Non-perturbative results for two- and three-point functions of Landau gauge Yang-Mills theory

Markus Q. Huber, Axel Maas, Lorenz von Smekal

Institute of Nuclear Physics, Technical University Darmstadt

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Gent University

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Our view of the world in terms of particles

The standard model:

Quarks	d	s	b	g	Bosons
	u	c	t	γ	
Leptons	e	μ	τ	Z	
	ν_e	ν_μ	ν_τ	$W^\pm$	

Higgs?

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~~Higgs?~~

Which?

Our view of the world in terms of particles

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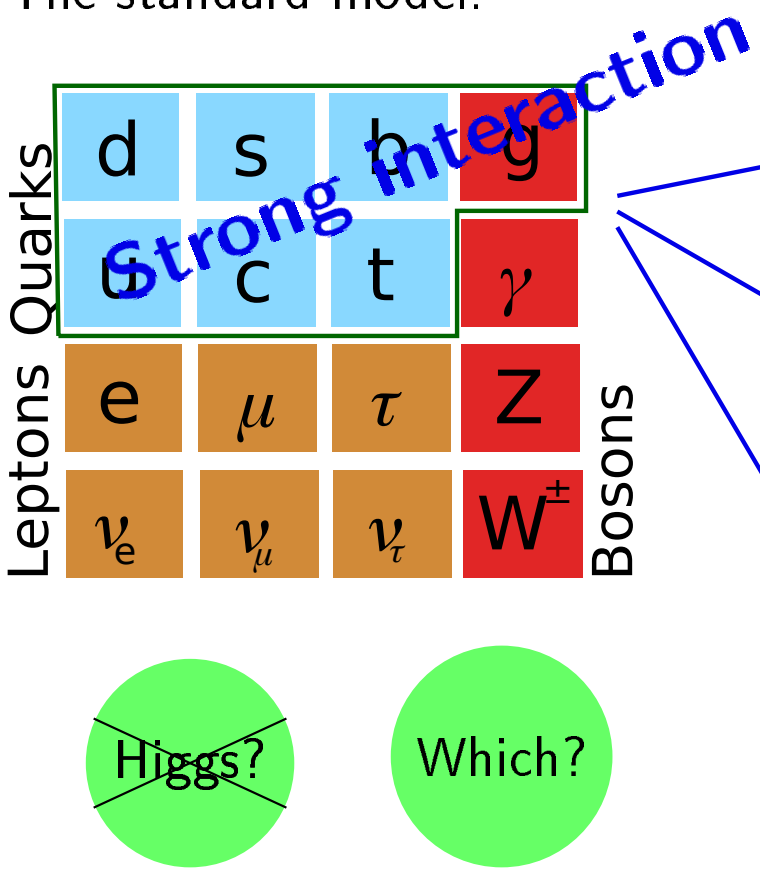
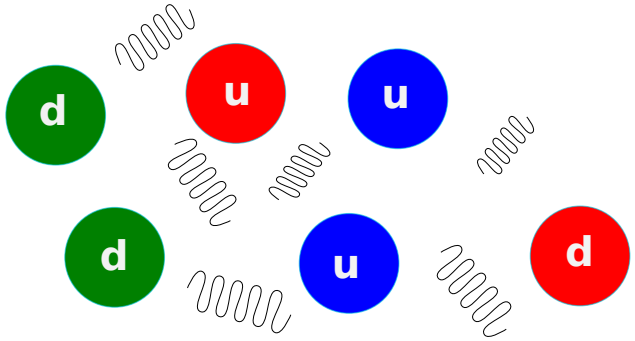
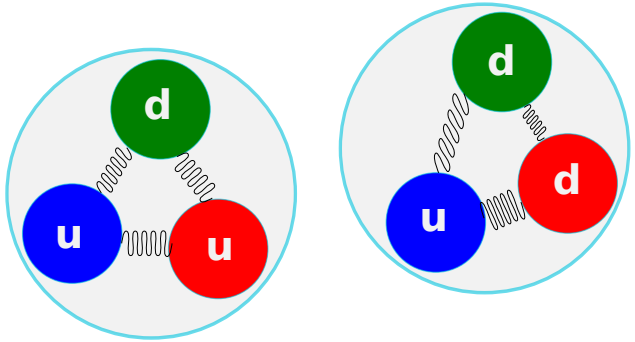


fundamental fields: quarks and gluons

u, d quarks are light $\sim$ MeV



physical degrees of freedom: hadrons



Hadrons are bound states of quarks,
but they cannot be split into their building blocks.

Strong interaction

Asymptotic freedom at high energies

$\longrightarrow$ perturbation theory applicable.

Many interesting phenomena at
low energies/non-zero temperature/non-zero density:

- Confinement
- Chiral symmetry breaking $\longrightarrow$ mass generation
- Phase transitions

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Non-perturbative methods required!

effective theories, lattice, functional methods, duality, ...

$\downarrow$

Close relationship to other areas, e.g. condensed matter, gravity, beyond the standard model, ...

Applications of Landau gauge Green functions

Landau gauge Green functions:

- Information about confinement
- Input for phenomenological calculations, e.g., bound states, QCD phase diagram

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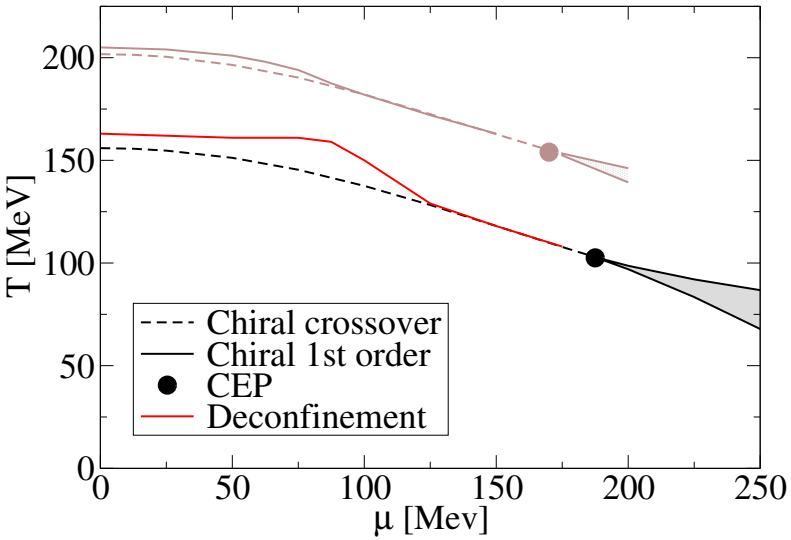
Example:

QCD phase diagram, $N_f = 2, 2+1$:



[Fischer, Lücker, 1206.5191]

- (De)confinement and chiral transitions can be calculated.
- No sign problem!
  $\longrightarrow$ Complement Monte-Carlo simulations
- Challenge: Infinite system of eqs.
  $\longrightarrow$ combine lattice and functional methods

Landau Gauge Yang-Mills theory

Gluonic sector of quantum chromodynamics: Yang-Mills theory

$$\mathcal{L} = \frac{1}{2} F^2 + \mathcal{L}_{gf} + \mathcal{L}_{gh}$$
$$F_{\mu\nu} = \partial_\mu \boldsymbol{A}_\nu - \partial_\nu \boldsymbol{A}_\mu + i\, g\, [\boldsymbol{A}_\mu, \boldsymbol{A}_\nu]$$

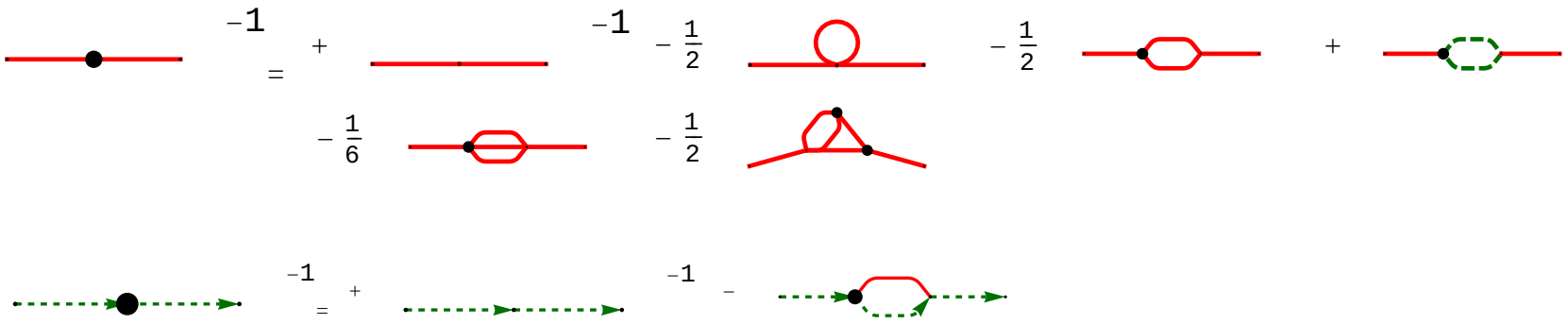
Propagators and vertices are gauge dependent
$\rightarrow$ choose any gauge, ideally one that is convenient.

Landau gauge

- simplest one for functional equations
- $\partial_\mu \boldsymbol{A}_\mu = 0$: $\quad \mathcal{L}_{gf} = \frac{1}{2\xi} (\partial_\mu \boldsymbol{A}_\mu)^2, \quad \xi \rightarrow 0$
- requires ghost fields: $\quad \mathcal{L}_{gh} = \bar{\boldsymbol{c}} \left(-\Box + g\, \boldsymbol{A} \times \right) \boldsymbol{c}$
- 2 fields, 3 vertices

Dyson-Schwinger equations: Propagators

Dyson-Schwinger equations (DSEs) of gluon and ghost propagators:



- Equations of motion of correlation functions:
  Describe how fields propagate and interact non-perturbatively!
- Infinite tower of coupled integral equations.
- Derivation straightforward, but tedious
  $\longrightarrow$ automated derivation with *DoFun* [MQH, Braun, CPC183]
- Contain three-point and four-point functions:
  ghost-gluon vertex , three-gluon vertex , four-gluon vertex

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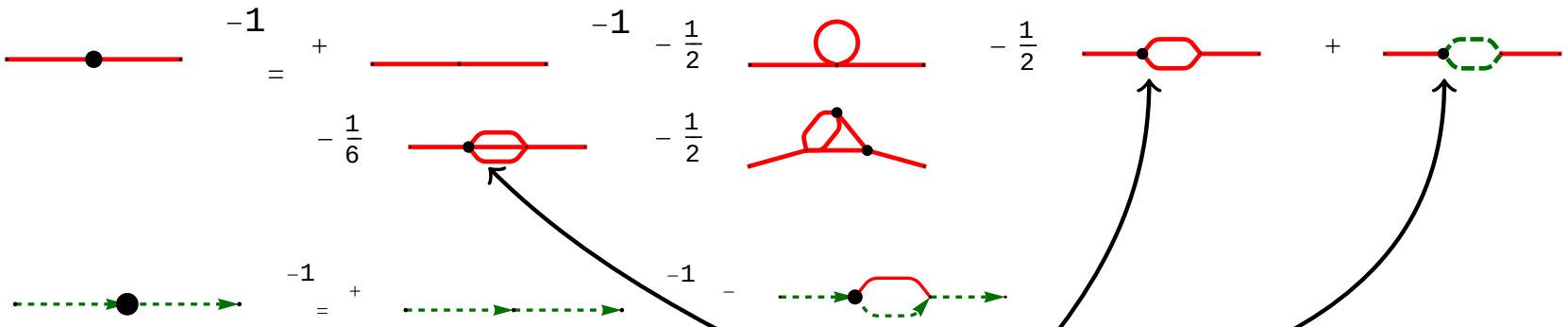

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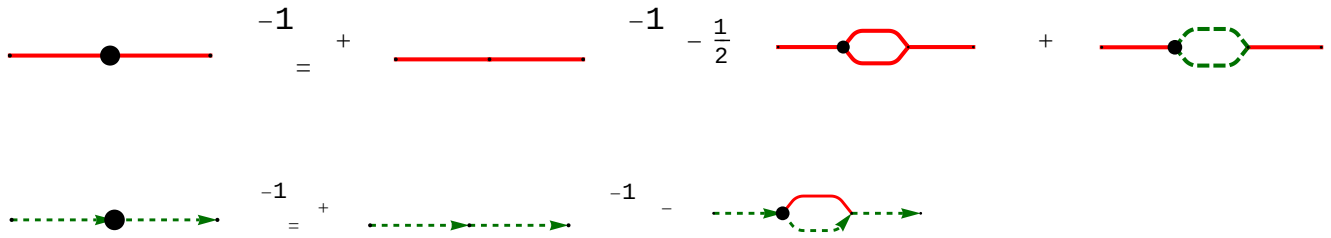
Truncated propagator Dyson-Schwinger equations

Standard truncation:



using bare ghost-gluon vertex and three-gluon vertex model

Influence of dynamic ghost-gluon vertex?

Truncated propagator Dyson-Schwinger equations

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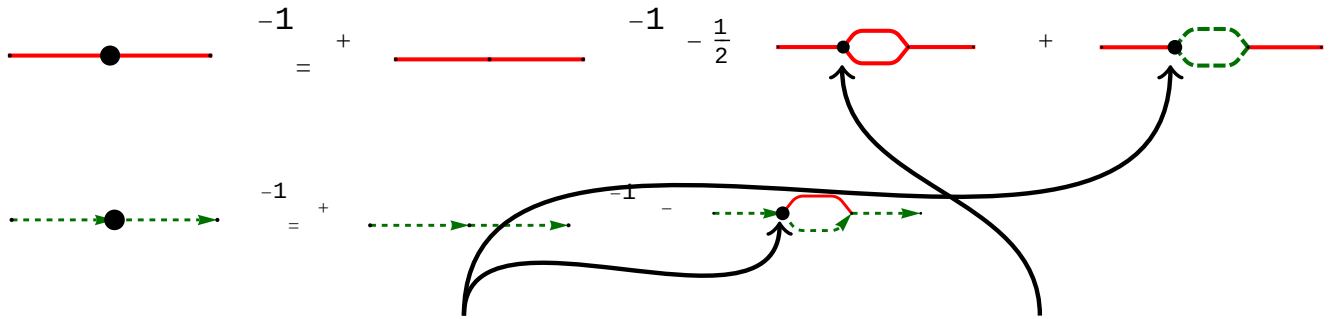

using bare ghost-gluon vertex and three-gluon vertex model

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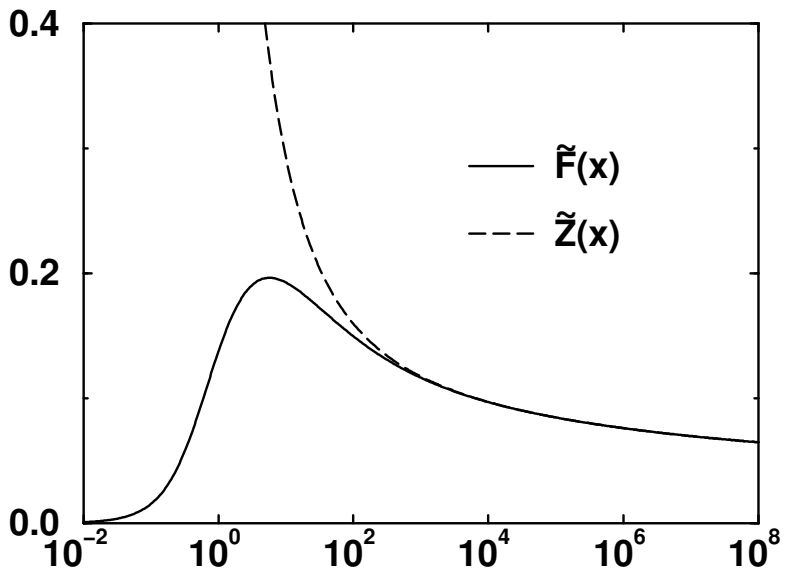
Truncating Dyson-Schwinger equations

gluon	ghost	gh-gl	3-gl	4-pt.	ref.
✓	0	0	model	0	[Mandelstam, PRD20, 1979]



[Hauck, von Smekal, Alkofer, CPC 112]

$$D_{gl,\mu\nu}(p) = \left(g_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}\right) \frac{\tilde{Z}(p^2)}{p^2}$$

- gluon dressing $\tilde{Z}(p^2)$ IR divergent $\rightarrow$ IR slavery

Truncating Dyson-Schwinger equations

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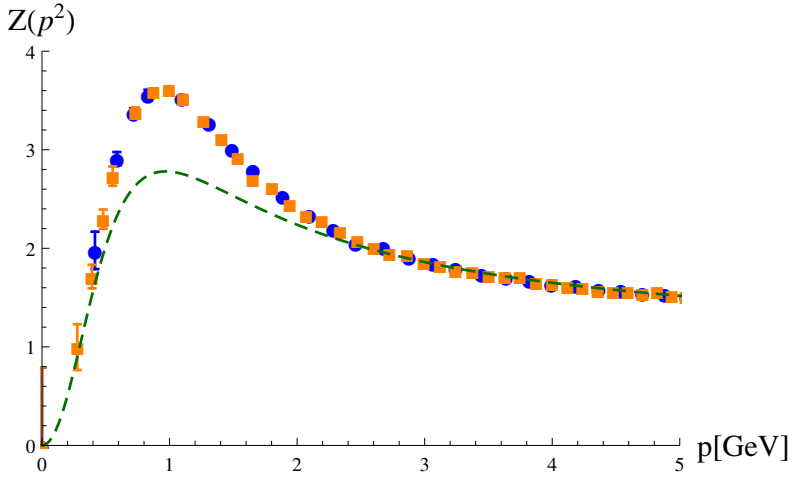

[MQH, von Smekal, 1211.6092; Sternbeck, hep-lat/0609016]

$$D_{gl,\mu\nu}(p) = \left(g_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}\right) \frac{Z(p^2)}{p^2}$$

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- gluon dressing $Z(p^2)$ IR vanishing
- deviations from lattice results in mid-momentum regime

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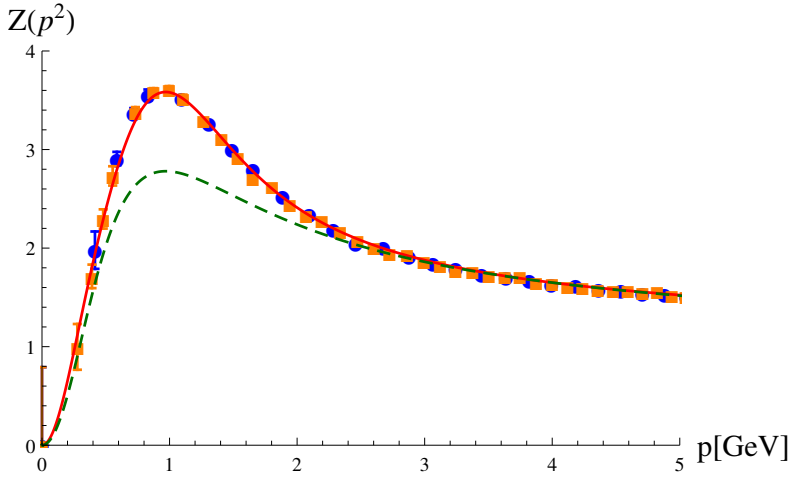

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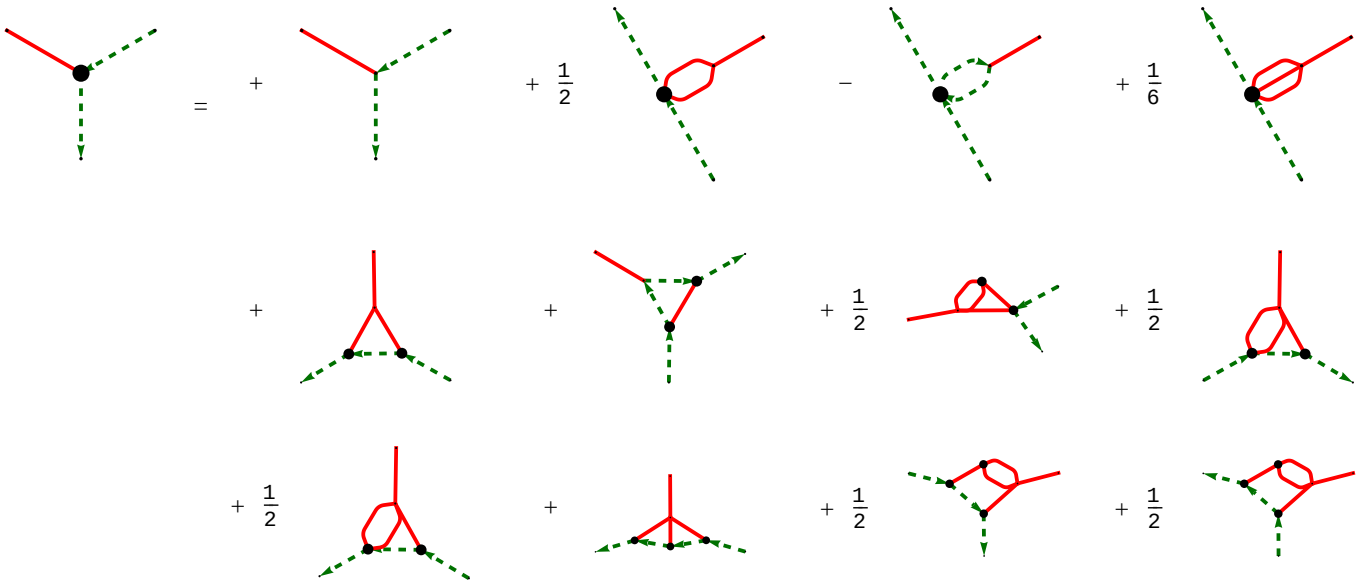
- gluon dressing $Z(p^2)$ IR vanishing
- improved mid-momentum behavior

Improved truncations necessary for quantitative results and also extensions, e.g., non-zero temperature [Fister, Pawlowski, 1112.5440].

Ghost-gluon vertex DSE

Full DSE:



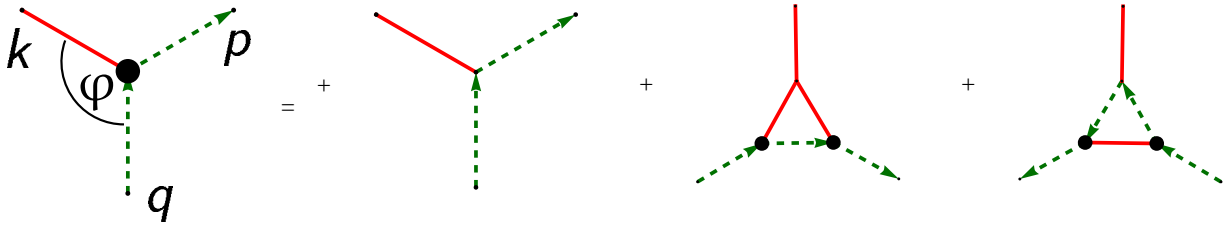
- Lattice results [Cucchieri, Maas, Mendes, PRD77; Ilgenfritz et al., BJP37]
- OPE analysis [Boucaud et al., JHEP 1112]
- Modeling via ghost DSE [Dudal, Oliveira, Rodriguez-Quintero, PRD86]
- Semi-perturbative DSE analysis [Schleifenbaum et al., PRD72]

Ghost-gluon vertex

$$\Gamma_\mu^{A\bar{c}c,abc}(k;p,q) := i\,g\,f^{abc}\,(p_\mu A(k;p,q) + k_\mu B(k;p,q))$$

Note:
$B(k;p,q)$ is irrelevant in Landau gauge, but it is not the pure longitudinal part. Taylor argument applies only to longitudinal part (it's an STI).

<u>IR and UV consistent truncation:</u>



Only unfixed quantity in complete system of eqs.: three-gluon vertex.

Three-gluon vertex: Ultraviolet

Ansatz that reproduces the correct UV behavior of the gluon propagator [Fischer, Alkofer, Reinhardt, PRD65]:

$$D^{A^3}(x,y,z) = \frac{1}{Z_1} \frac{[G(y)G(z)]^{1-a/\delta-2a}}{[Z(y)Z(z)]^{1+a}}$$

y and z are the momenta in the gluon loop, i.e., not Bose symmetric in y, z *and* x. Also wrong anomalous dimension.

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Bose symmetrized version:

$$D^{A^3,UV}(x,y,z) = G\left(\frac{x+y+z}{2}\right)^{\alpha} Z\left(\frac{x+y+z}{2}\right)^{\beta}$$

Fix α and β:

1. UV behavior of three-gluon vertex
2. IR behavior of three-gluon vertex?

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Fix α and β:

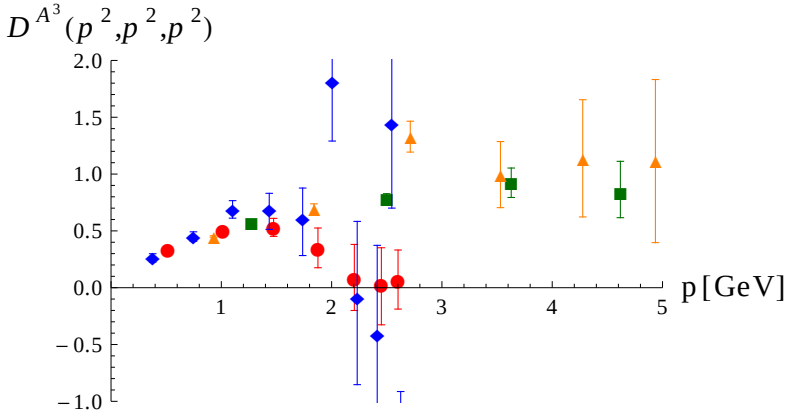
1. UV behavior of three-gluon vertex
2. IR behavior of three-gluon vertex $\rightarrow$ yes, but ...

Three-gluon vertex: Infrared

Hints from lattice data [Cucchieri, Maas, Mendes, PRD77]:

Three-gluon vertex might have a zero crossing.

($d = 2, 3$: zero crossing seen [Cucchieri, Maas, Mendes, PRD77; Maas, PRD75])



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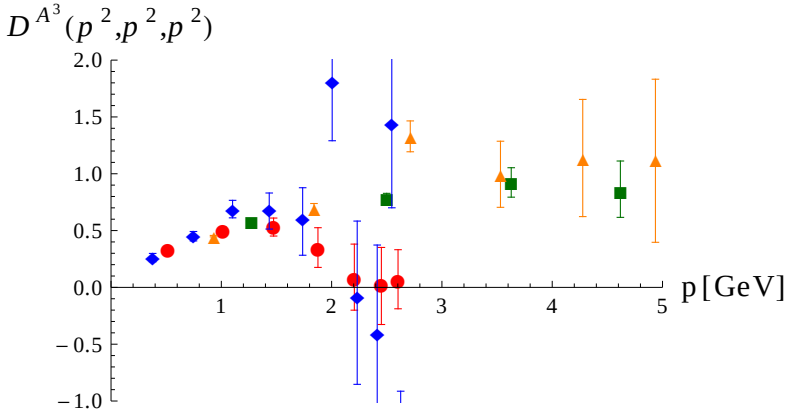
$$D^{A^3,IR}(x,y,z) = h_{IR}\, G(x+y+z)^3 (f^{3g}(x) f^{3g}(y) f^{3g}(z))^4$$

IR damping function $f^{3g}(x) := \dfrac{\Lambda_{3g}^2}{\Lambda_{3g}^2 + x}$

Parameter for zero crossing: $h_{IR} < 0$



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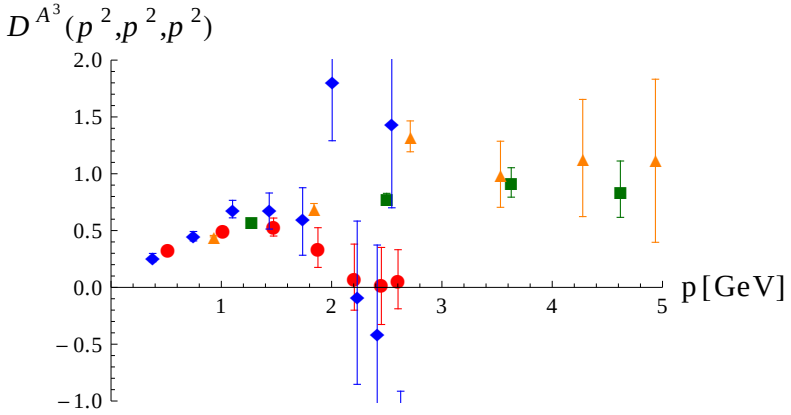
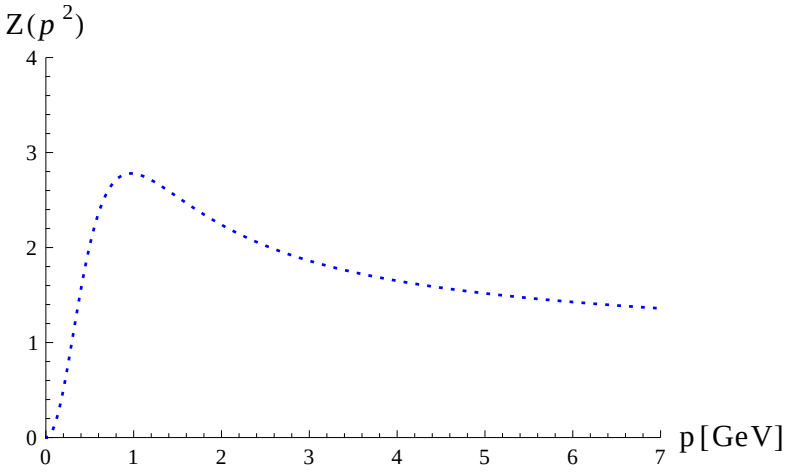


New three-gluon vertex:

$$D^{A^3}(x,y,z) = D^{A^3,IR}(x,y,z) + D^{A^3,UV}(x,y,z)$$

Influence of the three-gluon vertex

ghost-gluon vertex: bare



original three-gluon vertex

Influence of the three-gluon vertex

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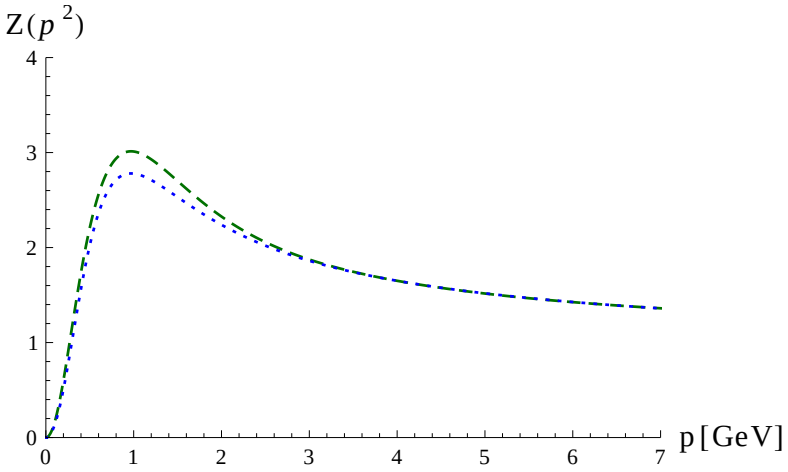

original three-gluon vertex
Bose symmetric three-gluon vertex

Influence of the three-gluon vertex

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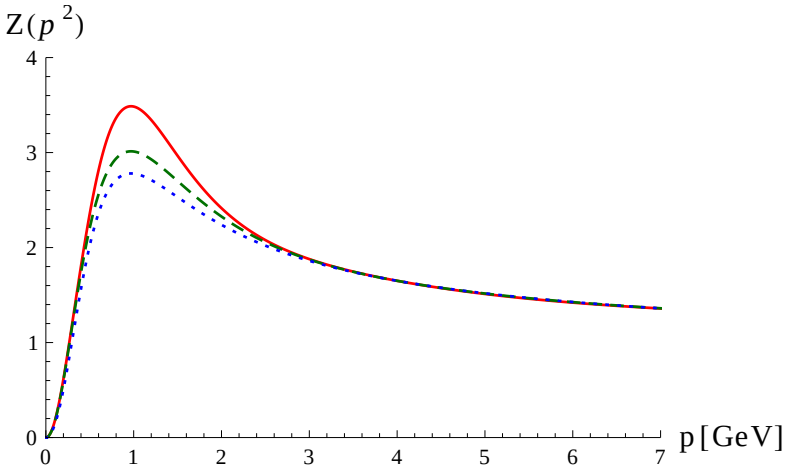
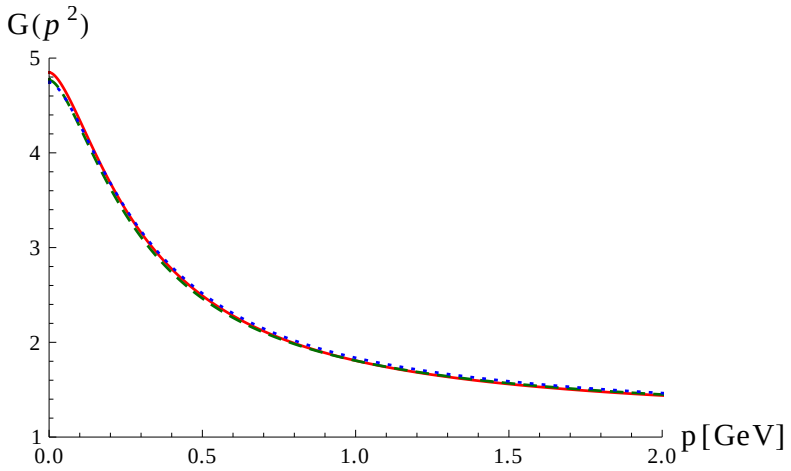
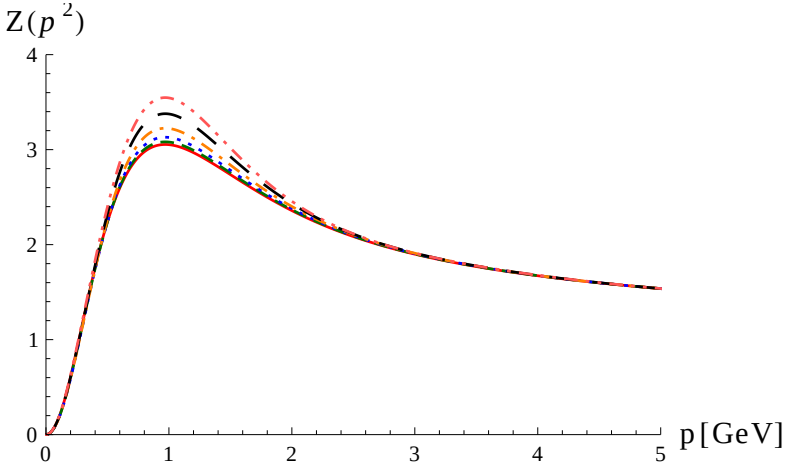


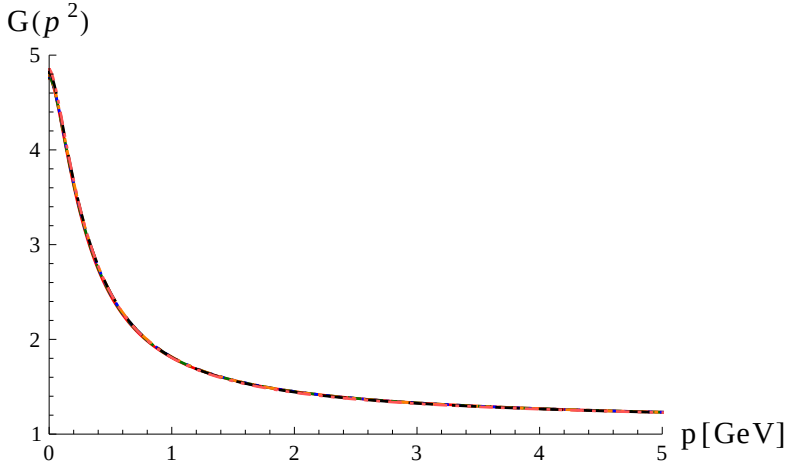


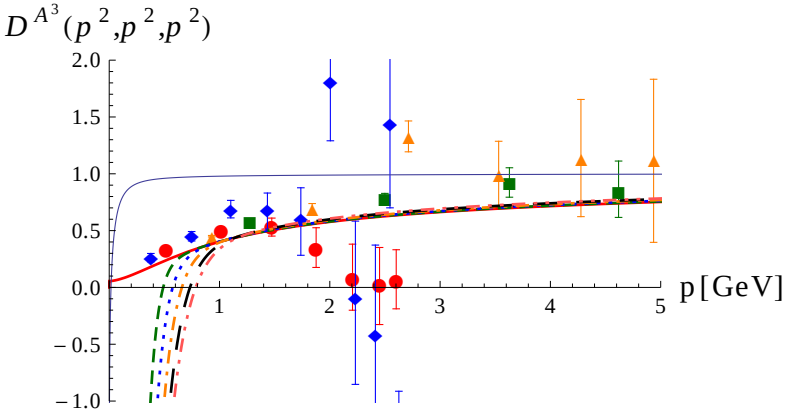
original three-gluon vertex
Bose symmetric three-gluon vertex
Bose symmetric three-gluon vertex with IR part

$\Rightarrow$ Improved three-gluon vertex adds additional strength in the mid-momentum regime.

Influence of the three-gluon vertex

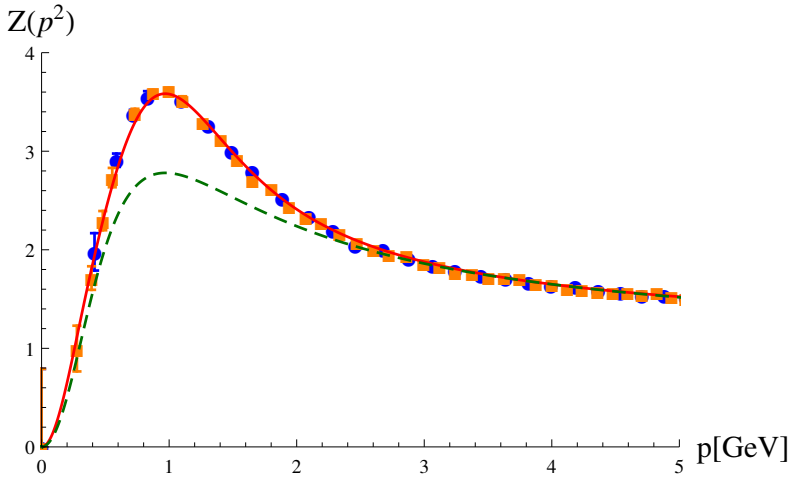


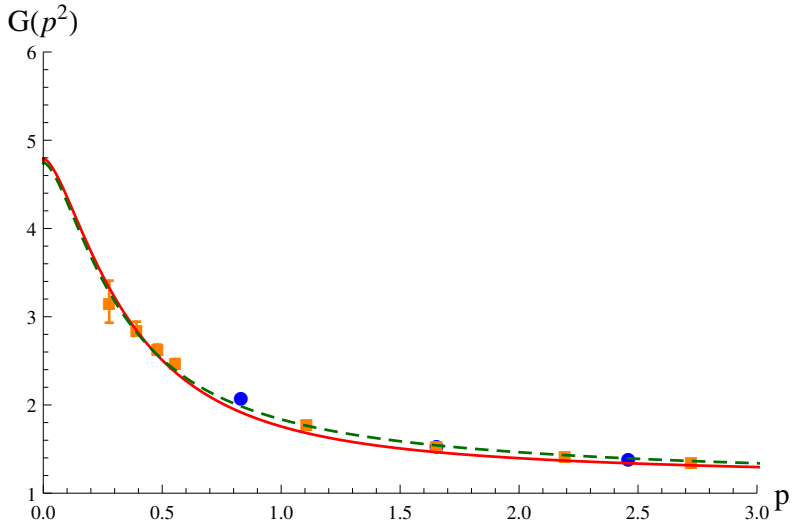




- Vary $\Lambda_{3g} \longrightarrow$ vary mid-momentum strength
- Ghost almost unaffected
- Thin line: Leading IR order <u>DSE calculation for three-gluon vertex</u> $\Rightarrow$ zero crossing

Dynamic ghost-gluon vertex: Propagator results





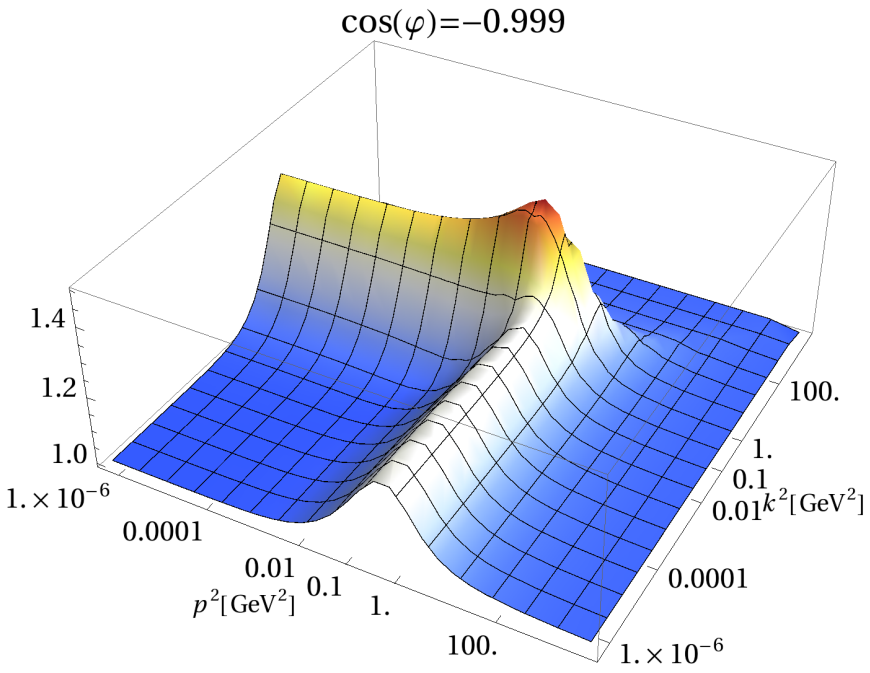
- Ghost-gluon vertex: dynamic
- Old three-gluon vertex model
- Three-gluon vertex: optimized effective model, i.e. optimized overlap with lattice results

- Good quantitative agreement of both ghost and gluon dressing

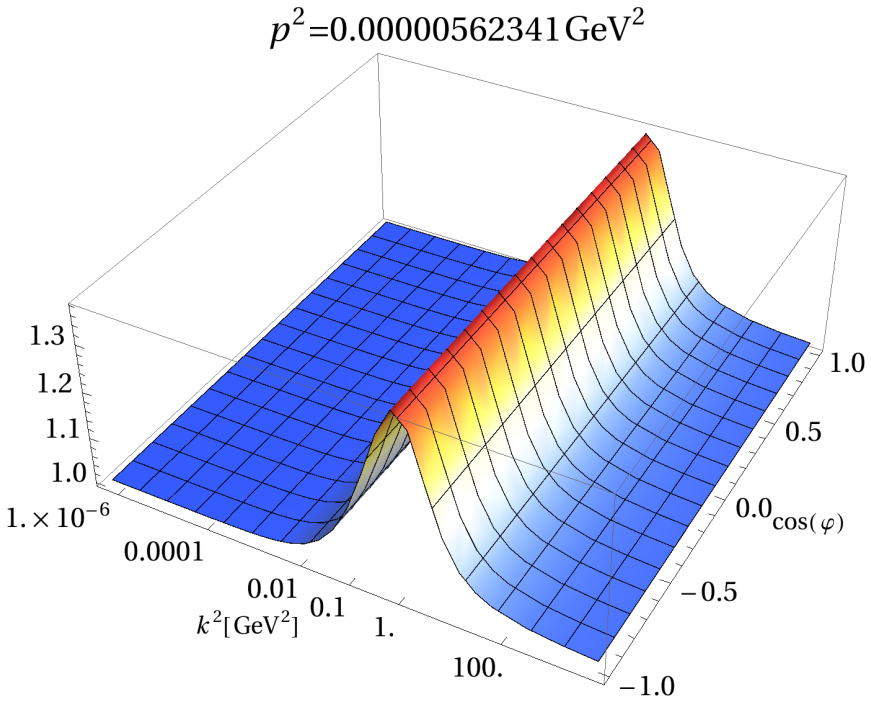
Ghost-gluon vertex: Selected configurations

$$\Gamma_\mu^{A\bar{c}c,abc}(k;p,q) := i\,g\,f^{abc}\,(p_\mu A(k;p,q) + k_\mu B(k;p,q))$$

Fixed angle:

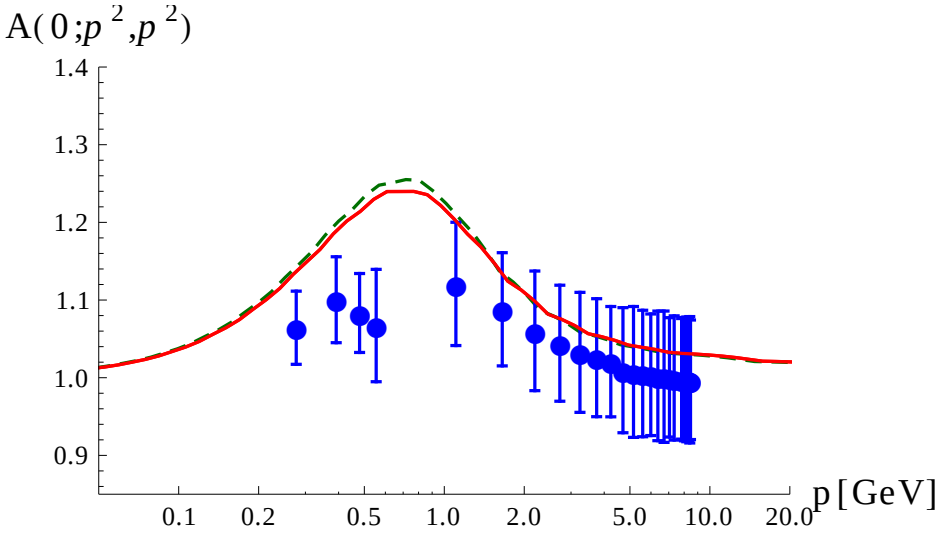


Fixed anti-ghost momentum:



Ghost-gluon vertex: Comparison with lattice data

Orthogonal configuration $k^2 = 0$, $q^2 = p^2$:



lattice data: [Sternbeck, hep-lat/0609016]

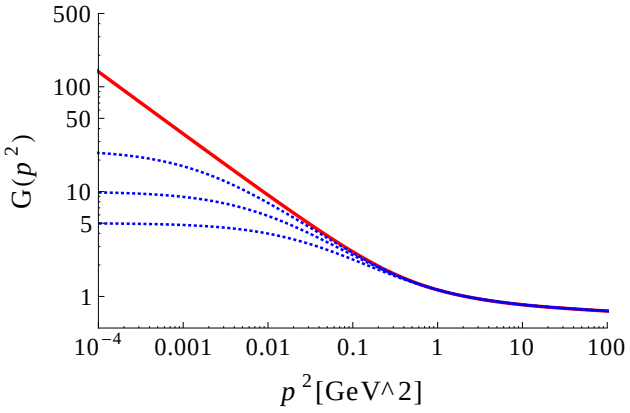
- constant in the IR
- relatively insensitive to changes of the three-gluon vertex (red/green lines: different three-gluon vertex models)

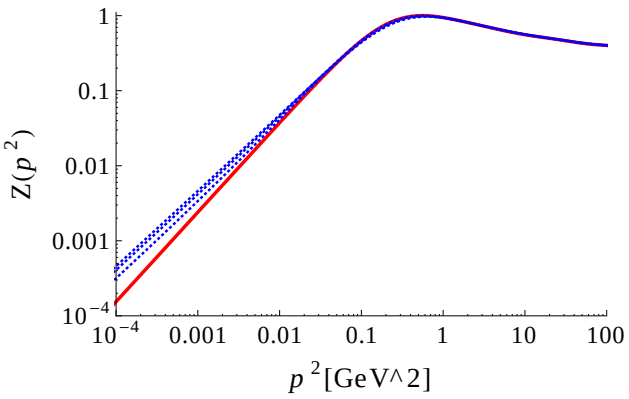
Solutions of DSEs: Decoupling and scaling

- Two types of solutions with functional methods that differ only in deep IR [Boucaud et al., JHEP 0806, 012; Fischer, Maas, Pawlowski, AP 324]:
  scaling [von Smekal, Alkofer, Hauck PRL97],
  decoupling [Aguilar, Binosi, Papavassiliou PRD78]
- Lattice calculations find only decoupling type solution for $d = 3, 4$ and scaling for $d = 2$
- Decoupling emerges also from Refined Gribov-Zwanziger framework [Dudal, Sorella, Vandersickel, Verschelde, PRD77]

Decoupling and scaling solutions

DSEs: Vary ghost boundary condition





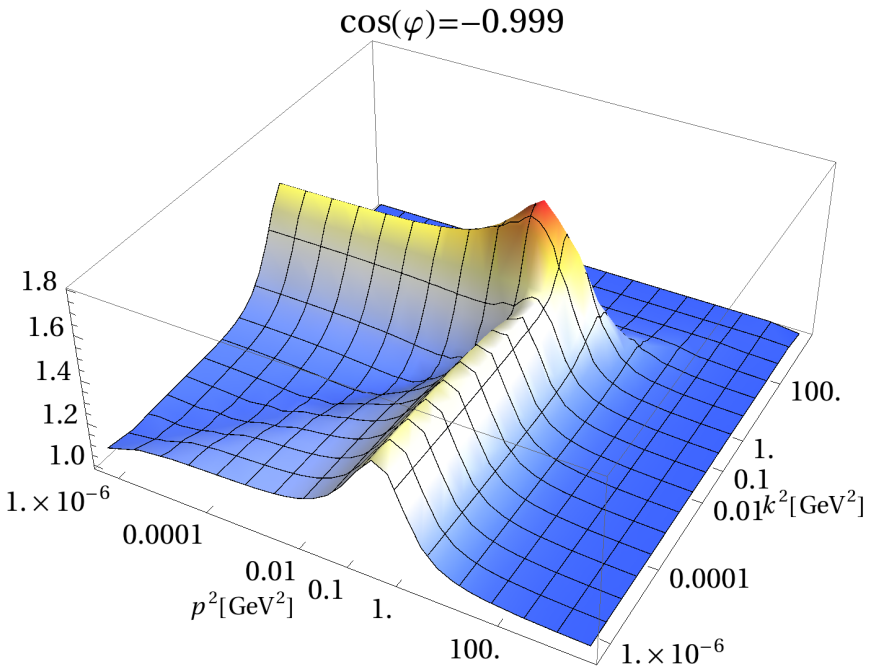
Realization on lattice?

- Dependence of propagators on Gribov copies, e.g., Bogolubsky, Burgio, Müller-Preussker, Mitrjushkin, PRD 74; Maas, 1106.3942
- First hints from Sternbeck, Müller-Preussker, 1211.3057: choosing Gribov copies by the lowest eigenvalue of the Faddeev-Popov operator $\longrightarrow$ modification of both dressings

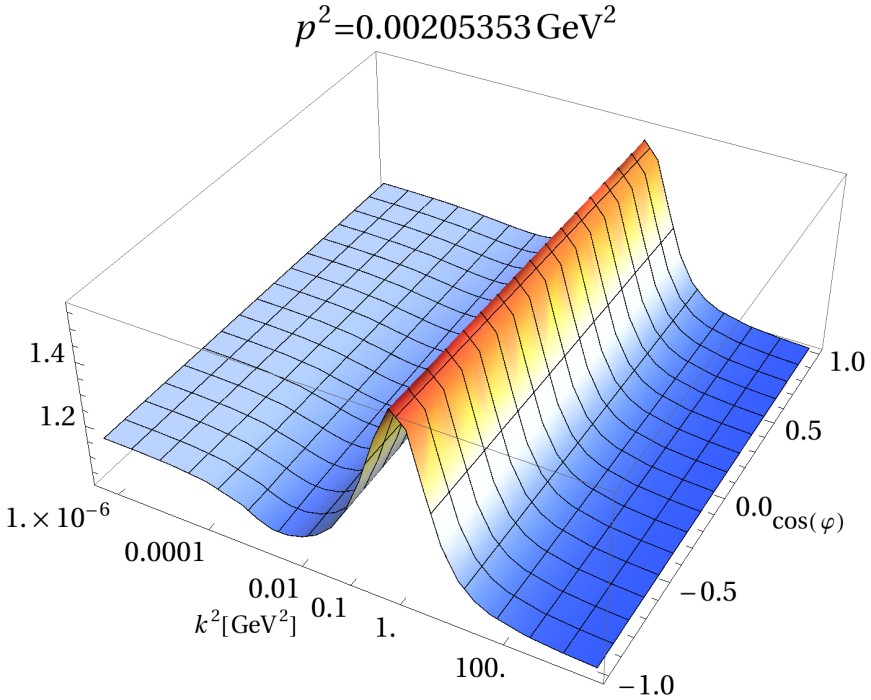
$d = 2$: Analytic and numerical arguments from DSEs for scaling only [Cucchieri, Dudal, Vandersickel, PRD85; MQH, Maas, von Smekal, JHEP1211] as well as from analysis of Gribov region [Zwanziger, 1209.1974].

Ghost-gluon vertex and scaling solution

Fixed angle:



Fixed momentum:



- Dressing not 1 in the IR ← Contributions from loop corrections (for decoupling they are suppressed)
- Scaling/decoupling also seen in ghost-gluon vertex

Two dimensions

Why are two dimensions interesting?

- Ambiguity of solutions?
- larger lattices $\longrightarrow$ lower momenta
  lower dimensions require (much) less computer power, e.g.:
  $d = 4$: 128^4 $(L \approx 27\,fm)$ [Cucchieri, Mendes, Pos LAT2007, 297],
  $d = 2$: 2560^2 $(L \approx 460\,fm)$ [Cucchieri, Mendes, AIP CP 1343, 185]
- $\longrightarrow$ good lattice results exist even for three-point functions
- Gribov problem also present

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Yang-Mills theory for $d = 2$

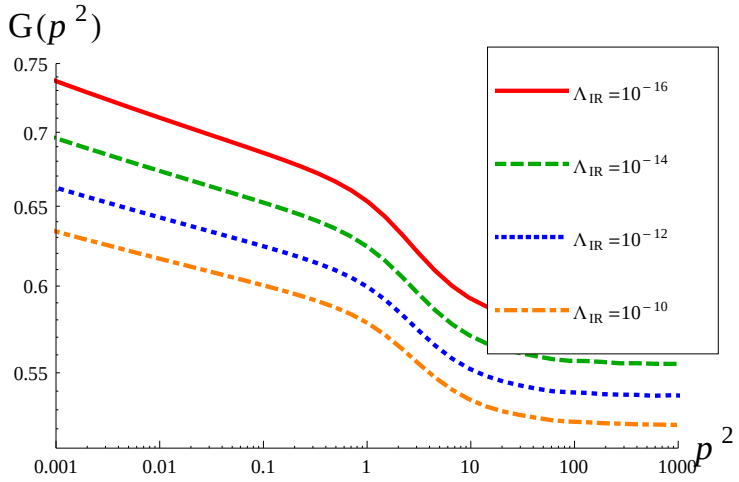
- Perturbation theory does not work because of IR divergences.
- Gluons have no transverse polarization $\longrightarrow$ no physical degrees of freedom, but we can investigate correlation functions, the Gribov problem, ...

Existence of decoupling solution

- Analytical:
  For $d = 2, 3, 4$ two possible scaling solutions, of which one is unphysical.
  Specific to $d = 2$: One can show analytically which one is unphysical. Coincides with decoupling type.
- Numerical:
  Ghost equation contains IR singularities for decoupling type.

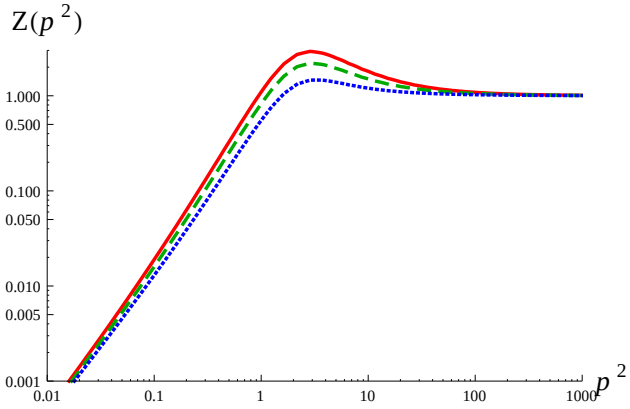
$\Rightarrow$ No decoupling type solution in two dimensions.

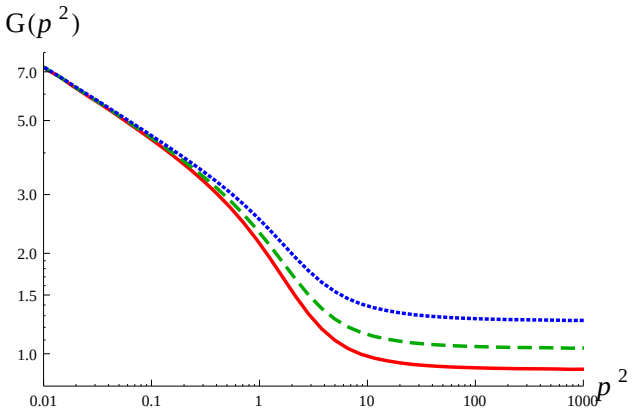
In agreement with Cucchieri, Dudal, Vandersickel, PRD85; Zwanziger, 1209.1974.



Aspects of $d = 2$ Dyson-Schwinger equations

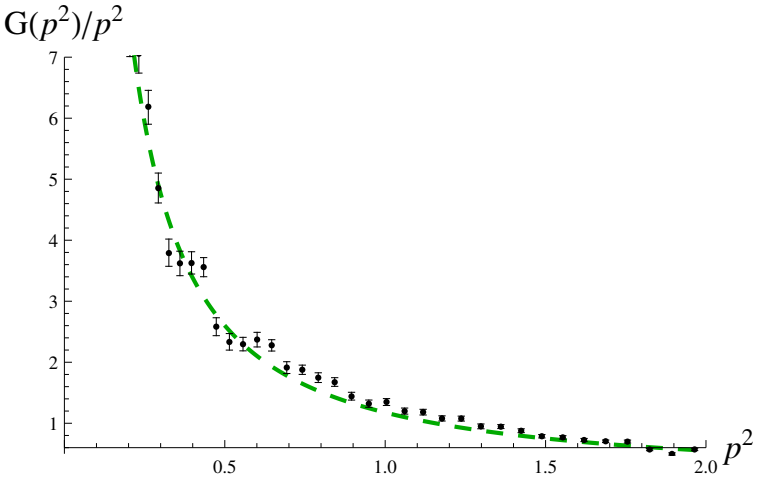
- Different momentum regimes mix,
  e.g., mid-momentum influences UV.

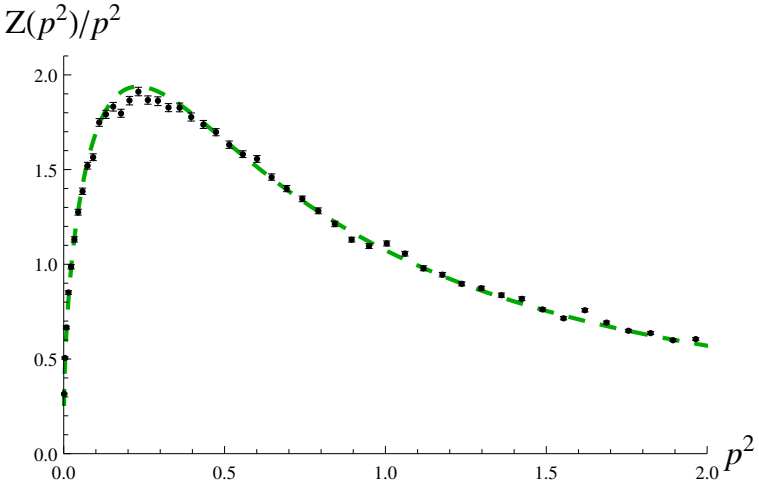




- Ghost dressing must approach 1 in the UV,
  but difficult to achieve due to mixing.
  $\longrightarrow$ Increased vertex dependence.
- Remaining logarithmic divergences:
  Different subtraction methods available.

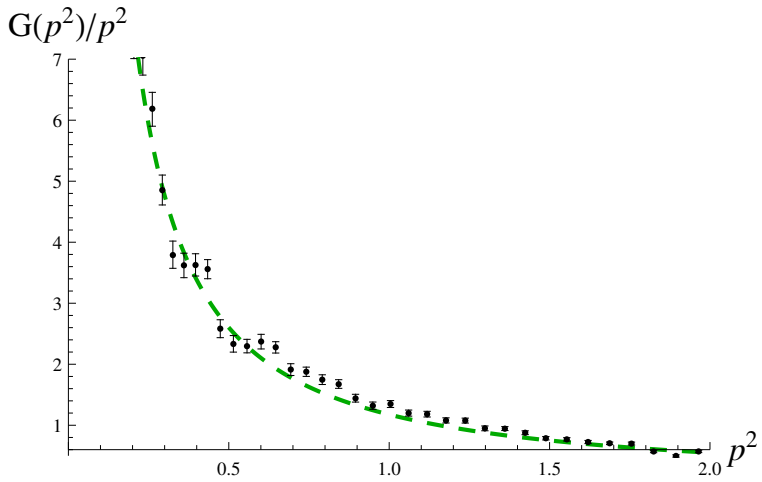
Propagator results

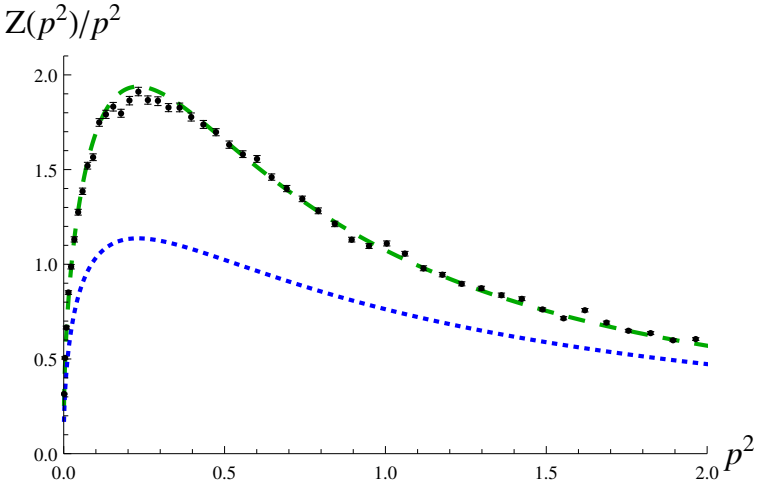




- bare ghost-gluon vertex, three-gluon vertex ansatz (1 parameter),
- lattice results [Maas, 1106.3942]

Propagator results





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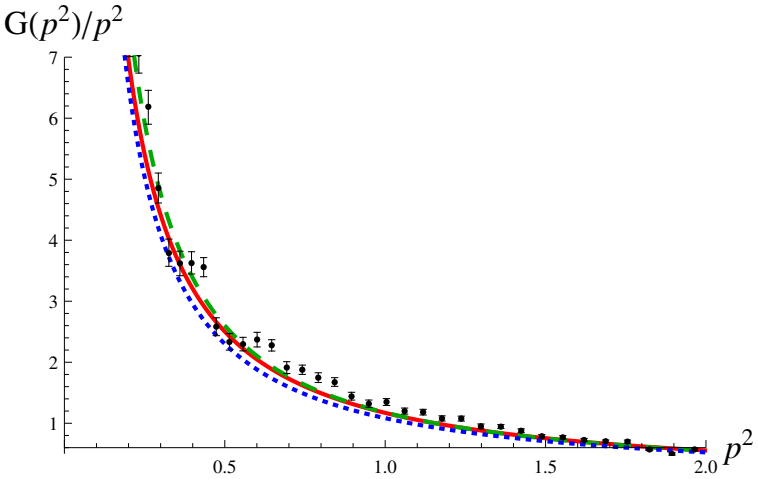
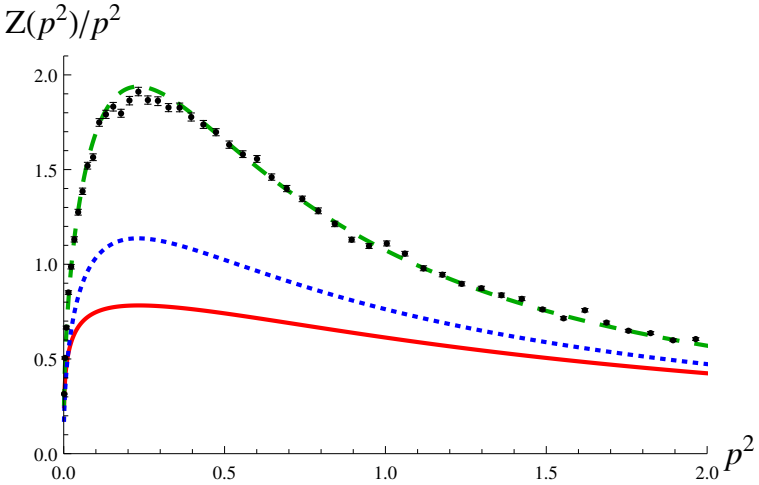
Propagator results



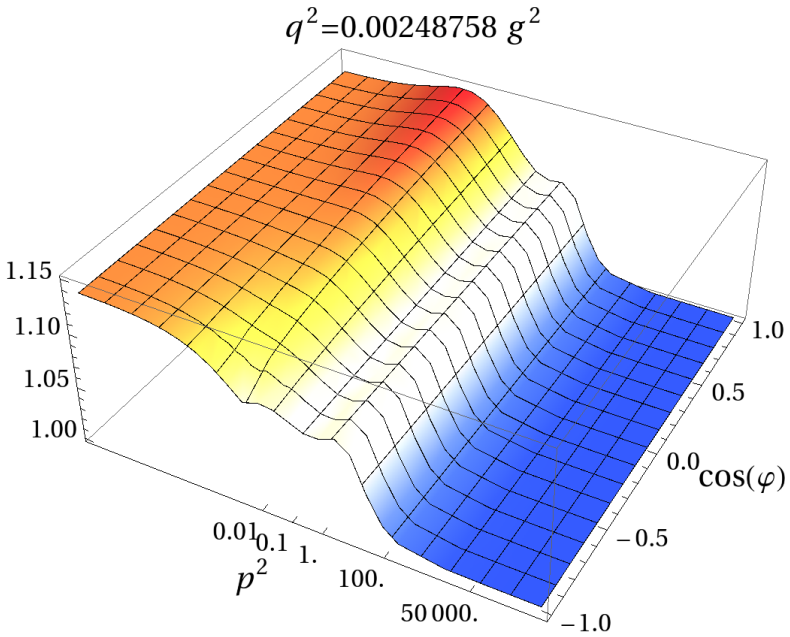


- bare ghost-gluon vertex, three-gluon vertex ansatz (1 parameter),
- lattice results [Maas, 1106.3942]
- lattice inspired models for both vertices,
- dynamic ghost-gluon vertex, lattice inspired three-gluon vertex

$\Rightarrow$ Good agreement with lattice can be obtained,
but strong dependence on vertices.

Ghost-gluon vertex results: Selected configurations

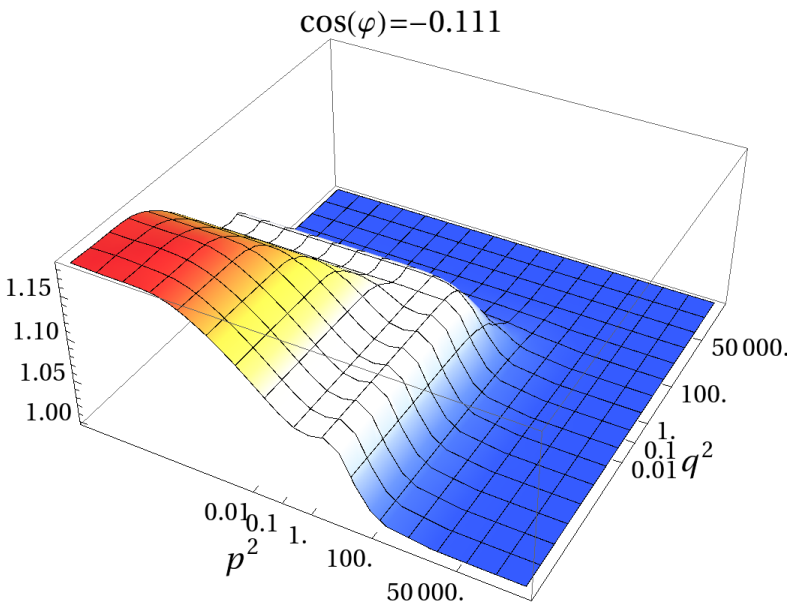
Fixed ghost momentum:



→ 1 in the UV

→ IR constant

Fixed angle:

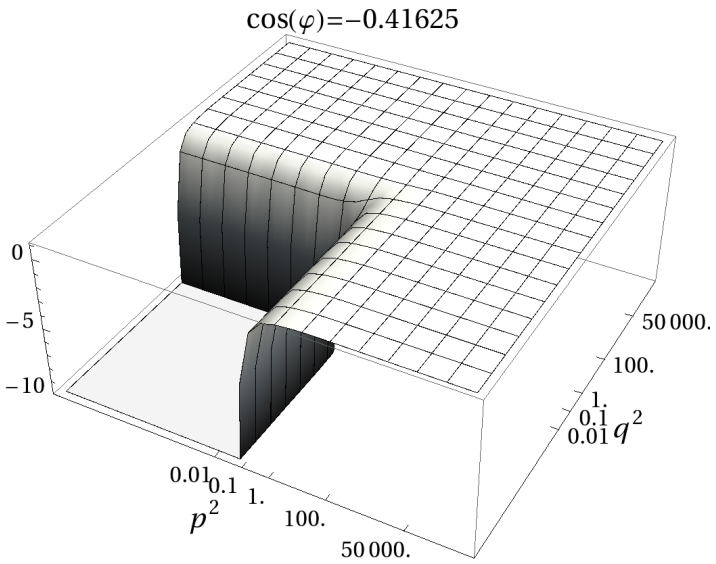


→ Almost no dependence on angle

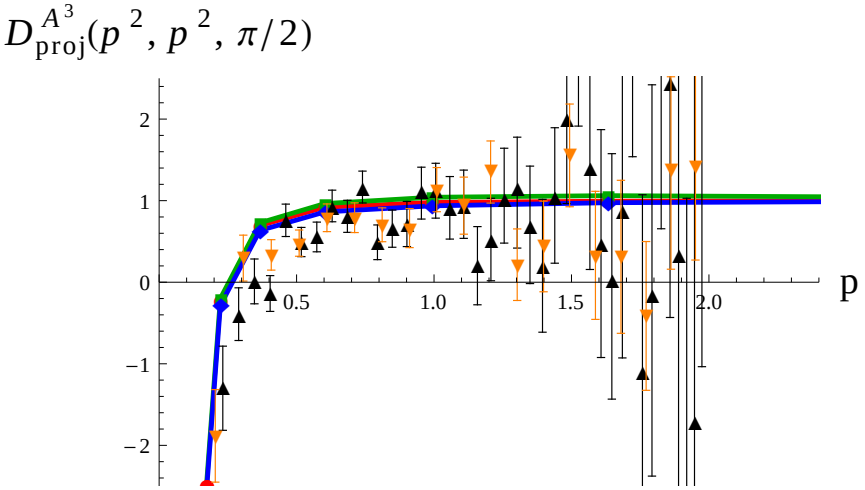
Three-gluon vertex

Three-gluon vertex from propagators and ghost-gluon vertex:

Fixed angle:



Orthogonal configuration:



red, green, blue: DSE calculation with different truncations
black (orange): lattice with $L = 21(12)fm^{-1}$ [Maas, PRD75]

$\Rightarrow$ Leading diagrams reproduce lattice results.

Summary & conclusions: 2 dimensions

- No decoupling solution in 2 dimensions.
- First study of influence of three-point functions.
- Ghost UV behavior very sensitive to truncations
  $\longrightarrow$ restrictions on vertices.
- Mixing of different momentum regimes.
- Quantitative importance of two-loop diagrams.
- First full momentum dependent calculation of three-gluon vertex, in good agreement with lattice data.

Summary & conclusions

- Systematic improvement of truncations of DSEs possible.
- Newest step: Inclusion of three-point functions
  - Required for quantitative results and
  - likely also for some aspects of non-zero temperature and density calculations [Fister, Pawlowski, 1112.5440].
  - Reproduction of lattice data possible.
- Automatization tools:
  DoFun [Alkofer, MQH, Schwenzer, CPC180; MQH, Braun, CPC183]
  CrasyDSE [MQH, Mitter, CPC183]
- While functional equations were at the beginning especially helpful to understand the qualitative behavior, we arrive now at quantitative results.

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Thank you very much for your attention.