Infrared Behavior of Vertex Functions in d-Dimensional Yang-Mills Theory

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Table of Contents

Infrared Regime

- Infrared (IR) is different from UV,
  where perturbation theory works due to asymptotic freedom.
- IR phenomena:
  dynamical chiral symmetry breaking and confinement.
- Complementing methods are needed:
  Lattice vs. functional methods as Dyson-Schwinger equations (DSEs) and renormalization group
- A first step in understanding confinement is to consider only ghosts and gluons, i.e. Yang-Mills (YM) theory.
- Confinement in YM theory (Landau gauge):
  Gribov-Zwanziger and Kugo-Ojima scenario

Dyson-Schwinger Approach

Dyson-Schwinger equations

- Equations of motion for Green functions
- Describe the theory completely, including non-perturbative effects
- Infinite tower of coupled integral equations
- Propagators: can be calculated using approximations for the vertices

In the IR a solution for the propagators and vertices in form of power laws is possible.

Infrared Behavior

- Gluon and ghost propagators: dressing functions show power-like behavior

$$D_{\mu\nu}(p) = \left(\delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}\right)\frac{Z(p)}{p^2}, \qquad D_G(p) = -\frac{G(p)}{p^2}$$

$$Z(p) = A\cdot(p^2)^\alpha, \qquad G(p) = B\cdot(p^2)^\beta$$

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- Vertices: dressing functions show power-like behavior, but more complicated tensor structure

$$\Gamma_{\mu\nu\ldots}(p_1, p_2, \ldots) = \sum_{i=0}^{n} H_i(p_1, p_2, \ldots)\tau^{(i)}_{\mu\nu\ldots}$$

$$H_i(p_1, p_2, \ldots) = C_i \cdot (p^2)^\gamma$$

Comparison of Different Methods

	Lattice	Continuum
Ghost-gluon vertex	constant	
Ghost propagator	diverging	diverging
Gluon propagator	vanishing?	vanishing
Other vertex functions	no data for IR	prediction: power-like (Alkofer et al., PLB 611)

Investigation of finite size effects via DSEs on a 4-dimensional torus → much bigger lattices needed (Fischer et al., Annals Phys. 2007)

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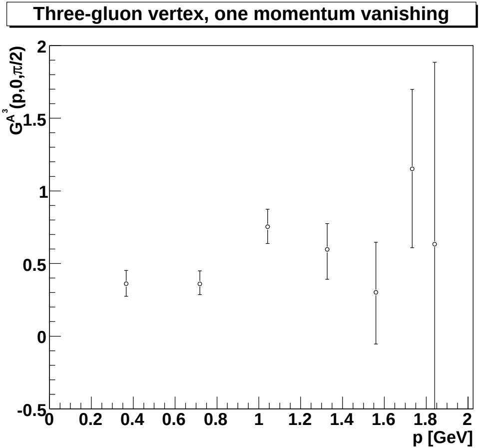
Continuum results ($\kappa = 0.5953\ldots$)

- ghost-gluon-vertex: $(p^2)^0 \rightarrow$ *constant*
- ghost propagator: $(p^2)^{-\kappa}$
- gluon propagator: $(p^2)^{2\kappa}$
- 3-gluon vertex: $(p^2)^{-3\kappa}$

Vertices: Available Lattice Data

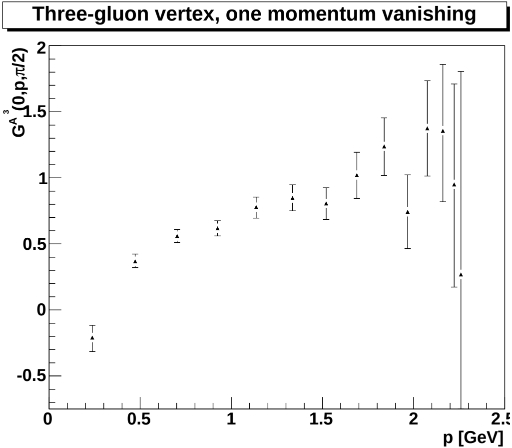


4 dimensions

Maas et al., Braz. J. Phys. 37N1B, 2007

no power-like behavior

Vertices: Available Lattice Data

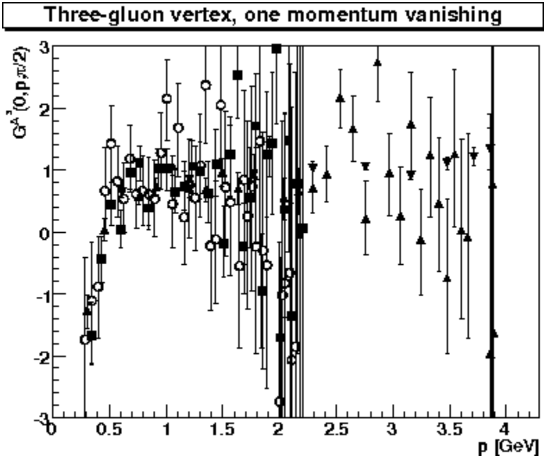


3 dimensions

Cucchieri et al., PRD74, 2006

change of sign

Vertices: Available Lattice Data



2 dimensions

Maas, arXiv:0704.0722, 2007

IR exponent can be extracted

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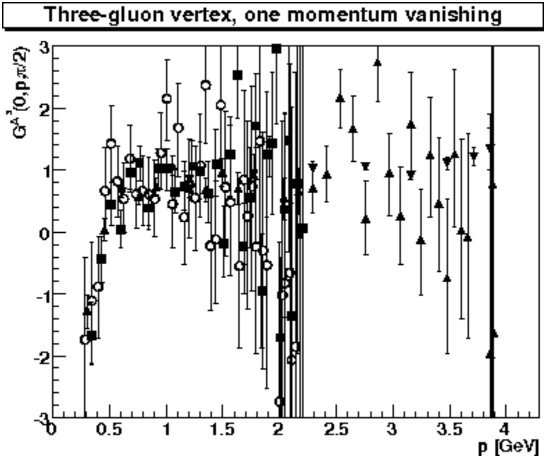

2 dimensions

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IR exponent can be extracted

Qualitatively the same results in 2, 3 and 4 dimensions?

The Bare Ghost-Gluon Vertex

Starting point is transversal gluon propagator in Landau gauge:

$$k_\mu D_{\mu\nu}(k) = k_\mu \frac{Z(k)}{k^2}\left[\delta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}\right] = 0$$

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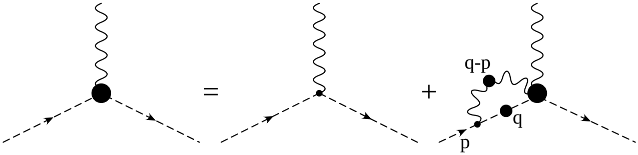
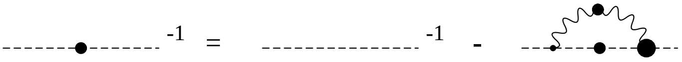


Propagators

- IR behavior can be determined from the ghost DSE:



- Simple power counting gives a relation between ghost and gluon propagator:

$$1 - \beta = \frac{d}{2} + \alpha - 1 + \beta - 1 + \frac{1}{2} + \frac{1}{2}$$

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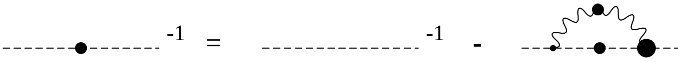

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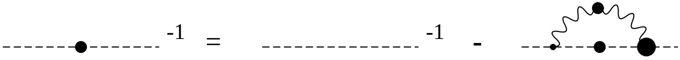

- Simple power counting gives a
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$$\alpha = -2\beta + 2 - \frac{d}{2}$$

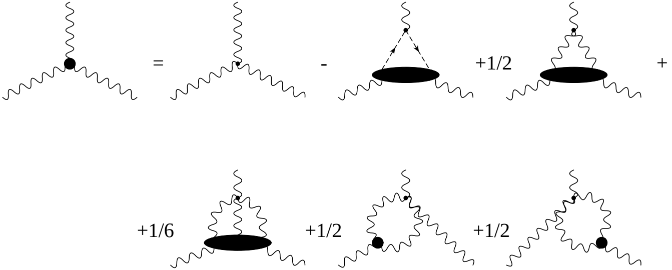
$$\beta = -\kappa$$

$$G(p^2) = (p^2)^{-\kappa}, \quad Z(p^2) = (p^2)^{2\kappa + 2 - \frac{d}{2}}$$

Skeleton Expansion

DSEs for n-point functions contain even higher n-point functions.

E.g. the three-gluon DSE:



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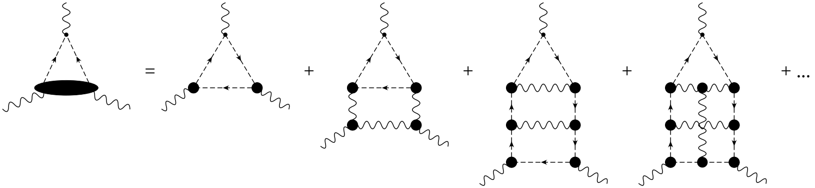
Skeleton expansion of the ghost-gluon scattering kernel:



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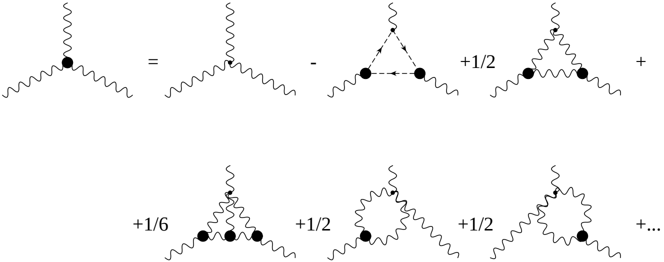
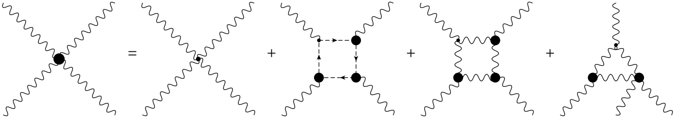
First order of skeleton expansion of 3-gluon vertex:



Simple power counting is possible again $\longrightarrow (p^2)^{-3\kappa + \frac{d}{2} - 2}$

Four-Gluon Vertex

First order of skeleton expansion is



Ghost loop is dominant again: $\rightarrow (p^2)^{-4\kappa + \frac{d}{2} - 2}$

Building Blocks

For the calculation of an arbitrary vertex function we can use these building blocks:

object	number of objects	scaling dimension
loop	l	$d/2$
internal ghost line	n_i	$\delta_{2,0} - 1 = -\kappa - 1$
internal gluon line	m_i	$\delta_{0,2} - 1 = 2\kappa + 1 - d/2$
ghost-gluon vertex	$v_{2,1}$	$1/2$
bare 3-gluon vertex	$v_{0,3}^b$	$1/2$
dressed 3-gluon vertex	$v_{0,3}$	$\delta_{0,3} + \frac{1}{2} = -3\kappa + d/2 - 3/2$
bare 4-gluon vertex	$v_{0,4}^b$	0
dressed 4-gluon vertex	$v_{0,4}$	$\delta_{0,4} = -4\kappa + d/2 - 2$

$2n$ Ghosts, m Gluons

Employing a skeleton expansion the calculation of the IR exponent of an arbitrary vertex function with $2n$ external ghosts and m external gluons is possible:

$$\rho_{2n,m} = (n-m)\kappa + (1-n)(\frac{d}{2} - 2).$$

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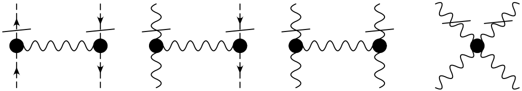
Higher orders of the skeleton expansion

Derivation is independent of order

$\rightarrow$ all orders have the same IR exponent

Alternatively: Show that the insertions for creating higher orders give no additional contribution.



Qualitative Behavior

The dimension dependence of the IR behavior of YM Green functions.

Dimension		4		3		2	
κ		$0.5953\ldots \approx 0.6$	1	$0.3976\ldots \approx 0.4$	0.5	0.2	0
Ghost	$-\kappa - 1$	-1.6	-2	-1.4	-1.5	-1.2	-1

- Ghost is divergent in all three values of space-time dimension.

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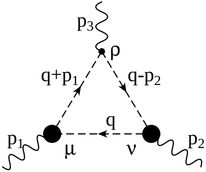
3-gluon	$-3\kappa + \frac{d}{2} - \frac{3}{2}$	-1.3	-2.5	-1.2	-1.5	-1.1	-0.5
4-gluon	$-4\kappa + \frac{d}{2} - 2$	-2.4	-4	-2.1	-2.5	-1.8	-1

- Ghost is divergent in all three values of space-time dimension.
- Gluon is vanishing in all three values of space-time dimension, except one case.
- Vertices have the same qualitative behavior in all three values of space-time dimension.

The Ghost Triangle

Analytic calculation of the ghost triangle confirms power-like behavior.



$$\Gamma^{gh-\Delta,IR}_{\mu\nu\rho}(p_1,p_2,p_3) =$$
$$\frac{N_c\,B^3\,g^3}{2}\int\frac{d^d q}{(2\pi)^d}\frac{(q+p_1)_\mu(q-p_2)_\rho q_\nu}{((q+p_1)^2)^{\kappa+1}((q-p_2)^2)^{\kappa+1}(q^2)^{\kappa+1}}$$

Used Method: Negative Dimension Integration (NDIM)

$\rightarrow$ Appell's function F_4

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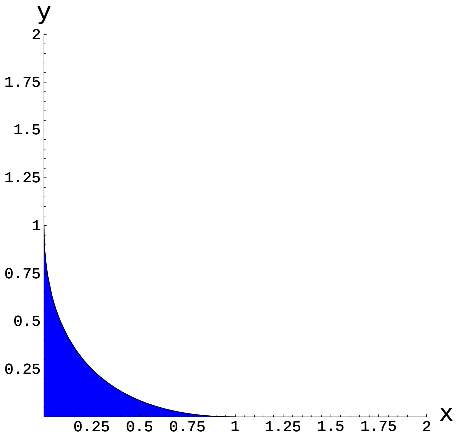

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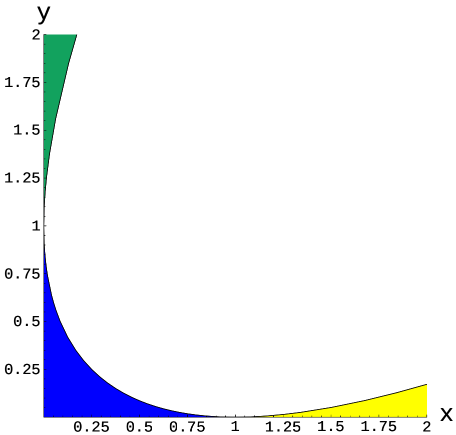

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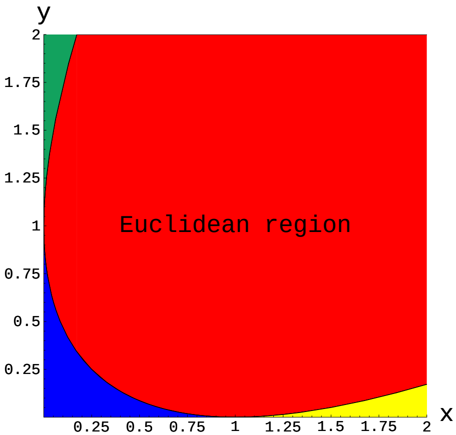
Solution for Euclidean region consists of several hypergeometric series.



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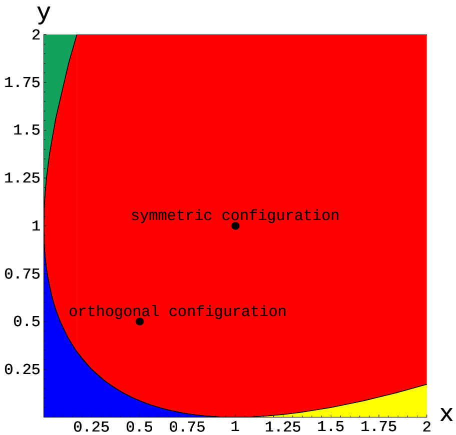
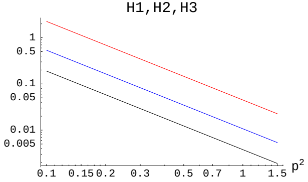
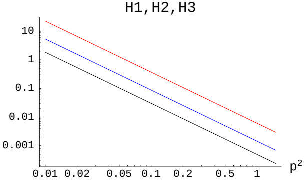


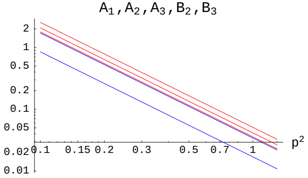
Tensor Components of the Ghost Triangle



d=3, symmetric point,
tensor basis of Celmaster-Gonsalves



d=4, symmetric point,
tensor basis of Celmaster-Gonsalves



d=3, orthogonal configuration,
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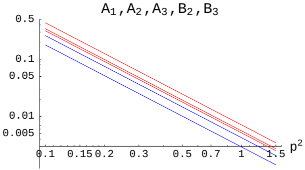

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Dependence on Infrared Exponent

How much influence has the numerical value of κ on the ghost triangle?

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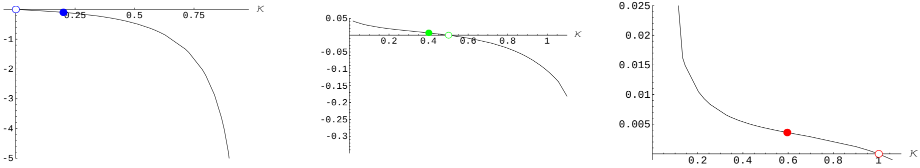
Overlap of the tree-level tensor with the ghost-triangle for $d = 2, 3$ and 4:



Dependence on κ is only weak

$\rightarrow$ ghost dominance seems to be a robust mechanism

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Gribov-Zwanziger confinement scenario
confirmed in 2 and 3 dimensions!!!