Infrared analysis of Yang-Mills theory in the maximally Abelian gauge and the Landau gauge

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FWF Der Wissenschaftsfonds.

SIC!QFT

Contents of the talk

- Infrared of Yang-Mills theory: What can we learn from it?
- Maximally Abelian gauge: Why do we need this complicated gauge, anyway? And what is its infrared behavior?
- Landau gauge: Does (partly) solving the Gribov problem change the infrared behavior?
- Non-perturbative tool: Dyson-Schwinger equations; is there an easy way to derive them?

Confinement of quarks and gluons

- Confinement is a long-range $\leftrightarrow$ IR phenomenon: We do not see individual $\sim$ infinitely separated quarks or gluons.
  What's the mechanism behind it?
- One expects that the property of being confined is encoded in the particles' propagators.
- Different confinement criteria for the propagators:
  - Positivity violations: negative norm contributions $\rightarrow$ not a particle of the physical state space
  - Kugo-Ojima: quartet mechanism, e. g. Gupta-Bleuler formalism in QED: time-like and longitudinal photon cancel each other.
    Landau gauge Yang-Mills, $p^2 \rightarrow 0$: $D_{gluon} \rightarrow 0$, $p^2 D_{ghost} \rightarrow \infty$
  - Gribov-Zwanziger (Landau gauge, Coulomb gauge): IR suppression of the gluon propagator due to Gribov horizon $\rightarrow$ no long-distance propagation.
    Already manifest at perturbative level with Gribov-Zwanziger Lagrangian!

Dyson-Schwinger equations (DSEs) for investigating QCD

Equations of motion of Green functions

Infinitely large tower of equations

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Pros:

- Exact equations
  $\longrightarrow$ non-perturbative regime accessible
- Continuum, different scales accessible
  $\longrightarrow$ complement lattice method

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Cons?:

- Truncations
- Gauge-dependent

Dyson-Schwinger equations (DSEs) for investigating QCD

Equations of motion of Green functions

Infinitely large tower of equations

Pros:

- Exact equations
  $\longrightarrow$ non-perturbative regime accessible
- Continuum, different scales accessible
  $\longrightarrow$ complement lattice method

Cons?:

- Truncations (not for all tasks)
- Gauge-dependent
  $\longrightarrow$ Exploit advantages of different gauges

Infrared regime of Yang-Mills theory in Landau gauge I

Scaling solution [Alkofer, Fischer, Gies, Maas, Pawlowski, von Smekal, ...]

- Dressing functions obey power laws. $\rightarrow$ Qualitative information provided by IR exponents.
- Qualitative IR solution of ALL correlation functions is known.
- Picture of confinement: IR vanishing gluon ($\rightarrow$ gluon confinement) and IR enhanced ghost propagator ($\rightarrow$ long-range force to confine quarks).
- Horizon condition/Kugo-Ojima $\leftrightarrow$ IR enhanced ghost.
- Gluon propagator violates positivity.
- Confining Polyakov loop potential [Braun, Gies, Pawlowski, arXiv:0708.2413].
- Method transferable to some other gauges ($\rightarrow$ MAG, lin. covariant gauge $\xi \neq 0$?).

Infrared regime of Yang-Mills theory in Landau gauge II

Decoupling solution

- Gluon massive, ghost tree-level like.
- Seen in most lattice calculations [Bogolubsky, Bornyakov, Cucchieri, Ilgenfritz, Maas, Mendes, Müller-Preussker, Pawlowski, Spielmann, Sternbeck, von Smekal, ...]. $\rightarrow$ Proof of unique solution?
- Adding condensates to the Gribov-Zwanziger action $\rightarrow$ refined Gribov-Zwanziger scenario [Dudal et al.]
- DSEs, FRGEs [Boucaud et al., Aguilar et al., Fischer et al.]
- Different renormalization of the ghost propagator $\Rightarrow$ tree-level like. $\leftrightarrow$ boundary condition for DSEs [Fischer et al., Ann. Phys. 324; Maas, 0907.5185]
- Gluon propagator violates positivity.
- Confining Polyakov loop potential [Braun, Gies, Pawlowski, arXiv:0708.2413].
- What is the mechanism for confinement?

Hypothesis of Abelian dominance

Dual superconductor picture of confinement [Mandelstam, 't Hooft]

- Picture a conventional superconductor, where the electric charges condense and force the magnetic flux into vortices.
- Change "electric" and "magnetic" components and you get a dual superconductor, where condensed magnetic monopoles squeeze the electric flux into flux tubes.
- QCD: No free chromoelectric charges. Are they confined by condensed magnetic monopoles?

Hypothesis of Abelian dominance [Ezawa, Iwazaki, PRD 25 (1981)]:
Magnetic monopoles live in Abelian part of the theory. $\longrightarrow$ Abelian part dominates in the IR?

Lattice results on Abelian IR dominance

Quenched QCD, linear rising potential between two quarks:

$$V(r) \sim \sigma\, r.$$

- σ_{Abel} (calculated from the Abelian part) is almost the same as σ.
- Suzuki et al. [PRD 80]: Without gauge fixing σ_{Abel} was extracted and agreed exactly with σ.
  Maybe MAG is a simple way to get monopoles?
- Available lattice results of MAG [Cucchieri, Mendes, Mihara, 2008]: all propagators massive, Abelian field has lowest mass
  $\Rightarrow$ other fields decouple. Realization of Abelian dominance.

Definition of the maximally Abelian gauge

Look for dominance of Abelian part. What is the Abelian part?
Gauge field components:

$$A_\mu = \boldsymbol{A}^i_\mu \boldsymbol{T}^i + \boldsymbol{B}^a_\mu \boldsymbol{T}^a, \quad i = 1, \ldots, N-1, \quad a = N, \ldots, N^2 - 1$$

Abelian subalgebra: $[T^i, T^j] = 0$, can be written as diagonal matrices

Abelian $\leftrightarrow$ diagonal fields $\boldsymbol{A}$,
non-Abelian $\leftrightarrow$ off-diagonal fields $\boldsymbol{B}$.

E.g. $T^1 = \frac{1}{2}\lambda^3$, $T^2 = \frac{1}{2}\lambda^8$ for $SU(3)$.
Which interactions are possible ($[T^r, T^s] = i f^{rst} T^t$)?

	$SU(2)$	$SU(N>2)$
f^{ijk}	0	0
f^{ija}	0	0
f^{iab}	✓	✓
f^{abc}	0	✓

$\Rightarrow$ 2 off-diagonal and 1 diagonal field can interact; 3 off-diagonal fields can only interact in $SU(N>2)$
$\rightarrow$ $SU(2)$ and $SU(3)$ different?

Gauge fixing condition

Stress role of diagonal fields $\Rightarrow$ minimize norm of off-diagonal field $\boldsymbol{B}$:

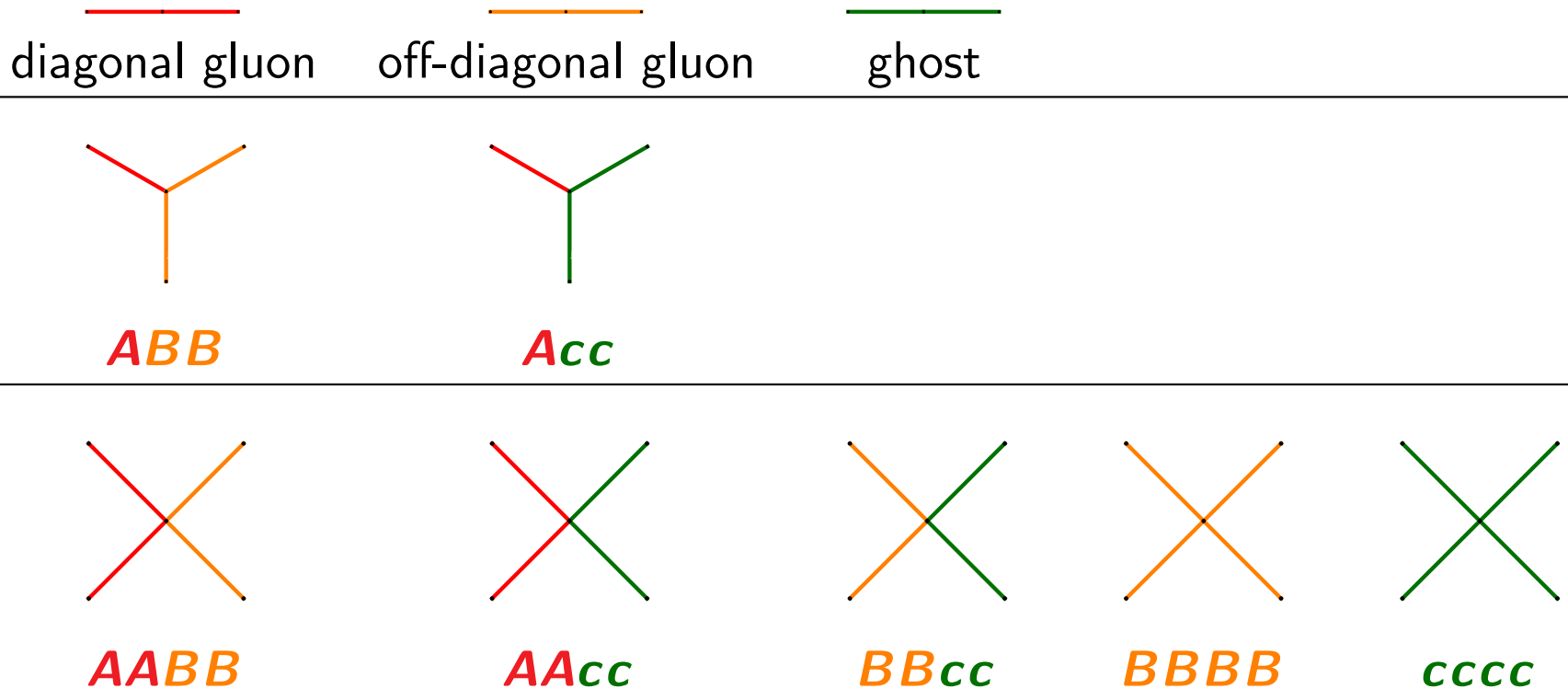
$$\|B_U\| = \int dx\, B_U^a B_U^a \rightarrow \text{minimize wrt. gauge transformations } U$$

$$D_\mu^{ab} \boldsymbol{B}_\mu^b = (\delta_{ab}\partial_\mu - g\, f^{abi} \boldsymbol{A}_\mu^i) \boldsymbol{B}_\mu^b = 0$$ non-linear gauge fixing condition!

Remaining symmetry of diagonal part: $U(1)^{N-1}$

Fix gauge of diag. gluon field $\boldsymbol{A}$ by Landau gauge condition: $\partial_\mu \boldsymbol{A}_\mu = 0$
$\Rightarrow$ diagonal ghosts decouple (like in QED).

Lagrangian for the MAG



$SU(N > 2)$

BBB *Bcc* *ABBB* *ABcc*

Deriving Dyson-Schwinger equations

Integral of a total derivative vanishes:

$$\int [D\phi] \frac{\delta}{\delta\phi} e^{-S+J\,\Phi} = \int [D\phi] \left(J - \frac{\delta S}{\delta\phi} \right) e^{-S+J\,\Phi} = 0.$$

$\Rightarrow$ DSEs for all Green functions (full, connected, 1PI) by further differentiations.

Doing it by hand?

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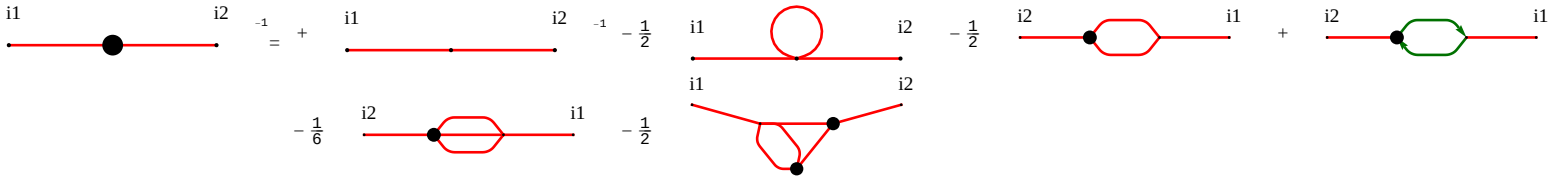
Doing it by hand?

Example: Landau gauge, only 2 propagators (***AA***, ***cc***), 3 interactions (***Acc***, ***AAA***, ***AAAA***)

Landau Gauge: Propagators

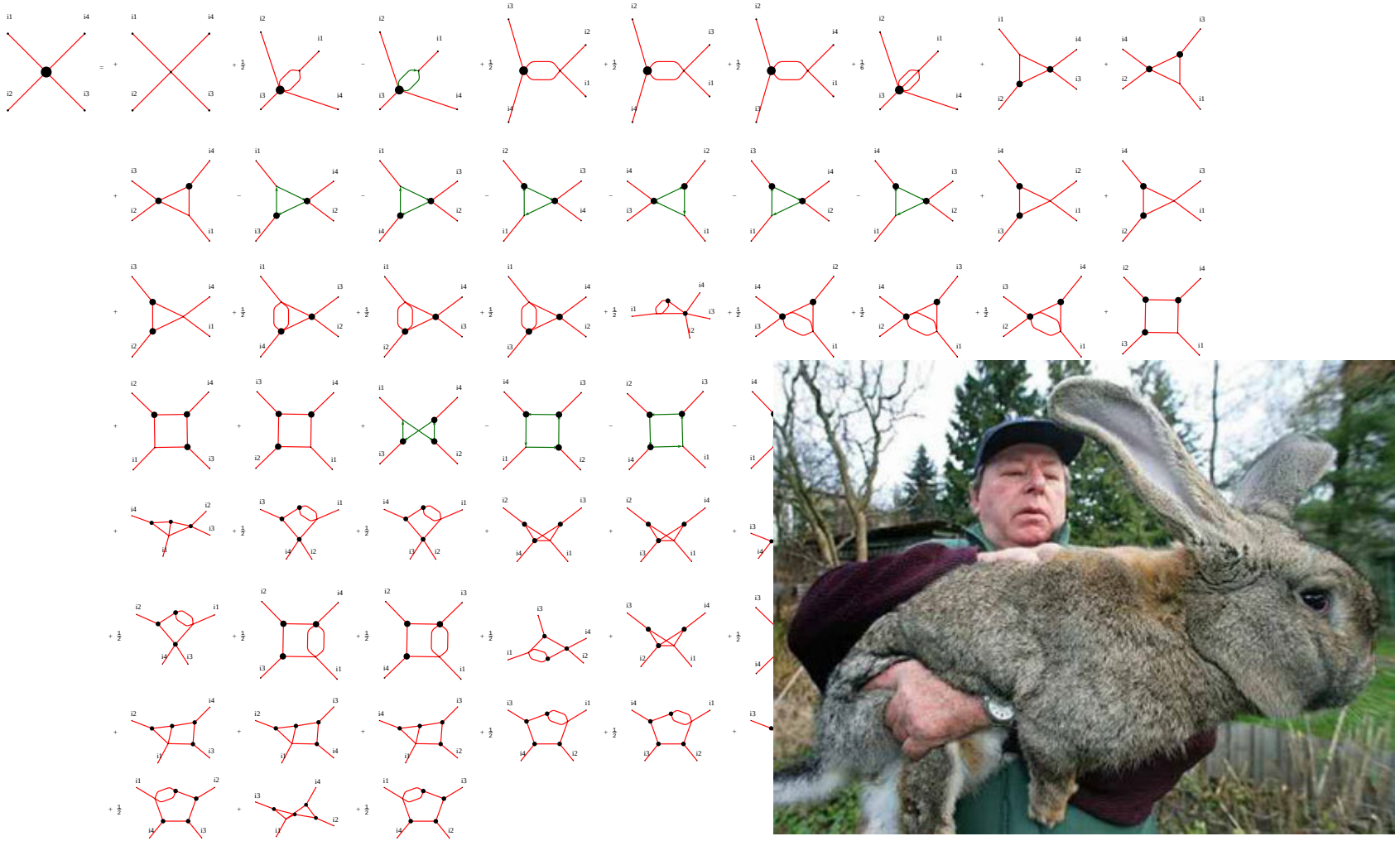
Gluon propagator:



Ghost propagator:

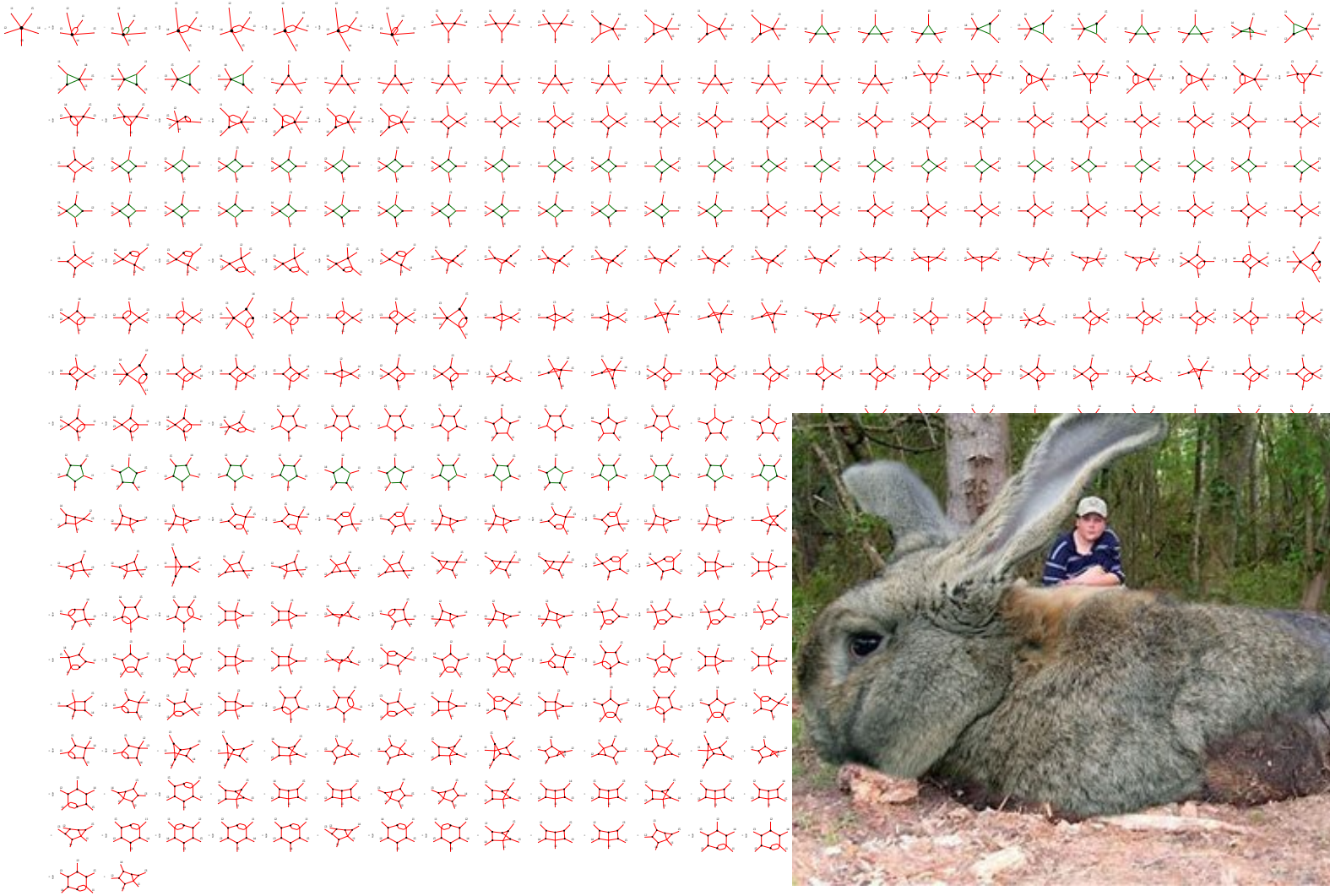
Landau Gauge: Four-Gluon Vertex

66 terms



Landau Gauge: Five-Gluon Vertex

434 terms



DoDSE

⇒ *DoDSE* [Alkofer, M.Q.H., Schwenzer, CPC 180 (2009)]

Given a structure of interactions, the DSEs are derived symbolically using *Mathematica*.

Example (Landau gauge):

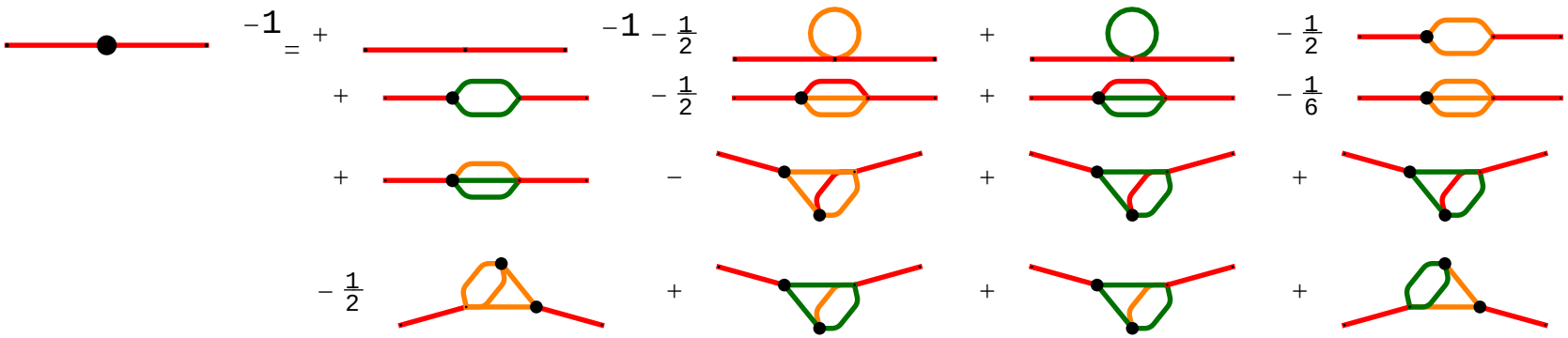
- only input: interactions in Lagrangian (AA, AAA, AAAA, cc, Acc)
- Which DSE do I want?

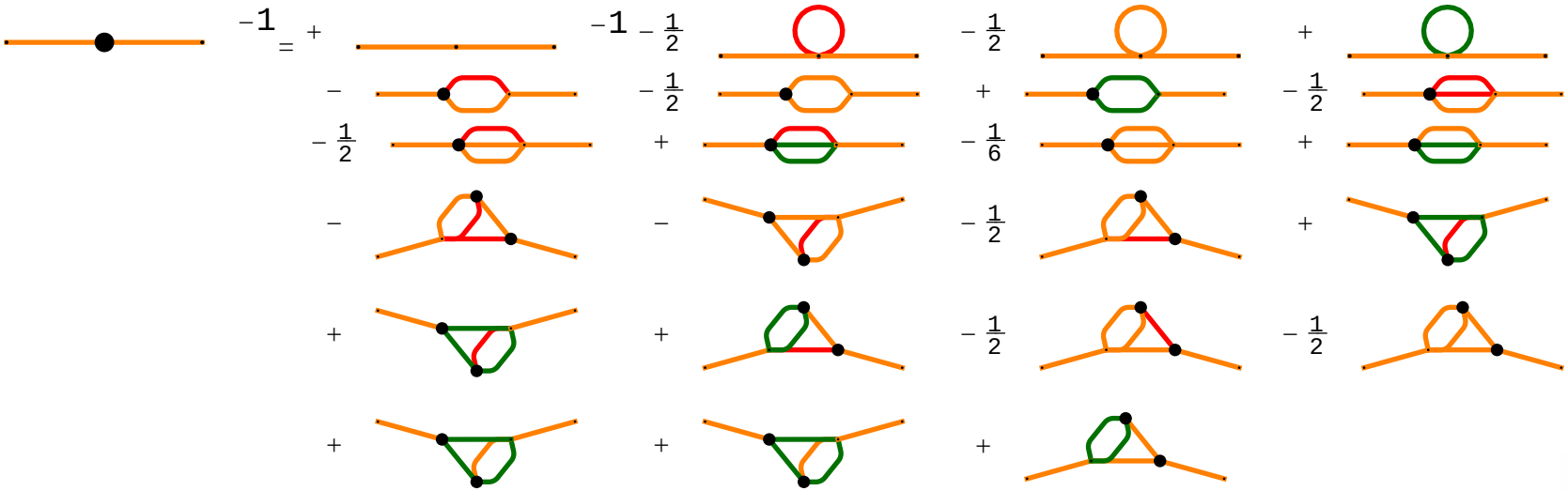
- Step-by-step calculations possible.
- Can handle mixed propagators (then there are really many diagrams; e. g. in Gribov-Zwanziger action).

Upgrade: *Symb2Alg*

Provide Feynman rules and get complete algebraic expressions.
→ E. g. calculate color algebra with *FORM* and integrals with *C*.

DSEs of the MAG





Infrared power counting

Generic propagator

$$T_{(ij)} \cdot \frac{D(p^2)}{p^2},$$

assume power law behavior at low p^2

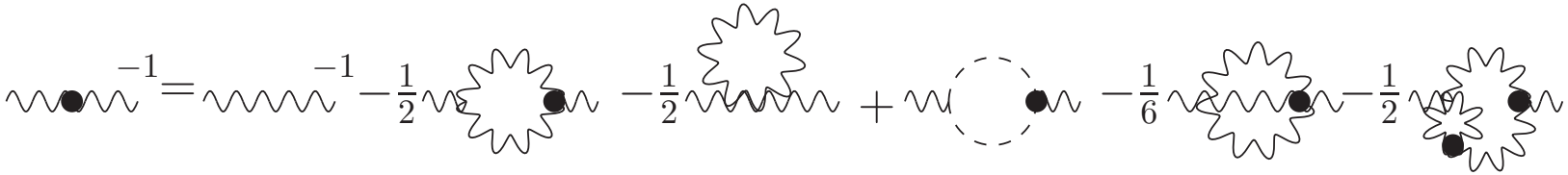
IR exponent

$$D^{IR}(p^2) = A \cdot (p^2)^{\delta}$$

- Vertices also assume power law behavior [Alkofer, Fischer, Llanes-Estrada, PLB 611 (2005) (skeleton expansion)].
- Limit of all momenta approaching zero simultaneously.
- Upon integration all momenta converted into powers of external momenta. $\Rightarrow$ Counting of IR exponents

System of inequalities

- IR exponent for every diagram
- lhs is dominated by at least one diagram on rhs and rhs cannot be more divergent than lhs. $\rightarrow \delta_{lhs} \leq \delta_{rhs,any\ diagram}$.
- Not knowing which diagram is leading on the rhs, we can write inequalities from all diagrams.



$$-\delta_{gl} \leq 2\delta_{gl} + \delta_{3g}, \qquad -\delta_{gl} \leq 2\delta_{gh} + \delta_{gg}, \qquad \ldots$$

That's the basic idea.

Still, for a large system a lot of work.

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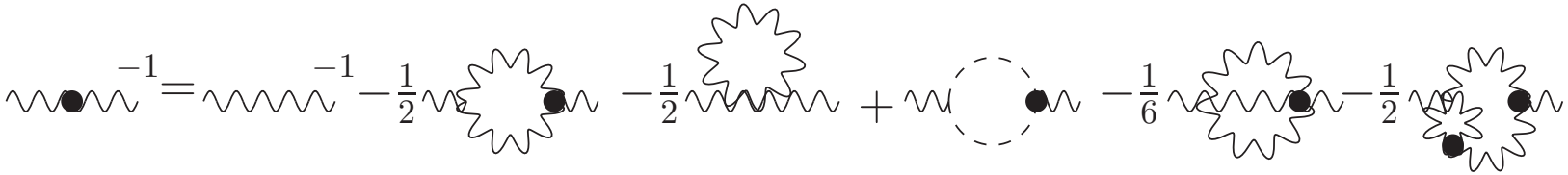

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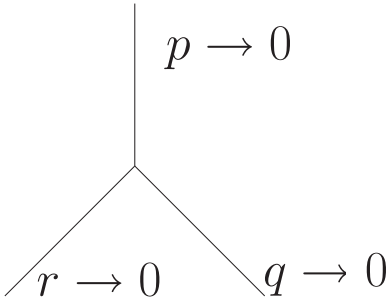
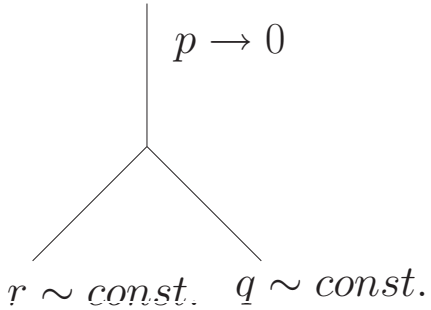
All inequalities relevant?

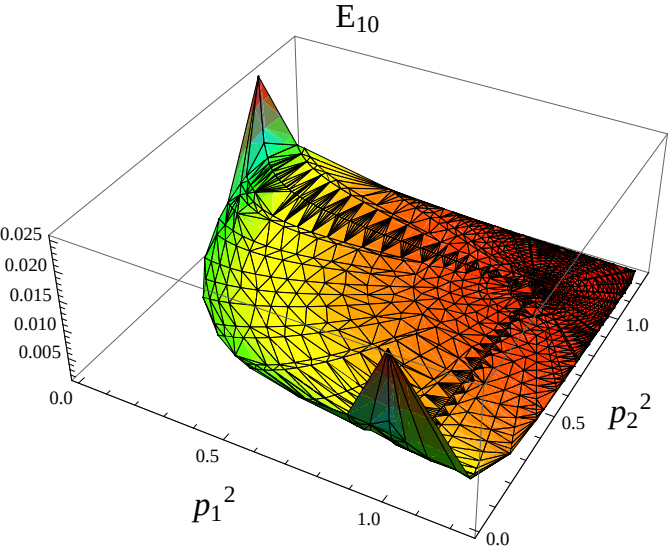
More IR exponents?



Uniform/global scaling: All external momenta go to zero simultaneously.

Kinematic scaling: Some external momenta go to zero simultaneously. $\longrightarrow$ Non-uniform dependence on momenta [Alkofer, M.Q.H., Schwenzer, EPJC 62 (2009)]





Additional singularities only in longitudinal parts!

[Alkofer, M.Q.H., Schwenzer, EPJC 62 (2009); Fischer, Pawlowski, PRD 80 (2009)]

Relevant inequalities

A closed form for all relevant inequalities can be derived from DSEs and FRGEs [Huber, Schwenzer, Alkofer, 0904.1873].

type		derived from	#
dressed vertices	$C_1 := \delta_{vertex} + \frac{1}{2} \sum_{\substack{\text{legs } j \text{ of} \\ \text{vertex}}} \delta_j \geq 0$	FRGEs	infinite
prim. div. vertices	$\boldsymbol{C_2 := \frac{1}{2} \sum_{\substack{\text{legs } j \text{ of} \\ \text{prim. div.} \\ \text{vertex}}} \delta_j \geq 0}$	DSEs+FRGEs	finite

Some inequalities are contained within others.
E. g. in MAG: $\delta_B \geq 0$ and $\delta_c \geq 0$ render $\delta_B + \delta_c \geq 0$ useless.

Scaling relations

General analysis of propagator DSEs

[M.Q.H., Schwenzer, Alkofer, arXiv:0804.1873]

- At least one inequality from a prim. divergent vertex has to be saturated, i. e. $\boxed{C_2^i = 0 \text{ for at least one } i}$.
- Necessary condition for a scaling solution.
- Related to bare vertices in DSEs: Fischer-Pawlowski consistency condition DSEs $\leftrightarrow$ FRGEs [Fischer, Pawlowski, PRD 75 (2007)].

$\Rightarrow$ One primitively divergent vertex is not IR enhanced.

The non-enhancement of at least one primitively divergent vertex is now established for all scaling type solutions.

How to obtain a scaling relation: Landau gauge

1. Look at all inequalities for primitively divergent vertices, i. e. at C_2^i.
2. Try all possibilities of $C_2^i = 0$.
3. Choose the non-trivial solutions.

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Application to Landau gauge:

1. $\delta_{gl} \geq 0$, $\delta_{gl} + 2\delta_{gh} \geq 0$
2. a $\delta_{gl} = 0$
   b $\delta_{gl} + 2\delta_{gh} = 0$
3. a $\cancel{\delta_{gl} = \delta_{gh} = 0}$
   b $\delta_{gl} + 2\delta_{gh} = 0$

Scaling relation of the Landau gauge: $\boxed{\frac{1}{2}\delta_{gl} = -\delta_{gh} = \kappa_{LG}}$

How to obtain a scaling relation: MAG

Many interactions $\Rightarrow$ many inequalities, but some of them are contained within others $\Rightarrow$ reduces number of possibilities.

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Application to the MAG:

1. $\delta_B \geq 0$, $\delta_c \geq 0$, $\delta_A + \delta_B \geq 0$, $\delta_A + \delta_c \geq 0$
2.
   a $\delta_B = 0$
   b $\delta_c = 0$
   c $\delta_A + \delta_B = 0$
   d $\delta_A + \delta_c = 0$
3.
   a $\cancel{\delta_A = \delta_B = \delta_c = 0}$
   b $\cancel{\delta_A = \delta_B = \delta_c = 0}$
   c $\delta_A + \delta_B = 0$
   d $\delta_A + \delta_c = 0$

Scaling relation of the MAG: $\boxed{\delta_B = \delta_c = -\delta_A = \kappa_{MAG} \geq 0}$

IR scaling solution of the MAG

$$\boxed{\delta_B = \delta_c = -\delta_A = \kappa_{MAG} \geq 0}$$

- The Abelian fields are IR enhanced. $\rightarrow$ Realization of Abelian dominance?
- Off-diagonal fields are IR suppressed.
- $SU(2)$ and $SU(N > 2)$ have the same solution.
- Qualitative solutions for tower of all Green functions.

Two-loop diagrams are IR leading (sunset, squint). $\rightarrow$ UV/IR preserving truncation?

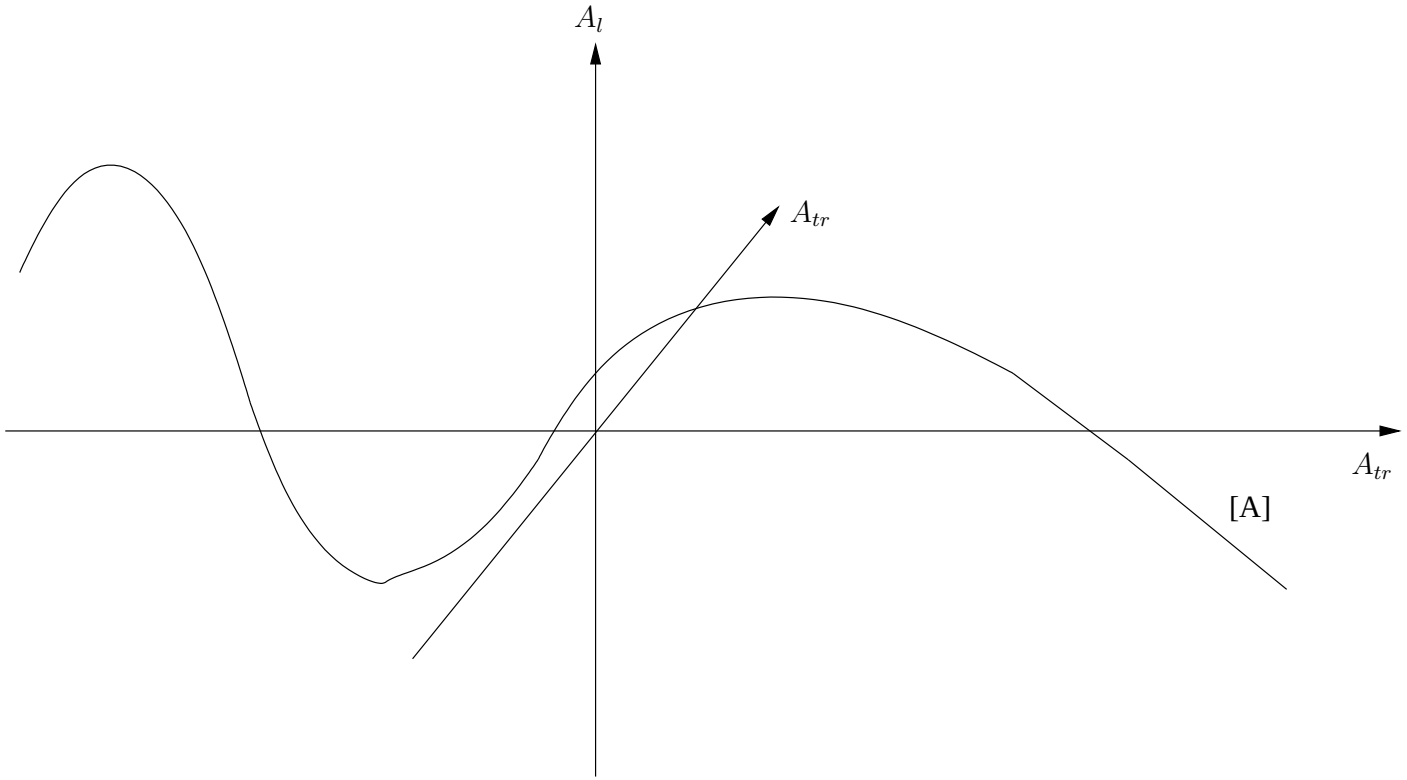
Relation Landau gauge & MAG

Landau gauge	maximally Abelian gauge
ghost dominance	Abelian (gluon) dominance
Gribov region bounded	Gribov region unbounded in diagonal direction [Capri et al., PRD79]

Greensite, Olejnik, Zwanziger, PRD78:

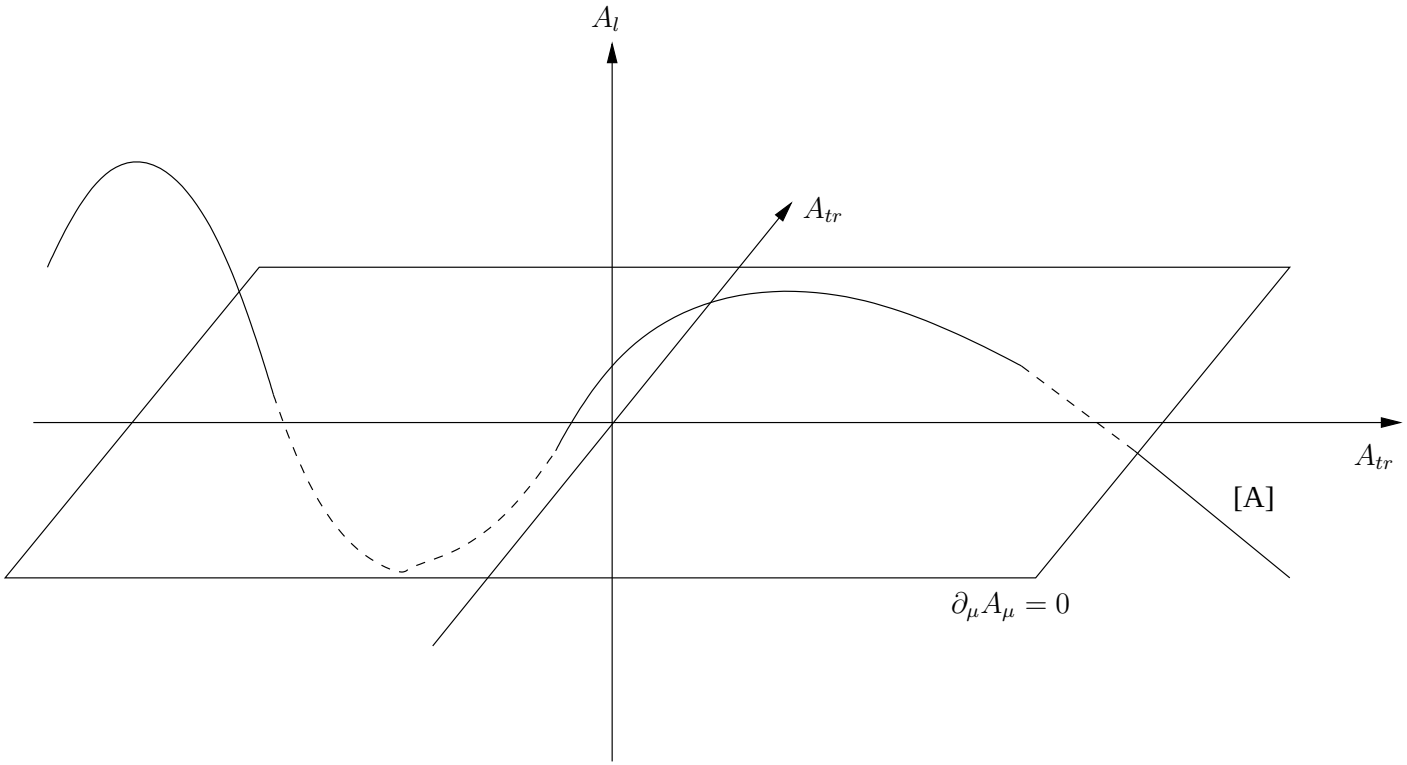
Abelian configurations $\xrightarrow{\text{Landau gauge}}$ on Gribov horizon

Gauge orbits and Gribov copies



Gauge equivalent configurations (gauge orbit $[A]$) $\Rightarrow$ integration in path integral is overcomplete.

Gauge orbits and Gribov copies



Faddeev and Popov: Restriction of integration to single representative of each gauge orbit possible? Gauge symmetry replaced by BRST symmetry!

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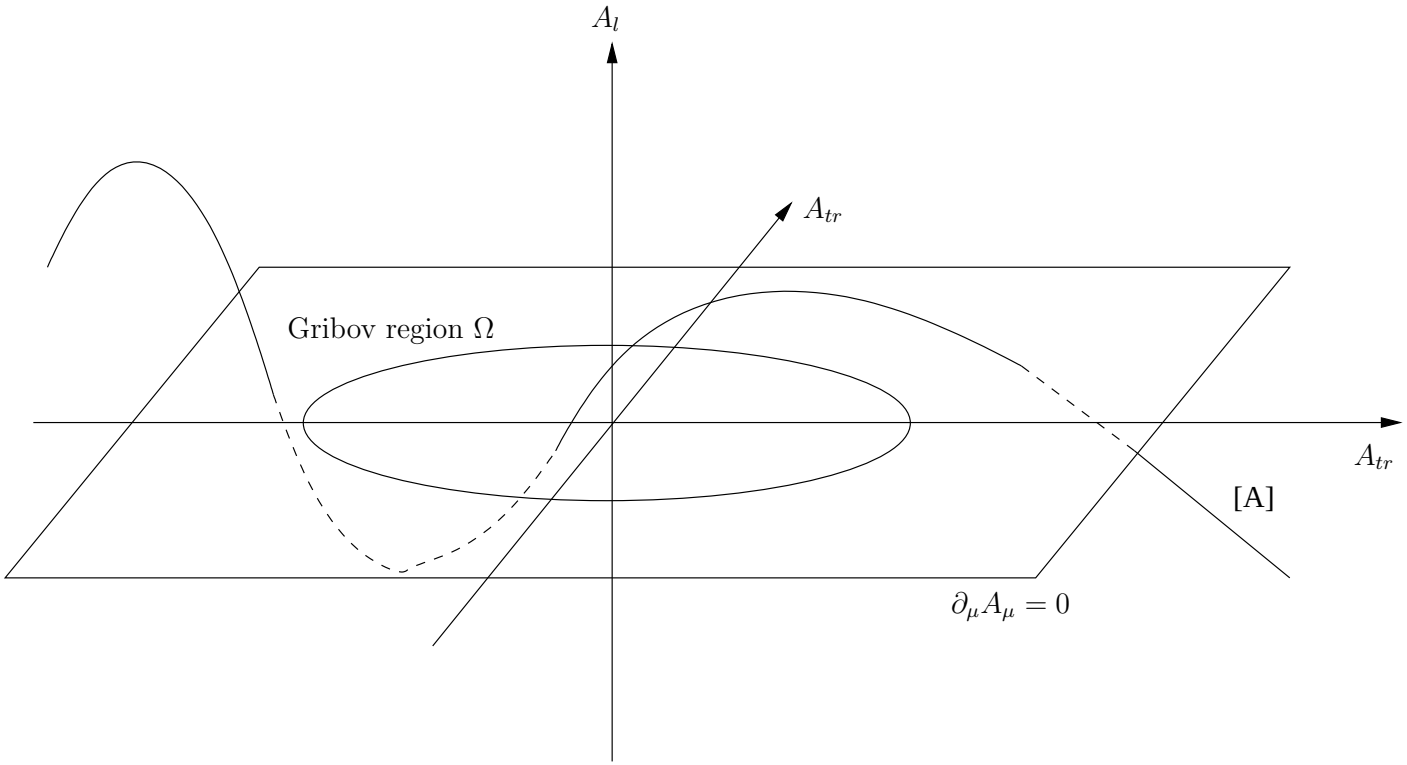
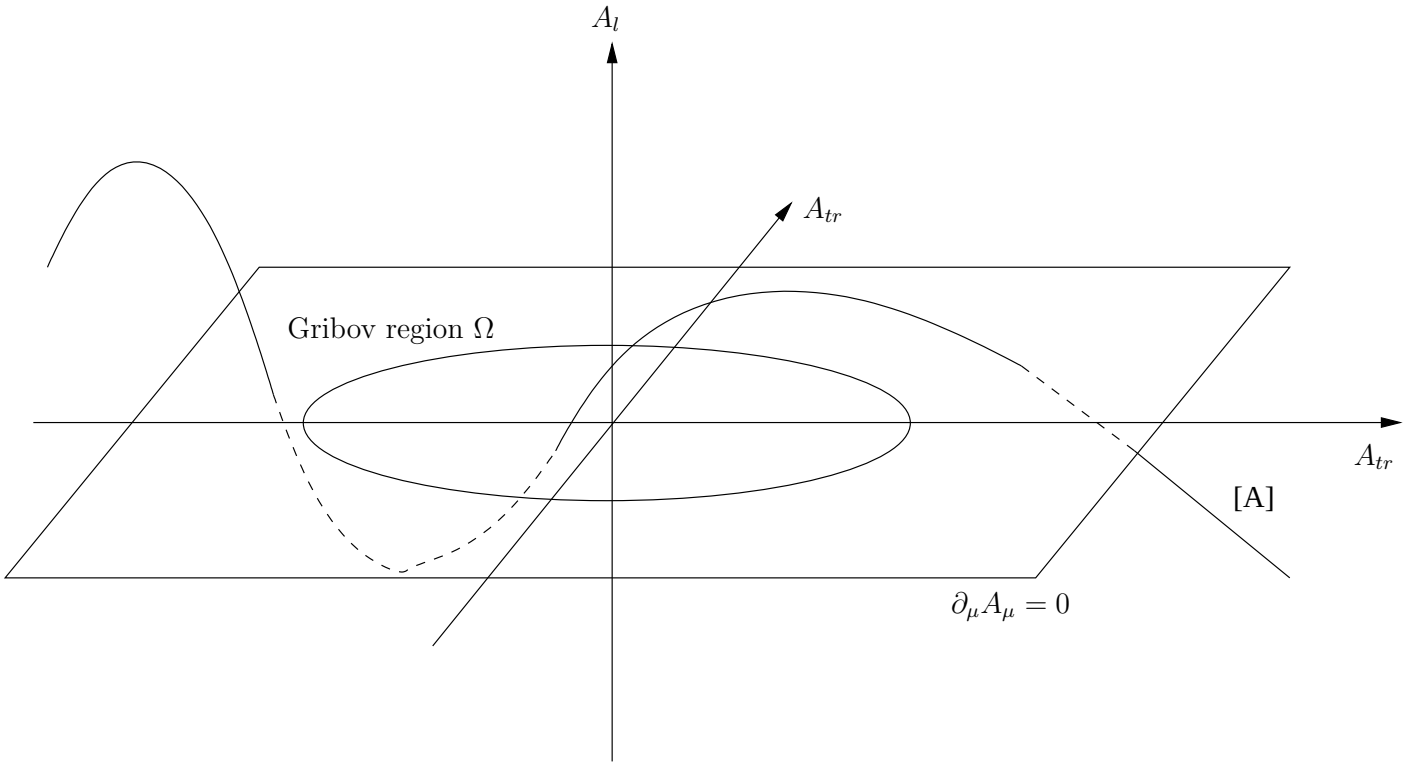


Restriction to Gribov horizon: almost unique gauge fixing.

Gauge orbits and Gribov copies



Restriction to Gribov horizon: almost unique gauge fixing.

Restriction to Gribov region is done via adding a non-local term to the Lagrangian. $\longrightarrow$ New parameter γ, determined by horizon condition.

How do DSEs usually deal with this?

Integral of a total derivative vanishes:

$$\int [D\phi] \frac{\delta}{\delta\phi} e^{-S+J\,\Phi} = \int [D\phi] \left(J - \frac{\delta S}{\delta\phi} \right) e^{-S+J\,\Phi} = 0.$$

$\Rightarrow$ DSEs for all Green functions (full, connected, 1PI) by further differentiations.

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Integral of a total derivative vanishes [Zwanziger, PRD65]:

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$$\int_{\Omega} [D\phi] \left(J - \frac{\delta S}{\delta\phi} \right) \delta(\partial \cdot A) \boldsymbol{det(M)} e^{-S_{YM}+J\,\Phi} = 0.$$

$$det(M)\Big|_{\Omega} = 0$$

Local renormalizable action

Non-local term can be localized with auxiliary fields $(\bar{\varphi}^{ab}_\mu,\ \varphi^{ab}_\mu,\ \bar{\omega}^{ab}_\mu,\ \omega^{ab}_\mu) \rightarrow$ local Gribov-Zwanziger action:

$$\mathcal{L}_{GZ} = \bar{\varphi}^{ac}_\mu M^{ab} \varphi^{bc}_\mu - \bar{\omega}^{ac}_\mu M^{ab} \omega^{bc}_\mu + \gamma^2 g\, f^{abc} A^a_\mu (\varphi^{bc}_\mu - \bar{\varphi}^{bc}_\mu) - \gamma^4 d(N^2-1)$$

Horizon condition in local form:

$$\langle g\, f^{abc} A^a_\mu (\varphi^{bc}_\mu - \bar{\varphi}^{bc}_\mu)\rangle = 2\gamma^2\, d(N^2-1),$$

- Restriction breaks BRST invariance.
- Mixing at the level of two-point functions, e. g. $\langle A^a_\mu \varphi^{bc}_\nu \rangle$.
  $\Rightarrow$ (3x3)-matrix relation between propagators and two-point functions:
  $$D^{\phi\phi} = (\Gamma^{\phi\phi})^{-1}, \qquad \phi \in \{A, \varphi, \bar{\varphi}\}$$

$\Rightarrow$ non-trivial relation between IR exponents of propagators and two-point functions

More fields ...

Simplify to (2x2)-matrix relation by splitting into real and imaginary part [Zwanziger, 0904.2380]:

$$\varphi = \frac{1}{\sqrt{2}}\,(U + i\,V)\,, \quad \bar{\varphi} = \frac{1}{\sqrt{2}}\,(U - i\,V)\,.$$

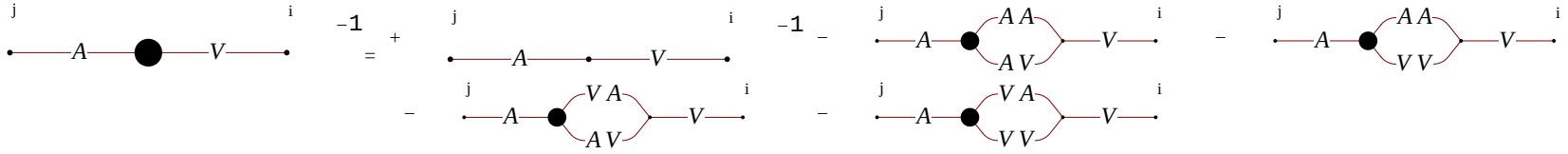
$$\begin{aligned}
\mathcal{L}'_{GZ} &= \mathcal{L}_U + \mathcal{L}_V + \mathcal{L}_{UV} - \bar{\omega}^{ac}_\mu M^{ab} \omega^{bc}_\mu\,, \\
\mathcal{L}_U &= \frac{1}{2} U^{ac}_\mu\, M^{ab}\, U^{bc}_\mu\,, \\
\mathcal{L}_V &= \frac{1}{2} V^{ac}_\mu\, M^{ab}\, V^{bc}_\mu + \boldsymbol{i\, g\, \gamma^2 \sqrt{2} f^{abc} A^a_\mu V^{bc}_\mu}\,, \\
\mathcal{L}_{UV} &= \frac{1}{2} i\, g f^{abc} U^{ad}_\mu V^{bd}_\mu \partial_\nu A^c_\nu \overset{LG}{=} 0,
\end{aligned}$$

Simplify even further:

$c,\ \bar{c},\ U,\ \omega,\ \bar{\omega} \longrightarrow \eta,\ \bar{\eta}$

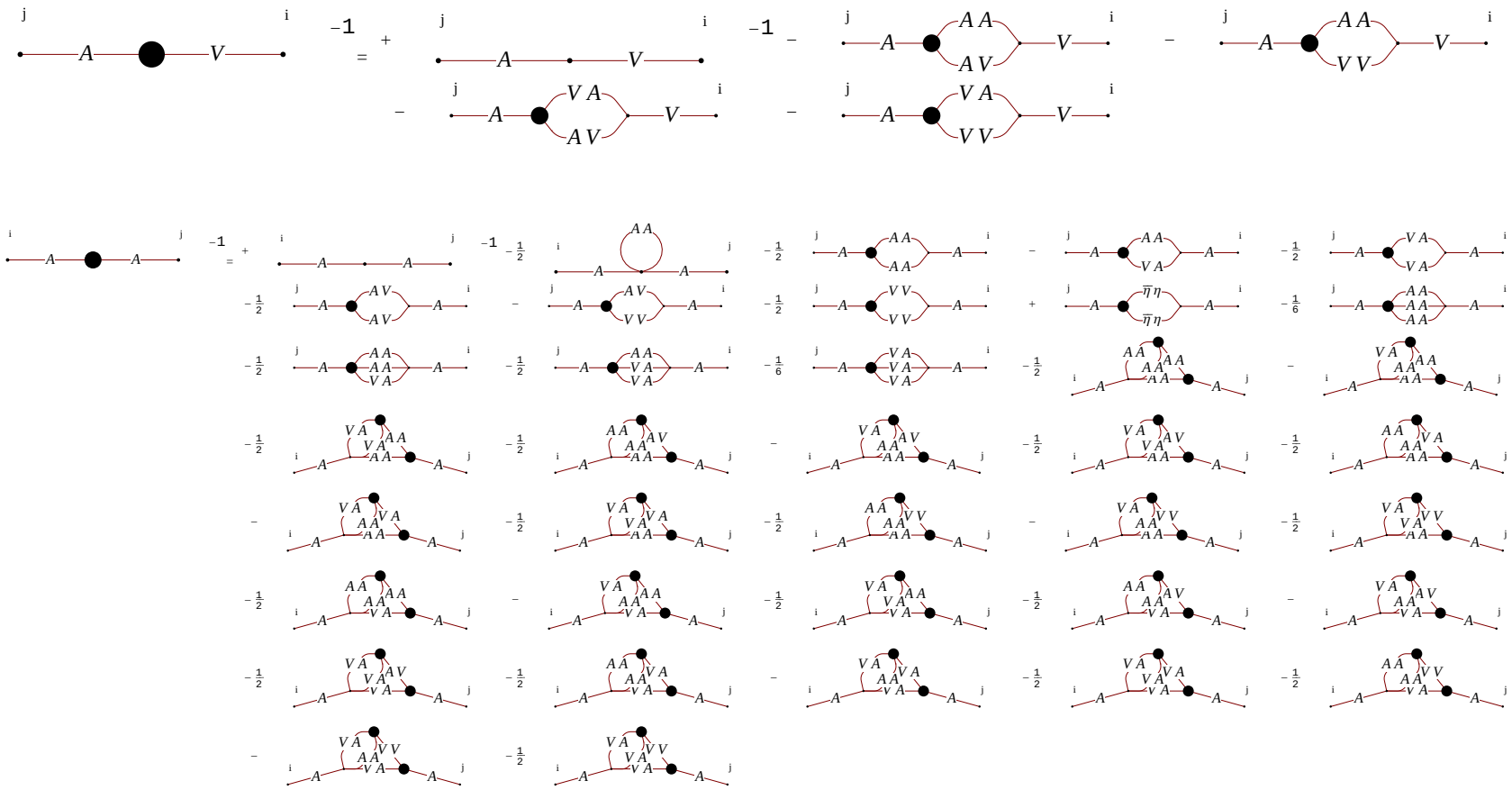
DSEs of Gribov-Zwanziger action

Just to give an impression:



DSEs of Gribov-Zwanziger action

Just to give an impression:



Complete analysis of all diagrams!

Truncation of tensors

Propagators $\langle A^a_\mu V^{bc}_\nu \rangle$ and $\langle V^{ab}_\mu V^{cd}_\nu \rangle$ can have many tensors with different dressing functions,
e. g. color space: f^{abc}; $\delta^{ab}\delta^{cd}$, $\delta^{ac}\delta^{bd}$, $\delta^{ad}\delta^{bc}$, $f^{abe}f^{cde}$, $f^{ace}f^{bde}$.

<u>Truncation:</u> Take only tree-level tensors of two-point functions.

$$\Gamma^{\phi\phi} = \begin{pmatrix} \Gamma^{AA} & \Gamma^{AV} \\ \Gamma^{VA} & \Gamma^{VV} \end{pmatrix},$$

$$\Gamma^{AA,ac}_{\mu\nu} = \delta^{ac} p^2 c_A^{\perp}(p^2) P_{\mu\nu} + \delta^{ac} \frac{1}{\xi} c_A^{\parallel}(p^2) p_\mu p_\nu,$$

$$\Gamma^{VV,abcd}_{\mu\nu} = \delta^{ac}\delta^{bd} p^2 c_V(p^2) g_{\mu\nu},$$

$$\Gamma^{AV,cab}_{\mu\nu} = f^{cab} i\, p^2 c_{AV}(p^2) g_{\mu\nu},$$

$c_{ij}(p^2)$ are dressing functions.

Propagators of the GZ action

$$D^{\eta\bar{\eta},ab}_{cd} = (\Gamma^{\eta\bar{\eta},ab}_{cd})^{-1} = -\delta^{ab}\delta^{cd}\frac{c_\eta(p^2)}{p^2}$$

D^{VV} has two tensors $\longrightarrow$ non-trivial truncation:

$$D^{AA,ab}_{\mu\nu} = \delta^{ab}\frac{1}{p^2}P_{\mu\nu}\frac{c_V(p^2)}{c_A^\perp(p^2)c_V(p^2)+2N\,c_{AV}^2(p^2)},$$

$$D^{VV,abcd}_{\mu\nu} = \frac{1}{p^2}\frac{1}{c_V(p^2)}\delta^{ac}\delta^{bd}g_{\mu\nu} - f^{abe}f^{cde}\frac{1}{p^2}P_{\mu\nu}\frac{2c_{AV}^2(p^2)}{c_A^\perp(p^2)c_V^2(p^2)+2N\,c_{AV}^2(p^2)c_V(p^2)},$$

$$D^{AV,abc} = -i\,f^{abc}\frac{1}{p^2}P_{\mu\nu}\frac{\sqrt{2}c_{AV}(p^2)}{c_A^\perp(p^2)c_V(p^2)+2N\,c_{AV}^2(p^2)}$$

Appearance of the determinant $c_A^\perp(p^2)c_V(p^2)+2N\,c_{AV}^2(p^2)$

The four possibilities

Which part of the determinant $c_A^{\perp}(p^2)c_V(p^2) + 2N\, c_{AV}^2(p^2)$ dominates in the IR?

$$c_{ij}(p^2) = d_{ij} \cdot (p^2)^{\kappa_{ij}}$$

I: $c_{AV}^2 > c_A c_V \leftrightarrow \kappa_A + \kappa_V > 2\kappa_{AV}$

II: $c_A c_V > c_{AV}^2 \leftrightarrow 2\kappa_{AV} > \kappa_A + \kappa_V$

III: $c_{AV}^2 \sim c_A c_V \leftrightarrow \kappa_A + \kappa_V = 2\kappa_{AV}$, no cancelations

IV: $c_{AV}^2 \sim c_A c_V \leftrightarrow \kappa_A + \kappa_V = 2\kappa_{AV}$, cancelations

Cancelations: Leading contributions cancel and some less dominant term takes over.

The four possibilities

Which part of the determinant $c_A^{\perp}(p^2)c_V(p^2) + 2N\, c_{AV}^2(p^2)$ dominates in the IR?

$$c_{ij}(p^2) = d_{ij} \cdot (p^2)^{\kappa_{ij}}$$

I: $\cancel{c_{AV}^2 > c_A c_V \leftrightarrow \kappa_A + \kappa_V > 2\kappa_{AV}}$

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III: $c_{AV}^2 \sim c_A c_V \leftrightarrow \kappa_A + \kappa_V = 2\kappa_{AV}$, no cancelations

IV: $\cancel{c_{AV}^2 \sim c_A c_V \leftrightarrow \kappa_A + \kappa_V = 2\kappa_{AV}}$, cancelations

Cancelations: Leading contributions cancel and some less dominant term takes over.

Two solutions lead to inconsistencies [M.Q.H., R. Alkofer, S. P. Sorella, 0910.5604].

Case II: Recovery of standard Landau gauge solution

$$c_A c_V > c_{AV}^2 \leftrightarrow \kappa_A + \kappa_V < 2\kappa_{AV}$$

- The VV-propagator becomes

$$\frac{1}{p^2}\frac{1}{c_V(p^2)}\delta^{ac}\delta^{bd}g_{\mu\nu} \qquad \longrightarrow \qquad \kappa_V = \kappa_\eta$$

  $\Rightarrow$ VV-propagator could be integrated out in the IR and the FP theory is recovered exactly.
- All contributions containing an AV-propagator are suppressed. $\Rightarrow$ DSEs reduce in the IR to the same system as in FP theory.
- Formula for IR exponent of arbitrary n-point functions is obtained.
- IR exponent of AV-2-point function is not fixed by scaling relation; calculated numerically.
  Several solutions: 0.0668776, 0.981386 and higher.

In the IR this is completely the same as FP theory!

Case III: The "strict" scaling solution

All IR exponents are connected by the scaling relations ($\kappa := \kappa_V = \kappa_\eta$):

$$\kappa_A + 2\kappa = \kappa + 2\kappa_{AV} = 0$$

Mixed propagator: $\delta_{AV} = \kappa/2 \Rightarrow$ Less pronounced IR suppression than in case II ($\delta_{AV} > \kappa/2$).

The determinant remains as it is. $\Rightarrow$ Non-linear relations between the coefficients of the dressing functions.

Summary Gribov-Zwanziger action

[M.Q.H., Alkofer, Sorella, 0910.5604, to appear in PRD]

Explicitly restricted integration to Gribov region by using the Gribov-Zwanziger action.

Mixed propagators complicate the analysis. Two candidates remain:

- All solutions have the same qualitative behavior.
- Mixed propagator IR suppressed.
- Scaling relation between FP ghost and gluon unaltered: $\kappa_A + 2\kappa_c = 0$.
- Input for numerical solution of the equations.

Both cases reproduce the
qualitative behavior of the Gribov-Zwanziger and Kugo-Ojima scenarios.

Summary maximally Abelian gauge [M.Q.H., Schwenzer, Alkofer, 0904.1873]

- Existence and form of scaling solutions can easily be obtained directly from the interactions.
- Fischer-Pawlowski consistency condition: one vertex remains bare in the IR.
- Scaling solution may exist in MAG:
  - Abelian gluon propagator is IR enhanced. $\rightarrow$ Support of hypothesis of Abelian dominance.
  - Complete numerical solution required. $\leftarrow$ Input from asymptotic behavior
  - Two-loop terms are IR leading $\leftrightarrow$ UV/IR preserving truncation?
  - Relation to chromomagnetic monopoles?

The end

Thank you very much for your attention.

IR Scaling solutions for other gauges

The analysis can be used also for other gauges. Beware: This corresponds to a naive application!

Linear covariant gauges	Ghost-antighost symmetric gauges
scaling solution only, if the longitudinal part of the gluon propagator gets dressed, but gauge fixing condition $\Rightarrow$ longitudinal part bare	quartic ghost interaction $\rightarrow \delta_{gh} \geq 0$ $\rightarrow$ with non-negative IREs only the trivial solution can be realized

This is valid for all possible dressings and agrees with the results from [Alkofer, Fischer, Reinhardt, v. Smekal, PRD 68 (2003)], where only certain dressings were considered.

$\Longrightarrow$

- Either the existence of a scaling solution is something special (?) or
- a more refined analysis (symmetries $\leftrightarrow$ cancelations) is needed in these cases.