complete analysis of all diagrams!

Analysis of big systems of functional equations

Big systems require shortcuts!

Two important methods/tools:

Automatization of calculations

- *DoFun*: derive DSEs and FRGEs in *Mathematica* [Alkofer, MQH, Schwenzer, CPC180 (2009); MQH, Braun, 1102.5307]
- *CrasyDSE*: provides framework for calculating DSEs, e.g. creates kernels automatically [MQH, Mitter, arXiv:111x.yzuv]

Combination of DSEs and FRGEs

- introduced by Fischer, Pawlowski for Landau gauge [Fischer, Pawlowski, PRD75 (2007)]
- scaling analysis can be generalized to generic systems [MQH, Schwenzer, Alkofer, EPJC68 (2010)]
  $\longrightarrow$Understanding of the generic structure of the equations with closed formulae for all important information.

Infrared power counting

Generic propagator

$$T_{(ij)} \cdot \frac{D(p^2)}{p^2},$$

assume power law behavior at low p^2

IR exponent

$$D^{IR}(p^2) = A \cdot (p^2)^{\delta}$$

- Vertices also assume power law behavior
  [e.g., Alkofer, Fischer, Llanes-Estrada, PLB 611 (2005) (skeleton expansion)].
- Limit of all momenta approaching zero simultaneously.
- Upon integration all momenta converted into powers of external momenta.

$\Rightarrow$ counting of IR exponents

System of inequalities

- lhs is dominated by at least one diagram on rhs and rhs cannot be more divergent than lhs:

$$\rightarrow \delta_{lhs} \leq \delta_{rhs, any\ diagram}$$

- Not knowing which diagram is leading on the rhs, we can write inequalities from all diagrams.

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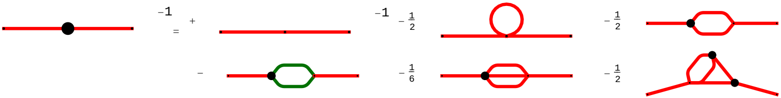

$$-\delta_{gl} \leq 2\delta_{gl} + \delta_{3g}, \qquad -\delta_{gl} \leq 2\delta_{gh} + \delta_{gg}, \qquad \ldots$$

That's the basic idea. Still, for a large system a lot of work.

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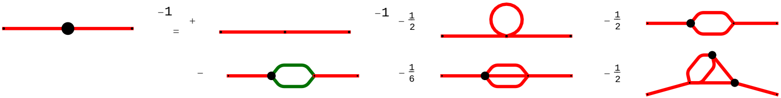

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That's the basic idea. Still, for a large system a lot of work.

All inequalities relevant?

IR analysis of large systems

Combining DSEs and FRGEs

$\Rightarrow$ All relevant inequalities can be written down in closed form.

We obtain the IR exponents of all Green functions!

$\rightarrow$ Qualitative behavior of all Green functions known for low momenta without truncations.

Typically all IR exponents exponents can be expressed with one variable.

for the MAG: κ_{MAG}

Infrared solution for the maximally Abelian gauge

Qualitative behavior of propagators at low momentum p ($\kappa_{MAG} \geq 0$):
[MQH, Schwenzer, Alkofer, EPJC68 (2010)]

canonical dim.

- Off-diagonal gluon propagator is IR suppressed $\sim (p^2)^{\kappa_{MAG}-1}$.
- Ghost propagator (off-diag.) is IR suppressed $\sim (p^2)^{\kappa_{MAG}-1}$.
- Diagonal gluon propagator is IR enhanced $\sim (p^2)^{-\kappa_{MAG}-1}$.
  $\rightarrow$ Diagrams with most diagonal gluons dominate in DSEs.
- Qualitative solution for the whole tower of Green functions.

$\rightarrow$ IR dominance of diagonal/"Abelian" degrees of freedom.

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($SU(N>2)$ has more interactions than $SU(2)$,
but the IR solution is the same.)

Value of the infrared exponent κ_{MAG}

Solution for κ_{MAG} is necessary but not sufficient.

Dressing functions of gluons and ghosts:

$$c_A(p^2) \overset{p^2 \to 0}{=} d_A \cdot (p^2)^{-\kappa_{MAG}} \qquad c_c(p^2) \overset{p^2 \to 0}{=} d_c \cdot (p^2)^{\kappa_{MAG}}$$

$$c_B(p^2) \overset{p^2 \to 0}{=} d_B \cdot (p^2)^{\kappa_{MAG}} \qquad 0 \le \kappa_{MAG} \le 1$$

$$\kappa_{MAG} \approx 0.74$$

$\rightarrow$ Infrared consistent solution exists.

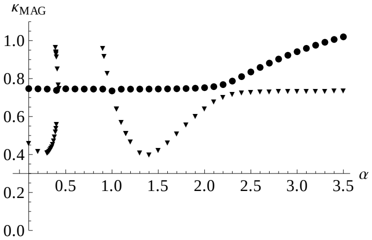
Gauge fixing condition:
$D_\mu A_\mu = 0 \leftrightarrow$ gauge fixing parameter α_{MAG}, MAG: $\alpha_{MAG} = 0$

(cf. linear covariant gauges: $\partial_\mu A_\mu = 0 \leftrightarrow \alpha_{LCG}$,
Landau gauge: $\alpha_{LCG} = 0$)

Gauge fixing parameter dependence

Extension to non-zero α_{MAG} easily possible,
not easy for Landau gauge and linear covariant gauges
[MQH, Alkofer, Schwenzer, PoS(FACESQCD) (2010)].

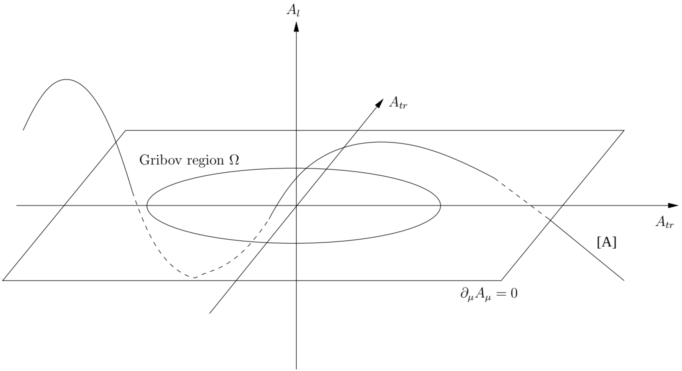
Two solution branches:



General linear covariant gauges: different from the Landau gauge
$\rightarrow$ The maximally Abelian is the first case where an IR solution is found that seems independent of the gauge fixing parameter.

Gauge orbits and Gribov copies

Neither Landau gauge nor MAG are complete gauges.
$\rightarrow$ Test influence of Gribov horizon for Landau gauge.



Restriction to Gribov region is done via adding a non-local term to the Lagrangian. $\rightarrow$ New parameter γ, determined by horizon condition.

Local Gribov-Zwanziger action

Add non-local horizon function h
to the Faddeev-Popov action [Zwanziger, NPB323 (1989)]:

$$\int dx\, \mathcal{L} = \int dx (\mathcal{L}_{FP} + \gamma^4 h)$$

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Localization with (anti)commuting fields $(\bar{\eta}_\mu^{ab}, \eta_\mu^{ab})$ V_μ^{ab}:

$$\mathcal{L}_{GZ} = \mathcal{L}'_{FP} - \bar{\eta}_\mu^{ac} \underbrace{M^{ab}}_{\text{Faddeev-Popov operator: } -\partial D^{ab}} \eta_\mu^{bc} + \frac{1}{2} V_\mu^{ac} M^{ab} V_\mu^{bc} + \boldsymbol{i\, g\, \gamma^2 \sqrt{2} f^{abc} A_\mu^a V_\mu^{bc}}$$

Connection to conventional fields:
$\varphi = \frac{1}{\sqrt{2}}(U + i\,V)\,, \quad \bar{\varphi} = \frac{1}{\sqrt{2}}(U - i\,V)$
$c,\ \bar{c},\ U,\ \omega,\ \bar{\omega} \longrightarrow \eta,\ \bar{\eta}$

Perturbative analysis of the Gribov-Zwanziger action

Gluon propagator:

$$\delta^{ab} P_{\mu\nu}(p) \frac{p^2}{p^4 + 2\,N\,g^2\,\gamma^4}$$

vanishes at zero momentum $\rightarrow$ violation of positivity, gluon confined

Ghost propagator:

$$\sim \frac{1}{k^4}$$

IR enhanced $\rightarrow$ ghost dominance

Gribov-Zwanziger confinement picture

Originally defined in Landau and Coulomb gauge:
[Gribov NPB139 (1989); Zwanziger NPB364 (1991)]

- Gluon propagator vanishes at zero momentum
  $\longrightarrow$ violation of positivity (indication of gluon being confined)
- Ghost propagator infrared enhanced
  $\longrightarrow$ ghost dominant degree of freedom ghost dominance
- Horizon condition $\longrightarrow$ boundary condition for functional equations

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Dominant configurations: close to the Gribov horizon

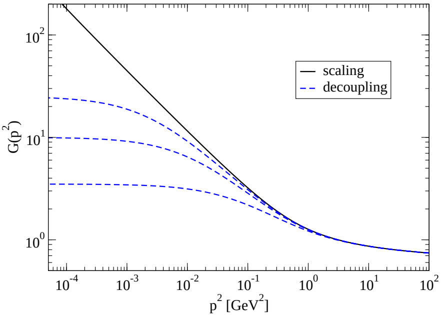
Supported by:

- Semi-perturbative analysis based on improved gauge fixing
  [Gribov NPB139 (1989); Zwanziger, NBP323 (1989); NBP399 (1993)]
- Non-perturbative results from functional equations [e. g. von Smekal, Hauck, Alkofer, PRL79 (1997); Pawlowski, Litim, Nedelko, von Smekal, PRL93 (2004)]

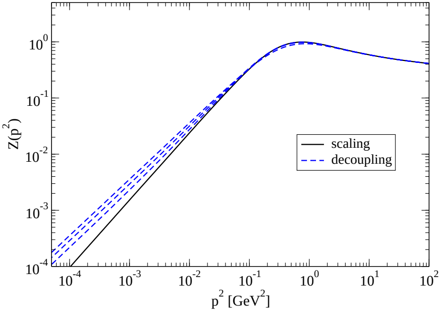
Ghost dominance from DSEs

Using Faddeev-Popov action: qualitatively the same solution (vanishing gluon and enhanced ghost propagators) + family of other solutions

Ghost dressing:



Gluon dressing:



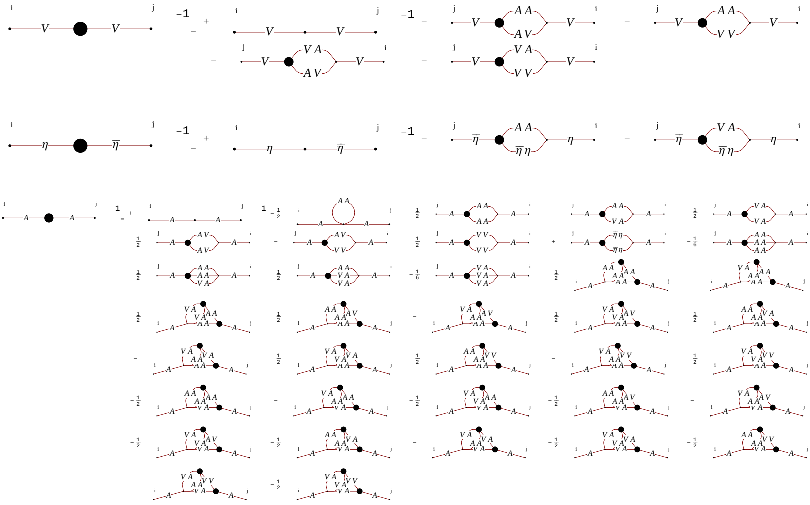
Fischer, Maas, Pawlowski, AP324, 2002

Here: Only scaling solution considered.

What changes when including the Gribov-Zwanziger terms?

DSEs of Gribov-Zwanziger action I

Just to give an impression:



DSEs of Gribov-Zwanziger action II

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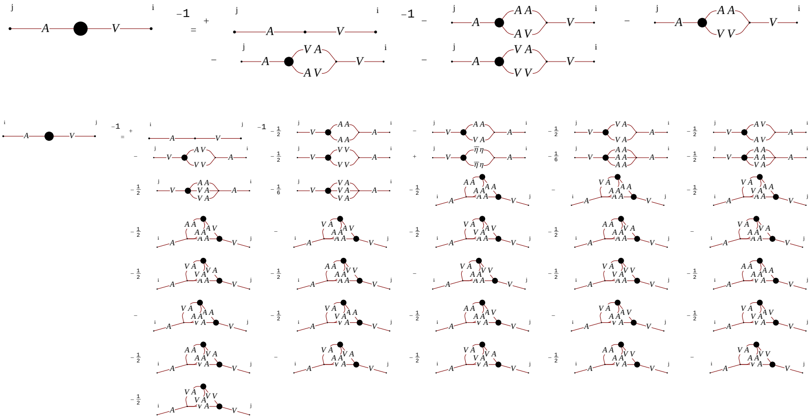

Complete analysis of all diagrams!

Propagators and two-point functions

Mixing at two-point level: $i\,g\,\gamma^2\sqrt{2}f^{abc}A_\mu^a V_\mu^{bc}$

$$D^{\phi\phi} = (\Gamma^{\phi\phi})^{-1}, \qquad \phi \in \{A, V\}$$

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Example: VV-two-point function,

$$\Gamma^{VV,abcd}_{\mu\nu} = \delta^{ac}\delta^{bd} p^2 c_V(p^2) g_{\mu\nu}$$

dressing function $c_V(p^2) \xrightarrow{p^2 \to 0} d_V \cdot (p^2)^{\kappa_V}$

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VV-propagator:

$$D^{VV,abcd}_{\mu\nu} = \frac{1}{p^2} \frac{1}{c_V(p^2)} \delta^{ac}\delta^{bd} g_{\mu\nu} - \\ - f^{abe} f^{cde} \frac{1}{p^2} P_{\mu\nu} \frac{2 c_{AV}^2(p^2)}{c_A^\perp(p^2) c_V^2(p^2) + 2N\, c_{AV}^2(p^2) c_V(p^2)}$$

IR solution of Gribov-Zwanziger action

Scaling relation of IR exponents: $\kappa_V = \kappa_c = -\kappa_A/2 = 0.595353$
[MQH, Alkofer, Sorella, PRD81 (2010); AIP CP1343 (2011)]

- System reduces *in the IR* to the Faddeev-Popov system.
- $\Rightarrow$ Same IR solution, i.e.,
  - IR vanishing gluon propagator,
  - IR enhanced ghost propagator,
  - qualitative behavior of all vertices known.
- Mixed propagator is IR suppressed.
- Auxiliary fields are IR enhanced, as in [Zwanziger, PRD81 (2010)].

Corroborates Zwanziger's argument on cutting the integral at $\partial\Omega$.

Adding new terms to action and recovering old action. $\rightarrow$ Reminiscent of renormalization group approach. Irrelevant operators?

The maximally Abelian and the Landau gauge

Structures of equations considerably different in the two gauges.

	Landau gauge (ghost dominance)	Maximally Abelian gauge (Abelian dominance)
dominant configurations	Gribov horizon ($\rightarrow$ ghosts)	diagonal

Abelian configurations $\xrightarrow{\text{gauge transf.}}$ Landau gauge Gribov horizon,
i.e., both on same gauge orbit.
[Greensite, Olejník, Zwanziger, PRD69 (2004)]

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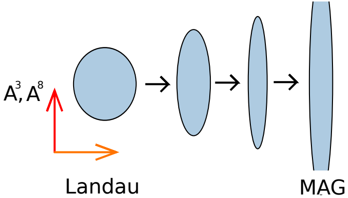
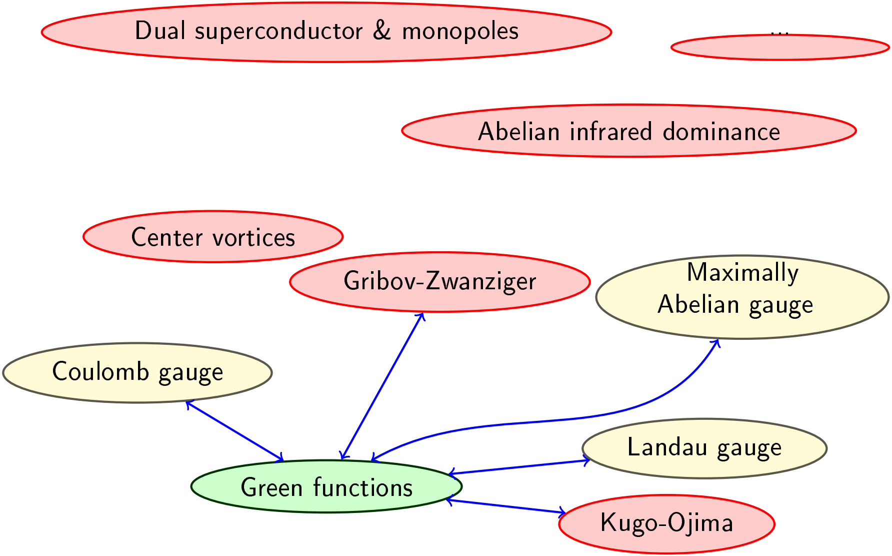
This relation is also reflected in the behavior of Green functions.
[MQH, Alkofer, Sorella, PRD81 (2010)]



Confinement scenarios and approaches: a sketch



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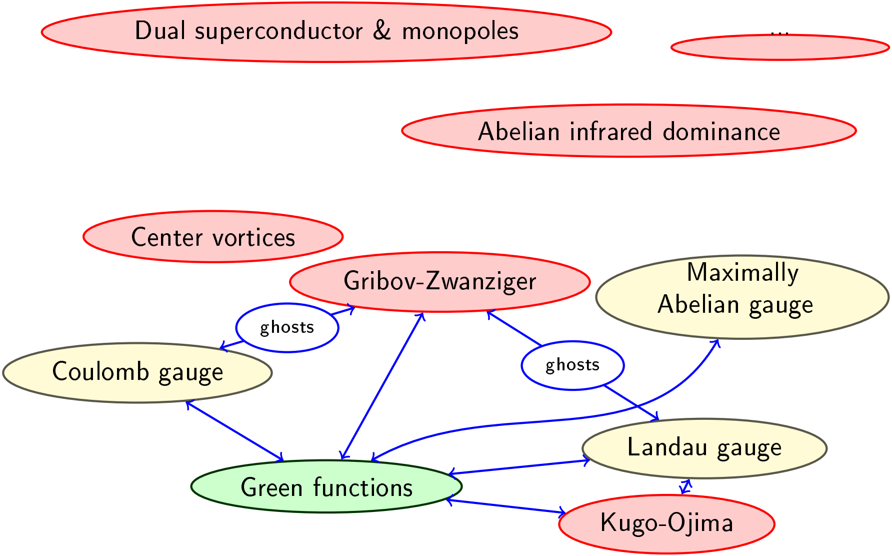

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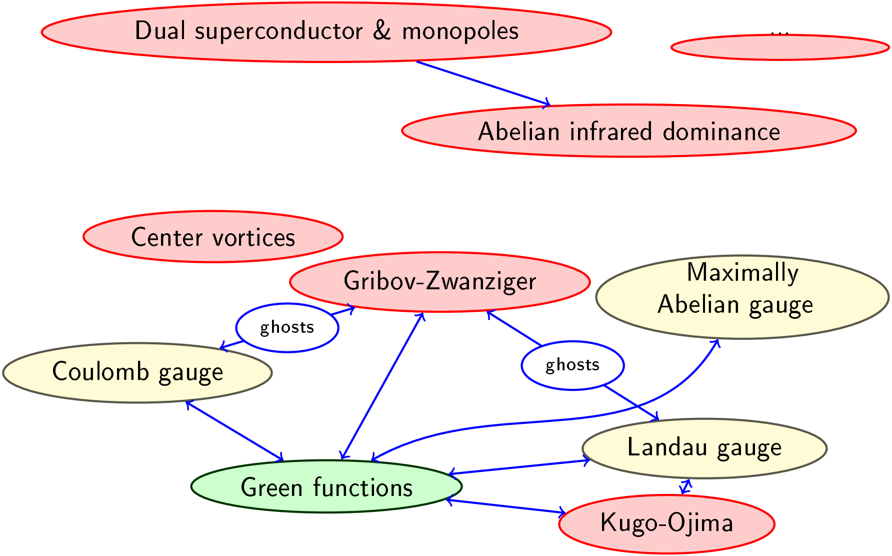

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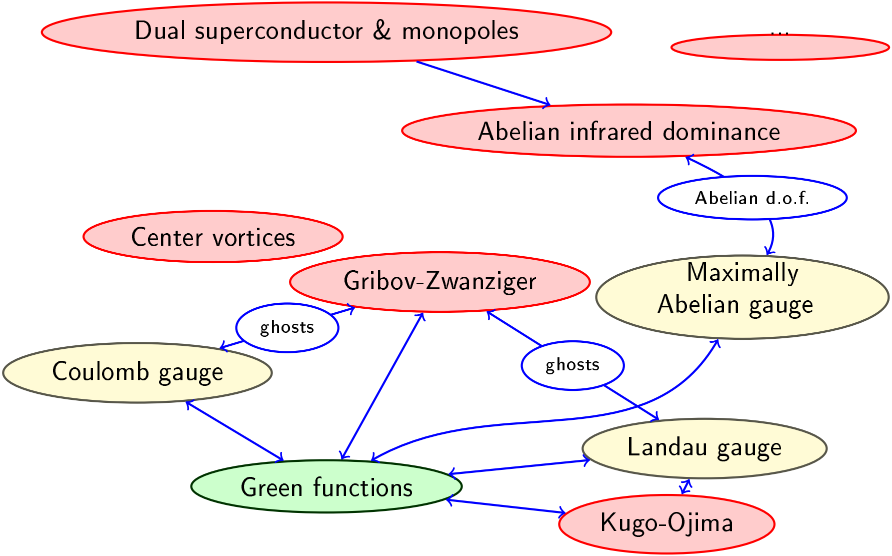

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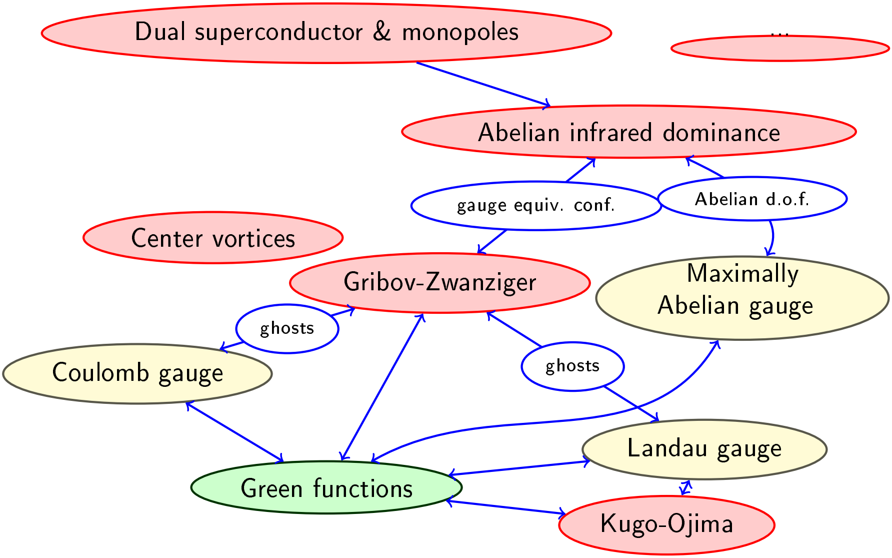

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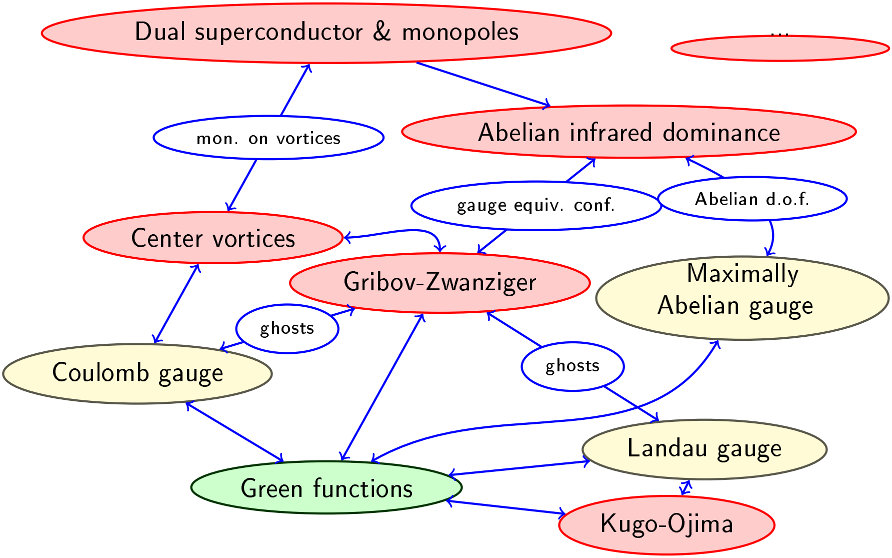

Summary

- Several confinement pictures on the market.
- Not mutually exclusive, but focus on different aspects, for example: Abelian degrees of freedom, structure of field configuration space, vortices, ...
- In the maximally Abelian gauge there may exist a solution that supports Abelian IR dominance motivated by dual superconductor picture.
- "Improving" the gauge fixing does not alter the non-perturbative scaling solution in the Landau gauge (ghost dominance).
- In order to understand confinement, one of the great challenges of particle physics, we need to understand the connections between different confinement scenarios.

The end

Thank you very much for your attention.