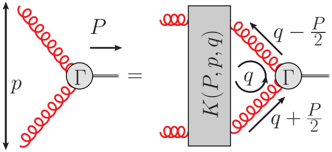
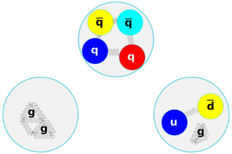
Exploring exotic hadrons with functional equations

Markus Q. Huber

Institute of Theoretical Physics, Giessen University





Group report

<u>Giessen:</u>
Christian S. Fischer
Stephan Hagel
Joshua Hoffer
Franziska Münster
<u>Graz:</u>
Gernot Eichmann

DPG Spring Meeting 2024

March 12, 2024

Giessen, Germany

Thursday, 16:45: J. Hoffer, Tetraquarks (HK 71.5)

JUSTUS-LIEBIG-UNIVERSITÄT GIESSEN

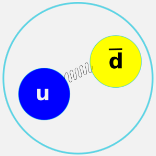
DFG Deutsche Forschungsgemeinschaft

Bound states of the strong interaction

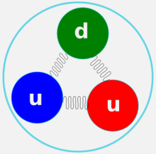
Quark model 1964:

- Solve Schrödinger equation with a given potential, e.g., Cornell:
  $V(r) = -\frac{4}{3}\frac{\alpha_s}{r} + \sigma\, r + \text{const.}$
- Abundance of states

Meson

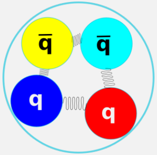


Baryon

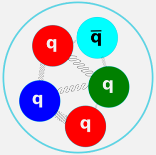


Exotics:

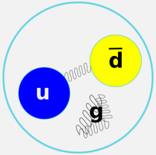
Tetraquark



Pentaquark



Hybrid



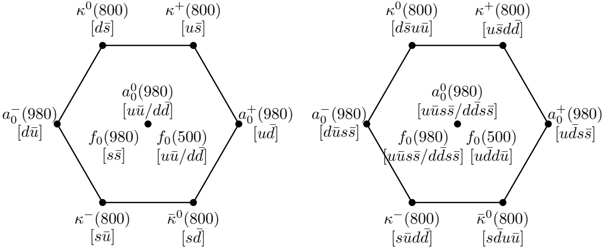
Glueball

Scalar sector

Classification not always easy, e.g., scalar sector $J^{PC} = 0^{++}$:

- $q\bar{q}$ mesons, tetraquarks: (inverted) mass hierarchy?

[Jaffe, Phys. Rev. D 15 (1977)]



Functional review:
[Eichmann, Fischer, Santowsky, Wallbott, Few-Body Syst.61 (2020)]

- Glueballs?

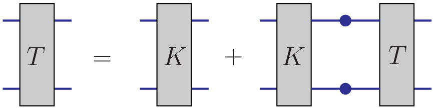
$f_0(500)$
$f_0(980)$
$f_0(1370)$
$f_0(1500)$
$f_0(1710)$

glueball candidates

Functional bound state equation

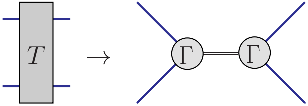
Dyson equation: nonperturbative resummation!

Compare: $f(x) = \dfrac{1}{1-x} = 1 + x + x^2 + \ldots = 1 + x\, f(x) = 1 + x + x^2\, f(x)$



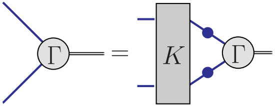
Scattering kernel K: interactions
Scattering matrix T
Bethe-Salpeter amplitude Γ

T contains bound states:



$$T \to \frac{\Gamma\, \overline{\Gamma}}{P^2 + M^2}$$

Plug into Dyson equation:
→ homogeneous Bethe-Salpeter equ.



[Review: Eichmann, Sanchis-Alepuz, Williams, Alkofer, Fischer, Prog.Part.Nucl.Phys. 91 (2016)]

Elements of a BSE

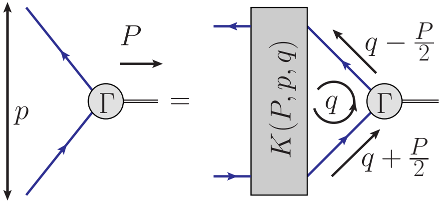
$$\Gamma = K\, G_0\, \Gamma$$

Input:

- Propagators G_0
- Kernel K

Output:

- Mass M: $M^2 = -P^2$
- Bethe-Salpeter amplitudes Γ



(quark-antiquark state)

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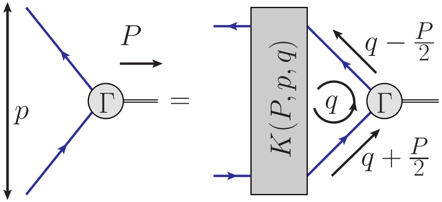

(quark-antiquark state)

Symmetry constraints: Propagators and kernels are not independent!

Relevant for QCD: Chiral symmetry in quark sector $\rightarrow$ axial-vector Ward-Takahashi identity

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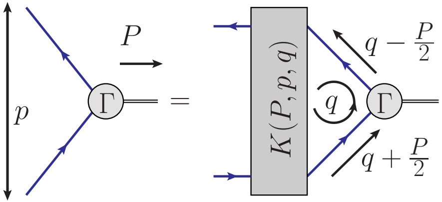

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Approximations: bottom-up $\longleftrightarrow$ top-down

Functional spectrum calculations: Bottom-up

Models, qualitative insight, quantitative results for some cases

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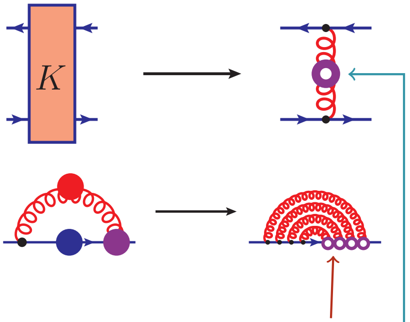


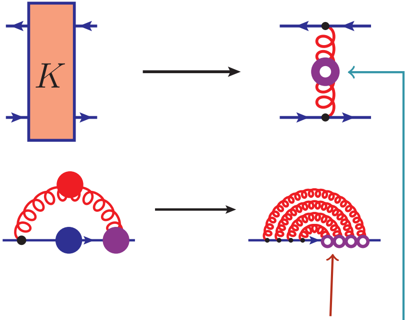
restricted structure of equations ($\boldsymbol{\Gamma}_\mu \to \gamma_\mu$)

IR strength + perturbative UV

Workhorse for more than 20 years: Rainbow-ladder truncation with an effective interaction, e.g., Maris-Tandy (or similar).

Functional spectrum calculations: Bottom-up

Models, qualitative insight, quantitative results for some cases



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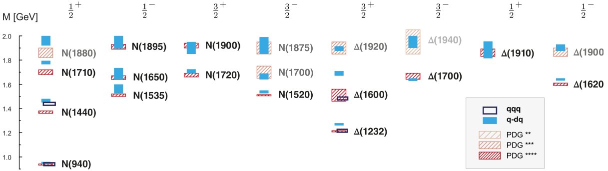
Example: Nucleon spectrum

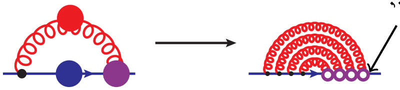


[Eichmann, Sanchis-Alepuz, Williams, Alkofer, Fischer, Prog.Part.Nucl.Phys. 91 (2016); Eichmann, Few Body Syst. 63 (2022)]

Bottom-up approximation: Rainbow

Need the gluon propagator ($\boldsymbol{Z(k^2)}$) and the quark-gluon vertex ($\boldsymbol{h_i(k;p,q)}$).

- $\boldsymbol{\Gamma^{a,\nu}(k;p,q)} \propto \gamma^\nu h_1(k;p,q)$
- $\frac{g^2}{4\pi} Z(k^2) h_1(k;p,q) \propto \boldsymbol{\alpha(k^2)} \longleftarrow$ 'model interaction'



Iteration $\to$ only 'rainbow-like' diagrams

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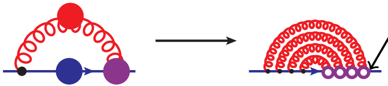

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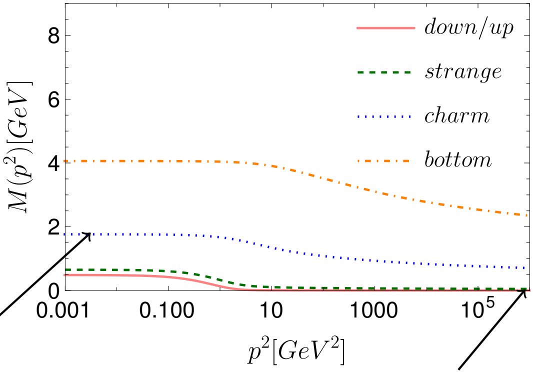


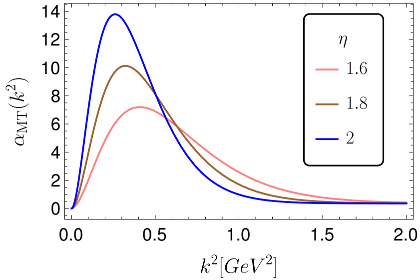
'constituent quark mass'

'current quark mass'

Example for a model: Maris-Tandy interaction

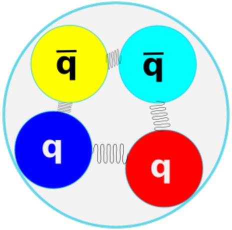
[Maris, Roberts, Tandy, Phys. Rev. C 56 (1997); Maris, Tandy, Phys. Rev. C 60 (1999)]:

$$\alpha(k^2) = \underbrace{\pi\,\eta^7 \left(\frac{k^2}{\Lambda^2}\right)^2 e^{-\eta^2 \frac{k^2}{\Lambda^2}}}_{\alpha_{\text{IR}}(k^2)} + \alpha_{\text{UV}}(k^2)$$



- Scale Λ from f_π
- Quark masses $m_u = m_d$, m_s from m_π, m_K
- Parameter η: window of small sensitivity (for meson masses and decay constants)
- α_{UV}: Phenomenologically irrelevant, provides correct perturbative running to quark propagator

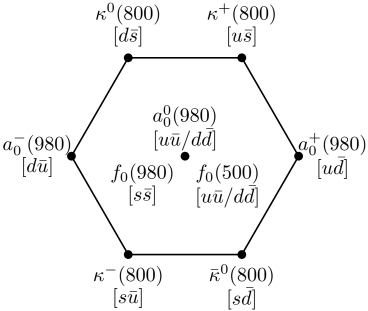
Tetraquarks



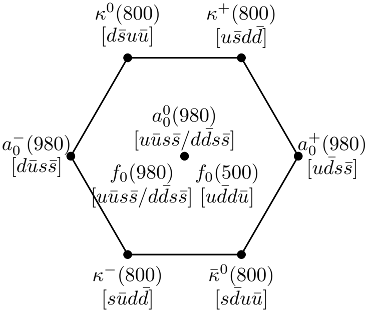
Tetraquarks

- Light scalar mesons: (inverted) mass hierarchy [Jaffe, PRD15 (1977)]?
  History of σ meson, lightest scalar nonet is incompatible with $q\,\bar{q}$ picture:

$q\,\bar{q}$:



$q\,\bar{q}\,q\,\bar{q}$:

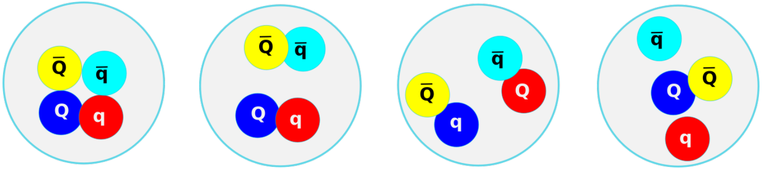


- Experimental discovery of exotic XYZ states $\rightarrow$ four-quark states?

Structure of four-quark states

Consider heavy-light system, e.g., $X(3872)$.

Possible clustering of states:

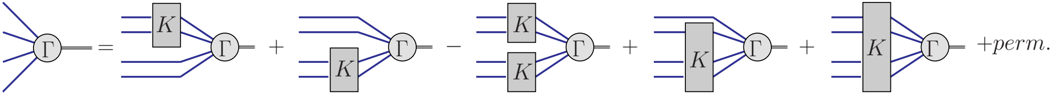


compact　　diquark/antidiquark　　meson molecule　　hadro-quarkonium

Not mutually exclusive: Superpositions!

Tetraquarks: Functional equations

[Eichmann, Fischer, Santowsky, Wallbott, Few-Body Syst.61 (2020)]



2-body interactions 3-body interaction 4-body interaction

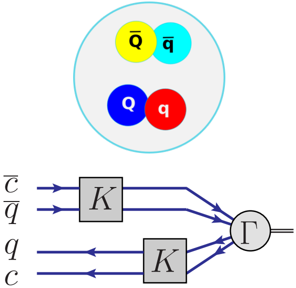
[Kvinikhidze, Khvedelidze, Theor. Math. Phys. 90 (1992); Heupel, Eichmann, Fischer, PLB 718 (2012); Eichmann, Fischer, Heupel, PLB 753 (2016)]

- Neglect 3- and 4-body interactions
- Complicated kinematics (4 momenta):
  - dressings f(9 Lorentz scalar)
  - $J = 0$: 256 tensors, $J = 1$: 768 tensors

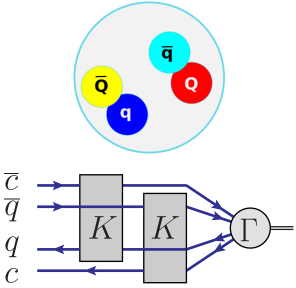
$\rightarrow$ Approximations necessary.

Clustering

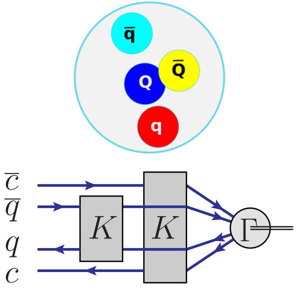
Two-body clusters in amplitudes [Eichmann, Fischer, Heupel, PLB 753 (2016); Wallbott, Eichmann, Fischer, PRD 102 (2020)]:



diquark-antidiquark



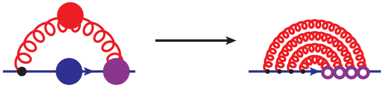
meson molecule



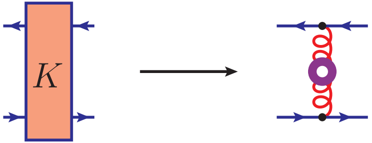
hadro-quarkonium

Rainbow-ladder truncation

Consistency between quark propagator and bound state equations:



Iteration $\rightarrow$ only 'rainbow-like' diagrams



Iteration $\rightarrow$ ladder

[Maris, Roberts, Tandy, Phys. Rev. C 56 (1997); Maris, Tandy, Phys. Rev. C 60 (1999)]:

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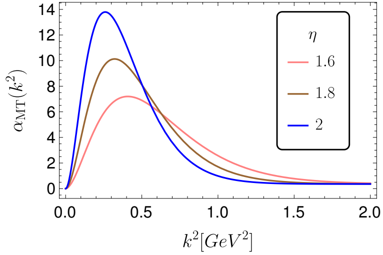

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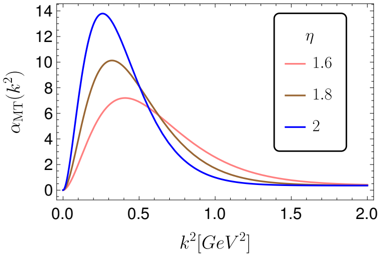

- Scale Λ from f_π
- Quark masses $m_u = m_d$, m_s, m_c, m_b from m_π, $m_{D^{(*)}}$, $m_{D_s^{(*)}}$, η_b, m_Υ
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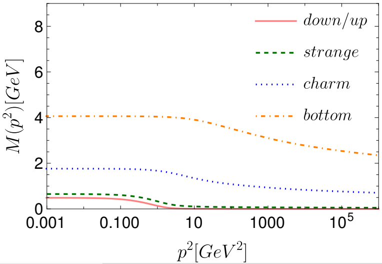
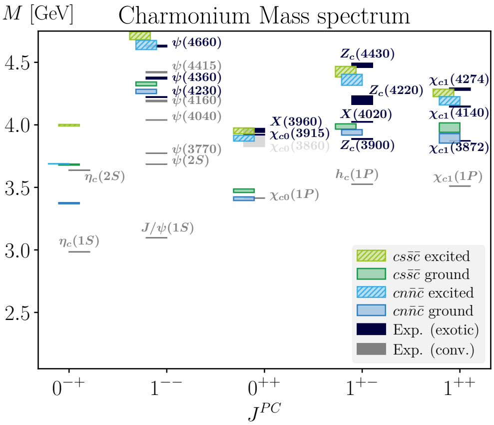
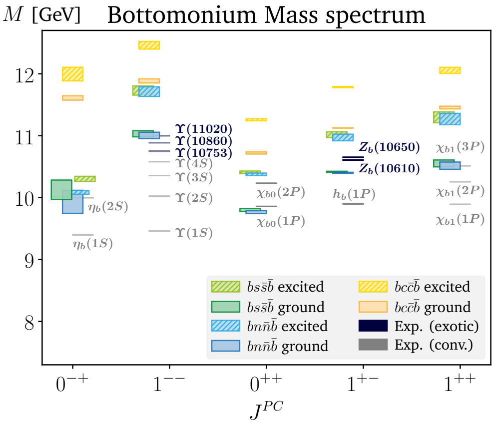


Hidden-flavor tetraquarks w/ charm and bottom quarks: Spectrum

Some teasers... → Full story: Thursday, 16:45: J. Hoffer, Tetraquarks (HK 71.5)

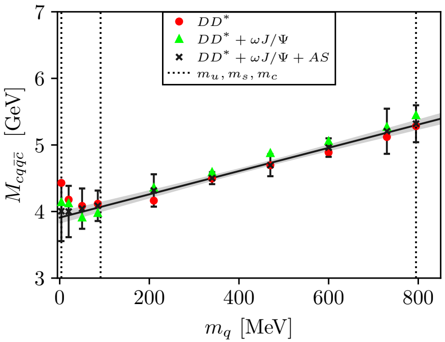




[Hoffer, Eichmann, Fischer, 2402.12830]

Hidden-flavor tetraquarks w/ charm and bottom quarks: Structure

Identify dominant components, e.g., $\chi_{c1}(3872)$ [$X(3872)$]:



- $D\,D^*$: $c\bar{q}$, $q\bar{c}$ (molecule)
- $\omega\,J/\psi$: $c\bar{c}$, $q\bar{q}$ (hadro-quarkonium)
- AS: cq, $\overline{cq}$ (diquark-antidiquark)

[Wallbott, Eichmann, Fischer, Phys. Rev. D 100 (2019)]

Hidden-flavor tetraquarks w/ charm and bottom quarks: Structure

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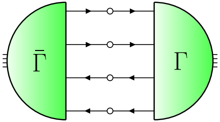
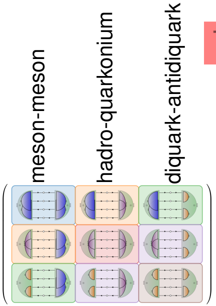
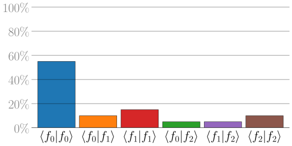
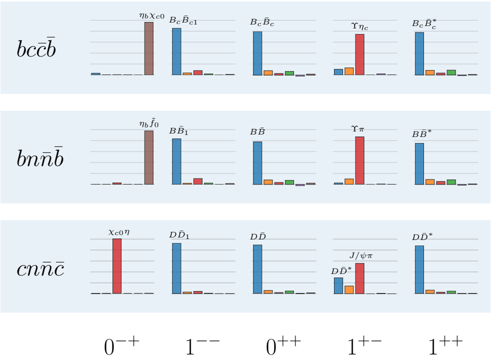
Norm contributions:







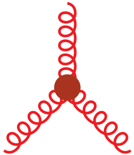


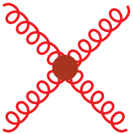
[Hoffer, Eichmann, Fischer, 2402.12830]

Glueballs

Glueballs

Non-Abelian nature of QCD $\rightarrow$ self-interaction of force fields.





Mass dynamically created from massless (due to gauge invariance) gluons.

Theory:

Glueballs from gauge inv. operators, e.g., $F_{\mu\nu}F^{\mu\nu}$.

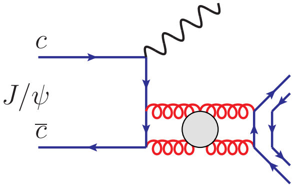
$\rightarrow$ Mixing of operators with equal quantum numbers.

Experiment:

Production in glue-rich environments, e.g., $p\bar{p}$ annihilation (PANDA), pomeron exchange in pp (central exclusive production), radiative J/ψ decays

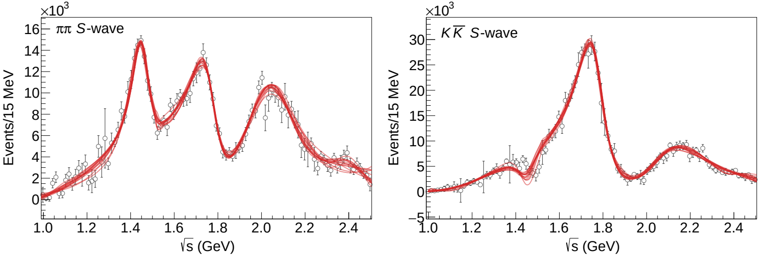
Reviews on glueballs: [Klempt, Zaitsev, Phys.Rept.454 (2007); Mathieu, Kochelev, Vento, Int.J.Mod.Phys.18 (2009); Crede, Meyer, Prog.Part.Nucl.Phys.63 (2009); Ochs, J.Phys.G40 (2013); Llanes-Estrada, EPJST 230 (2021); Vadacchino, 2305.04869]

Scalar glueballs from J/ψ decay



Coupled-channel analyses of exp. data (BESIII):

- +add. data, largest overlap with $f_0(1770)$ [Sarantsev, Denisenko, Thoma, Klempt, Phys. Lett. B 816 (2021)]
- largest overlap with $f_0(1710)$ [JPAC Coll., Rodas et al., Eur.Phys.J.C 82 (2022)]

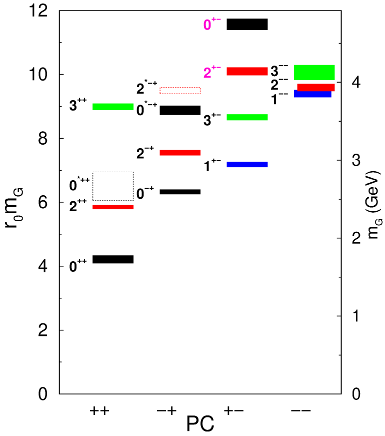


Glueball calculations: Lattice

Lattice methods

Pure gauge theory:
No dynamic quarks.
$\rightarrow$ “Pure” glueballs

- [Morningstar, Peardon, Phys. Rev. D60 (1999)]: standard reference
- [Athenodorou, Teper, JHEP11 (2020)]: improved statistics, more states



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"Real QCD":

- [Gregory et al., JHEP10 (2012)]
- [Brett et al., AIP Conf.Proc. 2249 (2020)]
- [Chen et al., 2111.11929]
- [Vadacchino, Lattice2022, 2305.04869]
- ...

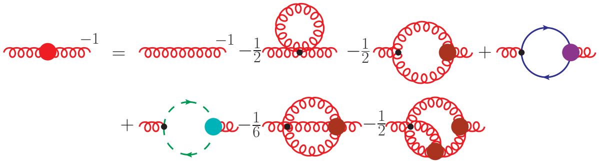
Challenging:

- Much higher statistics required (poor signal-to-noise ratio)
- Continuum extrapolation and inclusion of fermionic operators still to be done
- Mixing with $\bar{q}q$ challenging
- $m_\pi = 360\,\text{MeV}$
- Small unquenching effects found

No quantitative results yet.

Functional glueball calculations

Glueballs? Rainbow-ladder?



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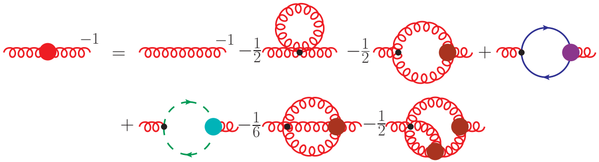

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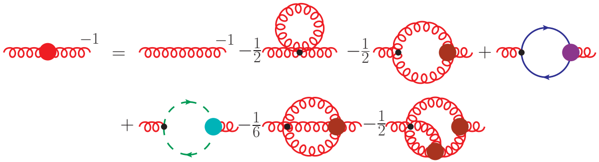

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<u>Model based</u> BSE calculations ($J = 0$):

- [Meyers, Swanson, Phys.Rev.D87 (2013)]
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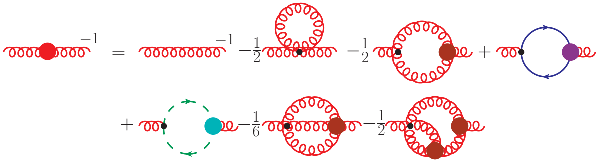

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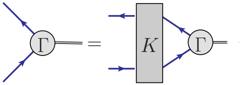
Alternative: Calculated input [MQH, Phys.Rev.D 101 (2020)]

- $J = 0$: [MQH, Fischer, Sanchis-Alepuz, Eur.Phys.J.C80 (2020)]
- $J = 0, 2, 3, 4$: [MQH, Fischer, Sanchis-Alepuz, Eur.Phys.J.C81 (2021)]

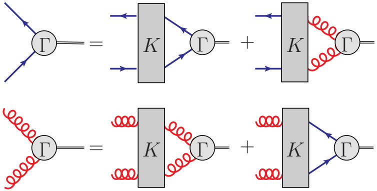
Extreme sensitivity on input!

Bound state equations for QCD



- Require scattering kernel K and propagator.

Bound state equations for QCD



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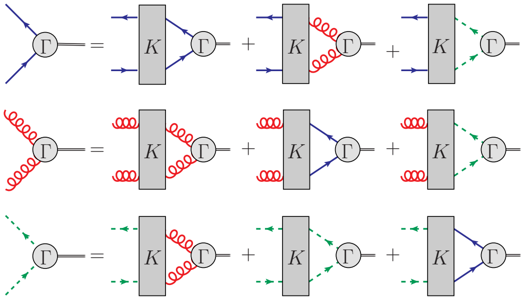

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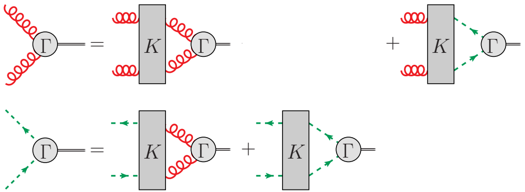
One framework

- Natural description of mixing.
- Similar equations for hadrons with more than two constituents

Bound state equations for QCD

Focus on pure glueballs.



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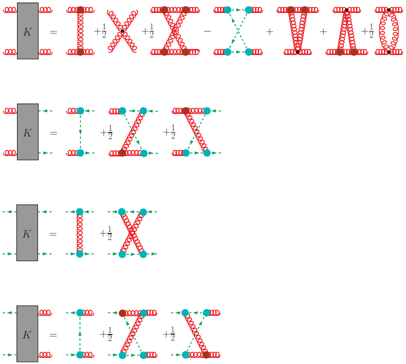
Kernels

Systematic derivation from 3PI effective action: [Berges, Phys. Rev. D 70 (2004); Carrington, Gao, Phys. Rev. D 83 (2011)]

Self-consistent treatment of 3-point functions requires 3-loop expansion.

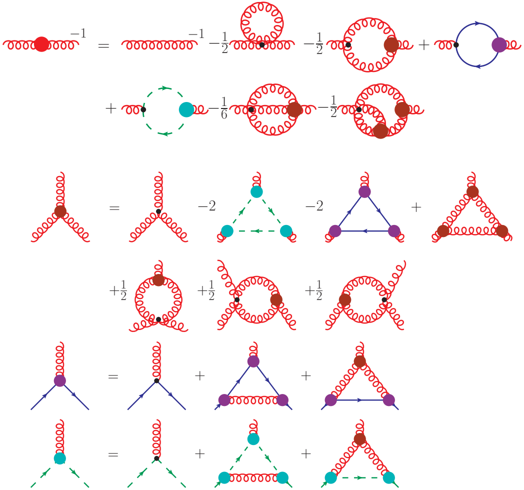


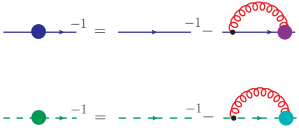
[Fukuda, Prog. Theor. Phys 78 (1987); McKay, Munczek, Phys. Rev. D 40 (1989); Sanchis-Alepuz, Williams, J. Phys: Conf. Ser. 631 (2015); MQH, Fischer, Sanchis-Alepuz, Eur.Phys.J.C80 (2020)]

Correlation functions of quarks and gluons

Equations of motion: 3-loop 3PI effective action

$\longrightarrow$ [Review: MQH, Phys.Rept. 879 (2020)]





- Conceptual and technical challenges: nonperturbative renormalization, two-loop diagrams, convergence, size of kernels, . . .
- Self-contained: Only parameters are the strong coupling and the quark masses!
- Long way, e.g., ghost-gluon vertex, three-gluon vertex, four-gluon vertex, . . .
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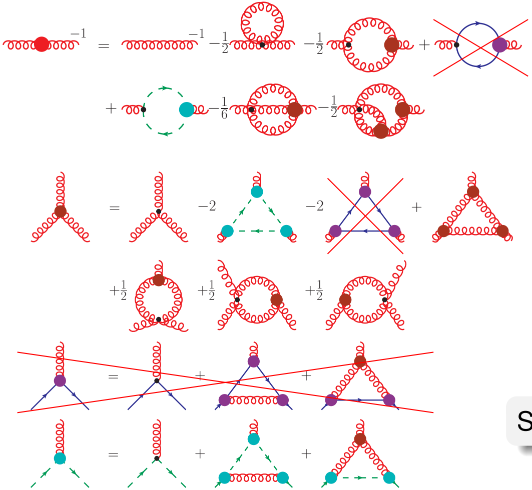

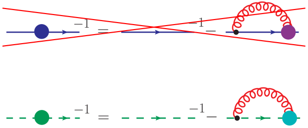

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Start with pure gauge theory.

Landau gauge propagators

Self-contained: Only external input is the coupling! $\rightarrow$ $\boxed{\text{Ab-initio!}}$ [MQH, Phys.Rev.D 101 (2020)]

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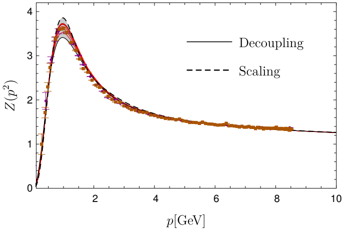
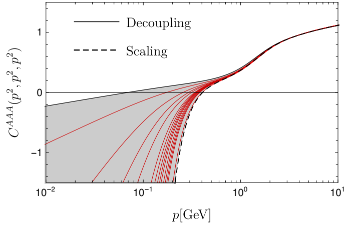
Gluon dressing function:

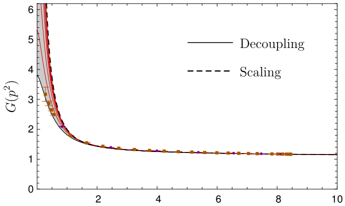


Three-gluon vertex:



Family of solutions [von Smekal, Alkofer, Hauck, PRL79 (1997); Aguilar, Binosi, Papavassiliou, Phys.Rev.D 78 (2008); Boucaud et al., JHEP06 (2008); Fischer, Maas, Pawlowski, Ann.Phys. 324 (2008); Alkofer, MQH, Schwenzer, Phys. Rev. D 81 (2010)]

Ghost dressing function:



Nonperturbative completions of Landau gauge [Maas, Phys. Lett. B 689 (2010)]?

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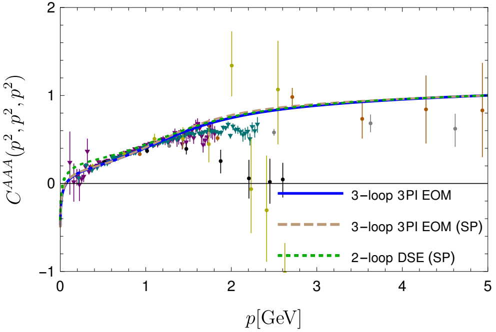

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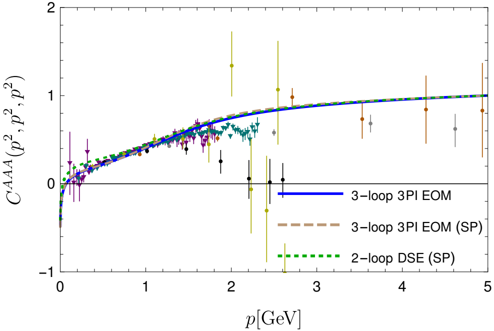
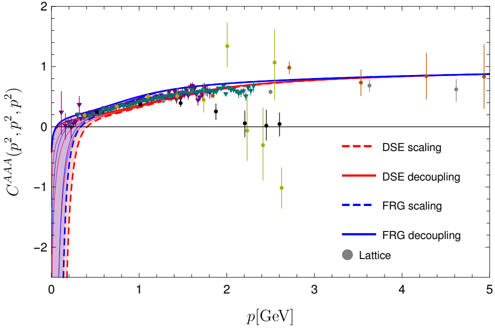


DSE vs. FRG:



[Cucchieri, Maas, Mendes, Phys.Rev.D77 (2008); Sternbeck et al., Proc.Sci. LATTICE2016 (2017); Cyrol et al., Phys.Rev.D 94 (2016); MQH, Phys.Ref.D101 (2020)]

Stability of the solution: Extensions

- Three-gluon vertex: Tree-level dressing dominant, others subleading [Eichmann, Williams, Alkofer, Vujinovic, Phys.Rev.D89 (2014); Pinto-Gómez et al., 2208.01020] ✓

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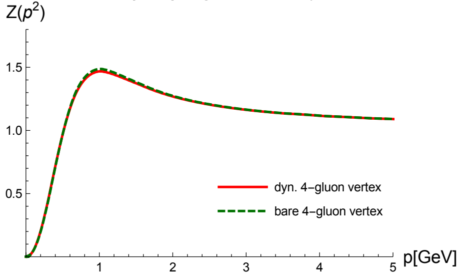

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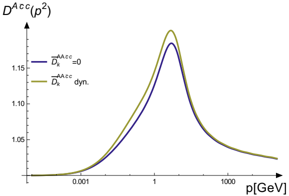
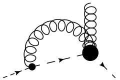
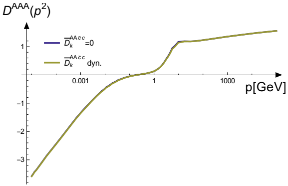
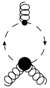
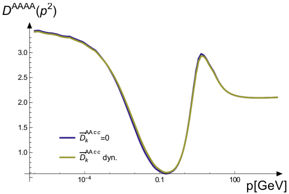
- Two-ghost-two-gluon vertex [MQH, Eur. Phys.J.C77 (2017)]: ✓ (FRG: [Corell, SciPost Phys. 5 (2018)])

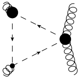








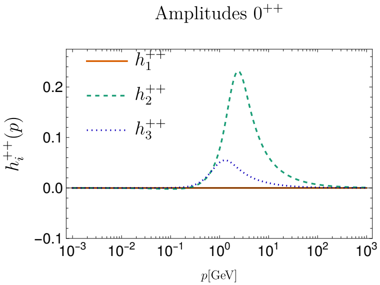




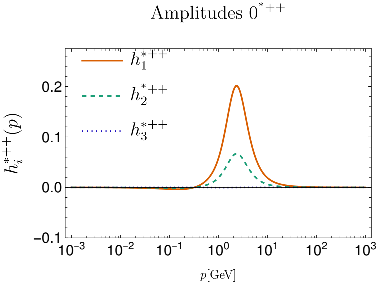
Amplitudes

Information about significance of single parts.

Ground state scalar glueball:



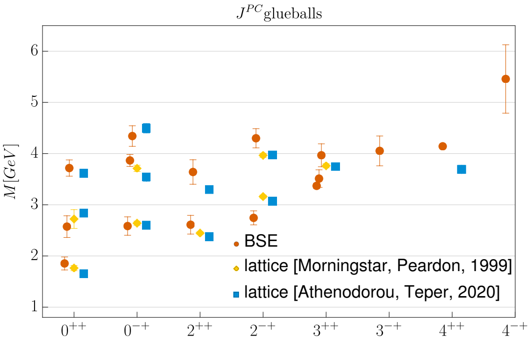
Excited scalar glueball:

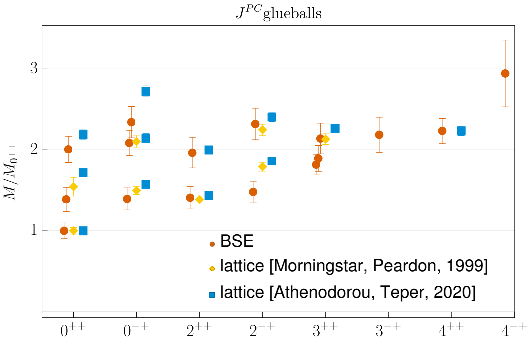


$\rightarrow$ Amplitudes have different behavior for ground state and excited state. Useful guide for future developments.

$\rightarrow$ Meson/glueball amplitudes: Information about mixing.

Glueball results

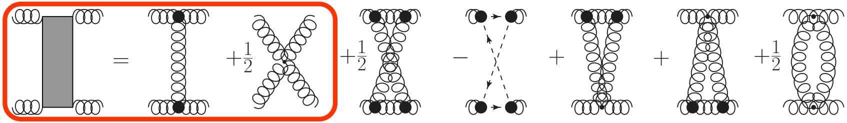




[MQH, Fischer, Sanchis-Alepuz, Eur.Phys.J.C81 (2021)]

- Agreement with lattice results
- New states: 0^{**++}, 0^{**-+}, 3^{-+}, 4^{-+}

Higher order diagrams



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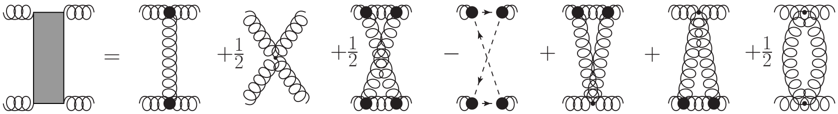

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Two-loop diagrams: subleading effects

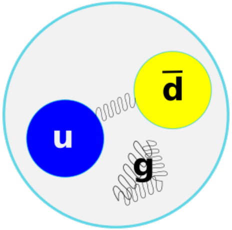
- 0^{-+}: none

  [MQH, Fischer, Sanchis-Alepuz, EPJ Web Conf. 258 (2022)]
- 0^{++}: $< 2\%$

  [MQH, Fischer, Sanchis-Alepuz, HADRON2021, arXiv:2201.05163]
- 2^{++}: none

  [MQH, Fischer, Sanchis-Alepuz, HADRON2023, arXiv:2312.12029]

Hybrids



Bound state equations for hybrids

[Münster, Fischer, MQH]

- (Anti)quarks + gluonic excitation
- Meson $\rightarrow$ three-body equation
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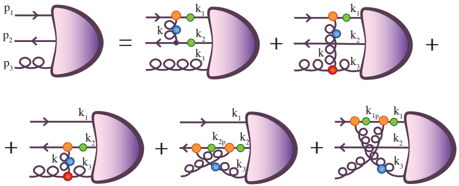
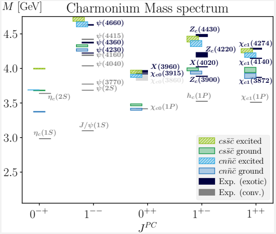


- $\pi_1(1600)$ (1^{-+}): 48 tensors
- Leading order of 3PI effective action: dressed quark-gluon and three-gluon interactions
- Preliminary results: diagrams with three-gluon vertices leading

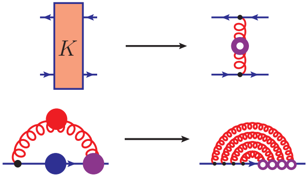
Summary and outlook

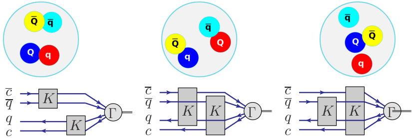
Tetraquarks: Bottom-up

- Structure: molecule/hadro-quarkonium/diquark-antidiquark



- Structure quantum number and flavor dependent!





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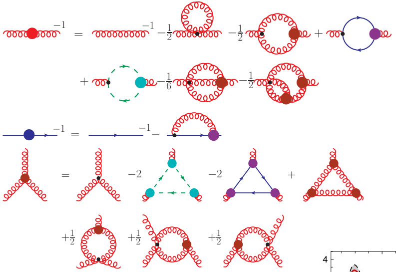
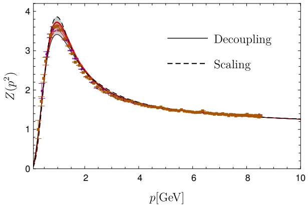
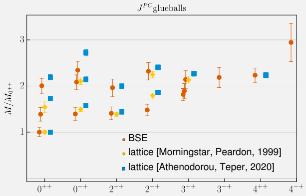




Glueballs: Top-down

- Self-contained input, parameters: coupling, (quark masses)
- Quantitative predictions
- Successful stability tests



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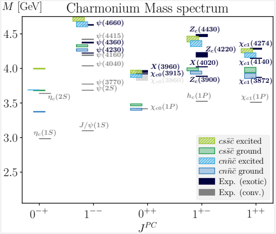

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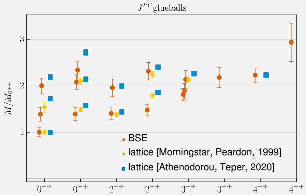

Future:

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- Open flavor tetraquarks
- Three-gluon glueballs
- Mixing glueballs/mesons

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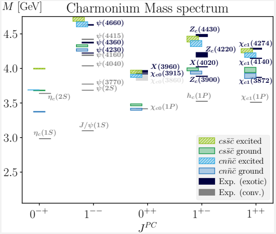

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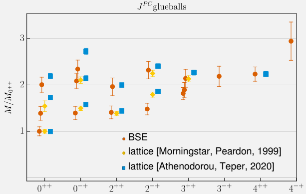

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Thank you for your attention!

Functional spectrum calculations: Top-down

Derivation of kernels and correlation functions from *n*PI effective actions [Fukuda, Prog.Theor.Phys. 78 (1987); Sanchis-Alepuz, Williams, J.Phys.Conf.Ser. 631 (2015)].

Loop expansion of *n*PI effective actions as reliable expansion in terms of nonperturbative quantities?

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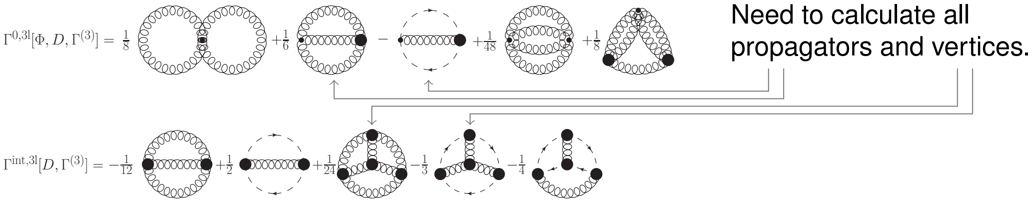
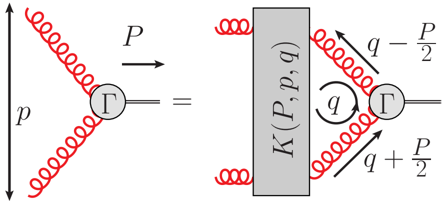
Example: 3-loop 3PI effective action [Berges, Phys. Rev. D 70 (2004); Carrington, Gao, Phys. Rev. D 83 (2011)]

$$\Gamma^{3l}[\Phi, D, \Gamma^{(3)}] = \Gamma^{0,3l}[\Phi, D, \Gamma^{(3)}] + \Gamma^{\text{int},3l}[\Phi, D, \Gamma^{(3)}]$$



Correlation functions for complex momenta



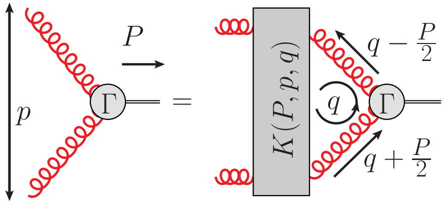
(pseudoscalar glueball)

$$\boldsymbol{\lambda(P)}\Gamma(P) = \mathcal{K} \cdot \Gamma(P)$$

$\rightarrow$ Eigenvalue problem for $\Gamma(P)$:

1. Solve for $\lambda(P)$.
2. Find P with $\lambda(P) = 1$.
   $\Rightarrow M^2 = -P^2$

Correlation functions for complex momenta



(pseudoscalar glueball)

$$\boldsymbol{\lambda(P)}\Gamma(P) = \mathcal{K}\cdot\Gamma(P)$$

$\rightarrow$ Eigenvalue problem for $\Gamma(P)$:

1. Solve for $\lambda(P)$.
2. Find P with $\lambda(P) = 1$.
   $\Rightarrow M^2 = -P^2$

However:

Propagators are probed at $\left(q \pm \frac{P}{2}\right)^2 = \frac{P^2}{4} + q^2 \pm \sqrt{P^2\, q^2}\cos\theta = -\frac{M^2}{4} + q^2 \pm \boldsymbol{i}\, M\sqrt{q^2}\,\cos\theta$

$\rightarrow$ Complex for $P^2 < 0$!

Time-like quantities ($P^2 < 0$) $\rightarrow$ Correlation functions for complex arguments.

Extrapolation of $\lambda(P^2)$

Extrapolation method

- Extrapolation to time-like P^2 using Schlessinger's continued fraction method (proven superior to default Padé approximants) [Schlessinger, Phys.Rev.167 (1968)]
- Average over extrapolations using subsets of points for error estimate

$$f(x) = \frac{f(x_1)}{1 + \frac{a_1(x-x_1)}{1+\frac{a_2(x-x_2)}{1+\frac{a_3(x-x_3)}{\cdots}}}}$$

Coefficients a_i can determined such that $f(x)$ exact at x_i.

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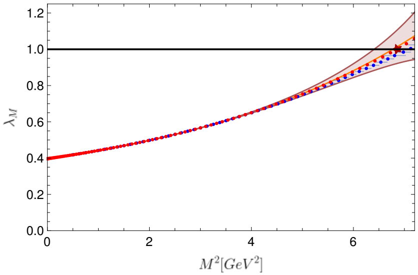
Test extrapolation for solvable system:
Heavy meson [MQH, Sanchis-Alepuz, Fischer, Eur.Phys.J.C 80 (2020)]



$J = 1$ glueballs

Landau-Yang theorem

Two-photon states cannot couple to $J^P = \mathbf{1}^{\pm}$ or $(2n+1)^-$

[Landau, Dokl.Akad.Nauk SSSR 60 (1948); Yang, Phys. Rev. 77 (1950)].

($\rightarrow$ Exclusion of $J = 1$ for Higgs because of $h \rightarrow \gamma\gamma$.)

Applicable to glueballs?

$\rightarrow$ Not in this framework, since gluons are not on-shell.

$\rightarrow$ Presence of $J = 1$ states is a dynamical question.

$J = 1$ not found here.

Glueballs as bound states

Hadron masses from correlation functions of color singlet operators.

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Example: For $J^{\mathrm{PC}} = 0^{++}$ glueball take $O(x) = F_{\mu\nu}(x)F^{\mu\nu}(x)$:

$$\boldsymbol{D(x-y)} = \langle O(x)O(y)\rangle$$

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Lattice: Mass exponential Euclidean time decay:

$$\lim_{t\to\infty}\langle O(x)O(0)\rangle \sim e^{-t\,M}$$

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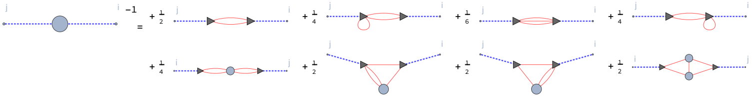
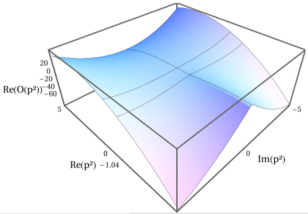
Functional approach: Complicated object in a diagrammatic language, 2-, 3- and 4-gluon contributions [MQH, Cyrol, Pawlowski, Comput.Phys.Commun. 248 (2020)]



+ 3-loop diagrams

Leading order:

[Windisch, MQH, Alkofer, Phys.Rev.D87 (2013)]



Glueballs as bound states

Hadron masses from correlation functions of color singlet operators.

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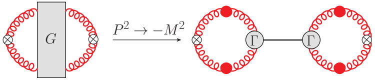
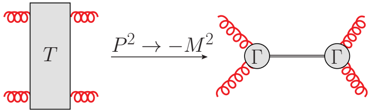
Put total momentum on-shell and consider individual 2-, 3- and 4-gluon contributions. $\rightarrow$ Each can have a pole at the glueball mass.

A^4-part of $D(x-y)$, total momentum on-shell:

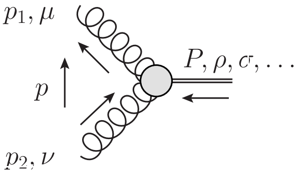




Glueball amplitudes for spin J

[MQH, Fischer, Sanchis-Alepuz, Eur.Phys.J.C81 (2021)]

$$\Gamma_{\mu\nu\rho\sigma\ldots}(p_1,p_2)=\sum\tau^i_{\mu\nu\rho\sigma\ldots}(p_1,p_2)h_i(p_1,p_2)$$



Increase in complexity:

- 2 gluon indices (transverse)
- J spin indices (symmetric, traceless, transverse to P)

Numbers of tensors:

J	P $=+$	P $=-$
0	2	1
1	4	3
>2	5	4

Low number of tensors, but high-dimensional tensors!

$\rightarrow$ Computational cost increases with J.

Charge parity

Transformation of gluon field under charge conjugation:

$$A_\mu^a \to -\eta(a) A_\mu^a$$

where

$$\eta(a) = \begin{cases} +1 & a = 1,3,4,6,8 \\ -1 & a = 2,5,7 \end{cases}$$

Color neutral operator with two gluon fields:

$$A_\mu^a A_\nu^a \to \eta(a)^2 A_\mu^a A_\nu^a = A_\mu^a A_\nu^a.$$

$\Rightarrow C = +1$

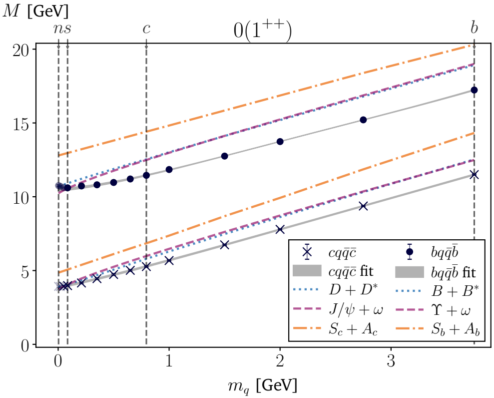
Negative charge parity, e.g.:

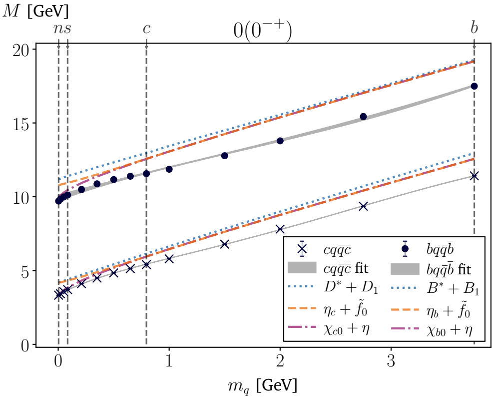
$$d^{abc} A_\mu^a A_\nu^b A_\rho^c \to -d^{abc}\eta(a)\eta(b)\eta(c) A_\mu^a A_\nu^b A_\rho^c = \\ -d^{abc} A_\mu^a A_\nu^b A_\rho^c.$$

Only nonvanishing elements of the symmetric structure constant d^{abc}: zero or two indices equal to 2, 5 or 7.

Hidden-flavor tetraquarks w/ charm and bottom quarks: Quark mass dependence

Some teasers… → Full story: Thursday, 16:45: J. Hoffer, Tetraquarks (HK 71.5)

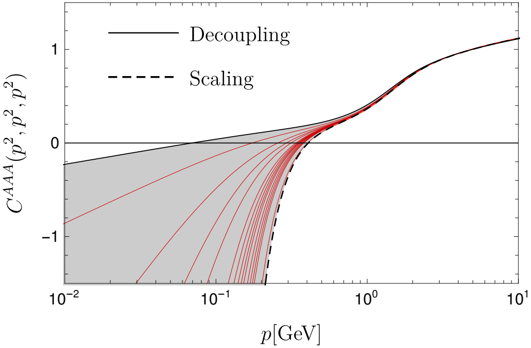


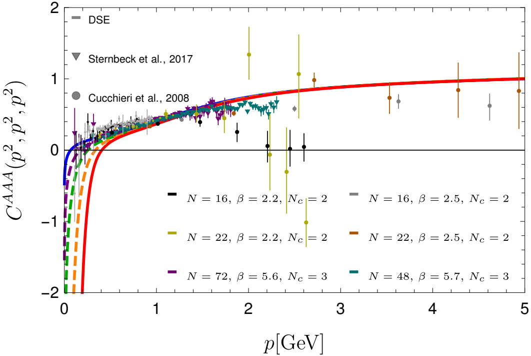


[Hoffer, Eichmann, Fischer, 2402.12830]

Three-gluon vertex

[Cucchieri, Maas, Mendes, Phys. Rev. D 77 (2008); Sternbeck et al., 1702.00612; MQH, Phys. Rev. D 101 (2020)]

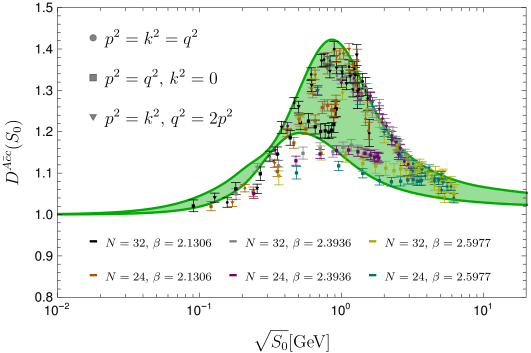




- Simple kinematic dependence of three-gluon vertex (only singlet variable of S_3)
- Large cancellations between diagrams

Ghost-gluon vertex

Ghost-gluon vertex:



[Maas, SciPost Phys. 8 (2019);

MQH, Phys. Rev. D 101 (2020)]

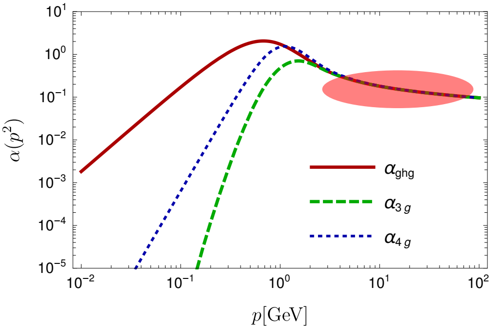
- Nontrivial kinematic dependence of ghost-gluon vertex
- Qualitative agreement with lattice results, though some quantitative differences (position of peak!).

Gauge invariance

[MQH, Phys. Rev. D 101 (2020)]

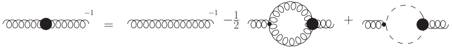
Couplings can be extracted from each vertex.

- Slavnov-Taylor identities (gauge invariance): Agreement perturbatively (UV) necessary.
  [Cyrol et al., Phys.Rev.D 94 (2016)]
- Difficult to realize: Small deviations $\rightarrow$ Couplings cross and do not agree.
- Here: Vertex couplings agree down to GeV regime (IR can be different).



Landau gauge propagators in the complex plane

Simpler truncation:



[Fischer, MQH, Phys.Rev.D 102 (2020)]

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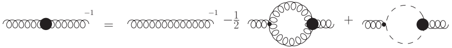
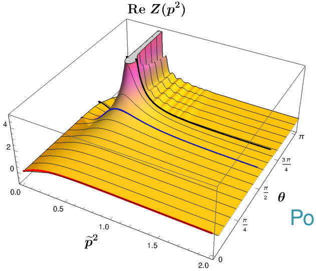
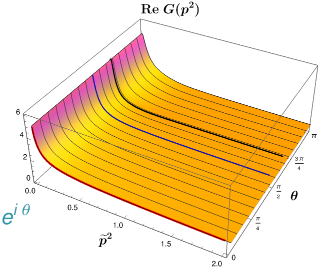


[Fischer, MQH, Phys.Rev.D 102 (2020)]

Ray technique for self-consistent solution of a DSE:



Polar coordinates: $p^2 = \tilde{p}^2 e^{i\,\theta}$



- Current truncation leads to a pole-like structure in the gluon propagator.
- Analyticity up to 'pole' confirmed by various tests (Cauchy-Riemann, Schlessinger, reconstruction)

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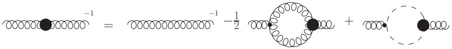

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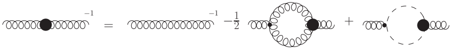

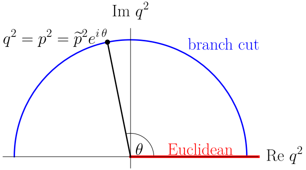

$\rightarrow$ Opening at $q^2 = p^2$.

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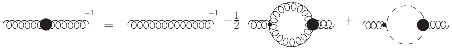
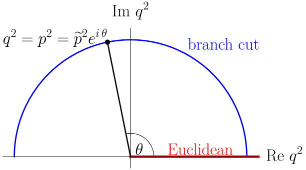
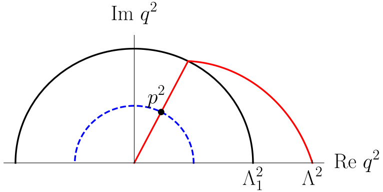




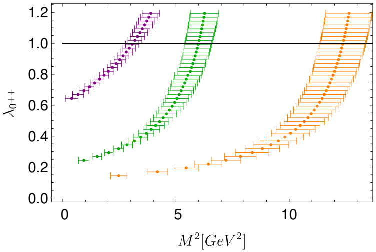


$\rightarrow$ Opening at $q^2 = p^2$.

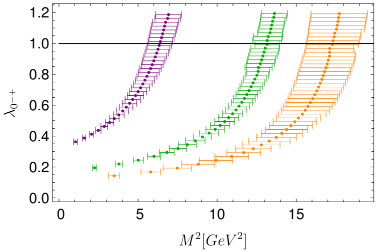
Appearance of branch cuts for complex momenta forbids integration directly to cutoff.

Extrapolation for glueball eigenvalue curves

0^{++}:



0^{-+}:



Several curves: ground state and excited states.