Infrared scaling solutions beyond the Landau gauge: The maximally Abelian gauge and Abelian infrared dominance

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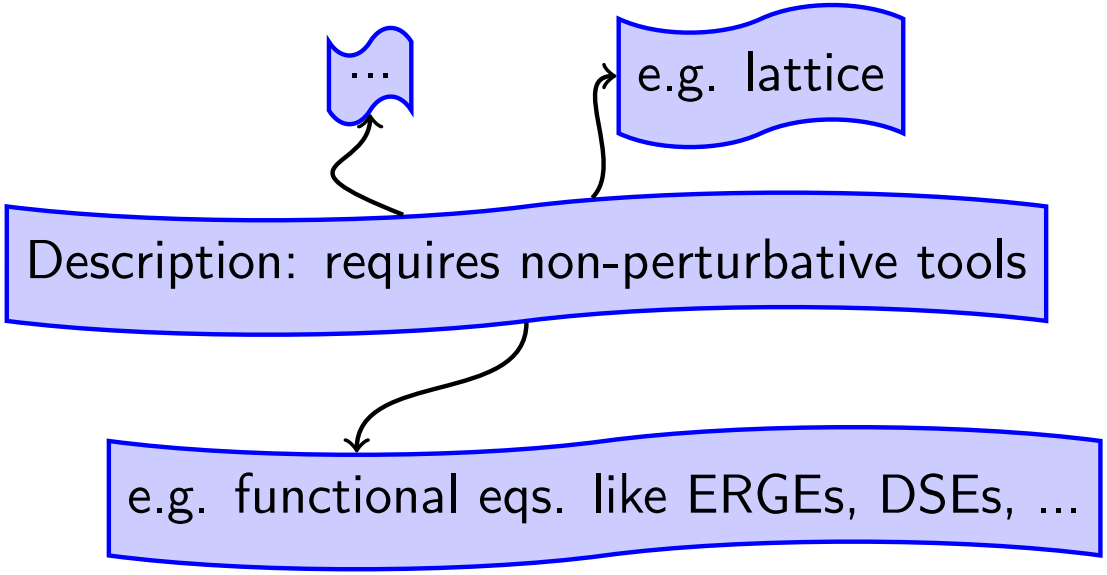
The many faces of QCD, Ghent

Confinement and functional methods



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...

e.g. lattice

Description: requires non-perturbative tools

e.g. functional eqs. like ERGEs, DSEs, ...

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Coulomb gauge

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dual superconductor

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Gribov-Zwanziger

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Confinement and functional methods

Contents of this talk:

Use functional equations to investigate the maximally Abelian gauge.

$\Rightarrow$ hints at dual superconductor/Abelian IR dominance?

dual superconductor

Kugo-Ojima

Gribov-Z

e.g. functional eqs. like ERGEs, DSEs

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Dyson-Schwinger equations (DSEs) for investigating QCD

DSEs describe non-perturbatively how particles propagate and interact.

Equations of motion of Green functions

Infinitely large tower of equations

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- Exact equations
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- Continuum, different scales accessible
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- Exact equations
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- Continuum, different scales accessible
  $\longrightarrow$ complement lattice method

Cons?:

- Truncations (not for all tasks)
- Gauge-dependent
  $\longrightarrow$ Exploit advantages of different gauges

Deriving Dyson-Schwinger equations

Integral of a total derivative vanishes:

$$\int [D\phi] \frac{\delta}{\delta\phi} e^{-S+J\,\Phi} = \int [D\phi] \left(J - \frac{\delta S}{\delta\phi} \right) e^{-S+J\,\Phi} = 0.$$

$\Rightarrow$ DSEs for all Green functions (full, connected, 1PI) by further differentiations.

Doing it by hand?

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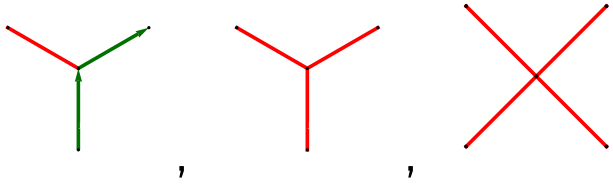
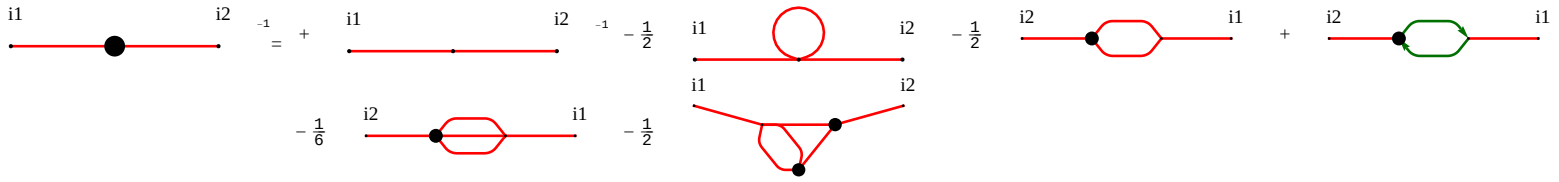
Example: Landau gauge

2 propagators (***AA***, ***cc***):

3 interactions (***Acc***, ***AAA***, ***AAAA***): 

Landau Gauge DSEs: Propagators

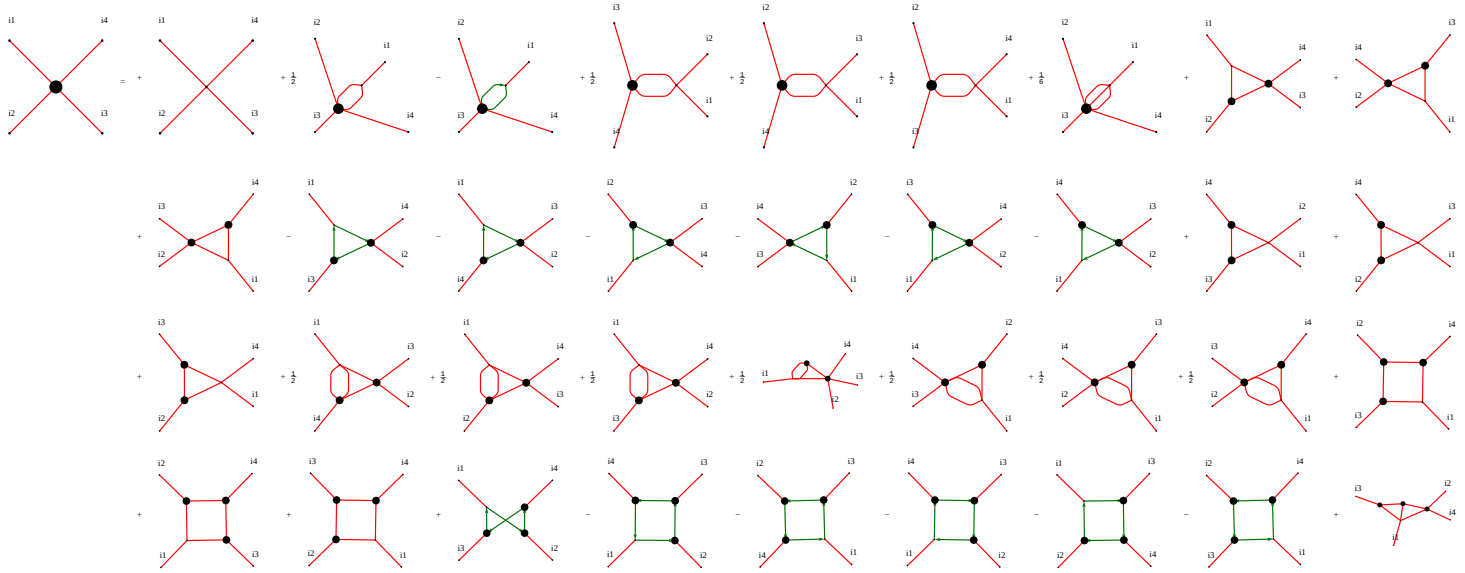
Gluon propagator:

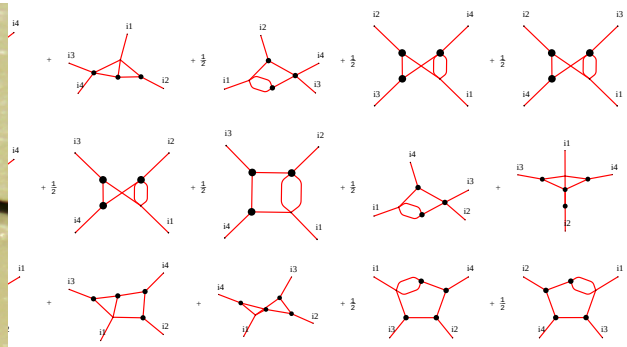


Ghost propagator:

Landau Gauge DSEs: Four-Gluon Vertex

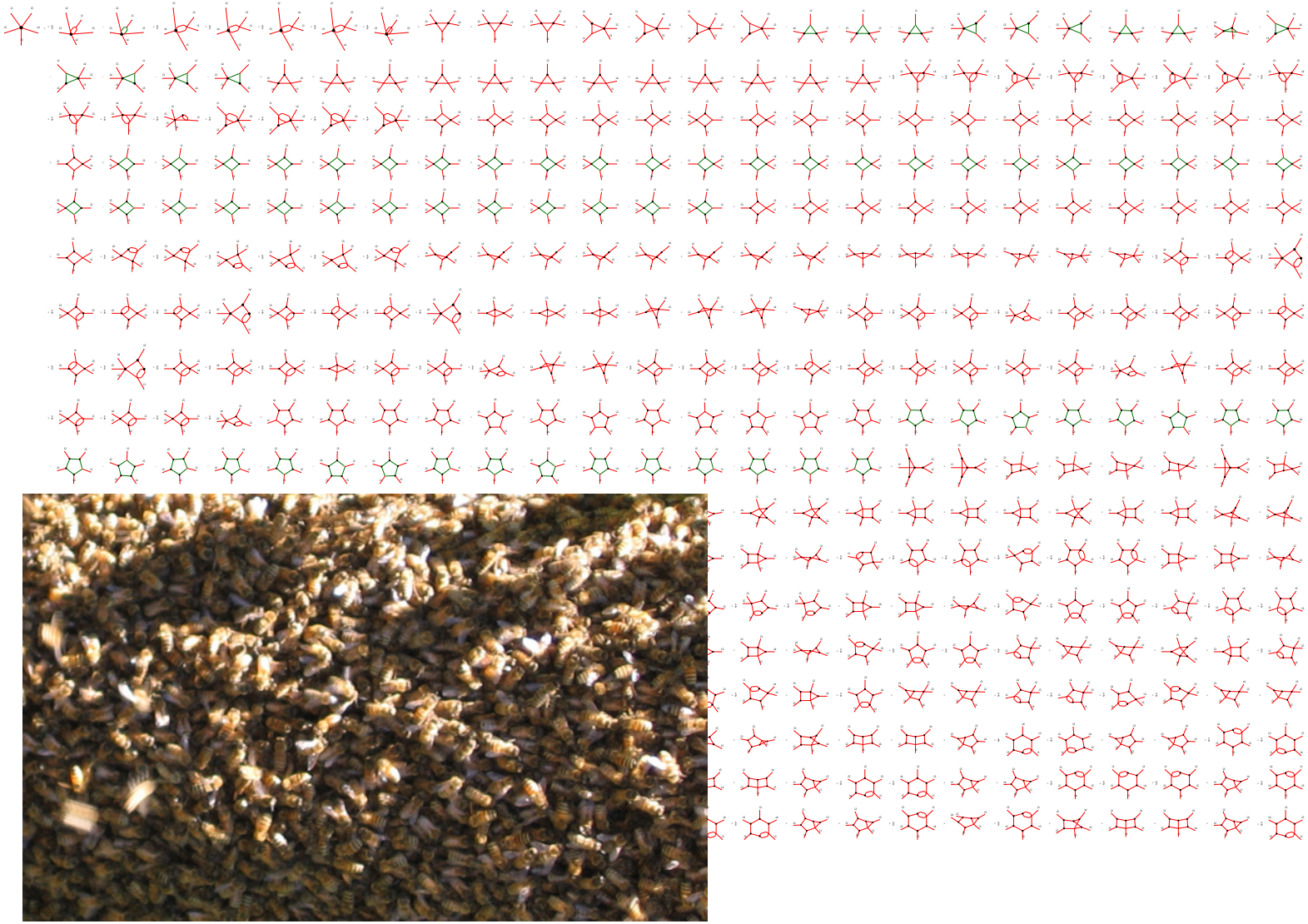
66 terms





Landau Gauge DSEs: Five-Gluon Vertex

434 terms



Derivation of DSEs (*DoDSE*)

⇒ *DoDSE* [Alkofer, M.Q.H., Schwenzer, CPC 180 (2009)]

Given a structure of interactions, the DSEs are derived symbolically using *Mathematica*.

Example (Landau gauge):

- only input: interactions in Lagrangian (AA, AAA, AAAA, cc, Acc)
- Which DSE do I want?

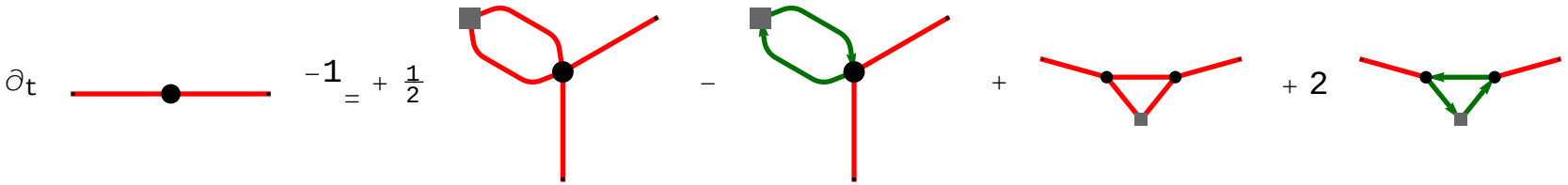
- Can handle mixed propagators (then there are really many diagrams; e. g. in Gribov-Zwanziger action).

Upgrades

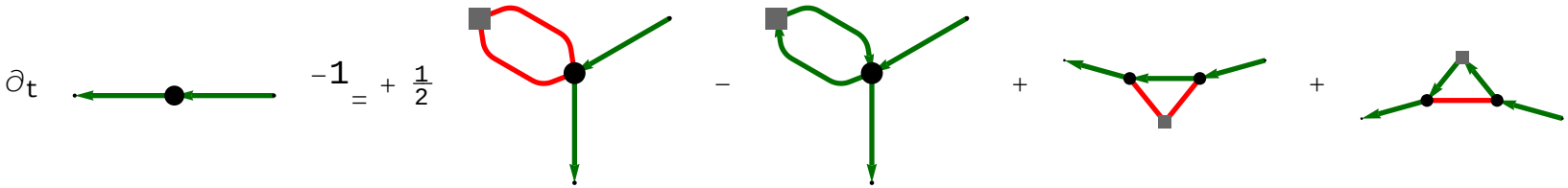
- Provide Feynman rules and get complete algebraic expressions.
  → E. g. calculate color algebra with *FORM* and integrals with *C*
- *DoRGE*: Calculate renormalization group equations.

Landau Gauge ERGEs: Propagators

Gluon propagator:



Ghost propagator:

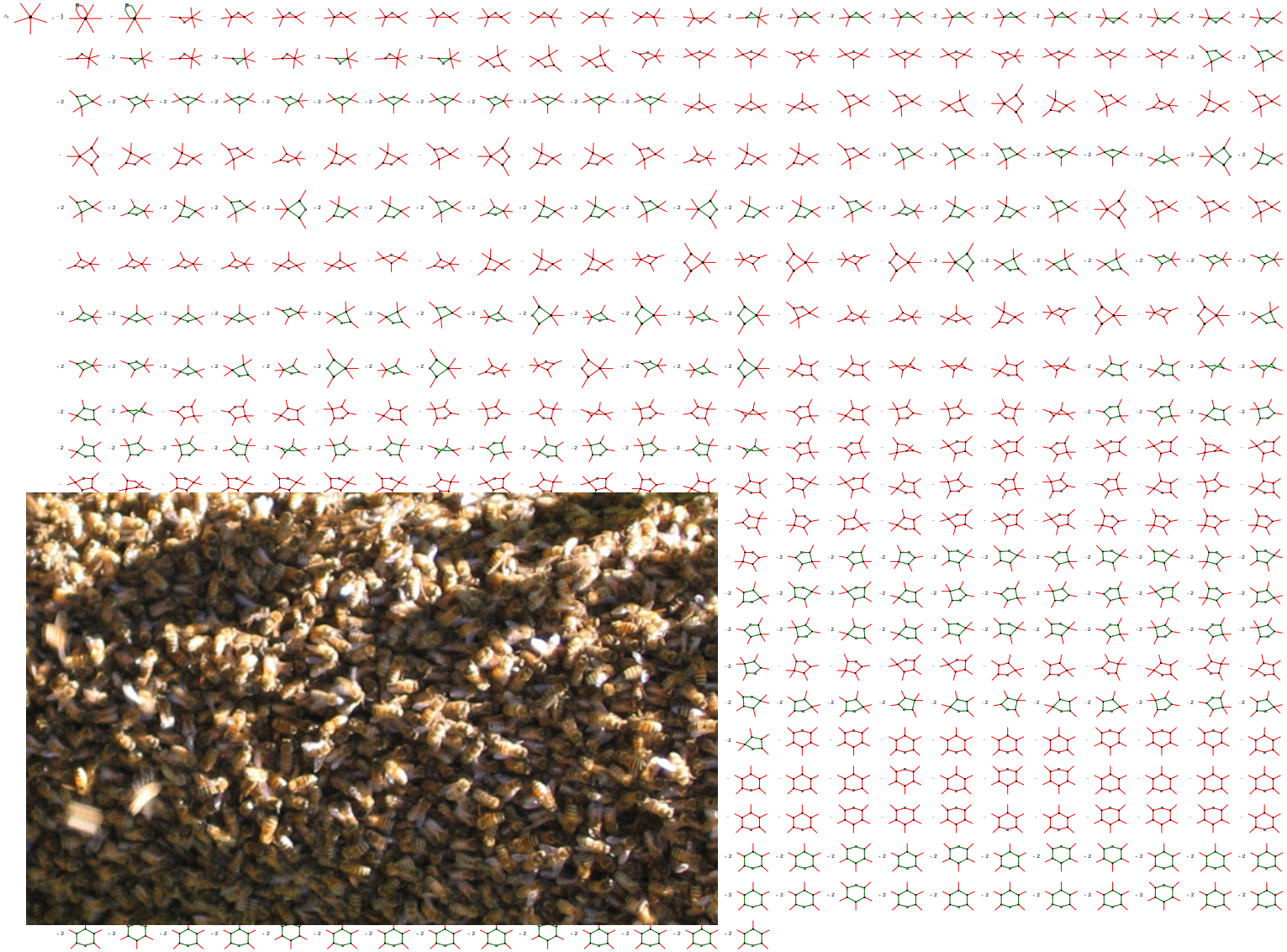


Landau Gauge ERGEs: Four-Gluon Vertex

76 terms

Landau Gauge ERGEs: Five-Gluon Vertex

542 terms



Hypothesis of Abelian IR dominance

Dual superconductor picture of confinement [Mandelstam, 't Hooft]

- Picture a conventional superconductor, where the electric charges condense and force the magnetic flux into vortices.
- Change "electric" and "magnetic" components and you get a dual superconductor, where condensed magnetic monopoles squeeze the electric flux into flux tubes.
- QCD: No free chromoelectric charges.

  Are they confined by condensed magnetic monopoles?

Hypothesis of Abelian IR dominance [Ezawa, Iwazaki, PRD 25 (1981)]:

Magnetic monopoles live in Abelian part of the theory.

$\longrightarrow$ Abelian part dominates in the IR?

Definition of the maximally Abelian gauge

Gauge field components:

$$A_\mu = \boldsymbol{A}^i_\mu \boldsymbol{T}^i + \boldsymbol{B}^a_\mu \boldsymbol{T}^a, \quad i = 1, \ldots, N-1, \quad a = N, \ldots, N^2 - 1$$

Abelian subalgebra: $[T^i, T^j] = 0$, can be written as diagonal matrices

Abelian $\leftrightarrow$ diagonal fields $\boldsymbol{A}$,
non-Abelian $\leftrightarrow$ off-diagonal fields $\boldsymbol{B}$.

E.g. $T^1 = \frac{1}{2}\lambda^3$, $T^2 = \frac{1}{2}\lambda^8$ for $SU(3)$.

Which interactions are possible ($[T^r, T^s] = i\, f^{rst}\, T^t$)?

	$SU(2)$	$SU(N>2)$
f^{ijk}	0	0
f^{ija}	0	0
f^{iab}	✓	✓
f^{abc}	0	✓

$\rightarrow$ $SU(2)$ and $SU(3)$ different?

Gauge fixing condition

Stress role of diagonal fields $\Rightarrow$ minimize norm of off-diagonal field $\boldsymbol{B}$:

$$\|B_U\| = \int dx\, \boldsymbol{B}^a_U \boldsymbol{B}^a_U \rightarrow \text{minimize wrt. gauge transformations } U$$

$$D^{ab}_\mu \boldsymbol{B}^b_\mu = (\delta_{ab}\partial_\mu - g\, f^{abi} \boldsymbol{A}^i_\mu)\boldsymbol{B}^b_\mu = 0$$

non-linear gauge fixing condition!

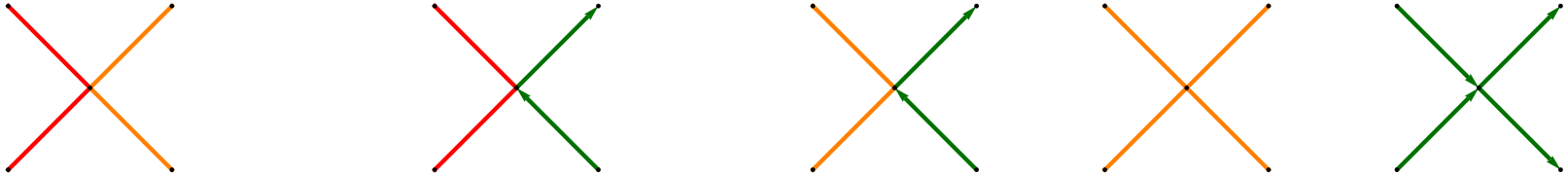
Remaining symmetry of diagonal part: $U(1)^{N-1}$

Fix gauge of diag. gluon field $\boldsymbol{A}$ by Landau gauge condition: $\partial_\mu \boldsymbol{A}_\mu = 0$
$\Rightarrow$ diagonal ghosts decouple (like in QED).

Lagrangian for the MAG

diagonal gluon | off-diagonal gluon | ghost

ABB | ***Acc***

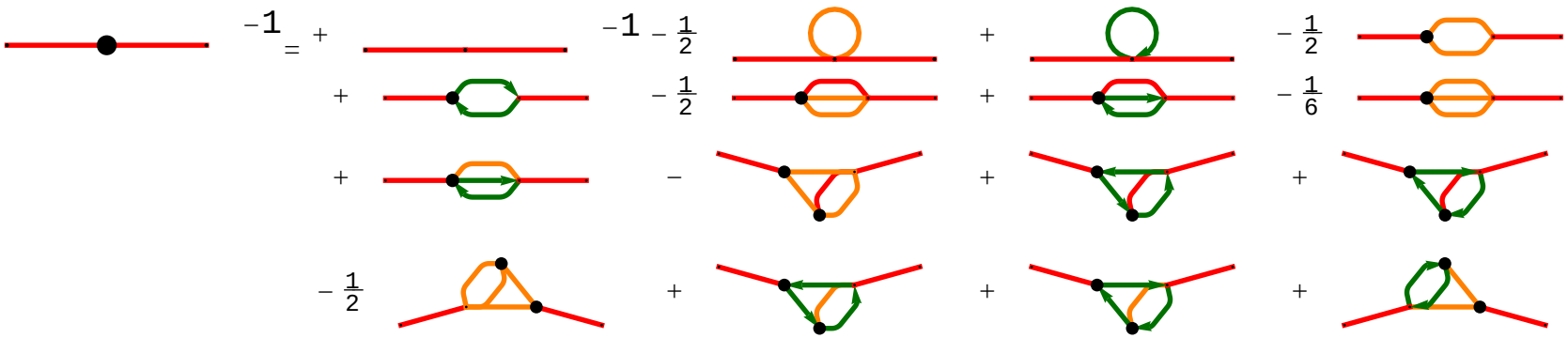


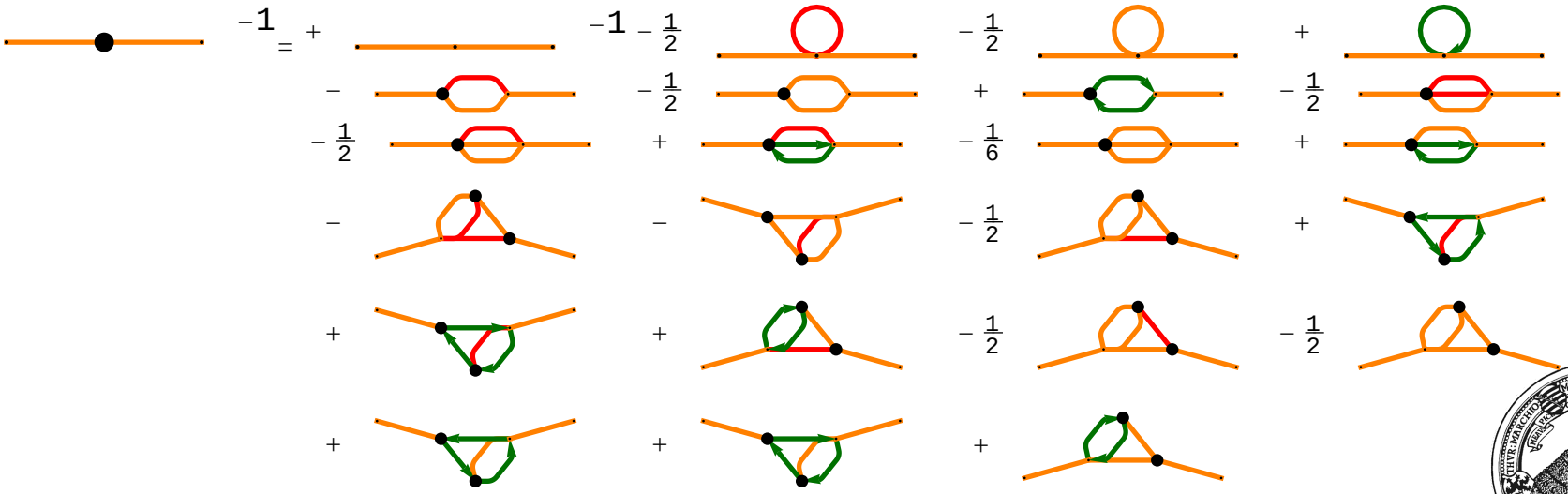
AABB | ***AAcc*** | ***BBcc*** | ***BBBB*** | ***cccc***

$SU(N > 2)$

BBB ***Bcc*** ***ABBB*** ***ABcc***

DSEs of the MAG





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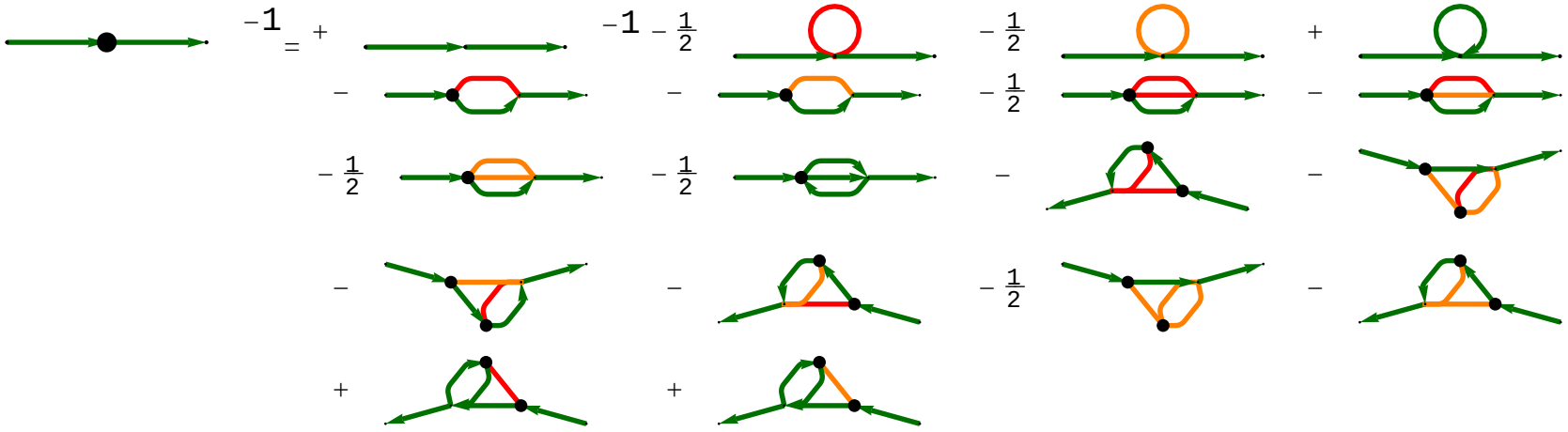

Complete analysis of all diagrams!

Available results in the MAG

- Available lattice results of MAG [Cucchieri, Mendes, Mihara, 2008]:
  All propagators massive, diagonal field has lowest mass.
  $\Rightarrow$ Other fields decouple. Realization of Abelian IR dominance.
- Refined Gribov-Zwanziger framework [Capri et al., PRD77, JPA43]:
  All propagators massive.

- $\rightarrow$ decoupling solution

$\rightarrow$ Existence of scaling solution?

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Reminder: Decoupling and scaling solutions in Landau gauge

In the deep IR two types of solutions emerge:

- massive gluon propagator
- finite ghost dressing function

- vanishing gluon propagator
- enhanced ghost propagator

Decided by boundary condition, e.g., value of the ghost dressing function at zero momentum [Fischer, Maas, Pawlowski, AP324; Maas, PLB689].

Infrared power counting

Generic propagator

$$T_{(ij)} \cdot \frac{D(p^2)}{p^2},$$

assume power law behavior at low p^2

IR exponent

$$D^{IR}(p^2) = A \cdot (p^2)^{\delta}$$

- Vertices also assume power law behavior
  [e.g., Alkofer, Fischer, Llanes-Estrada, PLB 611 (2005) (skeleton expansion)].
- Limit of all momenta approaching zero simultaneously.
- Upon integration all momenta converted into powers of external momenta.

$\Rightarrow$ counting of IR exponents

System of inequalities

- lhs is dominated by at least one diagram on rhs and rhs cannot be more divergent than lhs:

$$\rightarrow \delta_{lhs} \leq \delta_{rhs,any\ diagram}$$

- Not knowing which diagram is leading on the rhs, we can write inequalities from all diagrams.

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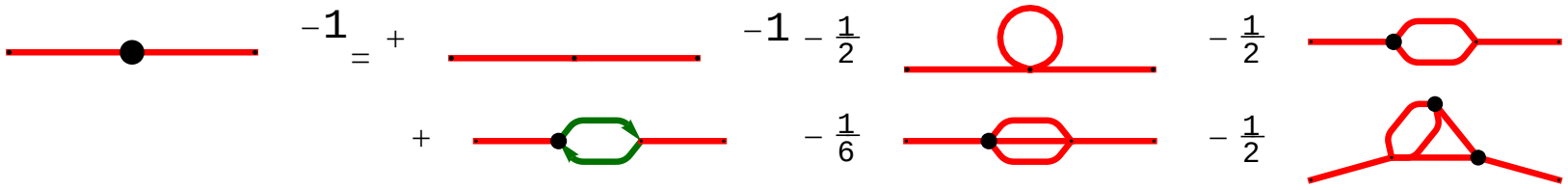

$$-\delta_{gl} \leq 2\delta_{gl} + \delta_{3g}, \qquad -\delta_{gl} \leq 2\delta_{gh} + \delta_{gg}, \qquad \ldots$$

That's the basic idea. Still, for a large system a lot of work.

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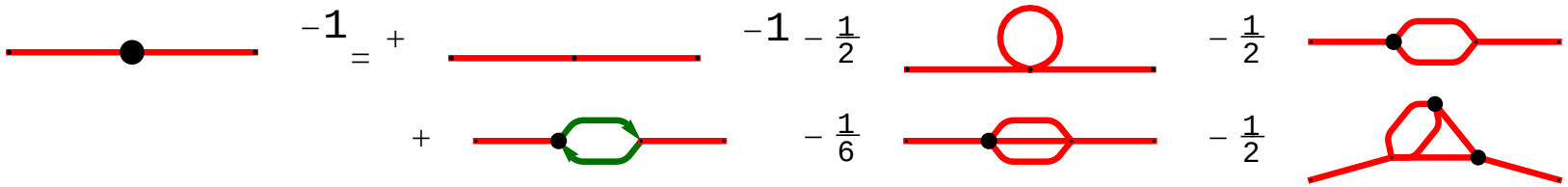

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All inequalities relevant?

Relevant inequalities

Closed form for all relevant inequalities
from 2 independent sets of funct. equations.

type		derived from	#
dressed vertices	$C_1 := \delta_{vertex} + \frac{1}{2} \sum_{\substack{\text{legs } j \text{ of} \\ \text{vertex}}} \delta_j \geq 0$	ERGEs	∞
prim. div. vertices	$\boldsymbol{C_2 := \frac{1}{2} \sum_{\substack{\text{legs } j \text{ of} \\ \text{prim. div.} \\ \text{vertex}}} \delta_j \geq 0}$	DSEs+ERGEs	a few

Relevant inequalities

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validity of skeleton expansion basic property of scaling solutions

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Shifting analysis to IR exponents $\rightarrow$ exact from this point on.

Analysis of propagator DSEs

at least for one i: $C_2^i := \frac{1}{2}\sum_j \delta_j = 0$

Example: $\delta_{gluon} + 2\delta_{ghost} = 0$ in Landau gauge [von Smekal, Alkofer, Hauck, PRL79]

- scaling relation
- one vertex always IR constant (no Taylor argument req.)
- related to bare vertices in DSEs, cf. [Fischer, Pawlowski PRD77]
- necessary condition for scaling solutions
- qualitative behavior of higher Green functions
- valid for a large number of systems

Very useful for complicated actions like the Gribov-Zwanziger action (results agree with standard DSE/ERGE calculations [M.Q.H., Alkofer, Sorella, PRD81]).

IR scaling solution of the MAG

Propagators:

$$D_A^{ij}(p^2) = \delta^{ij}\frac{c_A(p^2)}{p^2}\left(g_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}\right) + \xi\,\delta^{ij}\frac{p_\mu p_\nu}{p^2},$$

$$D_B^{ab}(p^2) = \delta^{ab}\frac{c_B(p^2)}{p^2}\left(g_{\mu\nu} - (1-\alpha)\frac{p_\mu p_\nu}{p^2}\right),$$

$$D_c^{ab}(p^2) = -\delta^{ab}\frac{c_c(p^2)}{p^2}$$

Power laws:

$$c_A(p^2) \overset{p^2\to 0}{=} d_A \cdot (p^2)^{\delta_A},$$

$$c_B(p^2) \overset{p^2\to 0}{=} d_B \cdot (p^2)^{\delta_B},$$

$$c_c(p^2) \overset{p^2\to 0}{=} d_c \cdot (p^2)^{\delta_c}$$

IR scaling solution of the MAG

$$\boxed{\delta_B = \delta_c = -\delta_A = \kappa_{MAG} \geq 0}$$

[M.Q.H., Schwenzer, Alkofer, EPJC 68]

- The diagonal field is IR enhanced.
  $\longrightarrow$ realization of Abelian IR dominance
- Off-diagonal fields are IR suppressed.
- $SU(2)$ and $SU(N > 2)$ have the same solution.
- Qualitative solutions for tower of Green functions.

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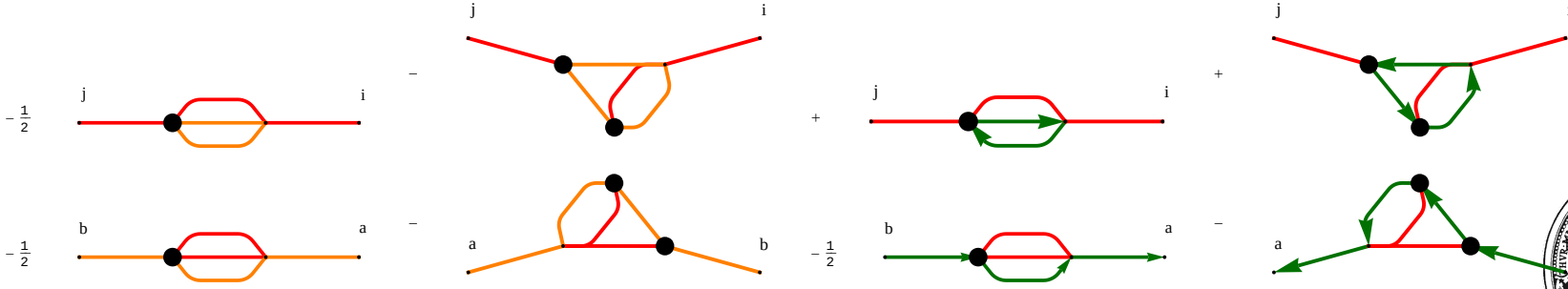
IR leading diagrams:



$\longrightarrow$ UV/IR preserving truncation?

Value of the IRE κ_{MAG}

Solution for κ_{MAG} is necessary but not sufficient.

Truncation: sunsets only

Dressing functions of gluons and ghosts:

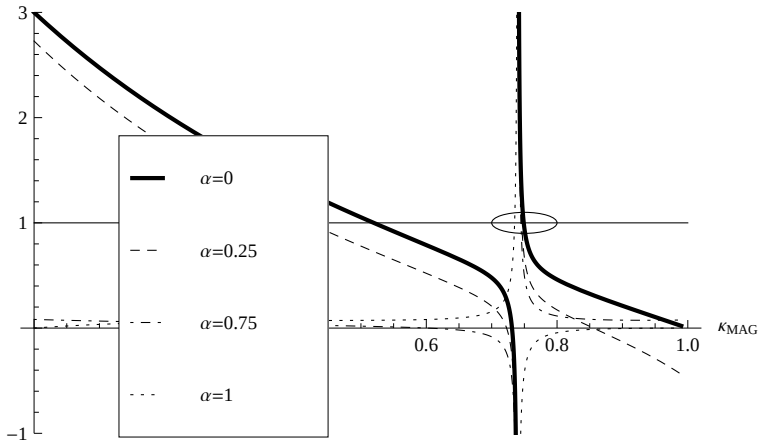
$$c_A(p^2) \overset{p^2 \to 0}{=} d_A \cdot (p^2)^{-\kappa_{MAG}}$$

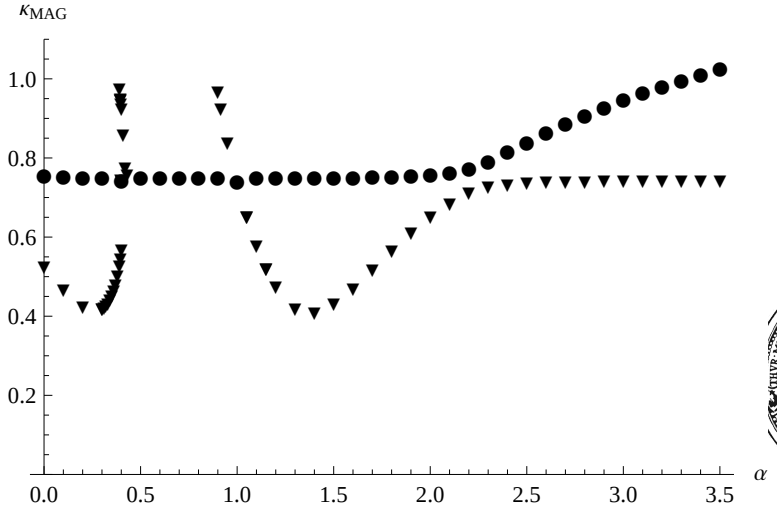
$$c_B(p^2) \overset{p^2 \to 0}{=} d_B \cdot (p^2)^{\kappa_{MAG}}$$

$$c_c(p^2) \overset{p^2 \to 0}{=} d_c \cdot (p^2)^{\kappa_{MAG}}$$

$$0 \leq \kappa_{MAG} \leq 1$$

Solution branch independent of gauge fixing parameter α.





Decoupling solutions in the MAG

Possible scenario for the connection between the decoupling and scaling solutions in the MAG

- Similar to ghost propagator in Landau gauge:
  via renormalization of diagonal gluon propagator
- IR divergent dressing: scaling solution
- IR massive dressing: decoupling

Note: If any of the three dressings is massive, the other two have to be massive too (tadpole diagrams).

Abelian configurations $\xrightarrow{\text{Landau gauge}}$ on Gribov horizon (ghost dominance)

[Greensite, Olejnik, Zwanziger, PRD78]

Summary

$\rightarrow$ Derivation of DSEs with *DoDSE*: useful for complicated systems.

$\rightarrow$ Possible scaling relations directly from Lagrangian.

Albeit additional interactions in $SU(3)$
$\rightarrow$ same IR behavior as in $SU(2)$.

Variant of Abelian IR dominance found:

- diagonal gluon propagator IR enhanced,
- off-diagonal degrees of freedom IR suppressed.

The end

Thank you very much for your attention.