Landau gauge Green functions from Dyson-Schwinger equations

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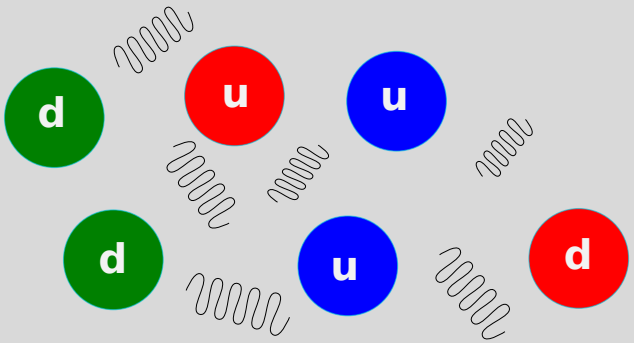
From QCD to hadrons

Quantum chromodynamics

Quarks, gluons:



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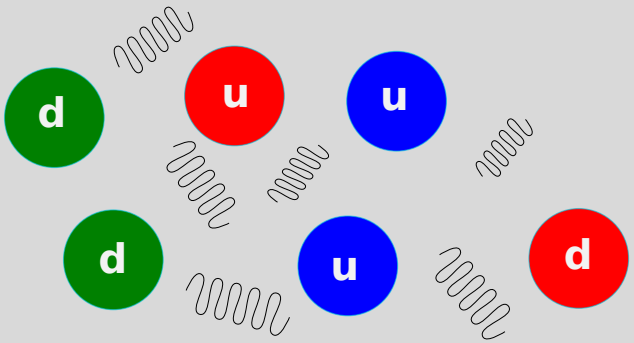

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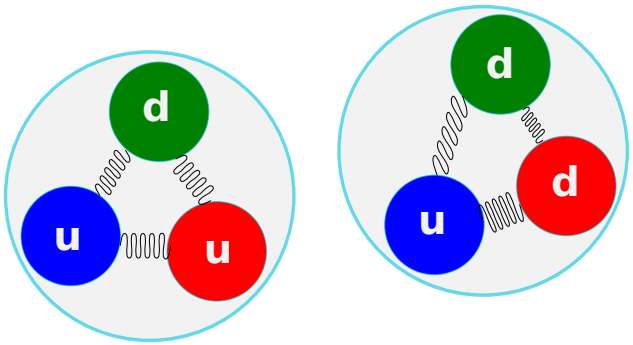

Description via

- models,
- effective theories,
- lattice,
- functional equations,
- ...

Ideally: $\mathcal{L}_{QCD} \rightarrow$ hadron spectrum

From QCD to hadrons

Quantum chromodynamics

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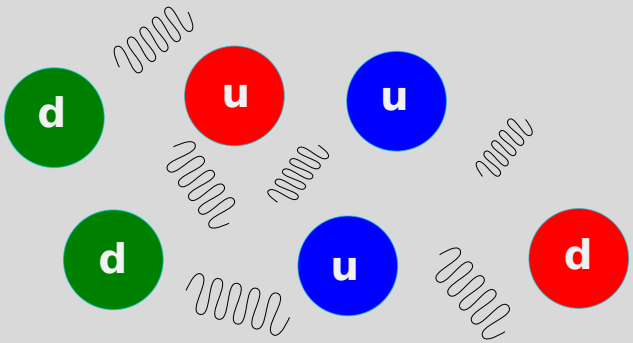

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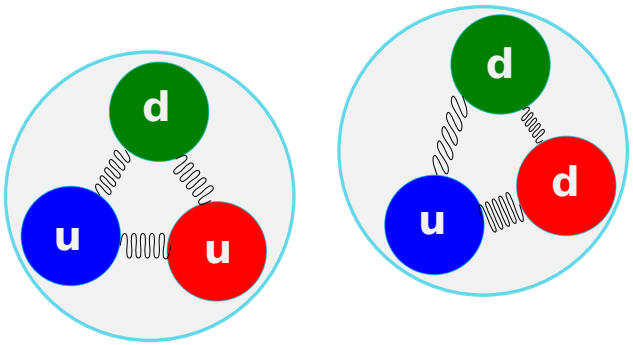

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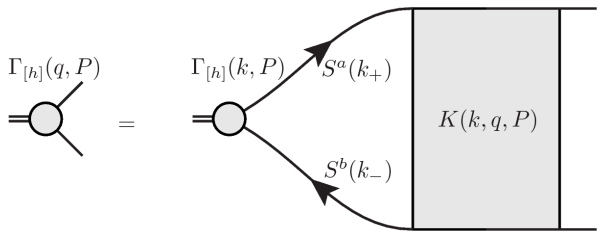
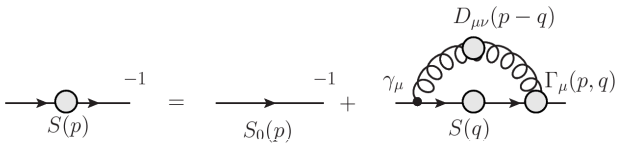
Functional equations for QCD

Bound state equations: Bethe-Salpeter/Faddeev eqs. (BSEs/FEs)

BSE:



Contains quark propagator S and kernel K.



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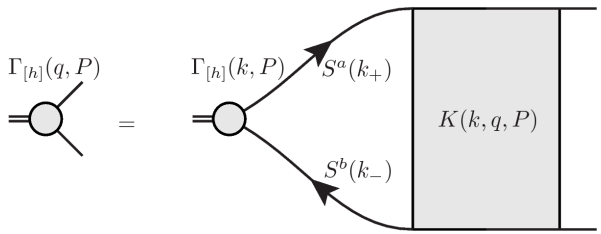

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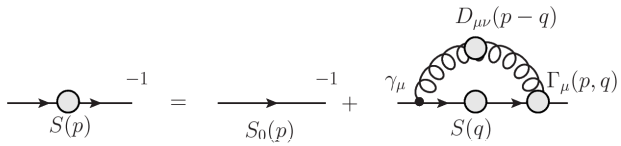


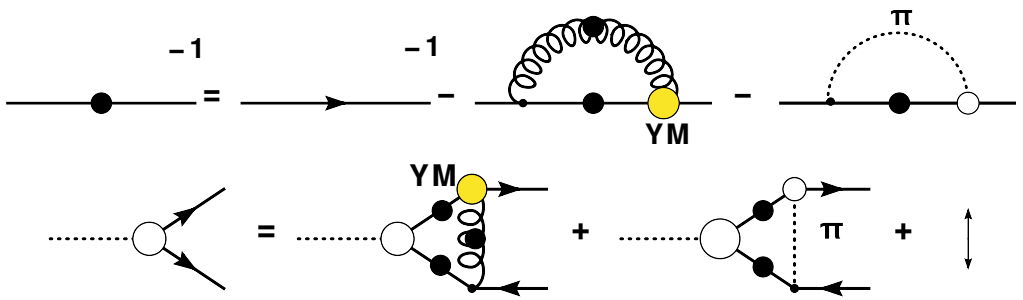
Standard truncation: rainbow-ladder (consistent with chiral symmetry)

- $K \longrightarrow$ dressed one gluon exchange
- effective gluon propagator
- bare quark-gluon vertex

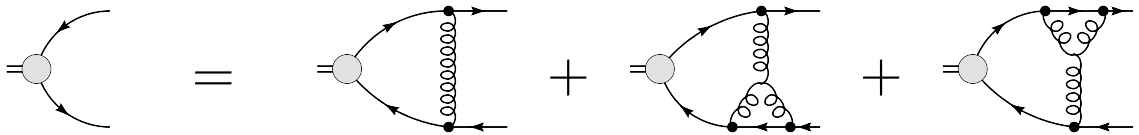
Beyond rainbow-ladder

For example:

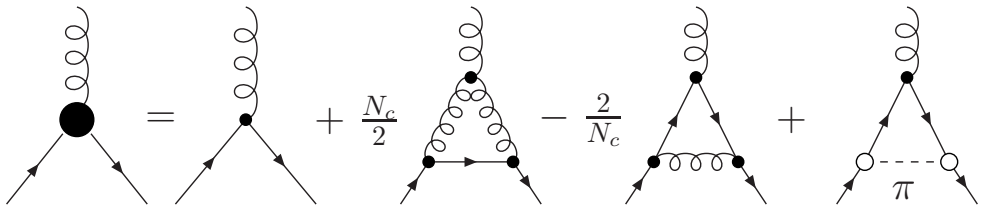
- Include pion back coupling effects [e.g., Fischer, Nickel, Wambach, PRD76 (2007); Fischer, Nickel, Williams, EPJC60 (2008); Fischer, Williams, PRD78 (2008)]:



- Include gluon self-interaction [e.g., Maris, Tandy, NPPS161 (2006); Fischer, Williams, PRL103 (2009)]:



- Solve quark-gluon vertex (12 tensors!)

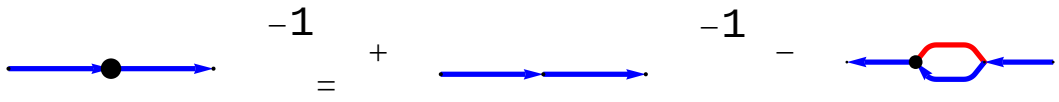


Required: gluon propagator, three-gluon vertex

Propagators

Calculate from Dyson-Schwinger equations

Quark:



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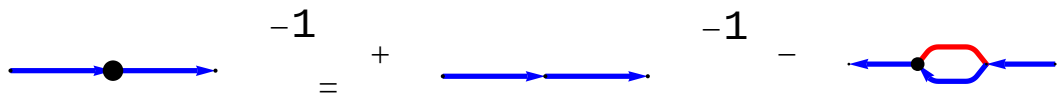
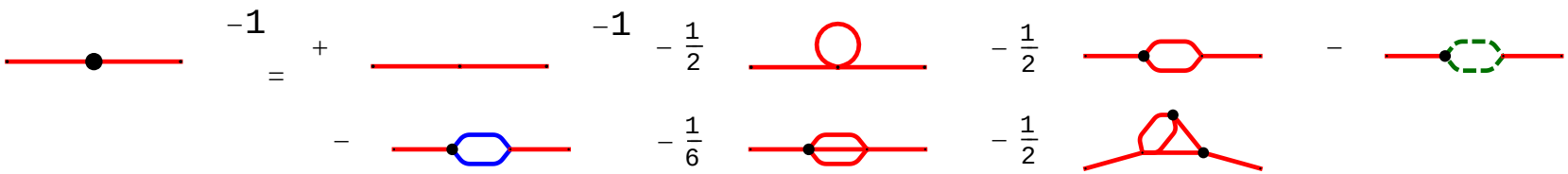


Gluon:



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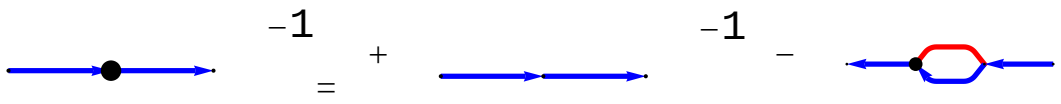

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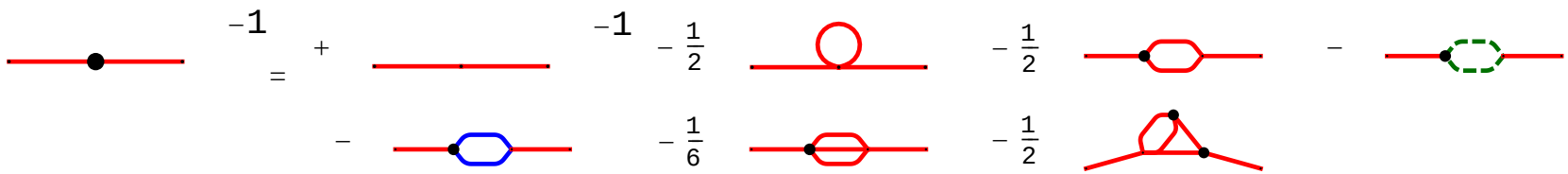

Ghost:

Propagators

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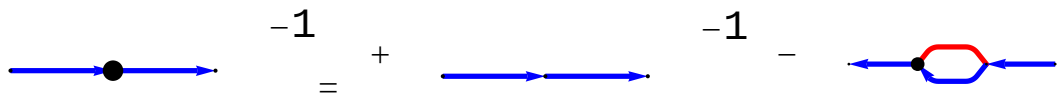

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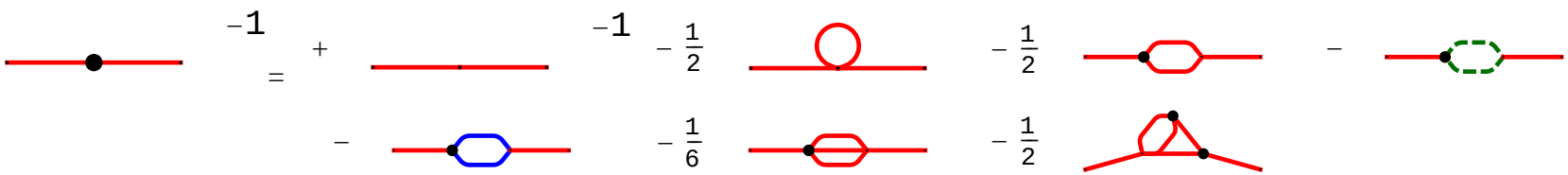


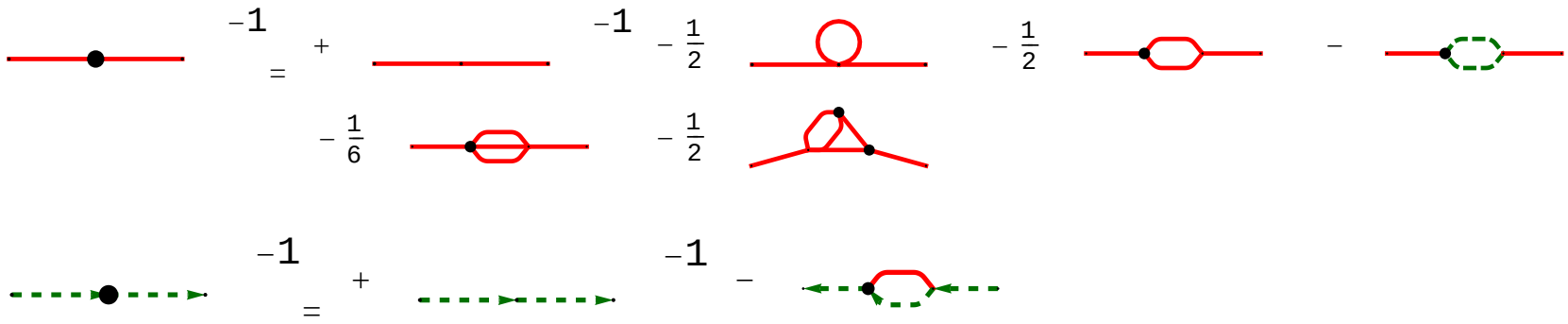
Ghost:

Required: three- and four-point functions

(or from flow equations or eqs. of motion from nPI effective action)

Dyson-Schwinger equations: Propagators

Dyson-Schwinger equations (DSEs) of gluon and ghost propagators:



- Infinite tower of coupled integral equations.
- Derivation straightforward, but tedious
  $\longrightarrow$ automated derivation with *DoFun* [MQH, Braun, CPC183 (2012)].
- Contain three-point and four-point functions:
  ghost-gluon vertex , three-gluon vertex , four-gluon vertex

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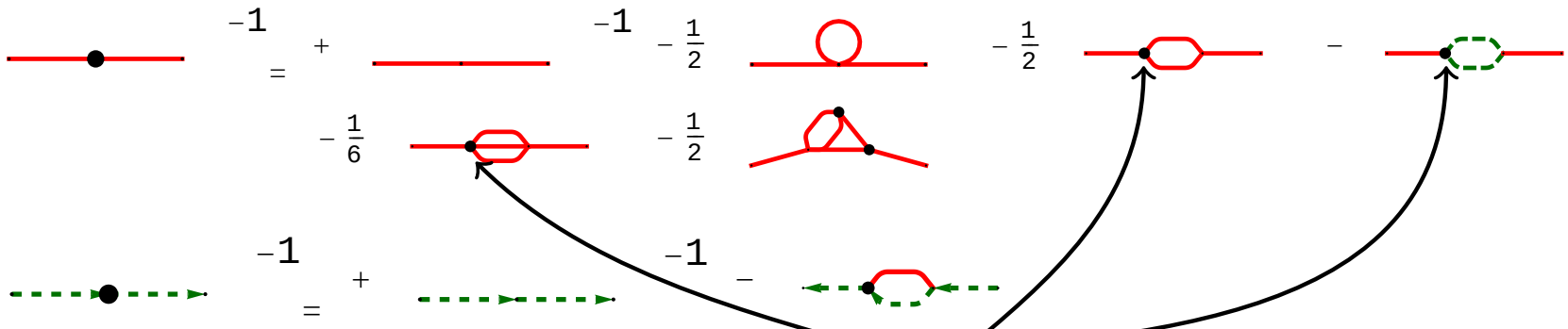

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Truncated propagator Dyson-Schwinger equations

Standard truncation:

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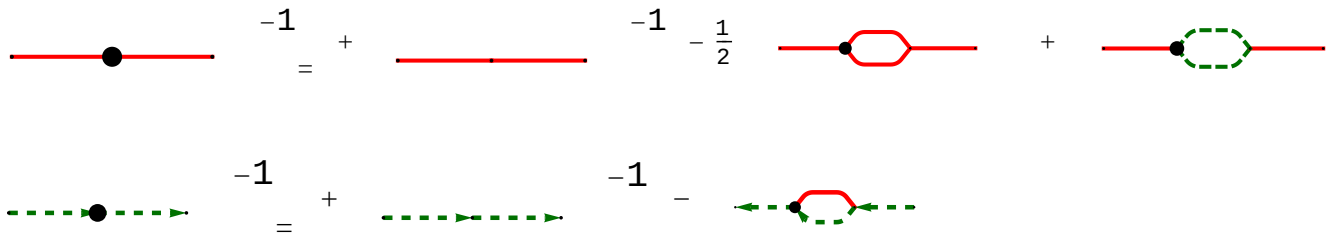

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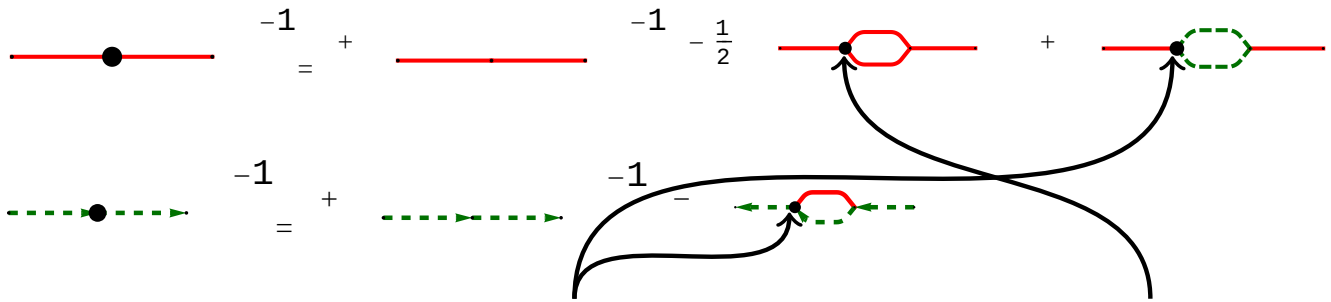

Standard: bare ghost-gluon vertex and three-gluon vertex model

Influence of three-point functions?

$$D^{ab}_{gl,\mu\nu}(p) = \left(g_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}\right) \frac{\boldsymbol{Z(p^2)}}{p^2} \delta^{ab}$$

$$D^{ab}_{gh}(p) = -\frac{\boldsymbol{G(p^2)}}{p^2} \delta^{ab}$$

Truncating Dyson-Schwinger equations

gluon	ghost	gh-gl	3-gl	4-pt.	ref.

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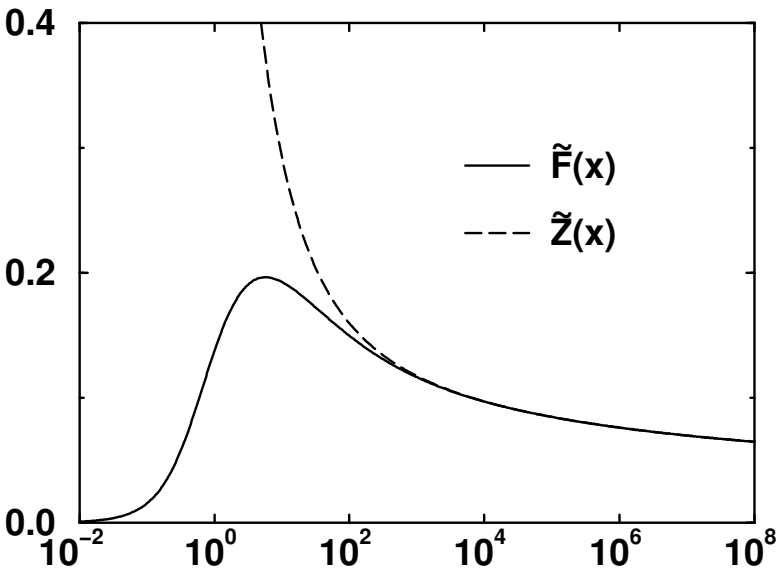

[Hauck, von Smekal, Alkofer, CPC 112 (1998)]

$$D_{gl,\mu\nu}(p) = \left(g_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}\right) \frac{\tilde{Z}(p^2)}{p^2}$$

- gluon dressing $\tilde{Z}(p^2)$ IR divergent $\rightarrow$ IR slavery

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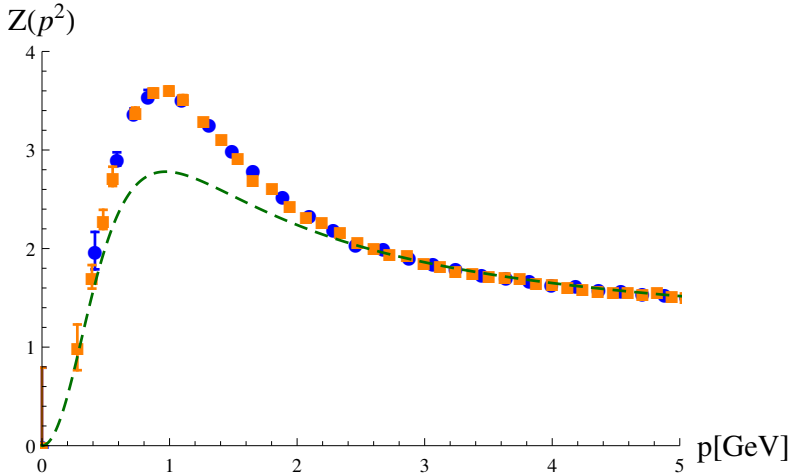

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- deviations from lattice results in mid-momentum regime

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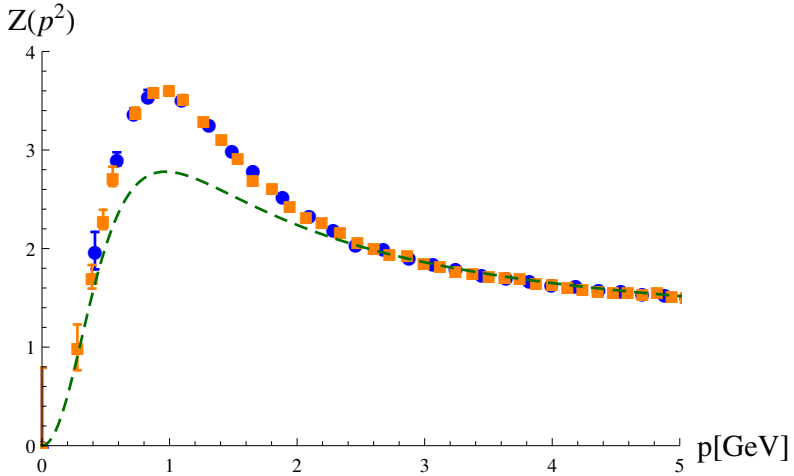

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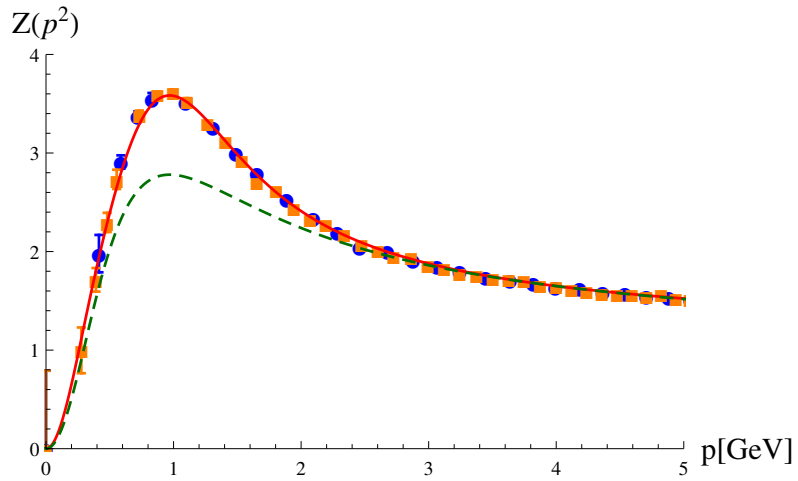

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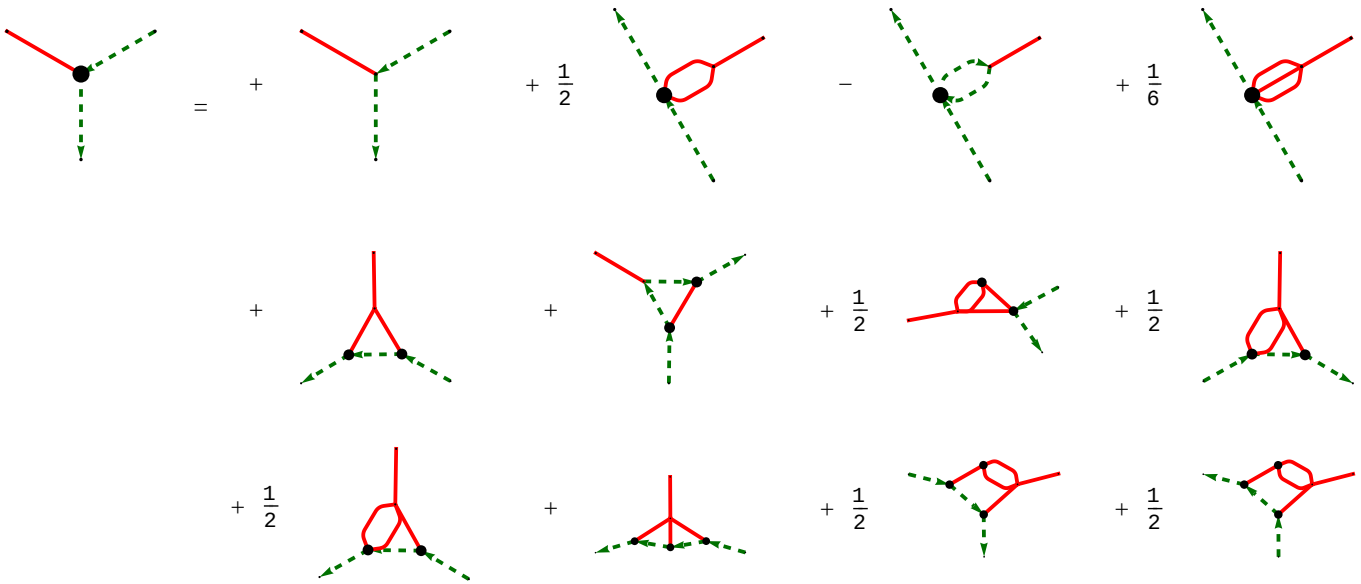
Ghost-gluon vertex DSE

Full DSE:



- Lattice results [Cucchieri, Maas, Mendes, PRD77 (2008); Ilgenfritz et al., BJP37 (2007)]
- OPE analysis [Boucaud et al., JHEP 1112 (2011)]
- Modeling via ghost DSE [Dudal, Oliveira, Rodriguez-Quintero, PRD86 (2012)]
- Semi-perturbative DSE analysis [Schleifenbaum et al., PRD72 (2005)]
- FRG [Fister, Pawlowski, 1112.5440]

Ghost-gluon vertex

$$\Gamma_\mu^{A\bar{c}c,abc}(k;p,q) := i\,g\,f^{abc}\,(p_\mu \boldsymbol{A(k;p,q)} + k_\mu B(k;p,q))$$

Note:

$B(k;p,q)$ is irrelevant in Landau gauge (but it is not the pure longitudinal part).

Taylor argument applies only to longitudinal part (it's an STI).

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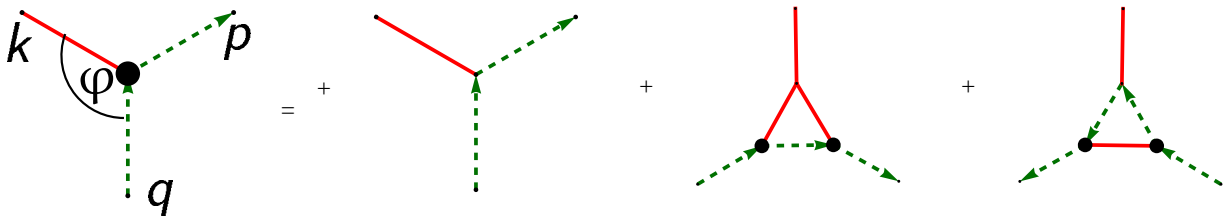
IR and UV consistent truncation:



System of eqs. to solve:
gluon and ghost propagators + ghost-gluon vertex

Only unfixed quantity in present truncation: three-gluon vertex.

Three-gluon vertex: Ultraviolet

Bose symmetric version:

$$D^{A^3,UV}(x,y,z) = G\left(\frac{x+y+z}{2}\right)^{\alpha} Z\left(\frac{x+y+z}{2}\right)^{\beta}$$

Fix α and β:

1. UV behavior of three-gluon vertex
2. IR behavior of three-gluon vertex?

Three-gluon vertex: Ultraviolet

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Fix α and β:

1. UV behavior of three-gluon vertex
2. IR behavior of three-gluon vertex $\rightarrow$ yes, but ...

Three-gluon vertex: Infrared

Three-gluon vertex might have a zero crossing.

$d = 2, 3$: seen on lattice

[Cucchieri, Maas, Mendes, PRD77 (2008); Maas, PRD75 (2007)],

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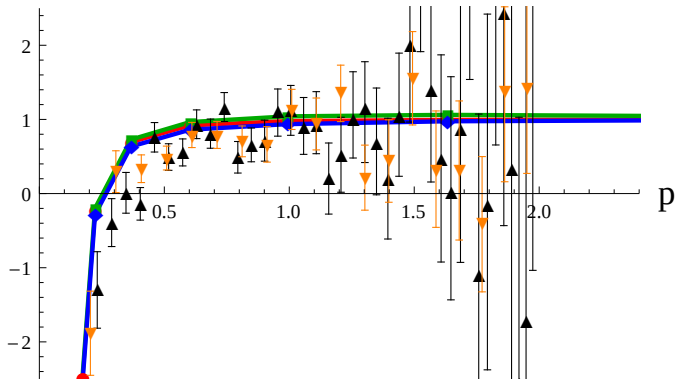
$D^{A^3}_{\text{proj}}(p^2, p^2, \pi/2)$



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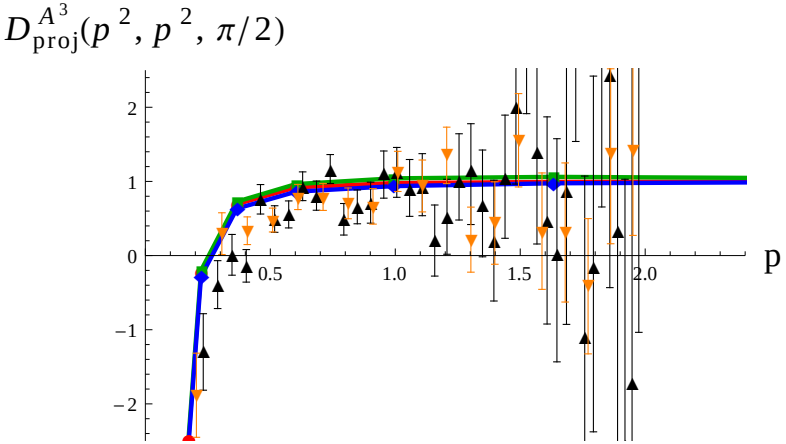

$d = 4$:

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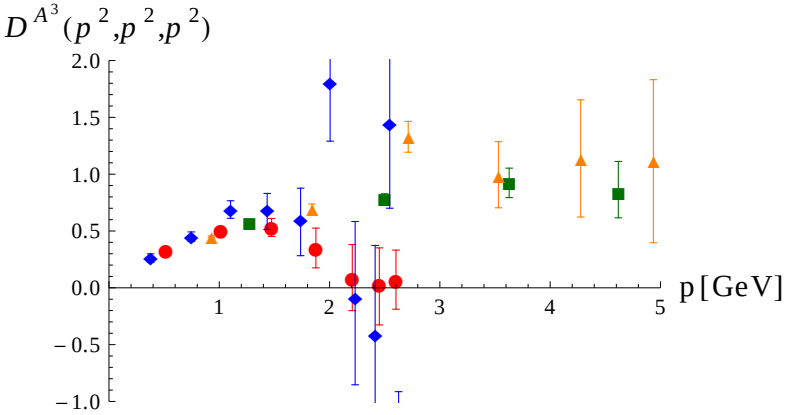

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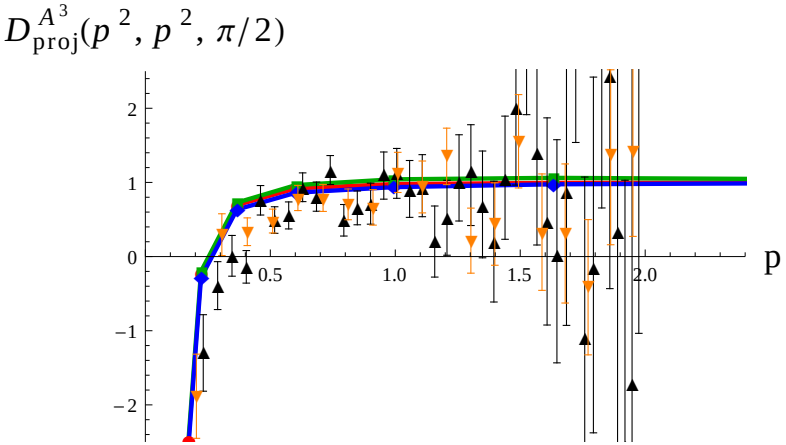

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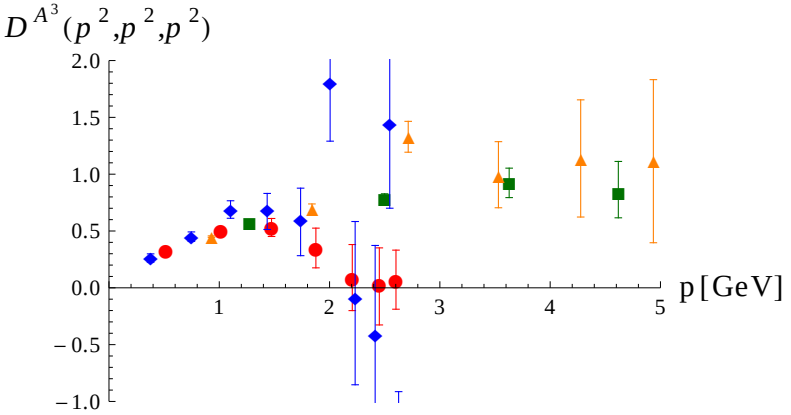
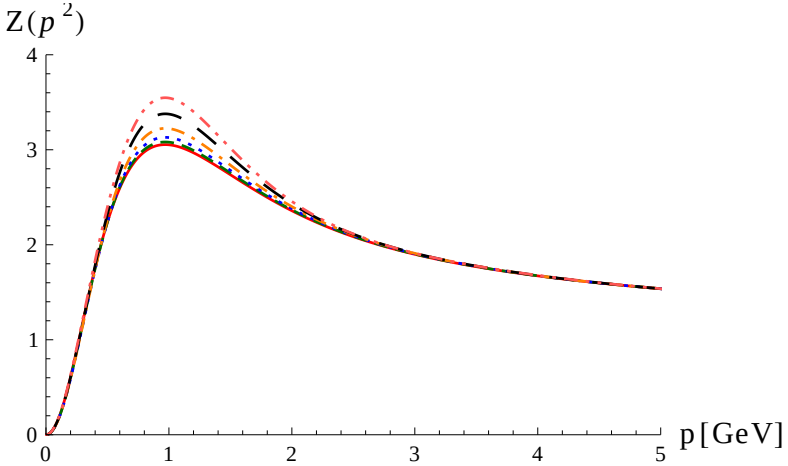
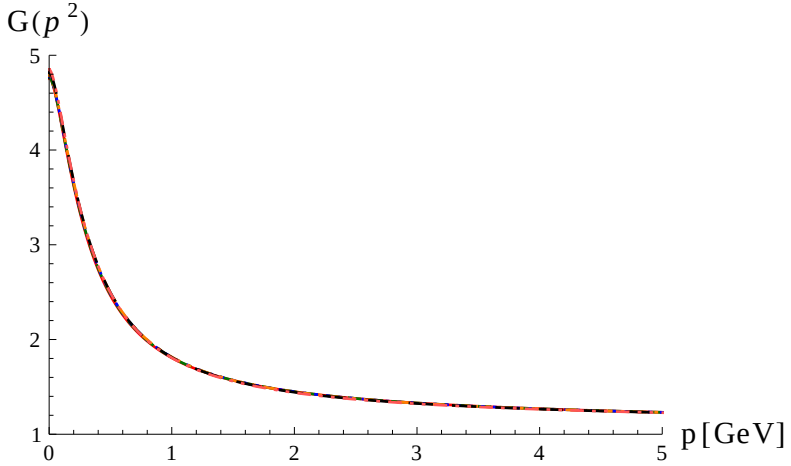


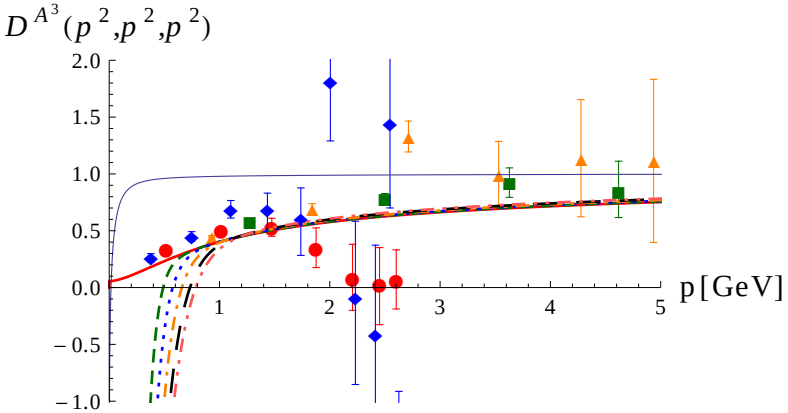
$$D^{A^3,IR}(x,y,z) = h_{IR}\, G(x+y+z)^3 (f^{3g}(x) f^{3g}(y) f^{3g}(z))^4$$

IR damping function $f^{3g}(x) := \dfrac{\Lambda_{3g}^2}{\Lambda_{3g}^2 + x}$

Influence of the three-gluon vertex







- Vary $\Lambda_{3g} \longrightarrow$ vary mid-momentum strength
- Ghost almost unaffected
- Thin line: Leading IR order DSE calculation for three-gluon vertex $\Rightarrow$ zero crossing

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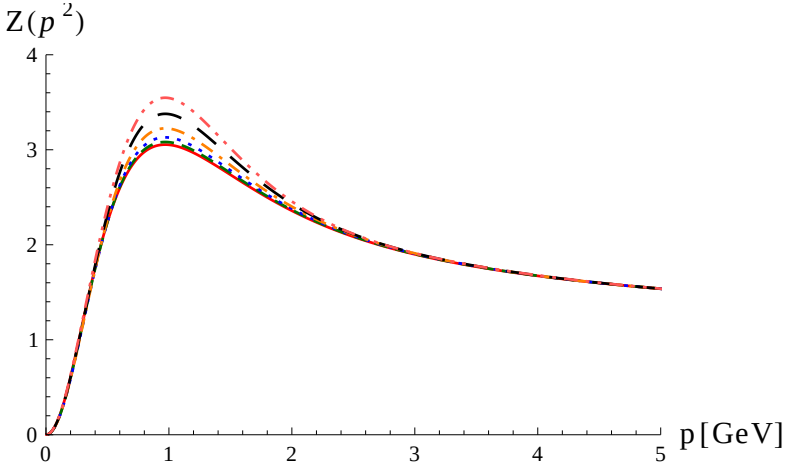

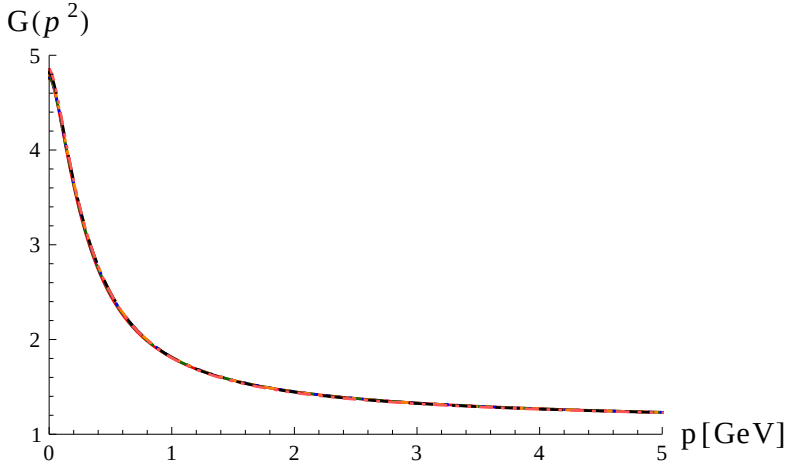

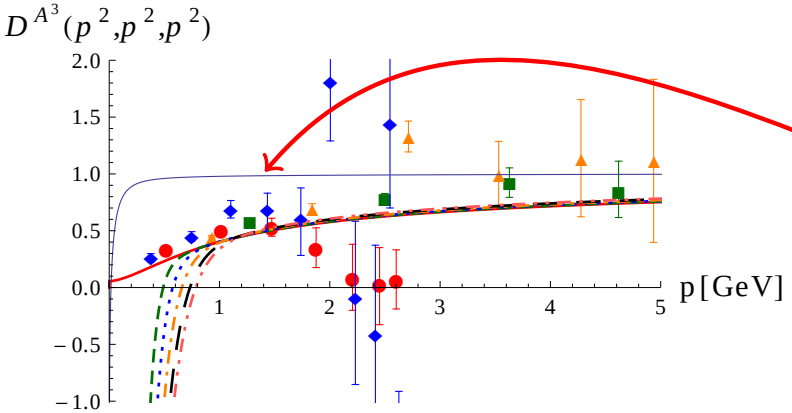

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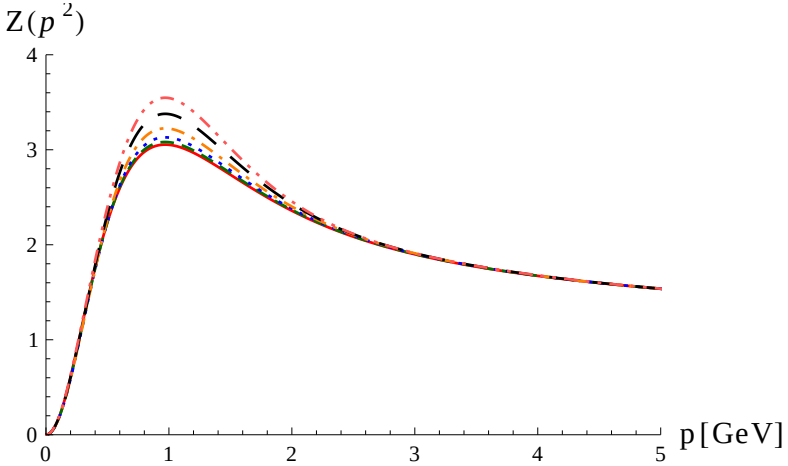
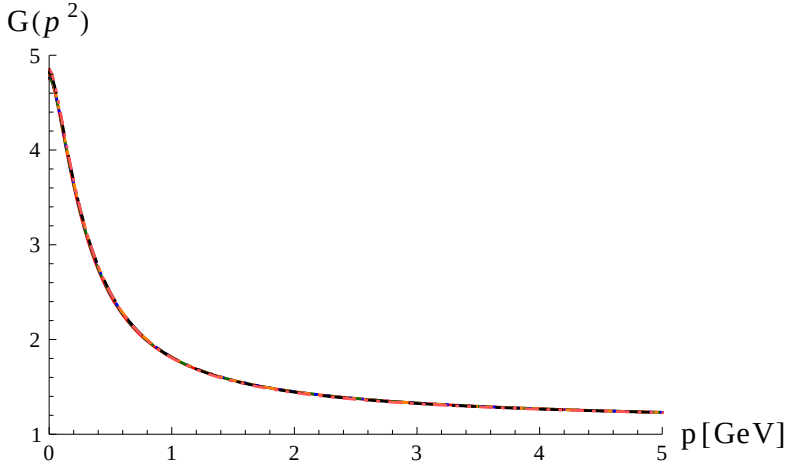
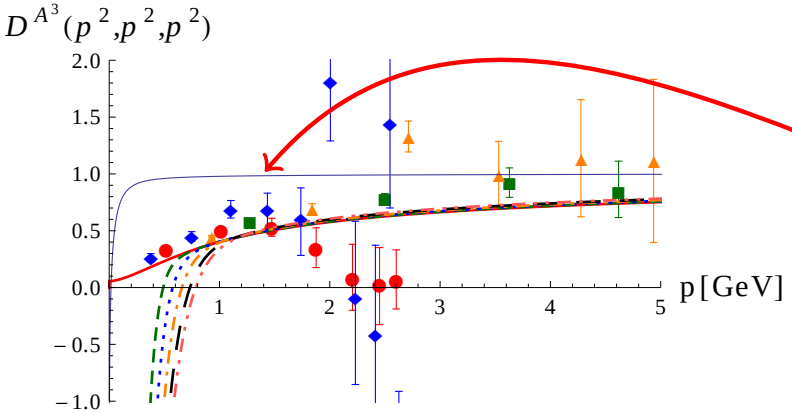






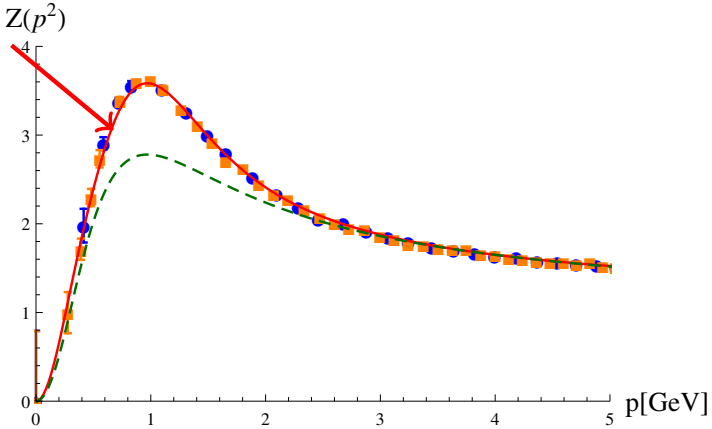
- Vary $\Lambda_{3g} \longrightarrow$ vary mid-momentum strength
- Ghost almost unaffected
- Thin line: Leading IR order DSE calculation for three-gluon vertex
  $\Rightarrow$ zero crossing

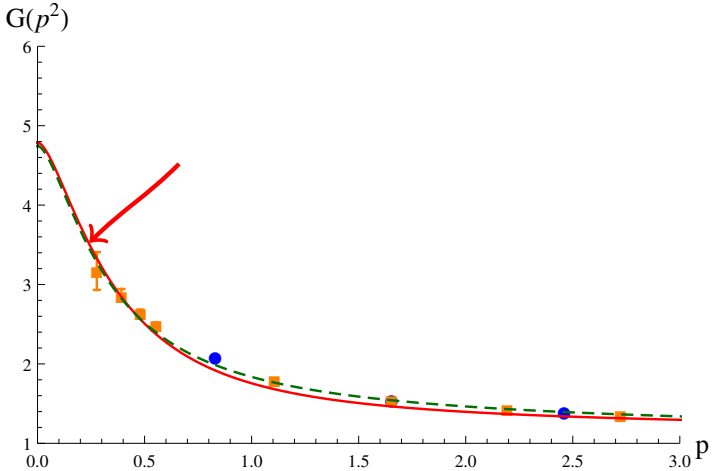
Optimized effective three-gluon vertex:
Choose Λ_{3g} where gluon dressing has best agreement with lattice results.

[MQH, von Smekal, JHEP (2013)]

Dynamic ghost-gluon vertex: Propagator results

Dynamic ghost-gluon vertex, opt. eff.
three-gluon vertex [MQH, von Smekal, JHEP (2013)]

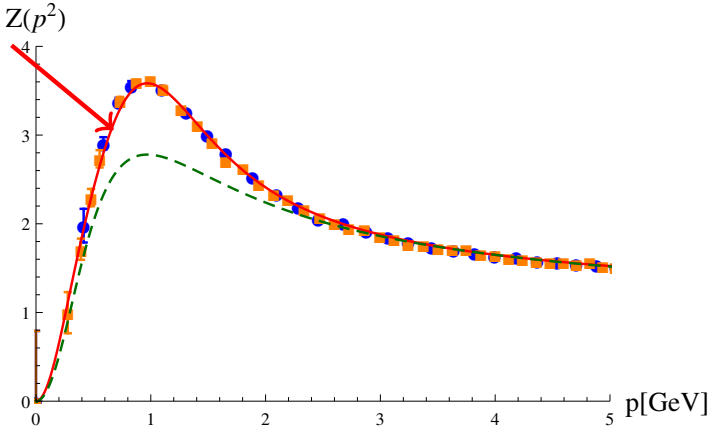


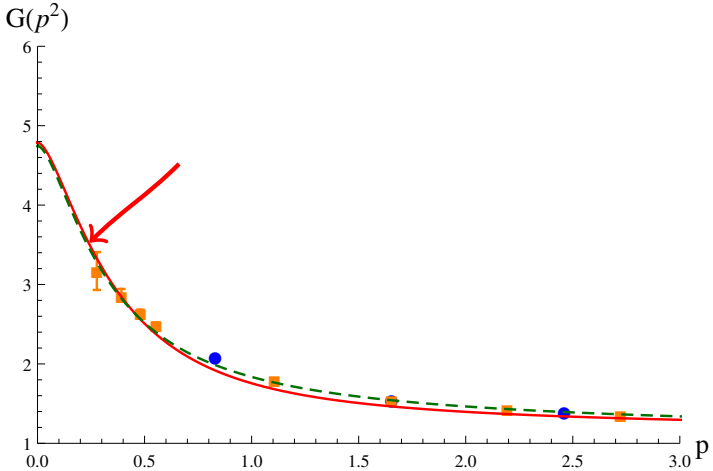


Good quantitative agreement for ghost and gluon dressings.

Dynamic ghost-gluon vertex: Propagator results

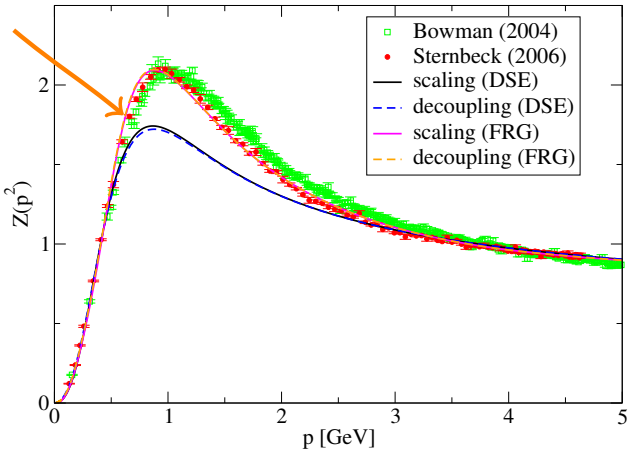
Dynamic ghost-gluon vertex, opt. eff. three-gluon vertex [MQH, von Smekal, JHEP (2013)]

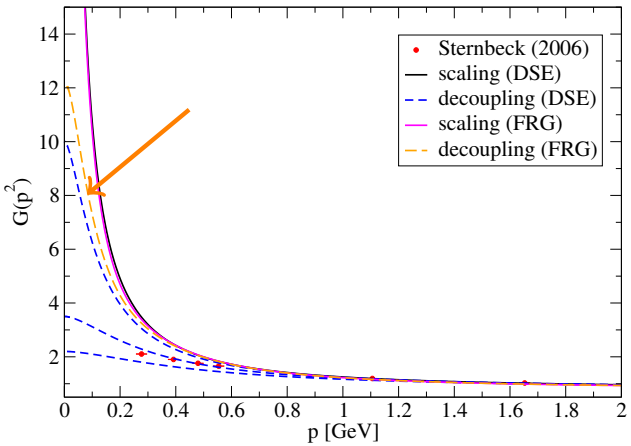




FRG results

[Fischer, Maas, Pawlowski, AP324 (2009)]



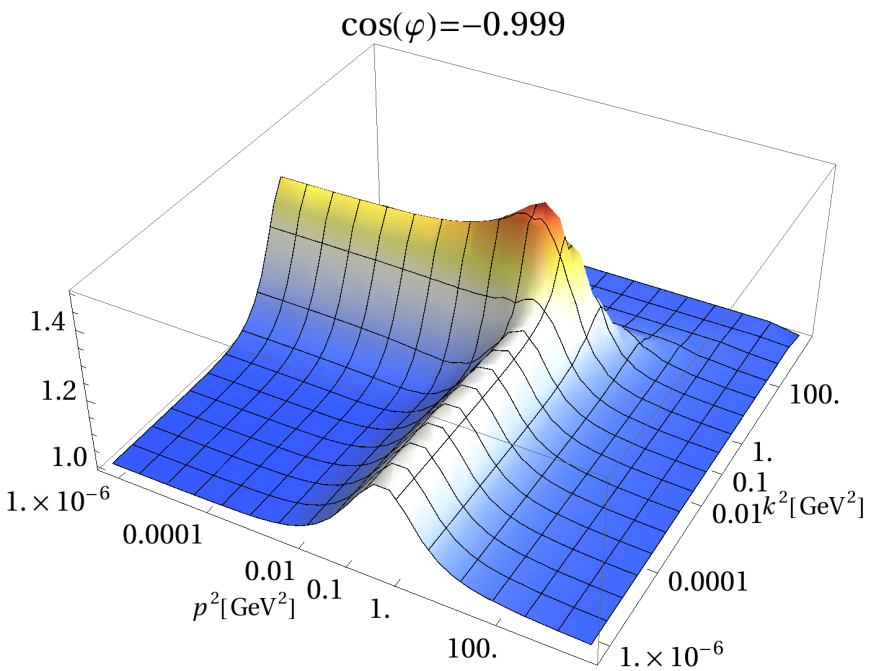


Good quantitative agreement for ghost and gluon dressings.

Ghost-gluon vertex: Selected configurations (decoupling)

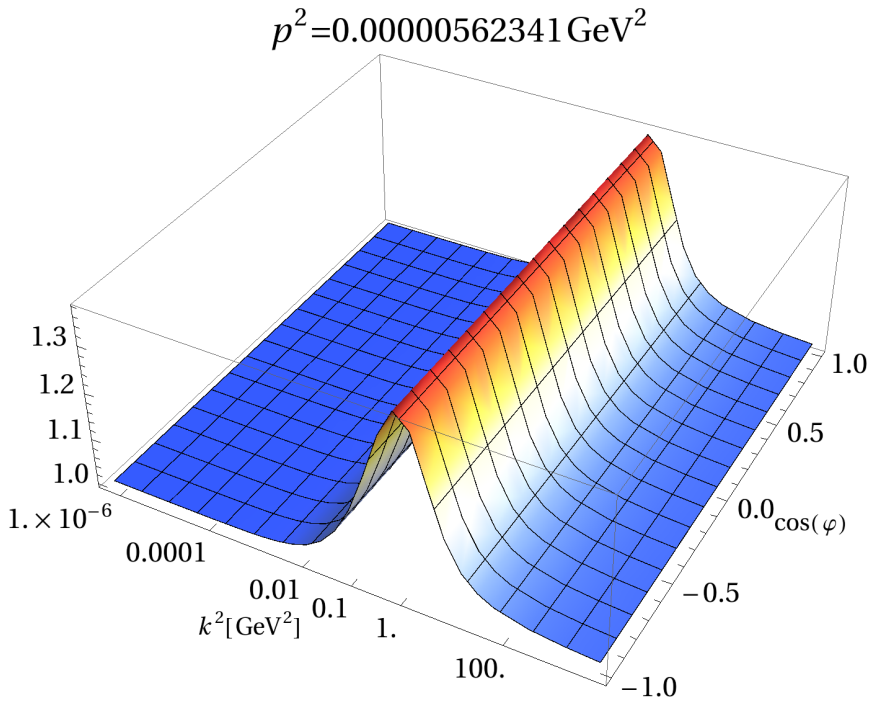
$$\Gamma^{A\bar{c}c,abc}_{\mu}(k;p,q) := i\,g\,f^{abc}\,(p_\mu A(k;p,q) + k_\mu B(k;p,q))$$

Fixed angle:



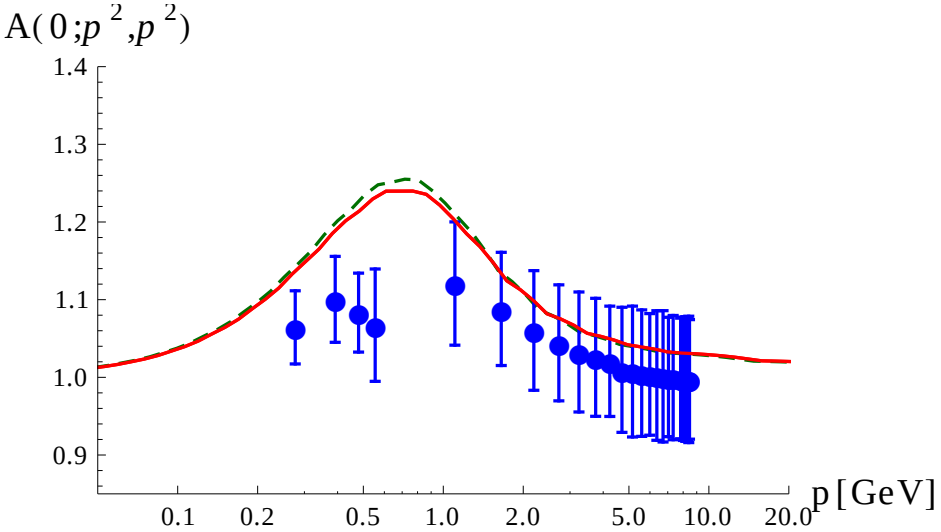
[MQH, von Smekal, JHEP (2013)]

Fixed anti-ghost momentum:



Ghost-gluon vertex: Comparison with lattice data

Orthogonal configuration $k^2 = 0$, $q^2 = p^2$:



- constant in the IR
- relatively insensitive to changes of the three-gluon vertex (red/green lines: different three-gluon vertex models)

DSE calculation: [MQH, von Smekal, JHEP (2013)]
lattice data: [Sternbeck, hep-lat/0609016]

Functional equations and lattice results

	functional equations	lattice
propagators	✓	✓
three-point functions	ghost-gluon vertex: ✓ 3-gluon vertex: in progress quark-gluon vertex: (✓)	limited mom. dependence
four-point functions	(✓)	not soon

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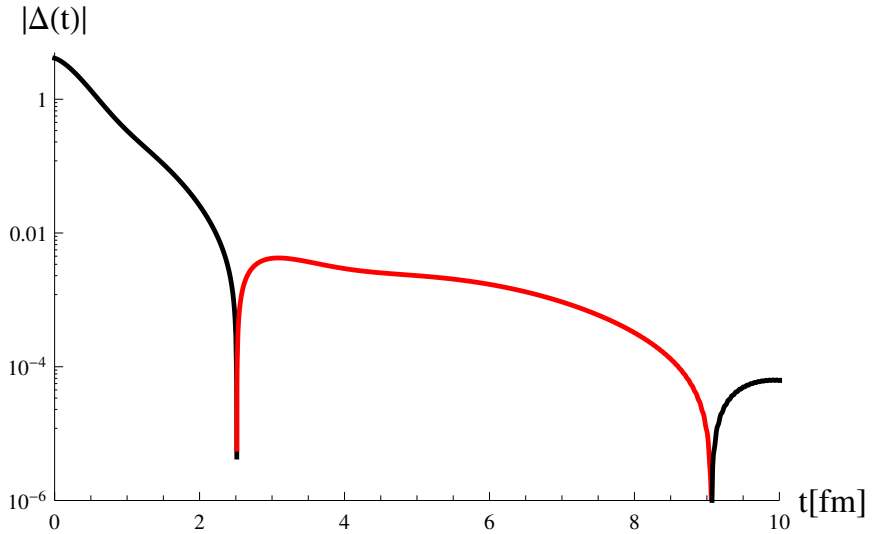
Schwinger function

Schwinger function $\Delta(t)$:

$$\Delta(t) = \frac{1}{\pi} \int dq \, \cos(q\,t) \frac{Z(q^2)}{q^2}$$



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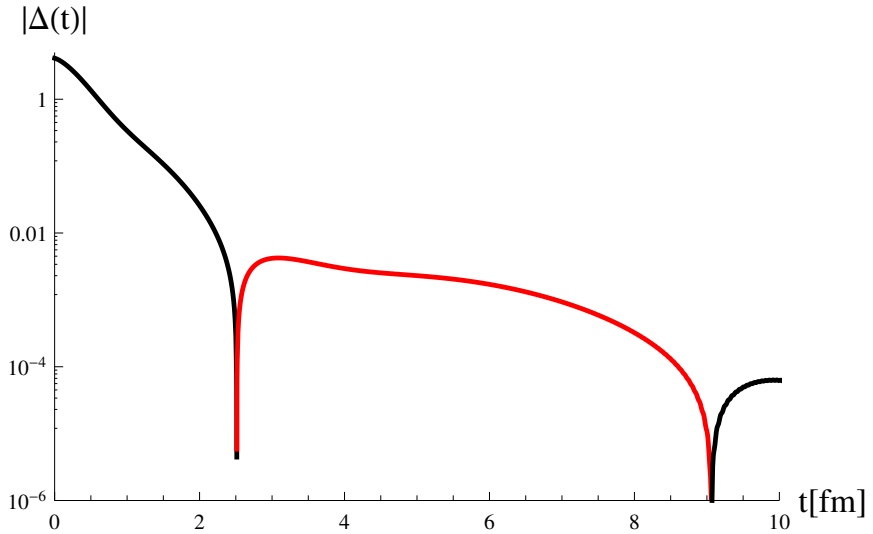


[MQH, von Smekal, PoS CONFX 062 (2013)]

$$\Delta(t) = \int_0^\infty d\nu\, \rho(\nu^2)e^{-\nu t} = \mathcal{L}(\rho)$$

ρ: spectral density, must be positive for physical particles

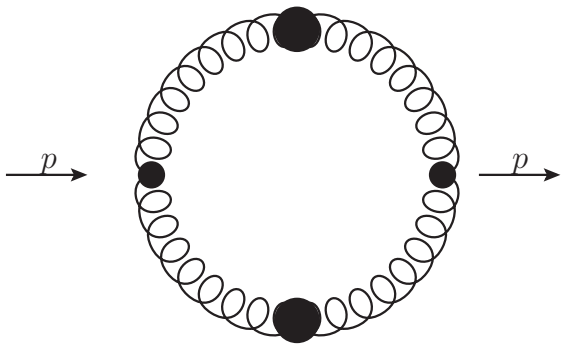
Positivity violation of propagators $\rightarrow$ confinement.

Glueballs

Glueball candidate: $G = F_{\mu\nu}F_{\mu\nu}$

- No positivity violation expected.
- Construction from positivity violating gluons?

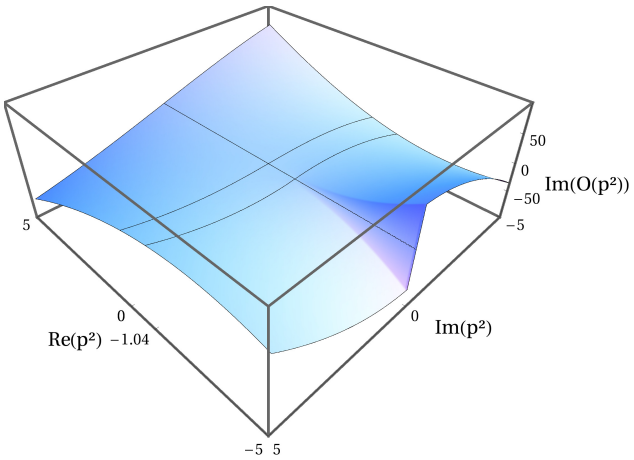
Correlation function $\langle G(x)G(y)\rangle$ in first order approximation (Born level):



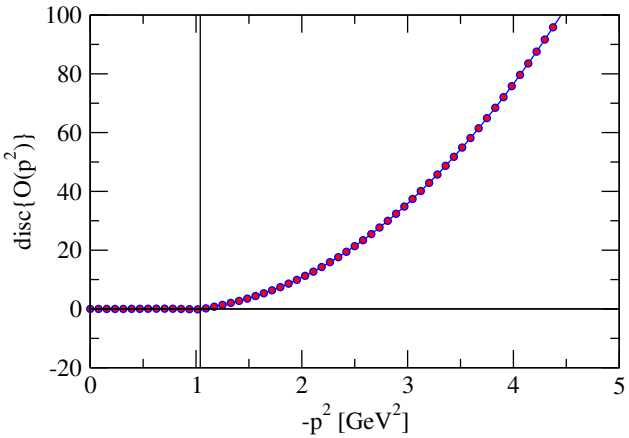
- Gluon propagators from fits to solutions of Yang-Mills systems.

Glueballs

- Calculation in complex plane.
- Extraction of spectral density.



[Windisch, Huber, Alkofer, PRD87 (2013)]



Propagators in complex plane: [Strauss, Fischer, Kellermann, PRL109 (2012)]

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  Inclusion of ghost-gluon vertex and
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  - Required for quantitative results.
  - Reproduction of lattice data possible.
- Automatization tools available:
  DoFun [Alkofer, MQH, Schwenzer, CPC180 (2009); MQH, Braun, CPC183 (2012)]
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Thank you for your attention!

Landau Gauge Yang-Mills theory

Gluonic sector of quantum chromodynamics: Yang-Mills theory

$$\mathcal{L} = \frac{1}{2} F^2 + \mathcal{L}_{gf} + \mathcal{L}_{gh}$$
$$F_{\mu\nu} = \partial_\mu \boldsymbol{A}_\nu - \partial_\nu \boldsymbol{A}_\mu + i\, g\, [\boldsymbol{A}_\mu, \boldsymbol{A}_\nu]$$

Propagators and vertices are gauge dependent
$\longrightarrow$ choose any gauge, ideally one that is convenient.

Landau gauge

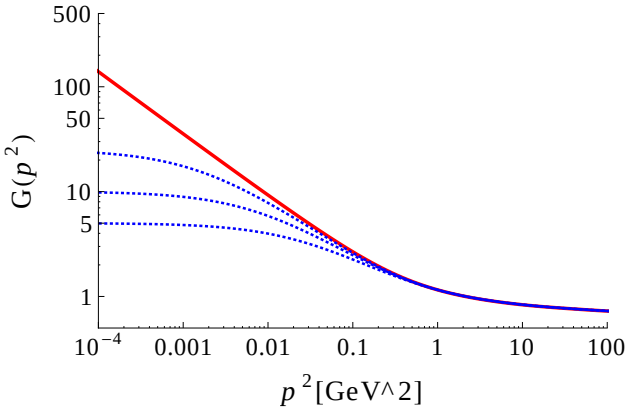
- simplest one for functional equations
- $\partial_\mu \boldsymbol{A}_\mu = 0$: $\mathcal{L}_{gf} = \frac{1}{2\xi} (\partial_\mu \boldsymbol{A}_\mu)^2, \quad \xi \to 0$
- requires ghost fields: $\mathcal{L}_{gh} = \bar{\boldsymbol{c}}\, (-\Box + g\, \boldsymbol{A} \times)\, \boldsymbol{c}$
- 2 fields: 3 vertices:

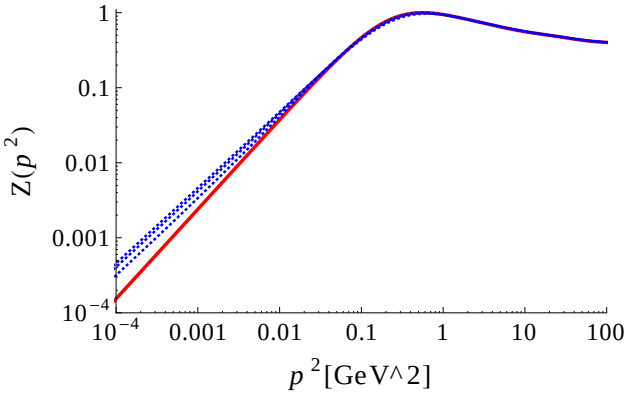
Solutions of functional equations: Decoupling and scaling

- Two types of solutions with functional methods that differ only in deep IR [Boucaud et al., JHEP 0806, 012; Fischer, Maas, Pawlowski, AP 324 (2009)]:
  scaling [von Smekal, Alkofer, Hauck PRL97],
  decoupling [Aguilar, Binosi, Papavassiliou PRD78; Fischer, Maas, Pawlowski, AP 324 (2009)]
- Lattice calculations find only decoupling type solution for $d = 3, 4$ and scaling for $d = 2$
- Decoupling emerges also from Refined Gribov-Zwanziger framework [Dudal, Sorella, Vandersickel, Verschelde, PRD77]

Decoupling and scaling solutions

DSEs: Vary ghost boundary condition [Fischer, Maas, Pawlowski, AP 324 (2009)]

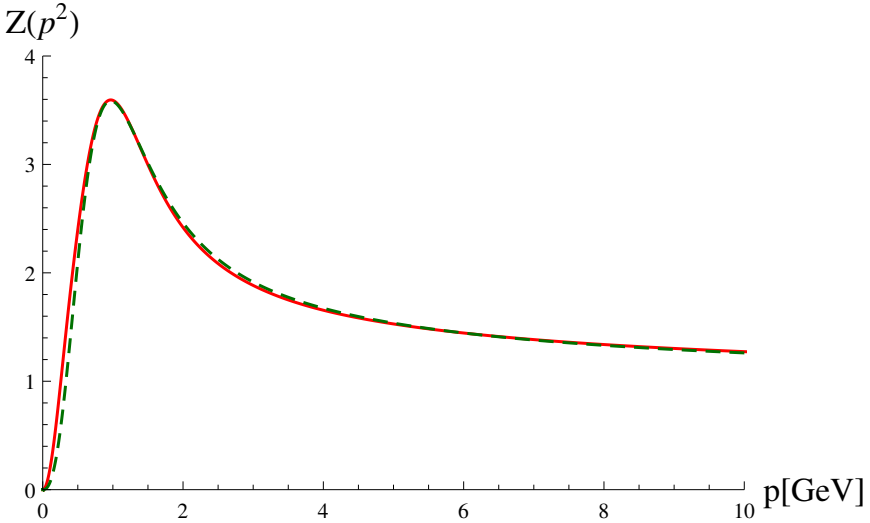


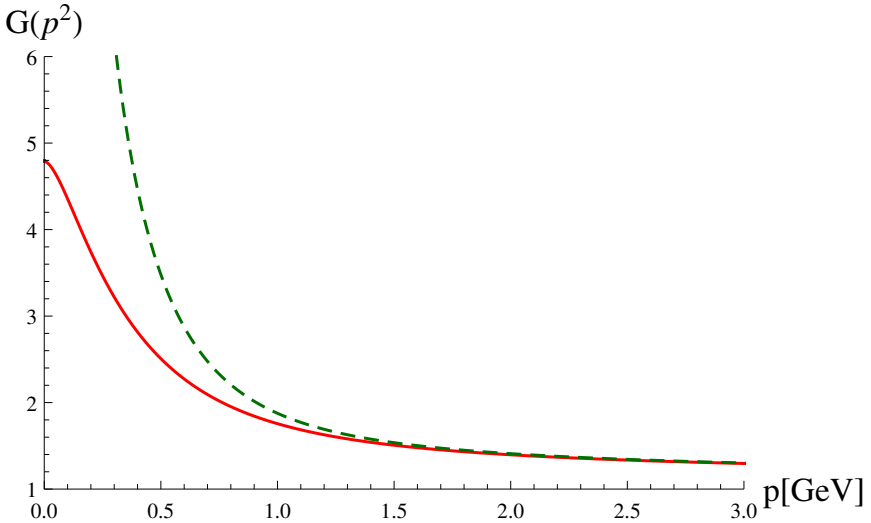


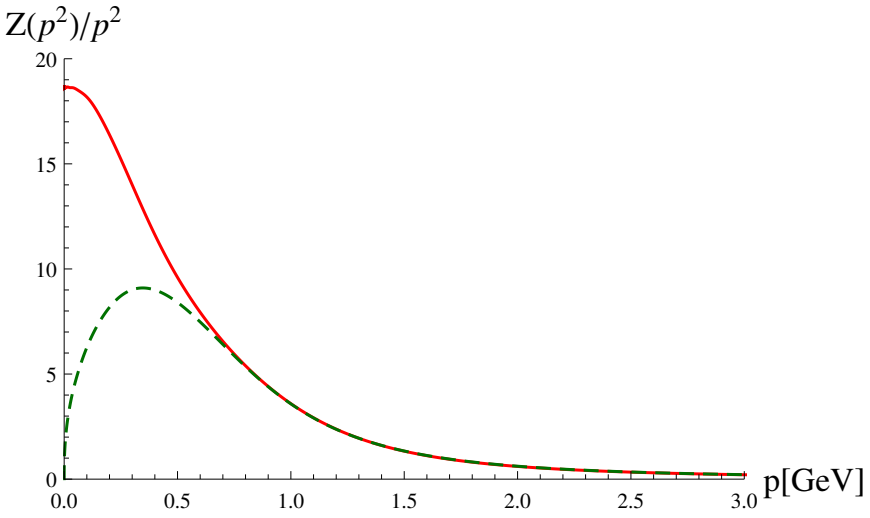
- Dependence of propagators on Gribov copies, e.g., [Bogolubsky, Burgio, Müller-Preussker, Mitrjushkin, PRD 74 (2006); Maas, PR 524 (2013)]
- Ideas:
  - [Sternbeck, Müller-Preussker, 1211.3057]: choose Gribov copies by lowest eigenvalue of the Faddeev-Popov operator
    $\longrightarrow$ modification of both dressings
  - [Maas, PLB689 (2010)]: choose Gribov copies by value of ghost propagator

$d = 2$: Analytic and numerical arguments from DSEs for scaling only [Cucchieri, Dudal, Vandersickel, PRD85 (2012); MQH, Maas, von Smekal, JHEP11 (2012)] as well as from analysis of Gribov region [Zwanziger, 1209.1974].

Scaling solution: Propagators



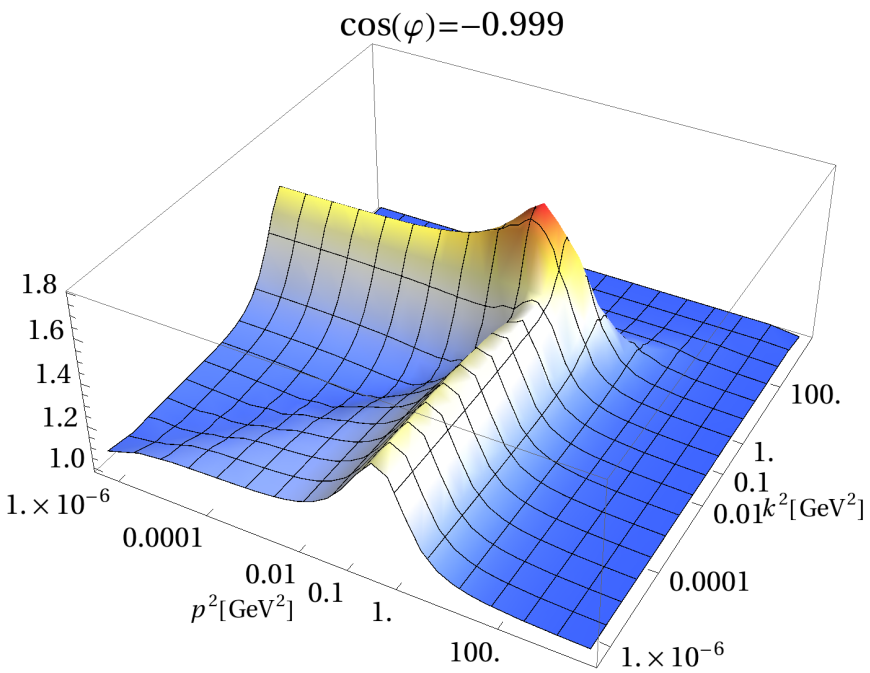




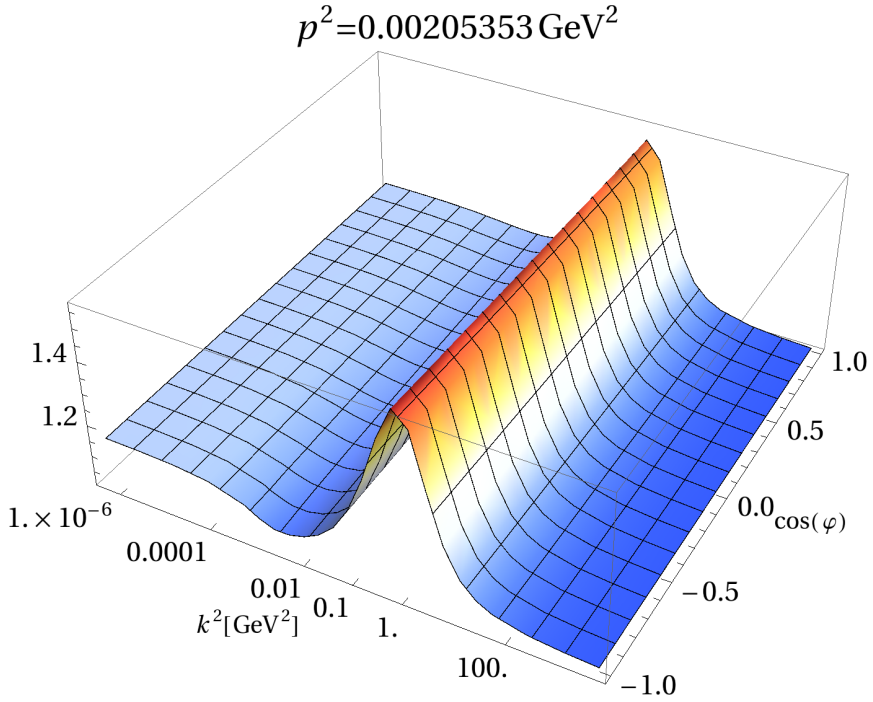
- Scaling solution
- Decoupling solution
- Differences only at low momenta.

Scaling solution: Ghost-gluon vertex

Fixed angle:



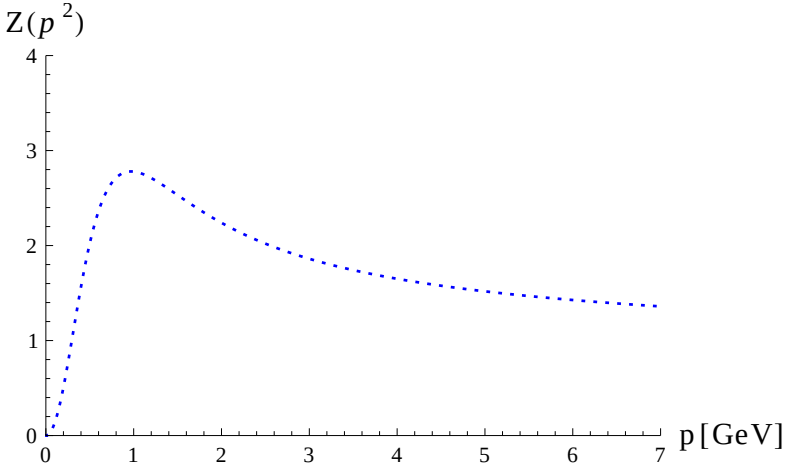
Fixed momentum:



- Dressing not 1 in the IR ← Contributions from loop corrections (for decoupling they are suppressed)
- Scaling/decoupling also seen in ghost-gluon vertex

Influence of the three-gluon vertex

ghost-gluon vertex: bare



original three-gluon vertex

Influence of the three-gluon vertex

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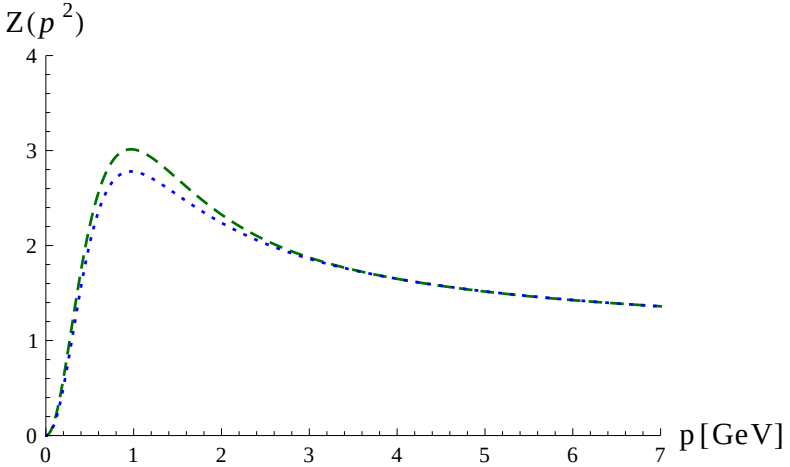

original three-gluon vertex

Bose symmetric three-gluon vertex

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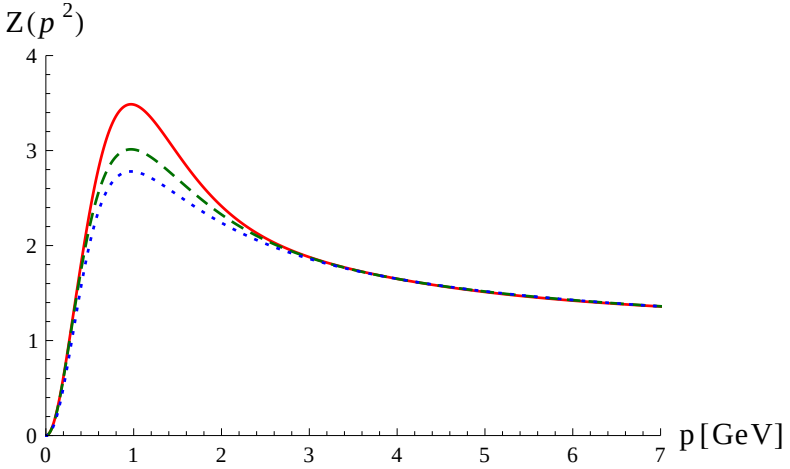

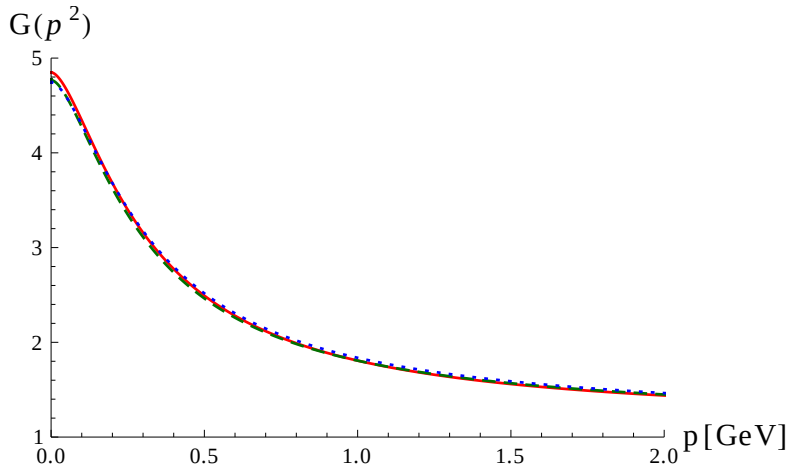

original three-gluon vertex
Bose symmetric three-gluon vertex
Bose symmetric three-gluon vertex with IR part

$\Rightarrow$ Improved three-gluon vertex adds additional strength in the mid-momentum regime.