Non-perturbative behavior of Yang-Mills Green functions

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Phenomenology of the strong interaction

Particles under the influence of the strong force:

hadrons, e. g. π, K, η, proton, neutron, Λ, Σ, ...

- High energy experiments:

  point-like particles inside the hadrons (quarks).
- Quarks only exist in bound states,

  never as free particles (confinement).
- Mediator of the strong force: gluons (also confined).
- Theory: Quantum Chromodynamics (QCD).
- At high energies QCD is asymptotically free, i. e. the coupling gets small and we can "observe" quarks (Nobel prize 2004).
- At lower energies non-perturbative methods are needed.

This talk

In this talk I will focus on the low energy behavior of Yang-Mills theory (gluonic part of QCD).

Contents of the talk

- Infrared of Yang-Mills theory:
  What can we learn from it?
- Maximally Abelian gauge:
  Why do we need this complicated gauge, anyway?
  And what is its infrared behavior?
- Landau gauge:
  Does (partly) solving the Gribov problem change the IR behavior?
- Non-perturbative tool: Dyson-Schwinger equations
  Is there an easy way to derive them?

Confinement of quarks and gluons

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Confinement of quarks and gluons

- One expects that the property of being confined is encoded in the particles' propagators.
- Different confinement criteria for the propagators:
  - Positivity violations: negative norm contributions
    $\longrightarrow$ not a particle of the physical state space
  - Kugo-Ojima: quartet mechanism, e. g., Gupta-Bleuler formalism in QED: time-like and longitudinal photon cancel each other.
  - Gribov-Zwanziger (Landau gauge, Coulomb gauge):
    infrared (IR) suppression of gluon propagator due to Gribov horizon
    $\longrightarrow$ no long-distance propagation
    Already manifest at perturbative level with Gribov-Zwanziger action!
  - KO, GZ in Landau gauge, $p^2 \longrightarrow 0$:
    $D_{gluon} \longrightarrow 0,\ p^2 D_{ghost} \longrightarrow \infty$

Dyson-Schwinger equations (DSEs) for investigating QCD

DSEs describe non-perturbatively how particles propagate and interact.

Equations of motion of Green functions

Infinitely large tower of equations

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  $\longrightarrow$ non-perturbative regime accessible
- Continuum, different scales accessible
  $\longrightarrow$ complement lattice method

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- Truncations
- Gauge-dependent

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Equations of motion of Green functions

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Pros:

- Exact equations
  $\longrightarrow$ non-perturbative regime accessible
- Continuum, different scales accessible
  $\longrightarrow$ complement lattice method

Cons?:

- Truncations (not for all tasks)
- Gauge-dependent
  $\longrightarrow$ Exploit advantages of different gauges

Infrared regime of Yang-Mills theory in Landau gauge I

Scaling solution [Alkofer, Fischer, Gies, Maas, Pawlowski, von Smekal, Zwanziger, ...]

- Dressing functions obey power laws.
  $\longrightarrow$ Qualitative information provided by IR exponents.
- Qualitative IR solution of ALL correlation functions is known.
- Picture of confinement:
  IR vanishing gluon ($\longrightarrow$ gluon confinement)
  IR enhanced ghost propagator ($\longrightarrow$ long-range force to confine quarks [Alkofer, Fischer, Llanes-Estrada, Schwenzer, AOP324]).
- Horizon condition/Kugo-Ojima $\leftrightarrow$ IR enhanced ghost.

Infrared regime of Yang-Mills theory in Landau gauge II

Decoupling solution

- Gluon massive, ghost tree-level like.
- Seen in most lattice calculations [Bogolubsky, Bornyakov, Cucchieri, Ilgenfritz, Maas, Mendes, Müller-Preussker, Pawlowski, Spielmann, Sternbeck, von Smekal, ...].
  $\longrightarrow$ Proof of unique solution?
- Adding condensates to the Gribov-Zwanziger action $\longrightarrow$ refined Gribov-Zwanziger scenario [Dudal, Gracey, Sorella, Vandersickel, Verschelde]
- DSEs, FRGEs [Boucaud et al., Aguilar et al., Fischer et al.]
- Vertices not IR enhanced [Alkofer, M.Q.H., Schwenzer, PRD81].

Common features

- Gluon propagator violates positivity.
- Confining Polyakov loop potential [Braun, Gies, Pawlowski, PLB684].
- Physical quantities independent on specific solution in the deep IR, e.g., Braun, Gies, Pawlowski, PLB684; Fischer, Mueller, PRD80.

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Functional methods: Understood where the two solutions come from.
$\longrightarrow$ different renormalization of the ghost propagator $\leftrightarrow$
boundary condition for DSEs [Fischer et al., AOP324]

Lattice:

- Scaling in 2d (?) and strong coupling limit (?: Sternbeck, von Smekal, 0811.4300; Cucchieri, Mendes, PRD81)
- Landau-B gauges [Maas, PLB689]

Hypothesis of Abelian dominance

Dual superconductor picture of confinement [Mandelstam, 't Hooft]

- Picture a conventional superconductor, where the electric charges condense and force the magnetic flux into vortices.
- Change "electric" and "magnetic" components and you get a dual superconductor, where condensed magnetic monopoles squeeze the electric flux into flux tubes.
- QCD: No free chromoelectric charges. Are they confined by condensed magnetic monopoles?

Hypothesis of Abelian dominance [Ezawa, Iwazaki, PRD 25 (1981)]:
Magnetic monopoles live in Abelian part of the theory.

$\longrightarrow$ Abelian part dominates in the IR?

Available results in the MAG

- Available lattice results of MAG [Cucchieri, Mendes, Mihara, 2008]:
  All propagators massive, Abelian field has lowest mass.
  $\Rightarrow$ Other fields decouple. Realization of Abelian dominance.
- Refined Gribov-Zwanziger framework [Capri et al., PRD77, JPA43]:
  All propagators massive
- $\rightarrow$ Decoupling solution
  $\rightarrow$ Existence of scaling solution?

Definition of the maximally Abelian gauge

Gauge field components:

$$A_\mu = \boldsymbol{A}^i_\mu \boldsymbol{T}^i + \boldsymbol{B}^a_\mu \boldsymbol{T}^a, \quad i = 1, \ldots, N-1, \quad a = N, \ldots, N^2 - 1$$

Abelian subalgebra: $[T^i, T^j] = 0$, can be written as diagonal matrices

Abelian $\leftrightarrow$ diagonal fields $\boldsymbol{A}$,
non-Abelian $\leftrightarrow$ off-diagonal fields $\boldsymbol{B}$.

E.g. $T^1 = \frac{1}{2}\lambda^3$, $T^2 = \frac{1}{2}\lambda^8$ for $SU(3)$.
Which interactions are possible ($[T^r, T^s] = i f^{rst} T^t$)?

	$SU(2)$	$SU(N>2)$
f^{ijk}	0	0
f^{ija}	0	0
f^{iab}	✓	✓
f^{abc}	0	✓

$\rightarrow$ $SU(2)$ and $SU(3)$ different?

Gauge fixing condition

Stress role of diagonal fields $\Rightarrow$ minimize norm of off-diagonal field $\boldsymbol{B}$:

$$\|B_U\| = \int dx\, B_U^a B_U^a \rightarrow \text{minimize wrt. gauge transformations } U$$

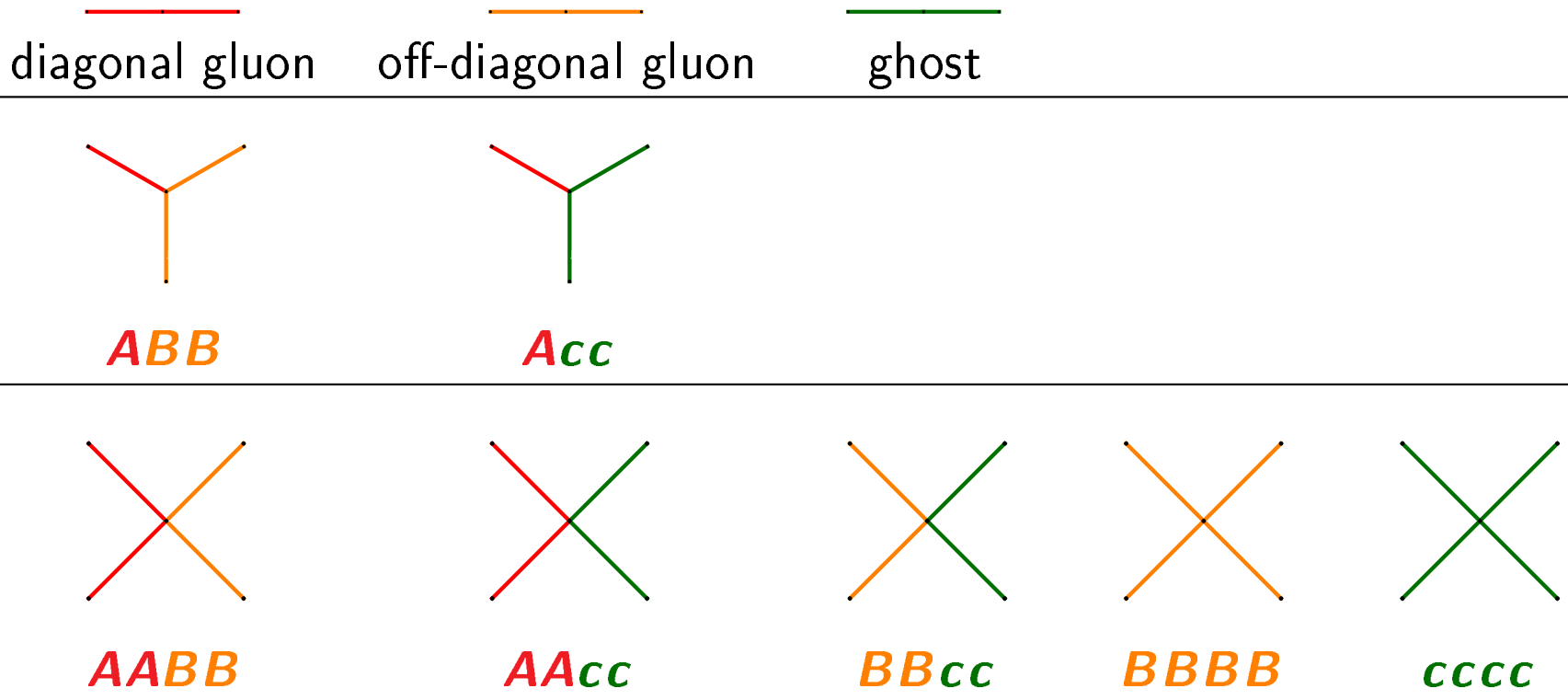
$$D_\mu^{ab} \boldsymbol{B}_\mu^b = (\delta_{ab}\partial_\mu - g\, f^{abi} \boldsymbol{A}_\mu^i) \boldsymbol{B}_\mu^b = 0$$

non-linear gauge fixing condition!

Remaining symmetry of diagonal part: $U(1)^{N-1}$

Fix gauge of diag. gluon field $\boldsymbol{A}$ by Landau gauge condition: $\partial_\mu \boldsymbol{A}_\mu = 0$
$\Rightarrow$ diagonal ghosts decouple (like in QED).

Lagrangian for the MAG



$SU(N > 2)$

BBB *Bcc* *ABBB* *ABcc*

Deriving Dyson-Schwinger equations

Integral of a total derivative vanishes:

$$\int [D\phi] \frac{\delta}{\delta\phi} e^{-S+J\,\Phi} = \int [D\phi] \left(J - \frac{\delta S}{\delta\phi} \right) e^{-S+J\,\Phi} = 0.$$

$\Rightarrow$ DSEs for all Green functions (full, connected, 1PI) by further differentiations.

Doing it by hand?

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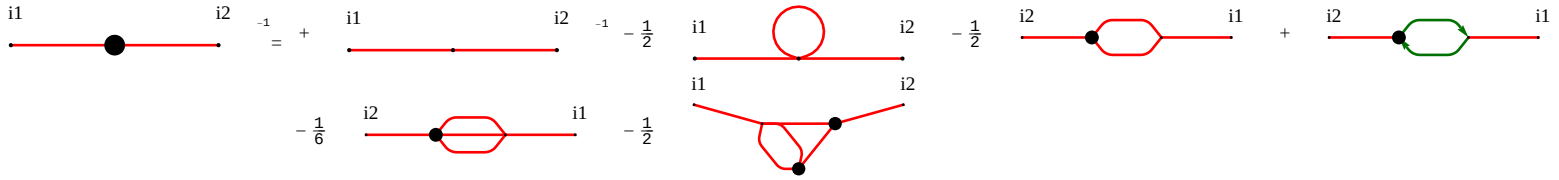
Doing it by hand?

Example: Landau gauge

2 propagators (***AA***, ***cc***)
3 interactions (***Acc***, ***AAA***, ***AAAA***)

Landau Gauge: Propagators

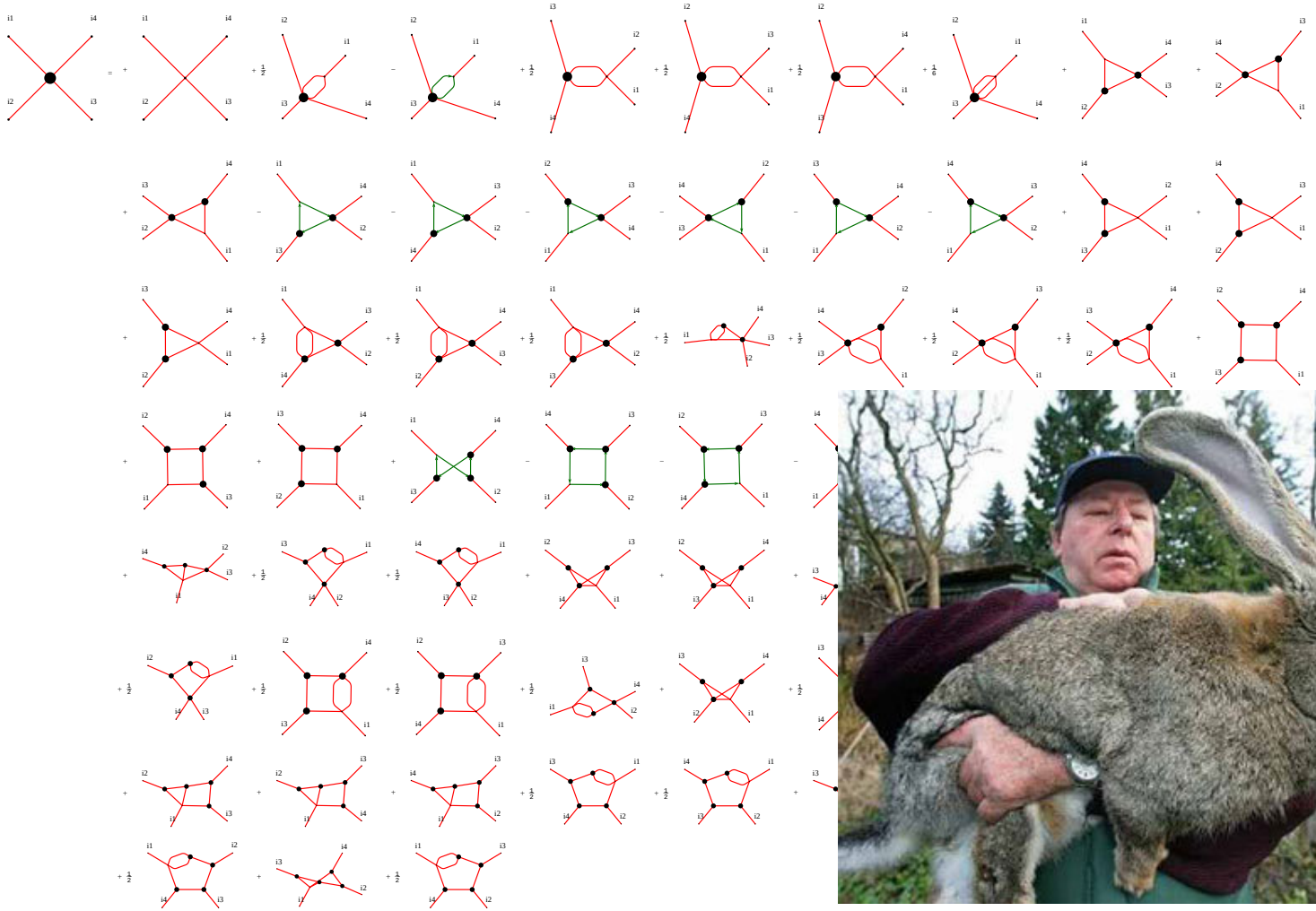
Gluon propagator:



Ghost propagator:

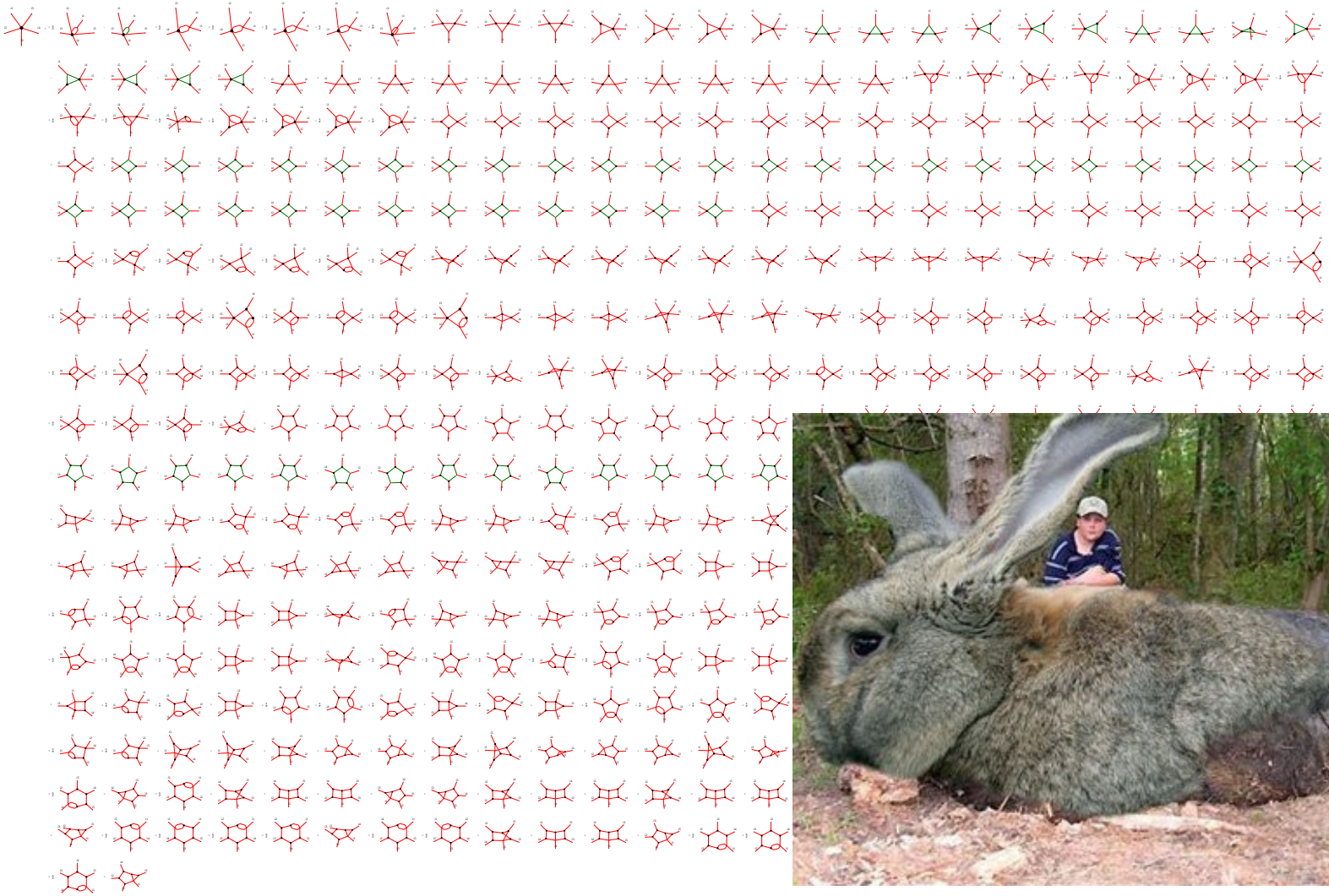
Landau Gauge: Four-Gluon Vertex

66 terms



Landau Gauge: Five-Gluon Vertex

434 terms



DoDSE

⇒ *DoDSE* [Alkofer, M.Q.H., Schwenzer, CPC 180 (2009)]

Given a structure of interactions, the DSEs are derived symbolically using *Mathematica*.

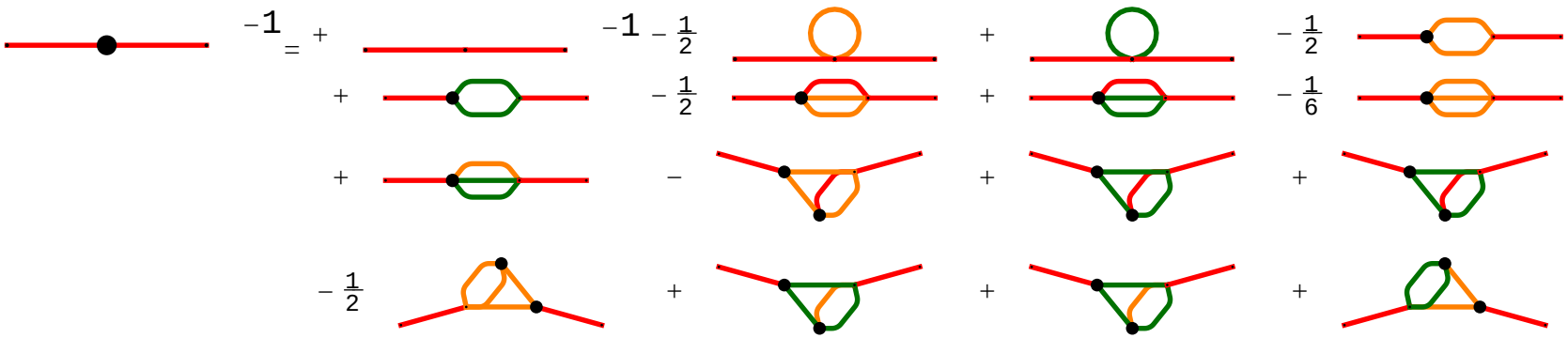
Example (Landau gauge):

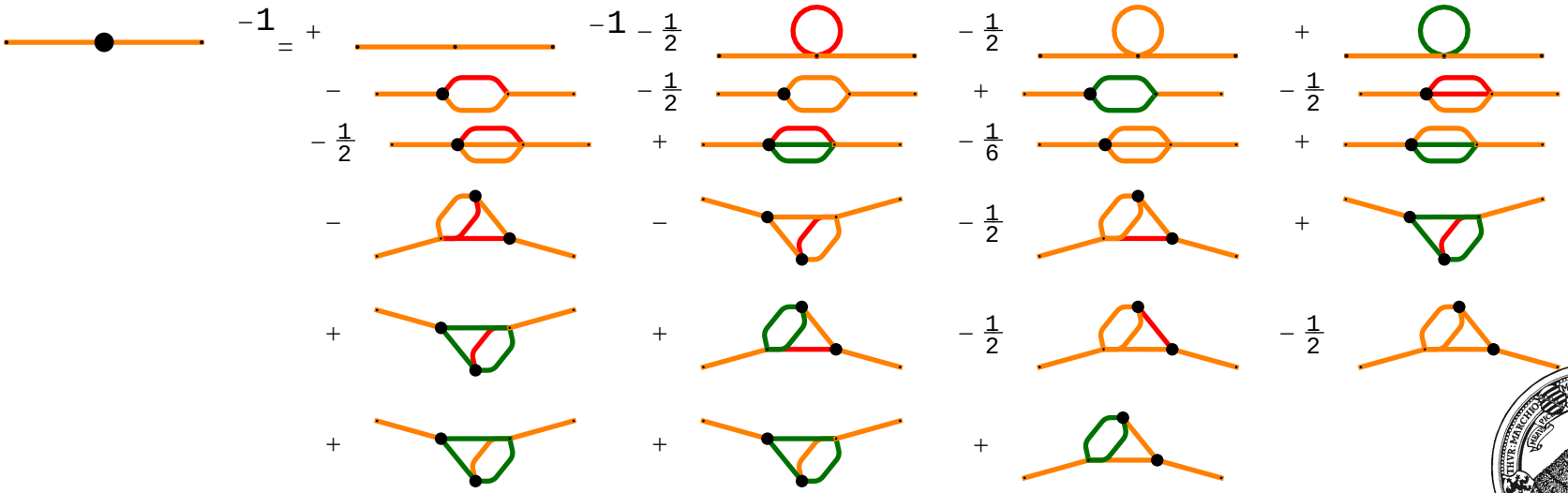
- only input: interactions in Lagrangian (AA, AAA, AAAA, cc, Acc)
- Which DSE do I want?

- Step-by-step calculations possible.
- Can handle mixed propagators (then there are really many diagrams; e. g. in Gribov-Zwanziger action).

Upgrade:
Provide Feynman rules and get complete algebraic expressions.
→ E. g., calculate color algebra with *FORM* and integrals with *C*.

DSEs of the MAG





Infrared power counting

Generic propagator

$$T_{(ij)} \cdot \frac{D(p^2)}{p^2},$$

assume power law behavior at low p^2

IR exponent

$$D^{IR}(p^2) = A \cdot (p^2)^{\delta}$$

- Vertices also assume power law behavior
  [e.g., Alkofer, Fischer, Llanes-Estrada, PLB 611 (2005) (skeleton expansion)].
- Limit of all momenta approaching zero simultaneously.
- Upon integration all momenta converted into powers of external momenta.

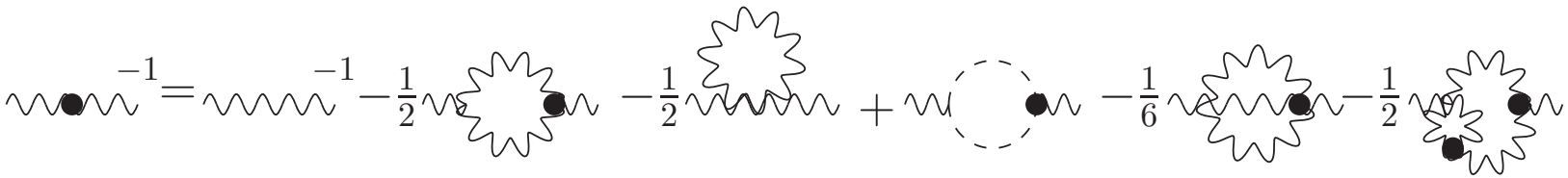
$\Rightarrow$ Counting of IR exponents

System of inequalities

- lhs is dominated by at least one diagram on rhs and rhs cannot be more divergent than lhs.

$$\rightarrow \delta_{lhs} \leq \delta_{rhs,any\ diagram}$$

- Not knowing which diagram is leading on the rhs, we can write inequalities from all diagrams.



$$-\delta_{gl} \leq 2\delta_{gl} + \delta_{3g}, \qquad -\delta_{gl} \leq 2\delta_{gh} + \delta_{gg}, \qquad \ldots$$

That's the basic idea. Still, for a large system a lot of work.

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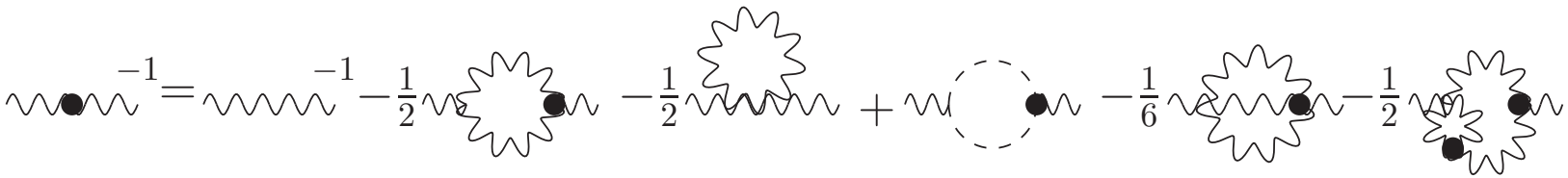

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All inequalities relevant?

Still, for a large system a lot of work.

Relevant inequalities

Closed form for all relevant inequalities
from 2 independent sets of functional equations
[Huber, Schwenzer, Alkofer, 0904.1873]

type		derived from	#
dressed vertices	$C_1 := \delta_{vertex} + \frac{1}{2} \sum_{\substack{\text{legs } j \text{ of} \\ \text{vertex}}} \delta_j \geq 0$	FRGEs	infinite
prim. div. vertices	$C_2 := \frac{1}{2} \sum_{\substack{\text{legs } j \text{ of} \\ \text{prim. div.} \\ \text{vertex}}} \delta_j \geq 0$	DSEs+FRGEs	finite

Shifting analysis to IR exponents $\rightarrow$ exact from this point on.

Analysis of propagator DSEs [Huber, Schwenzer, Alkofer, EPJC, t.b.p]

$\boxed{\text{At least one } C_2^i = 0 \text{ for one } i}$ $\Rightarrow$ scaling relation

$\Downarrow$

At least one vertex IR constant

- Necessary condition for a scaling solution
- Related to bare vertices in DSEs, cf. Fischer, Pawlowski PRD77
- Valid for a large number of systems
- Qualitative behavior of all Green functions determinable
- Generalization to actions with mixed propagators possible ($\rightarrow$ Gribov-Zwanziger action)

How to obtain a scaling relation: MAG

Many interactions
$\Rightarrow$ many inequalities, but some of them are contained within others
$\Rightarrow$ reduces number of possibilities

1. Look at all inequalities for primitively divergent vertices, i. e. at C_2^i.
2. Try all possibilities of $C_2^i = 0$.
3. Choose the non-trivial solutions.

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Application to the MAG:

1. $\delta_B \geq 0$, $\delta_c \geq 0$, $\delta_A + \delta_B \geq 0$, $\delta_A + \delta_c \geq 0$
2. a $\delta_B = 0$
   b $\delta_c = 0$
   c $\delta_A + \delta_B = 0$
   d $\delta_A + \delta_c = 0$
3. a $\cancel{\delta_A = \delta_B = \delta_c = 0}$
   b $\cancel{\delta_A = \delta_B = \delta_c = 0}$
   c $\delta_A + \delta_B = 0$
   d $\delta_A + \delta_c = 0$

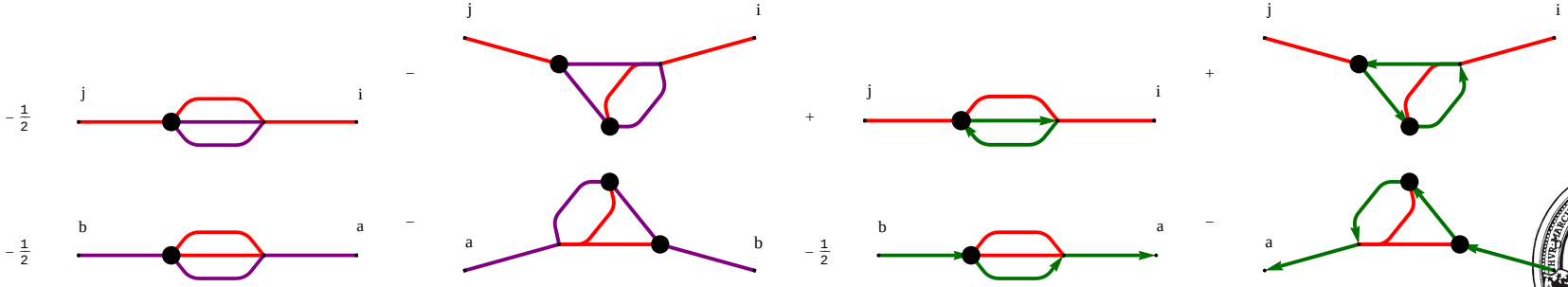
Scaling relation of the MAG: $\boxed{\delta_B = \delta_c = -\delta_A = \kappa_{MAG} \geq 0}$

IR scaling solution of the MAG

$$\boxed{\delta_B = \delta_c = -\delta_A = \kappa_{MAG} \geq 0}$$

- The Abelian fields are IR enhanced. $\longrightarrow$ Realization of Abelian dominance?
- Off-diagonal fields are IR suppressed.
- $SU(2)$ and $SU(N > 2)$ have the same solution.
- Qualitative solutions for tower of all Green functions.

IR leading diagrams:



$\longrightarrow$ UV/IR preserving truncation?

Decoupling solution in the MAG

Possible scenario for the connection between the decoupling and scaling solutions in the MAG

- Similar to ghost propagator in Landau gauge:
  via renormalization of diagonal gluon propagator
- IR divergent dressing: scaling solution
- IR massive dressing: decoupling

Note: If any of the three dressings is massive, the other two have to be massive too (tadpole diagrams).

Value of the IRE κ_{MAG}

Solution for κ_{MAG} is necessary but not sufficient.

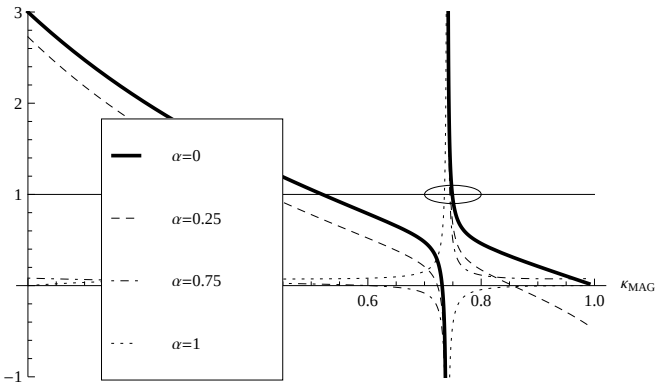
Dressing functions of gluons and ghosts:

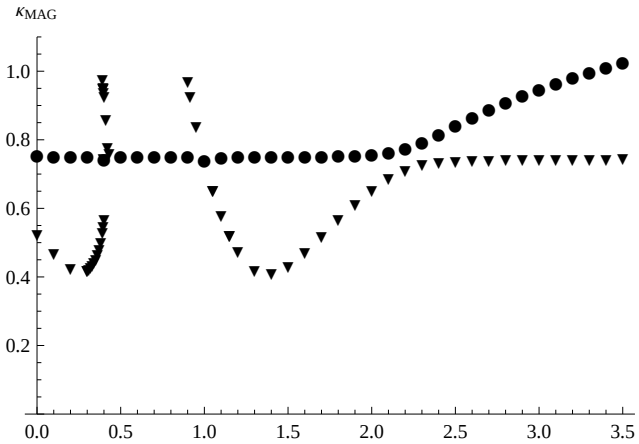
$$c_A(p^2) \overset{p^2 \to 0}{=} d_A \cdot (p^2)^{-\kappa_{MAG}}$$

$$c_B(p^2) \overset{p^2 \to 0}{=} d_B \cdot (p^2)^{\kappa_{MAG}}$$

$$c_c(p^2) \overset{p^2 \to 0}{=} d_c \cdot (p^2)^{\kappa_{MAG}}$$

$$0 \leq \kappa_{MAG} \leq 1$$





Solution branch independent of gauge fixing parameter α.

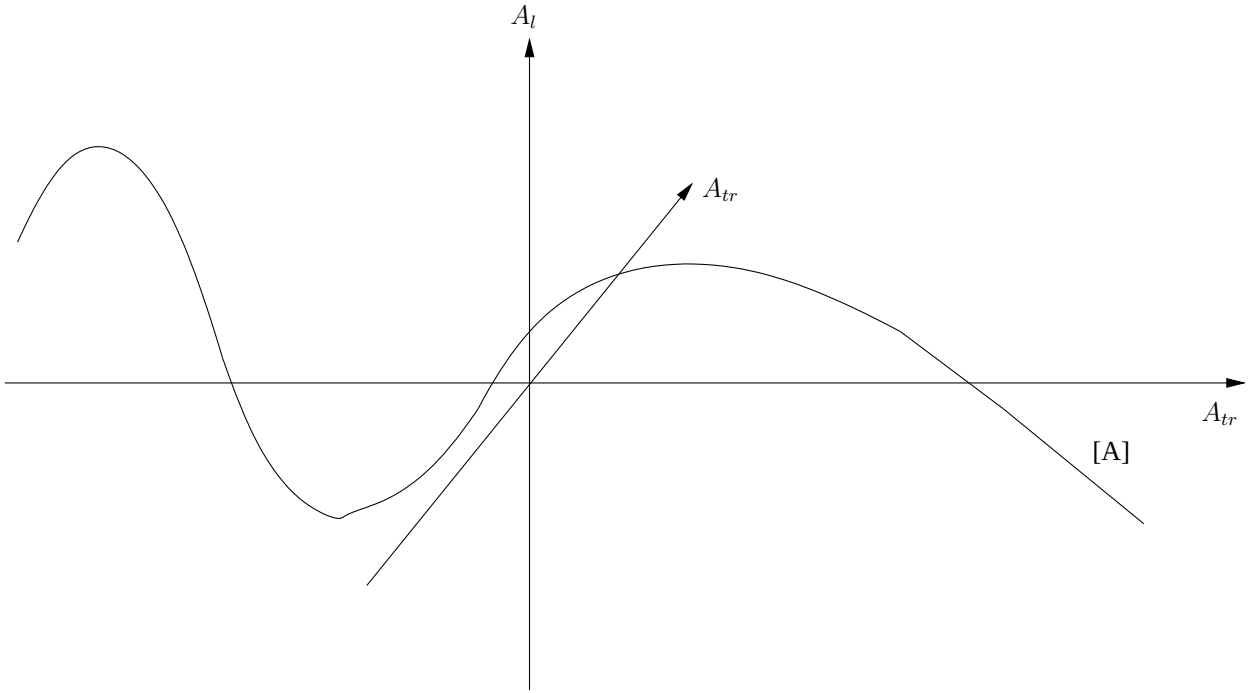
Relation Landau gauge & MAG

Landau gauge	maximally Abelian gauge
ghost dominance	Abelian (gluon) dominance
Gribov region bounded	Gribov region unbounded in diagonal direction [Capri et al., PRD79]

Greensite, Olejnik, Zwanziger, PRD78:

Abelian configurations $\xrightarrow{\text{Landau gauge}}$ on Gribov horizon

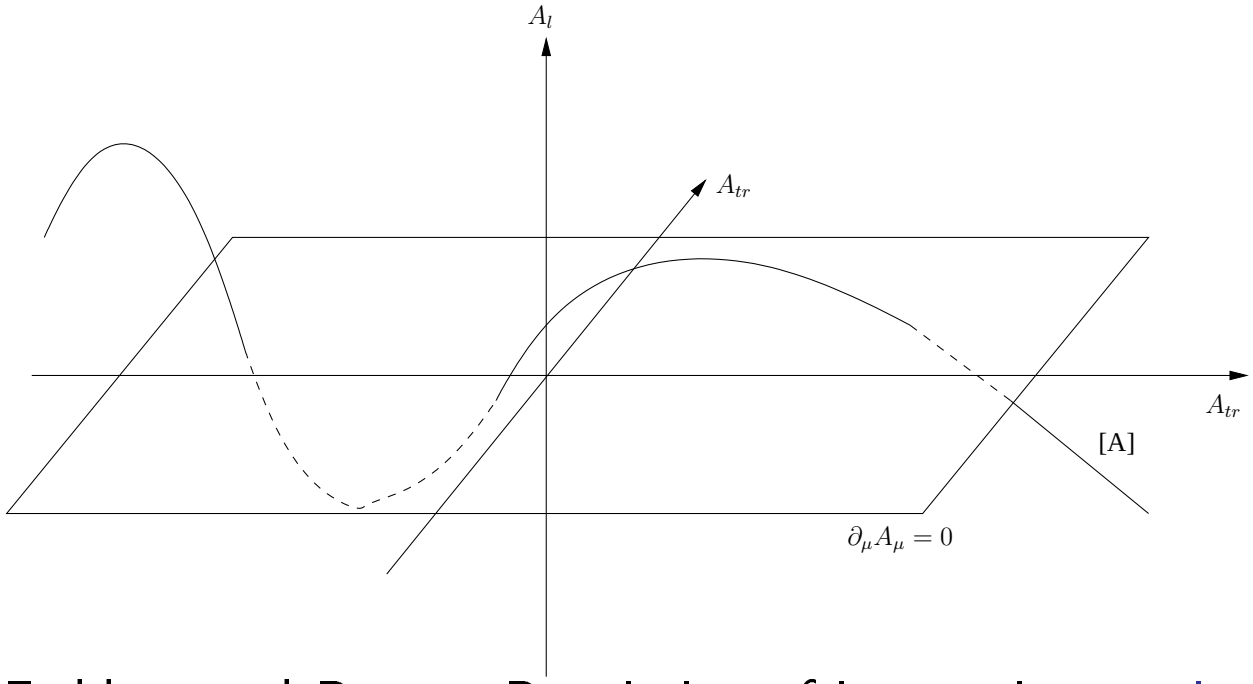
Gauge orbits and Gribov copies



Gauge equivalent configurations (gauge orbit $[A]$)
$\Rightarrow$ integration in path integral is overcomplete:

$$Z[J] = \int [DA] e^{-S+AJ}$$

Gauge orbits and Gribov copies

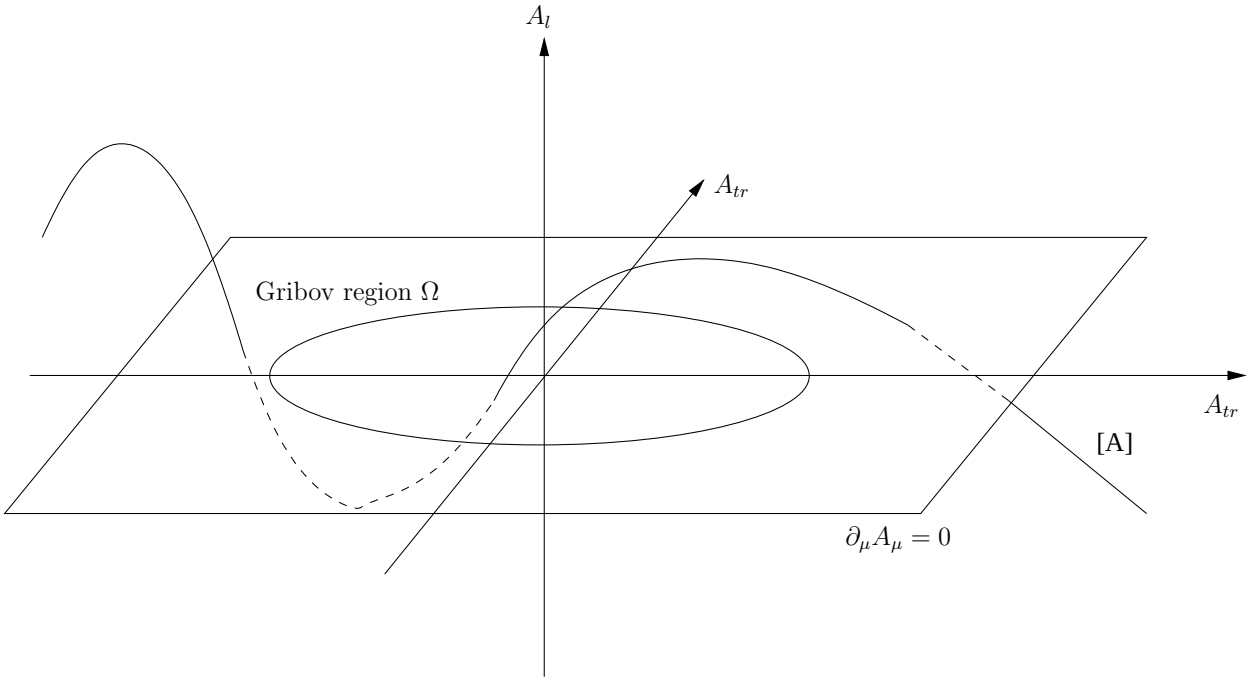


Faddeev and Popov: Restriction of integration to single representative of each gauge orbit possible?
Gauge symmetry replaced by BRST symmetry!

Faddeev-Popov operator

$$Z[J] = \int [DA]\delta(\partial_\mu A_\mu)\det M e^{-S+AJ}$$

Gauge orbits and Gribov copies



Restriction to Gribov region Ω: almost unique gauge fixing.

$$\Omega := \{A;\ \partial_\mu A_\mu = 0,\ M > 0\}$$

Restriction via a non-local term $\rightarrow$ Gribov-Zwanziger action.
New parameter γ, determined by horizon condition.

How do DSEs usually deal with this?

Integral of a total derivative vanishes:

$$\int [D\phi] \frac{\delta}{\delta\phi} e^{-S+J\,\Phi} = \int [D\phi] \left(J - \frac{\delta S}{\delta\phi} \right) e^{-S+J\,\Phi} = 0.$$

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$$\int_{\Omega} [D\phi] \left(J - \frac{\delta S}{\delta\phi} \right) \delta(\partial \cdot A) \boldsymbol{det(M)} e^{-S_{YM}+J\,\Phi} = 0.$$

$$det(M)\Big|_{\Omega} = 0$$

Can this be confirmed by an explicit analysis?

Local renormalizable Gribov-Zwanziger action

Auxiliary fields for localization: V_μ^{ab}, $\bar{\eta}_c^a$, η_c^a

$$\mathcal{L}_{GZ} = \mathcal{L}_{YM} + \mathcal{L}_{gf} + \mathcal{L}_\eta + \mathcal{L}_V,$$
$$\mathcal{L}_\eta = -\bar{\eta}_c^a M^{ab} \eta_c^b,$$
$$\mathcal{L}_V = \frac{1}{2} V_\mu^{ac} M^{ab} V_\mu^{bc} + i\, g\, \gamma^2 \sqrt{2} f^{abc} A_\mu^a V_\mu^{bc},$$

η and $\bar{\eta}$ fields:

Comprise Faddeev-Popov ghosts c and $\bar{c}$ and parts of the (original) auxiliary fields from localization

Properties of the Gribov-Zwanziger action

$$\mathcal{L}_V = \frac{1}{2} V_\mu^{ac} M^{ab} V_\mu^{bc} + i\, g\, \gamma^2 \sqrt{2} f^{abc} A_\mu^a V_\mu^{bc}$$

- Renormalizable

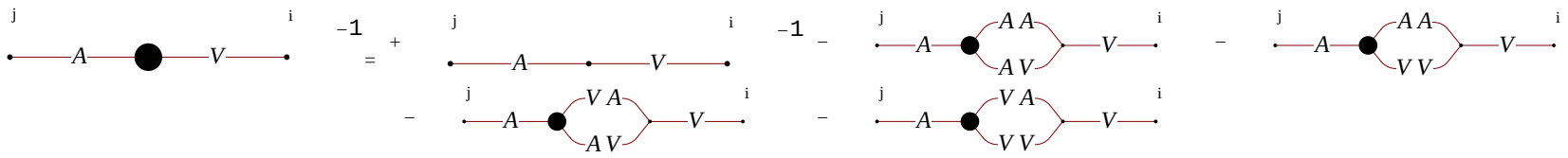
  [Zwanziger; Maggiore, Schaden; Dudal, Sobreiro, Sorella, Vandersickel, Verschelde]
- Not BRST invariant
- Mixing at the level of two-point functions: $\langle A_\mu^a V_\nu^{bc} \rangle$

$$D^{\Phi\Phi} = (\Gamma^{\Phi\Phi})^{-1}, \qquad \phi \in \{A, V\}$$

$\Rightarrow$ non-trivial relation between IR exponents of propagators and two-point functions

DSEs of Gribov-Zwanziger action

Just to give an impression:



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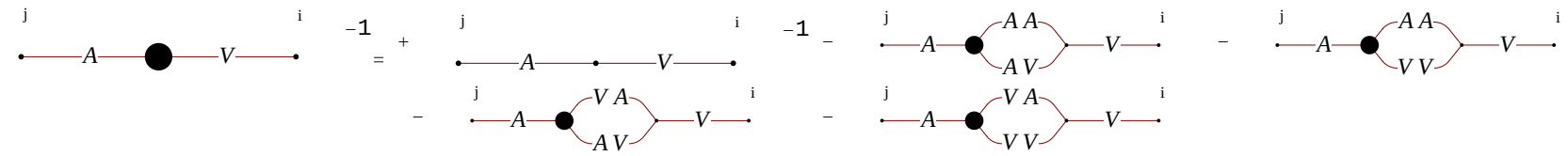

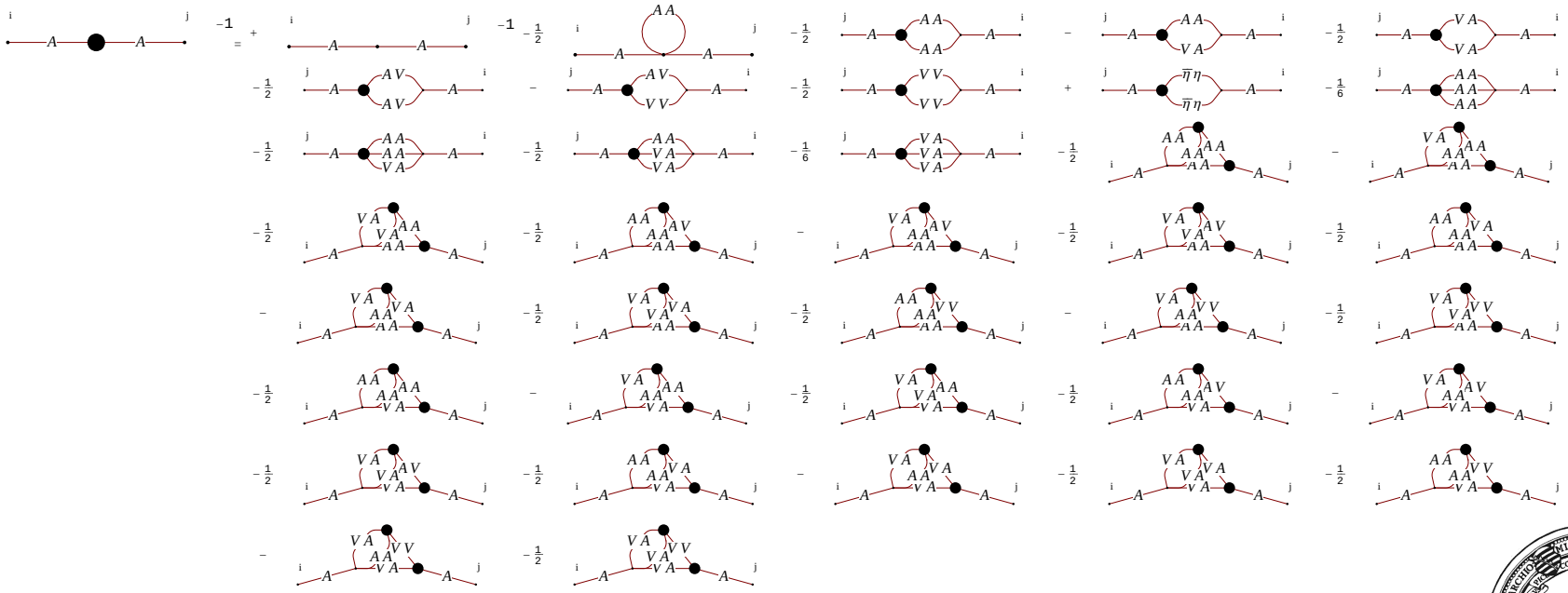

Complete analysis of all diagrams!

Propagators and two-point functions

Example: VV-two-point function

$$\Gamma^{VV,abcd}_{\mu\nu} = \delta^{ac}\delta^{bd} p^2 \boldsymbol{c_V(p^2)} g_{\mu\nu}$$

dressing function $c_V(p^2) \xrightarrow{p^2 \to 0} d_V \cdot (p^2)^{\boldsymbol{\kappa_V}}$

infrared exponent

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infrared exponent

VV-propagator:

$$D^{VV,abcd}_{\mu\nu} = \frac{1}{p^2}\frac{1}{c_V(p^2)}\delta^{ac}\delta^{bd} g_{\mu\nu} - f^{abe} f^{cde} \frac{1}{p^2} P_{\mu\nu} \frac{2c^2_{AV}(p^2)}{c^{\perp}_A(p^2) c^2_V(p^2) + 2N\, c^2_{AV}(p^2) c_V(p^2)}$$

The four possibilities

Which part of the determinant $c_A^{\perp}(p^2)c_V(p^2) + 2N\, c_{AV}^2(p^2)$ dominates in the IR?

$$c_{ij}(p^2) = d_{ij} \cdot (p^2)^{\kappa_{ij}}$$

I: $c_{AV}^2 > c_A c_V \leftrightarrow \kappa_A + \kappa_V > 2\kappa_{AV}$

II: $c_A c_V > c_{AV}^2 \leftrightarrow 2\kappa_{AV} > \kappa_A + \kappa_V$

III: $c_{AV}^2 \sim c_A c_V \leftrightarrow \kappa_A + \kappa_V = 2\kappa_{AV}$, no cancelations in determinant

IV: $c_{AV}^2 \sim c_A c_V \leftrightarrow \kappa_A + \kappa_V = 2\kappa_{AV}$, cancelations in determinant

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Two solutions lead to inconsistencies [M.Q.H., R. Alkofer, S. P. Sorella, PRD 81].

Qualitative IR behavior of the solutions [Huber, Alkofer, Sorella, PRD81]

- Scaling relation between FP ghost and gluon unaltered:

$$\kappa_A + 2\kappa_c = 0.$$

  - Gluon propagator is IR suppressed.
  - Propagators of ghost and auxiliary fields are IR enhanced.
  - Mixed propagators are IR suppressed.

- IR exponents of all vertices are obtained.

- Input for numerical solution of the equations.

- Qualitatively the IR behavior of Faddeev-Popov theory is reproduced (Case II corresponds in the IR *exactly* to the Faddeev-Popov theory.)

$\downarrow$ $\downarrow$ $\downarrow$

in agreement with scenarios of Gribov-Zwanziger and Kugo-Ojima

Summary

- New tool for deriving DSEs: *DoDSE*
- A generic method for the IR analysis of Green functions was developed applicable to many actions.
- Application to the Gribov-Zwanziger action:
  - Same qualitative IR behavior of all Green functions as in FP theory
  - Confirmation of Zwanziger's conjecture on improved gauge fixing
- Application to the maximally Abelian gauge:
  - Diagonal gluon propagator IR enhanced, off-diagonal ones suppressed
  - Confirmed Abelian IR dominance
  - IR Behavior of $SU(2)$ and $SU(N > 2)$ equal
- Asymptotic IR behavior
  - is a reliable starting point for the full solution of two-point DSEs.
  - gives information about confinement:
    Kugo-Ojima, Gribov-Zwanziger scenarios, Abelian IR dominance

The end

Thank you very much for your attention.

IR Scaling solutions for other gauges

The analysis can be used also for other gauges. Beware: This corresponds to a naive application!

Linear covariant gauges	Ghost-antighost symmetric gauges
scaling solution only, if the longitudinal part of the gluon propagator gets dressed, but gauge fixing condition $\Rightarrow$ longitudinal part bare	quartic ghost interaction $\rightarrow \delta_{gh} \geq 0$ $\rightarrow$ with non-negative IREs only the trivial solution can be realized

This is valid for all possible dressings and agrees with the results from [Alkofer, Fischer, Reinhardt, v. Smekal, PRD 68 (2003)], where only certain dressings were considered.

$\Longrightarrow$

- Either the existence of a scaling solution is something special (?) or
- a more refined analysis (symmetries $\leftrightarrow$ cancelations) is needed in these cases.

Propagators of the GZ action

$$D^{\eta\bar{\eta},ab}_{cd} = (\Gamma^{\eta\bar{\eta},ab}_{cd})^{-1} = -\delta^{ab}\delta^{cd}\frac{c_\eta(p^2)}{p^2}$$

D^{VV} has two tensors $\longrightarrow$ non-trivial truncation:

$$D^{AA,ab}_{\mu\nu} = \delta^{ab}\frac{1}{p^2}P_{\mu\nu}\frac{c_V(p^2)}{c_A^\perp(p^2)c_V(p^2)+2N\,c_{AV}^2(p^2)},$$

$$D^{VV,abcd}_{\mu\nu} = \frac{1}{p^2}\frac{1}{c_V(p^2)}\delta^{ac}\delta^{bd}g_{\mu\nu} - f^{abe}f^{cde}\frac{1}{p^2}P_{\mu\nu}\frac{2c_{AV}^2(p^2)}{c_A^\perp(p^2)c_V^2(p^2)+2N\,c_{AV}^2(p^2)c_V(p^2)},$$

$$D^{AV,abc} = -i\,f^{abc}\frac{1}{p^2}P_{\mu\nu}\frac{\sqrt{2}c_{AV}(p^2)}{c_A^\perp(p^2)c_V(p^2)+2N\,c_{AV}^2(p^2)}$$

Appearance of the determinant $c_A^\perp(p^2)c_V(p^2)+2N\,c_{AV}^2(p^2)$