On three-point correlations in pure Landau gauge QCD

JUSTUS-LIEBIG-UNIVERSITÄT GIESSEN

Markus Q. Huber
Adrian L. Blum, Mario Mitter, Lorenz von Smekal

Institut für Theoretische Physik, Justus-Liebig-Universität Gießen

DPG-Frühjahrstagung 2014, Frankfurt, 18. März 2014

Unterstützt von / Supported by
Alexander von Humboldt Stiftung/Foundation

HIC for FAIR
Helmholtz International Center

MARIE CURIE ACTIONS

Landau Gauge Yang-Mills theory

JUSTUS-LIEBIG-UNIVERSITÄT GIESSEN

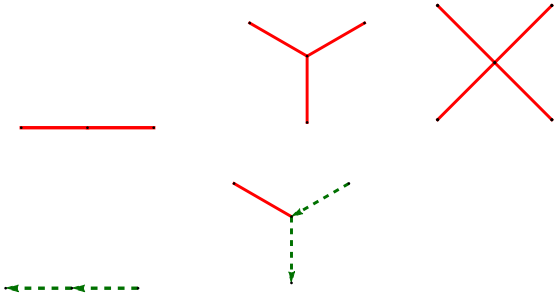
Gluonic sector of quantum chromodynamics: Yang-Mills theory

$$\mathcal{L} = \frac{1}{2} F^2 + \mathcal{L}_{gf} + \mathcal{L}_{gh}$$

$$F_{\mu\nu} = \partial_\mu \mathbf{A}_\nu - \partial_\nu \mathbf{A}_\mu + i\, g\, [\mathbf{A}_\mu, \mathbf{A}_\nu]$$

Landau gauge

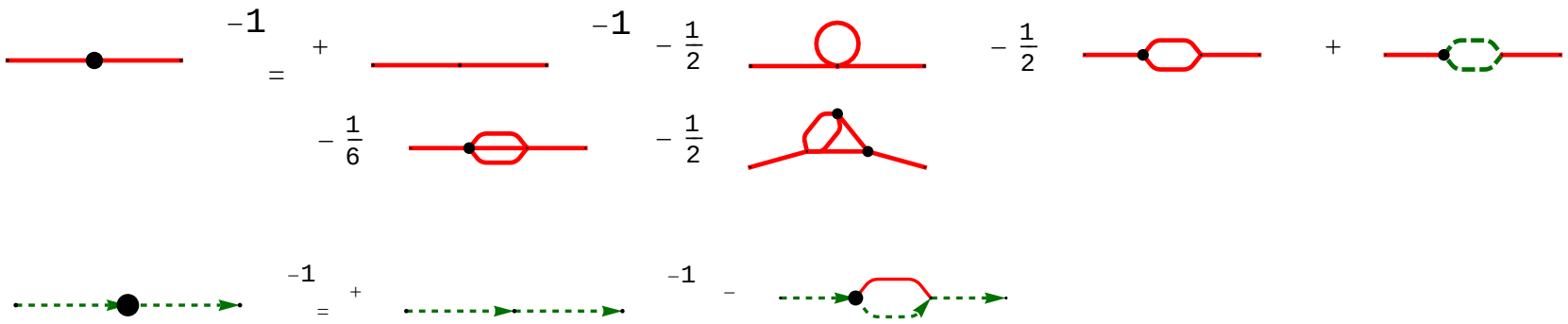
- simplest one for functional equations
- $\partial_\mu \mathbf{A}_\mu = 0$: $\quad \mathcal{L}_{gf} = \frac{1}{2\xi} (\partial_\mu \mathbf{A}_\mu)^2, \quad \xi \to 0$
- requires ghost fields: $\quad \mathcal{L}_{gh} = \bar{\mathbf{c}}\, (-\Box + g\, \mathbf{A}\, \times)\, \mathbf{c}$



Dyson-Schwinger equations: Propagators

JUSTUS-LIEBIG-UNIVERSITÄT GIESSEN

Dyson-Schwinger equations (DSEs) of gluon and ghost propagators:

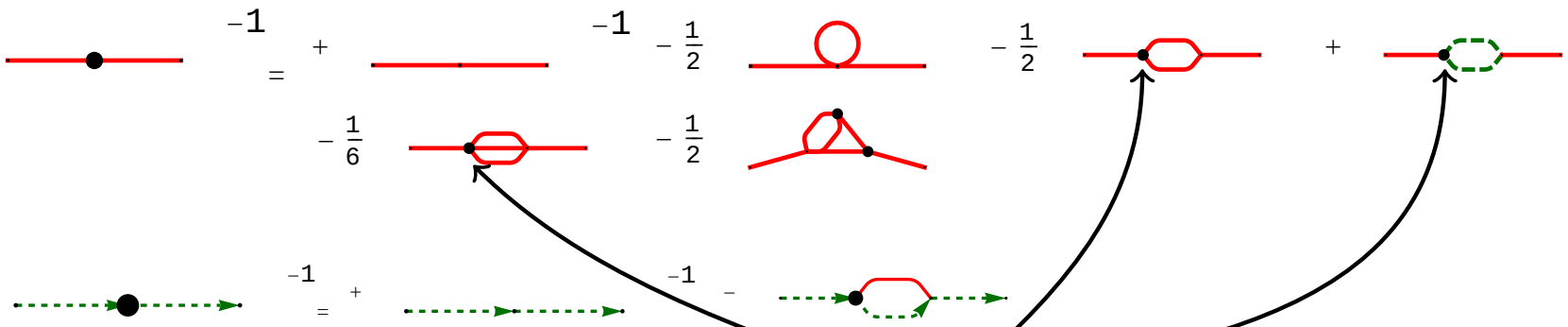


- Infinite tower of coupled integral equations.
- Derivation straightforward, but tedious
  $\rightarrow$ automated derivation with *DoFun* [MQH, Braun, CPC183 (2012); Alkofer, MQH, Schwenzer, CPC180 (2009)].
- Contain three-point and four-point functions:
  ghost-gluon vertex , three-gluon vertex , four-gluon vertex

Dyson-Schwinger equations: Propagators

JUSTUS-LIEBIG-UNIVERSITÄT GIESSEN

Dyson-Schwinger equations (DSEs) of gluon and ghost propagators:



- Infinite tower of coupled integral equations.
- Derivation straightforward, but tedious
  $\rightarrow$ automated derivation with *DoFun* [MQH, Braun, CPC183 (2012); Alkofer, MQH, Schwenzer, CPC180 (2009)].
- Contain three-point and four-point functions:
  ghost-gluon vertex , three-gluon vertex , four-gluon vertex

Truncated propagator Dyson-Schwinger equations

JUSTUS-LIEBIG-UNIVERSITÄT GIESSEN

Standard truncation:

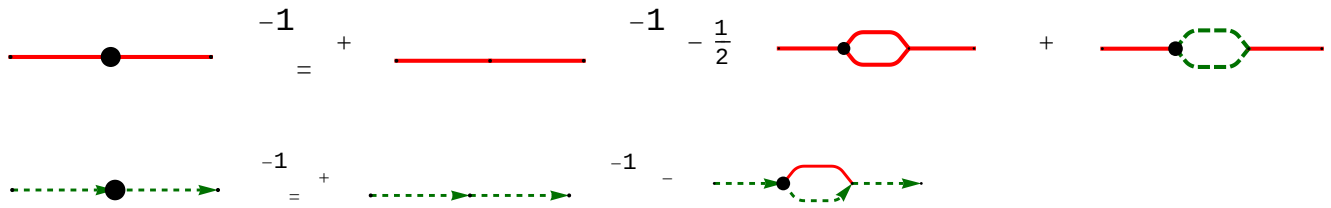
- No four-point interactions
- models for ghost-gluon and three-gluon vertices

Truncated propagator Dyson-Schwinger equations

JUSTUS-LIEBIG-UNIVERSITÄT GIESSEN

Standard truncation:

- No four-point interactions
- models for ghost-gluon and three-gluon vertices

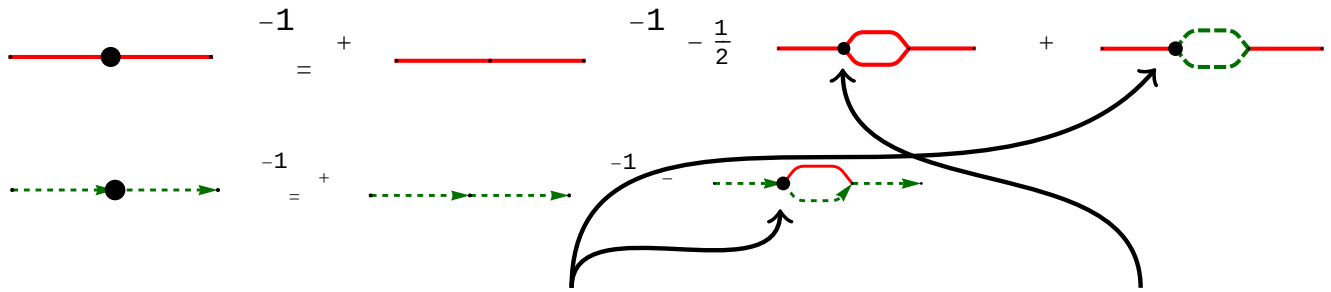


Truncated propagator Dyson-Schwinger equations

JUSTUS-LIEBIG-UNIVERSITÄT GIESSEN

Standard truncation:

- No four-point interactions
- models for ghost-gluon and three-gluon vertices



Standard: bare ghost-gluon vertex and three-gluon vertex model

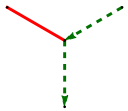
$$D^{ab}_{gl,\mu\nu}(p) = \left(g_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}\right) \frac{\mathbf{Z(p^2)}}{p^2} \delta^{ab}$$

Influence of three-point functions?

$$D^{ab}_{gh}(p) = -\frac{\mathbf{G(p^2)}}{p^2} \delta^{ab}$$

Improving truncations

JUSTUS-LIEBIG-UNIVERSITÄT GIESSEN

					
1979	✓	0	0	model	qual. wrong*
1997	✓	✓	model	model	qual. ok**
2012	✓	✓	✓	model	quant. improvement***
2014	✓	✓	✓	✓	****

* [Mandelstam, PRD20 (1979)]

** [von Smekal, Hauck, Alkofer, PRL79 (1997); Aguilar, Binosi, Papavassiliou PRD78 (2008); Fischer, Maas, Pawlowski, AP324 (2009); Pennington, Wilson, PRD84 (2001); Llanes-Estrada, Williams, PRD86 (2012); Strauss, Fischer, Kellermann, PRL109 (2012)]

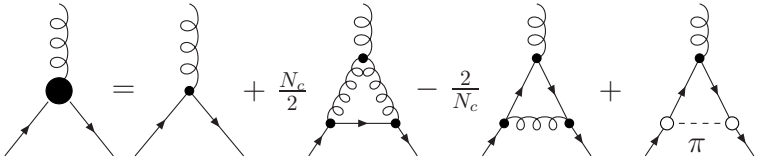
*** [MQH, von Smekal, JHEP04 (2013)]

**** [Blum, MQH, Mitter, von Smekal, to app. in PRD, 1401.0713]

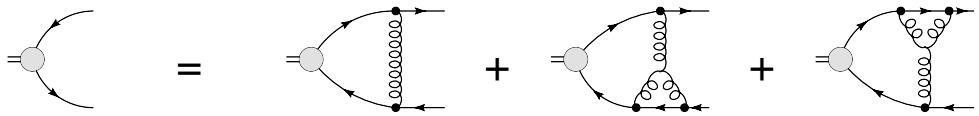
Three-gluon vertex

JUSTUS-LIEBIG-UNIVERSITÄT GIESSEN

- Solve quark-gluon vertex



- Bound state calculations:
  required for some ideas to go beyond rainbow-ladder. E. g.:
  - Include gluon self-interaction
    [e.g., Maris, Tandy, NPPS161 (2006); Fischer, Williams, PRL103 (2009)]:

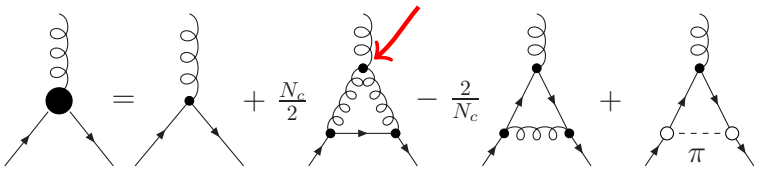


- Quantitatively important in mid-momentum regime.
  $\Rightarrow$ Quantitative impact at non-zero temperature and density.

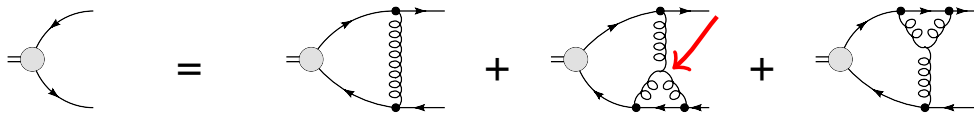
Three-gluon vertex

JUSTUS-LIEBIG-UNIVERSITÄT GIESSEN

- Solve quark-gluon vertex



- Bound state calculations:
  required for some ideas to go beyond rainbow-ladder. E. g.:
  - Include gluon self-interaction
    [e.g., Maris, Tandy, NPPS161 (2006); Fischer, Williams, PRL103 (2009)]:

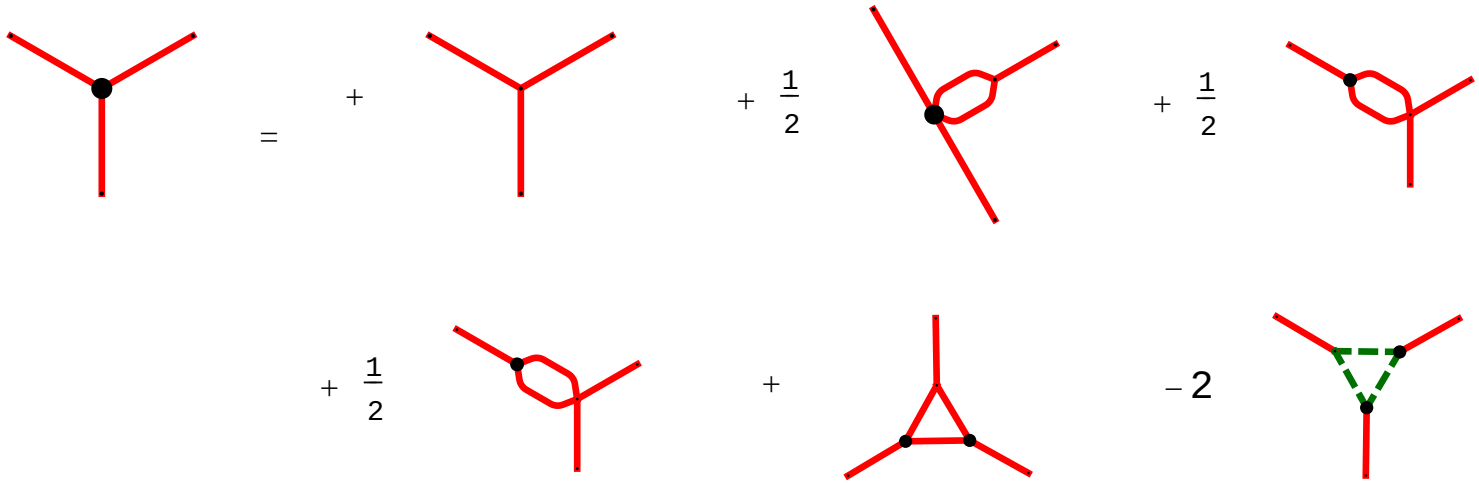


- Quantitatively important in mid-momentum regime.
  $\Rightarrow$ Quantitative impact at non-zero temperature and density.

Three-gluon vertex: Truncation

JUSTUS-LIEBIG-UNIVERSITÄT GIESSEN

- Restrict to tree-level tensor, viz. disregard three tensors.
  Solution with full transverse basis: [Eichmann, Williams, Alkofer, Vujinovic, 1402.1365]
  $\rightarrow$ Tree-level dominant.
- UV leading one-loop diagrams only: triangles and swordfish.
- Four-gluon vertex: model.



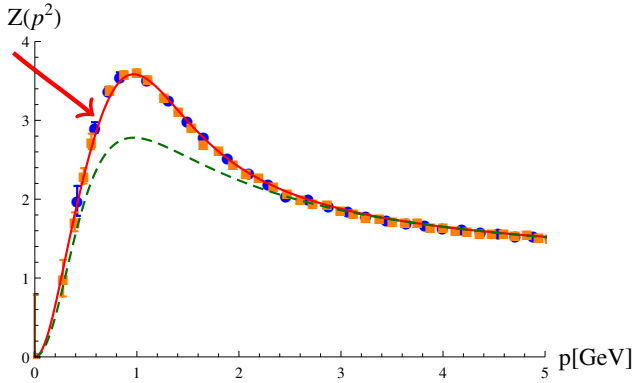
- Bose symmetrization.

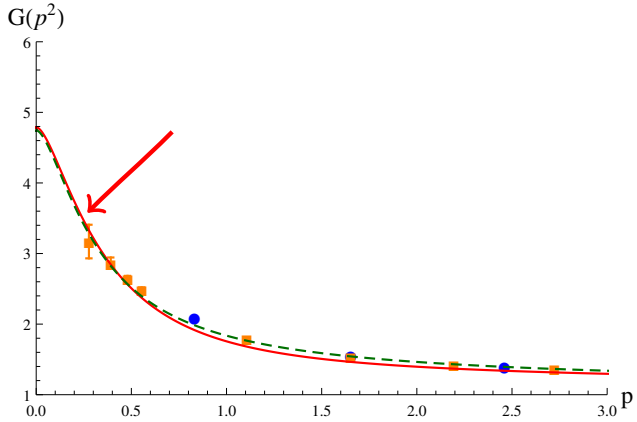
Propagator and ghost-gluon vertex input

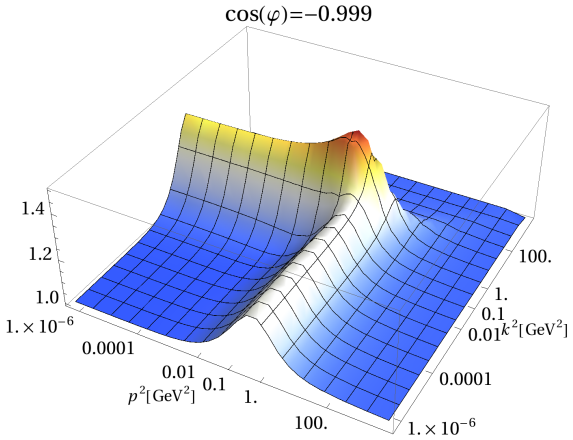
JUSTUS-LIEBIG-UNIVERSITÄT GIESSEN

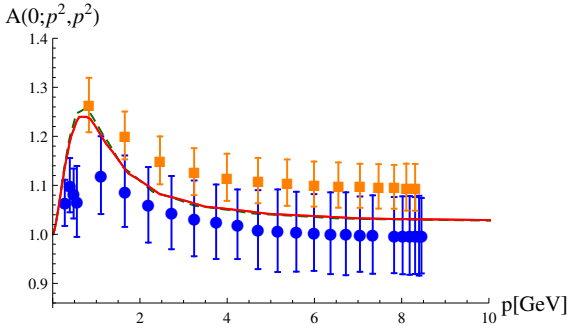
Dynamic ghost-gluon vertex, opt. eff. three-gluon vertex:

[MQH, von Smekal, JHEP04 (2013)]









ghost-gluon vertex lattice data: [Sternbeck (2006)]

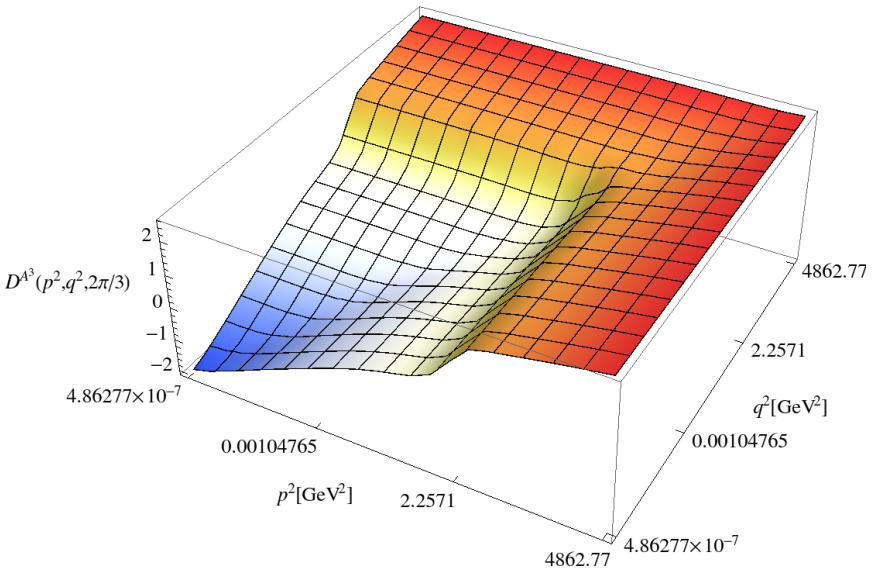
Good quantitative agreement for ghost *and* gluon dressings.

Results three-gluon vertex (standalone)

JUSTUS-LIEBIG-UNIVERSITÄT GIESSEN

$$\Gamma^{AAA,abc}_{\mu\nu\rho}(p,q,k) := i\,g\,f^{abc}\,D^{AAA}(p^2,q^2,\cos\theta)\Gamma^{AAA,(0)}_{\mu\nu\rho}(p,q,k)$$

Fixed angle:



- Correct UV behavior.
- Zero crossing (at least in this tensor).
- No angle dependence seen.

[Blum, MQH, Mitter, von Smekal, to app. in PRD, 1401.0713]

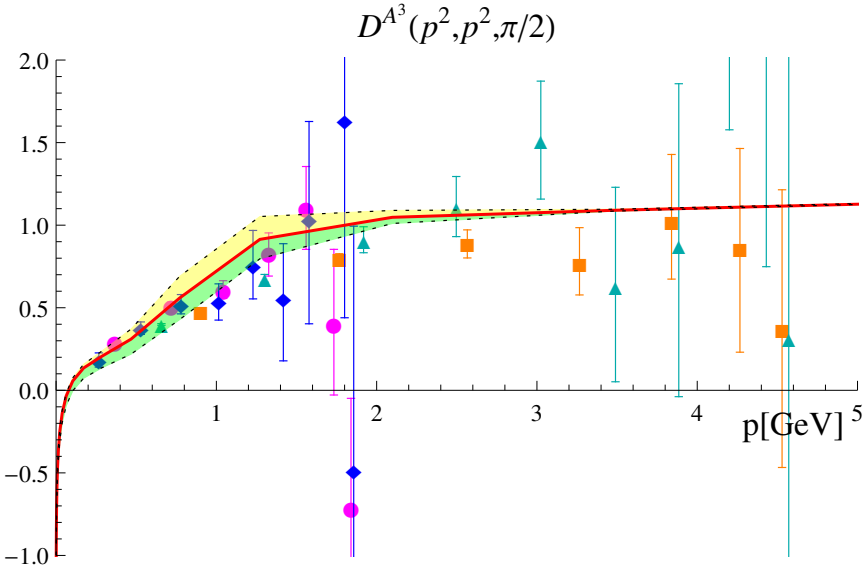
Results three-gluon vertex (standalone)

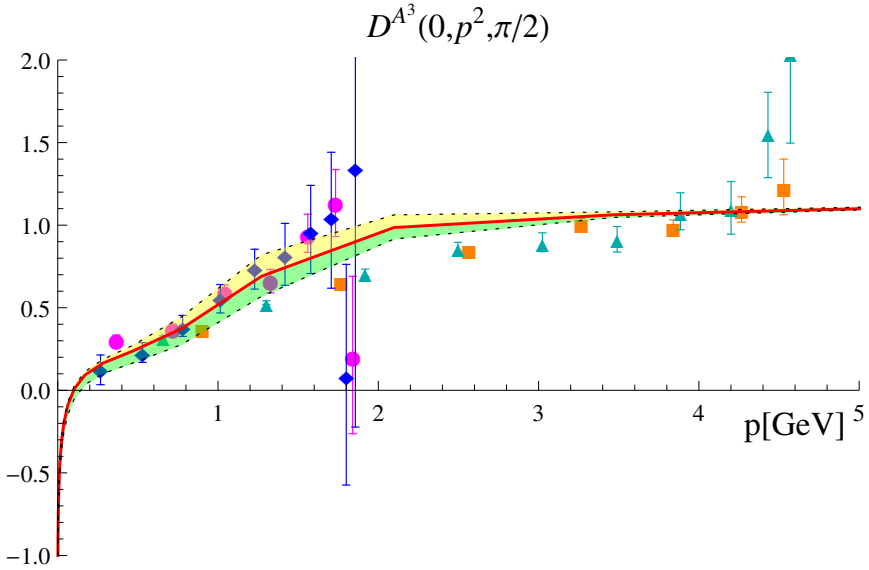
JUSTUS-LIEBIG-UNIVERSITÄT GIESSEN

Four-gluon vertex model:

$$D^{A^4}(p,q,r,s) = (a\tanh(b/\bar{p}^2)+1)\, D^{A^4}_{RG}(p,q,r,s)$$

$\rightarrow$ Test model dependence by varying a and b.





[Blum, MQH, Mitter, von Smekal, to app. in PRD, 1401.0713]; lattice: [Cucchieri, Maas, Mendes, PRD77]

$\rightarrow$ Truncation reliable.

Four-gluon vertex

JUSTUS-LIEBIG-UNIVERSITÄT GIESSEN

Input: Lattice equivalent propagators and three-point functions.

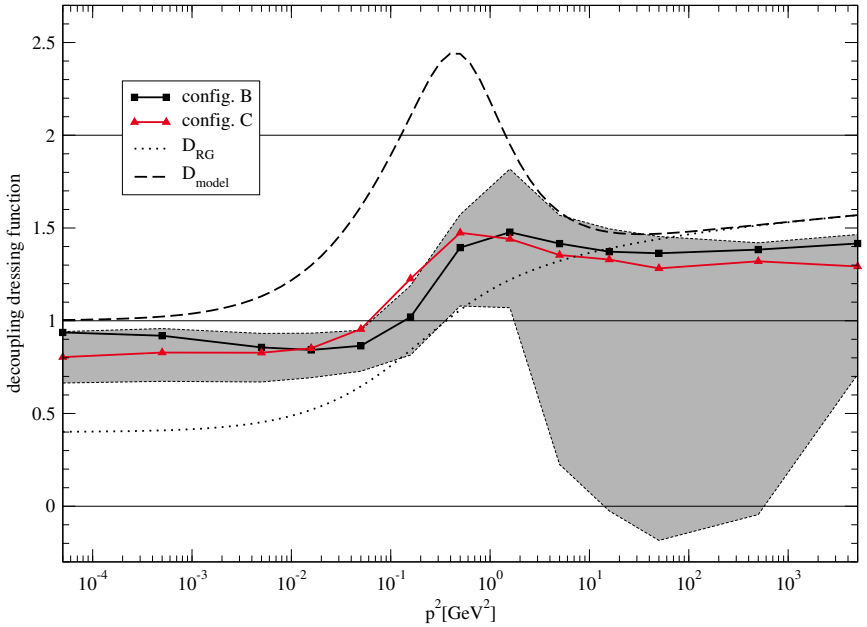
$\rightarrow$ No dependence on higher Green functions within truncation!

Four-gluon vertex

Input: Lattice equivalent propagators and three-point functions.

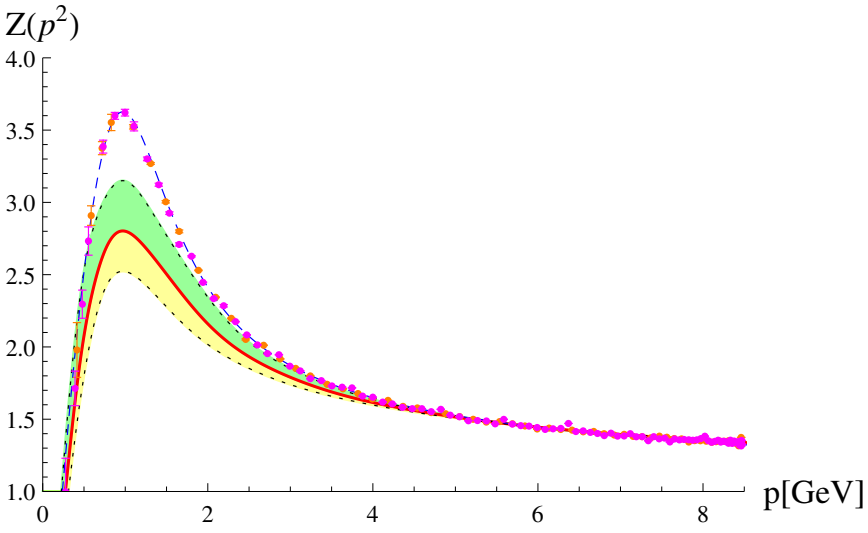
$\rightarrow$ No dependence on higher Green functions within truncation!

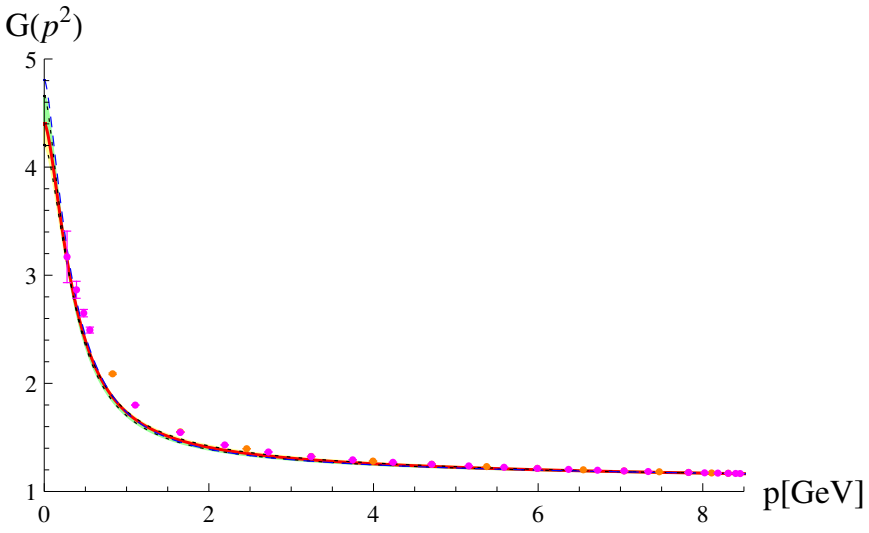
Comparison model and DSE result:



[Cyrol, MQH, von Smekal, in prep.]

Results of propagators with calculated three-gluon vertex





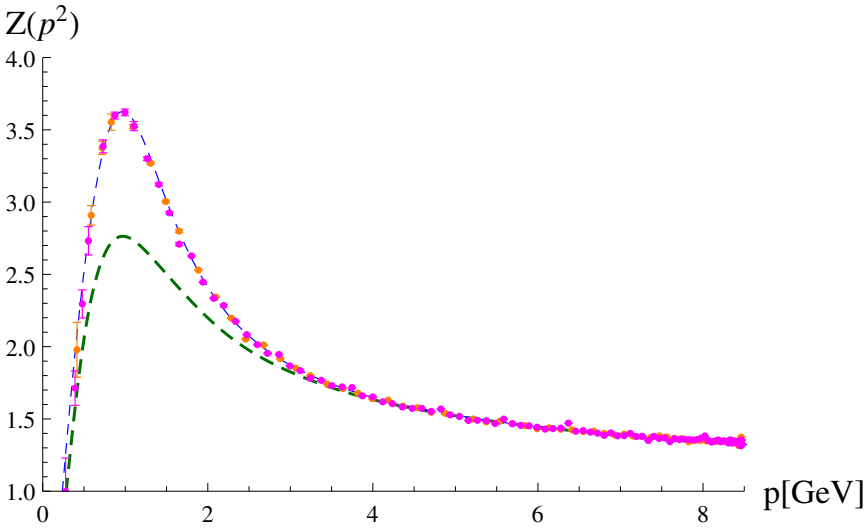
[Blum, MQH, Mitter, von Smekal, to app. in PRD, 1401.0713]

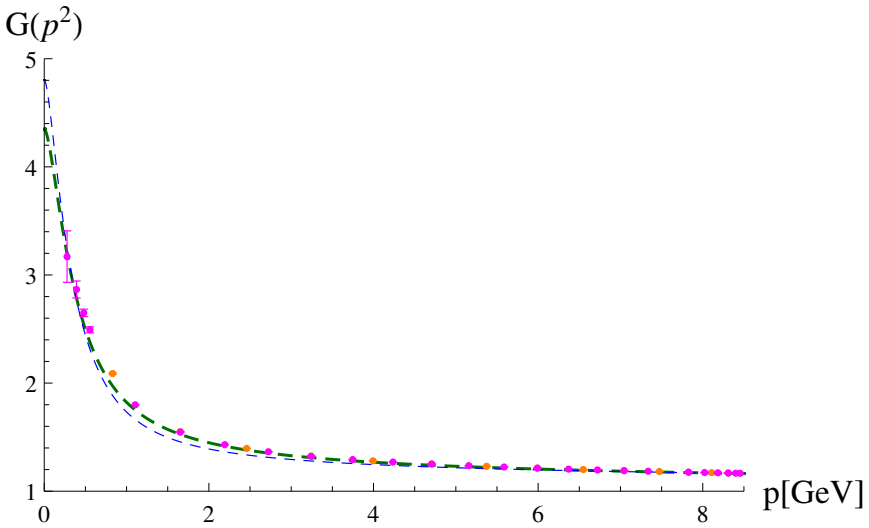
- Ghost almost unaffected.
- Gap in midmomentum regime must be due to missing two-loop diagrams!

Results of three-point closed calculation

JUSTUS-LIEBIG-UNIVERSITÄT GIESSEN

Calculate all four two- and three-point functions.





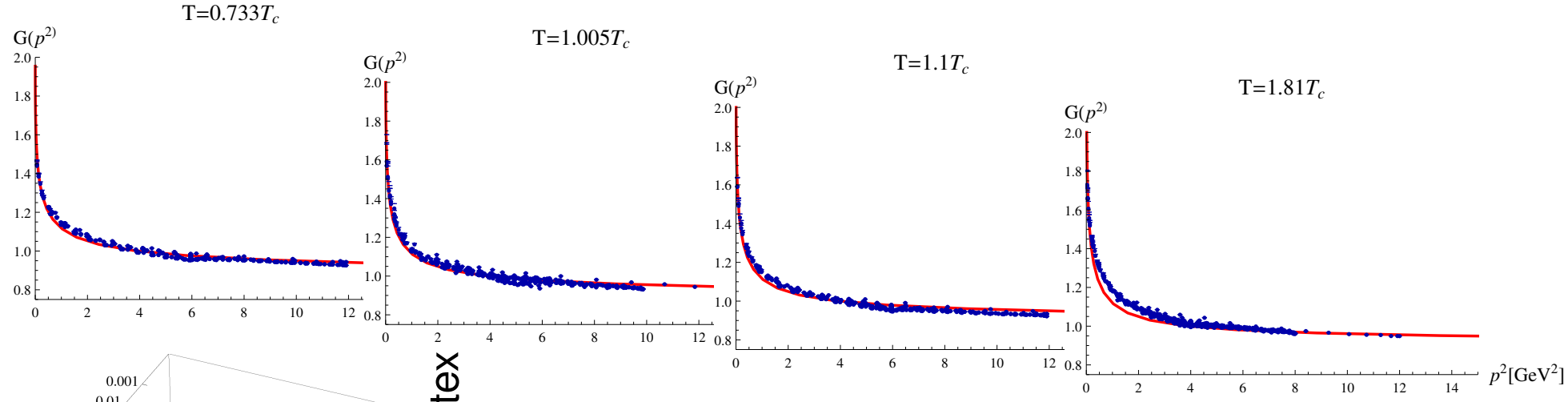
[Blum, MQH, Mitter, von Smekal, to app. in PRD, 1401.0713]

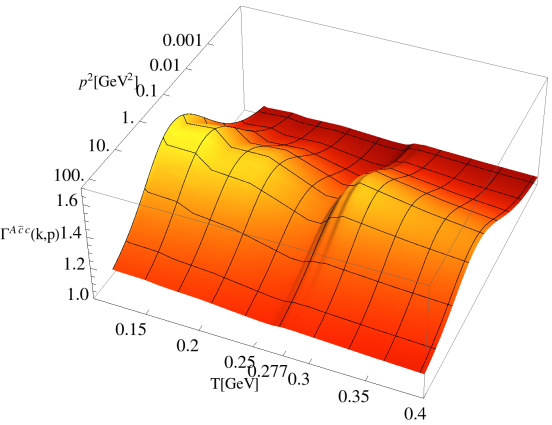
- Ghost almost unaffected.
- Close to previous solutions with modeled three-point functions.
- Gap in midmomentum regime due to missing two-loop diagrams.

A glimpse at non-zero temperature

JUSTUS-LIEBIG-UNIVERSITÄT GIESSEN

First steps towards full system:
Take some lattice input for gluon propagator [Fischer, Maas, Müller, EPJC68 (2010)]





Ghost-gluon vertex

Line: Ghost DSE [MQH, von Smekal, 1311.0702]

Lattice: [Maas, Pawlowski, Spielmann, von Smekal, PRD85 (2012)]

Summary

JUSTUS-LIEBIG-UNIVERSITÄT GIESSEN

- Systematically enlarging truncations of DSEs
  $\rightarrow$ (partially) quantitative improvement of Green functions
  $\rightarrow$ reduce dependence on external input
  $\rightarrow$ basis for further calculations where no direct crosschecks are possible, e. g., non-zero density.

Summary

JUSTUS-LIEBIG-UNIVERSITÄT GIESSEN

- Systematically enlarging truncations of DSEs
  $\rightarrow$ (partially) quantitative improvement of Green functions
  $\rightarrow$ reduce dependence on external input
  $\rightarrow$ basis for further calculations where no direct crosschecks are possible, e. g., non-zero density.

- Propagators + ghost-gluon vertex + improved three-gluon vertex model: Lattice equivalent results [MQH, von Smekal, JHEP04 (2013)] $\rightarrow$ good propagator input.

Summary [Blum, MQH, Mitter, von Smekal, to app. in PRD, 1401.0713]

JUSTUS-LIEBIG-UNIVERSITÄT GIESSEN

Pinpointed missing contributions in DSEs:

- Three-gluon vertex:
  - Good propagator input $\rightarrow$ good vertex results.
  - $\Rightarrow$ Truncation of DSE reliable.

Summary [Blum, MQH, Mitter, von Smekal, to app. in PRD, 1401.0713]

JUSTUS-LIEBIG-UNIVERSITÄT GIESSEN

Pinpointed missing contributions in DSEs:

- Three-gluon vertex:
  - Good propagator input $\rightarrow$ good vertex results.
  - $\Rightarrow$ Truncation of DSE reliable.
- Gluon propagator:
  - Good three-gluon input $\rightarrow$ bad propagator results.
  - $\Rightarrow$ Two-loop diagrams in gluon propagator DSE quantitatively important.

Summary [Blum, MQH, Mitter, von Smekal, to app. in PRD, 1401.0713]

JUSTUS-LIEBIG-UNIVERSITÄT GIESSEN

Pinpointed missing contributions in DSEs:

- Three-gluon vertex:
  - Good propagator input $\rightarrow$ good vertex results.
  - $\Rightarrow$ Truncation of DSE reliable.
- Gluon propagator:
  - Good three-gluon input $\rightarrow$ bad propagator results.
  - $\Rightarrow$ Two-loop diagrams in gluon propagator DSE quantitatively important.
- Three-point closed truncation feasible, but two-loop diagrams in gluon propagator DSE are crucial.

Summary [Blum, MQH, Mitter, von Smekal, to app. in PRD, 1401.0713]

JUSTUS-LIEBIG-UNIVERSITÄT GIESSEN

Pinpointed missing contributions in DSEs:

- Three-gluon vertex:
  - Good propagator input $\rightarrow$ good vertex results.
  - $\Rightarrow$ Truncation of DSE reliable.
- Gluon propagator:
  - Good three-gluon input $\rightarrow$ bad propagator results.
  - $\Rightarrow$ Two-loop diagrams in gluon propagator DSE quantitatively important.
- Three-point closed truncation feasible, but two-loop diagrams in gluon propagator DSE are crucial.

Thank you for your attention.