n-point functions in d dimensions

(of Landau gauge Yang-Mills theory)

Markus Q. Huber

Institute of Theoretical Physics, Giessen University
Institute of Physics, University of Graz

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"Hadrons in Vacuum, Nuclei and Stars"

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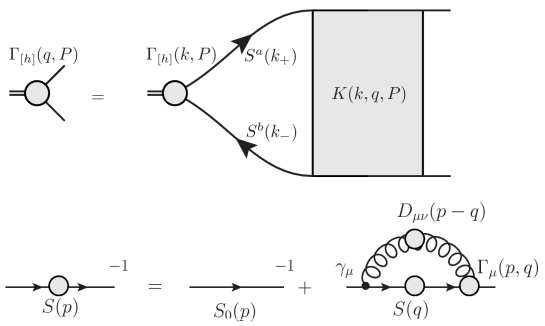
Motivation: Where Yang-Mills theory is important in QCD

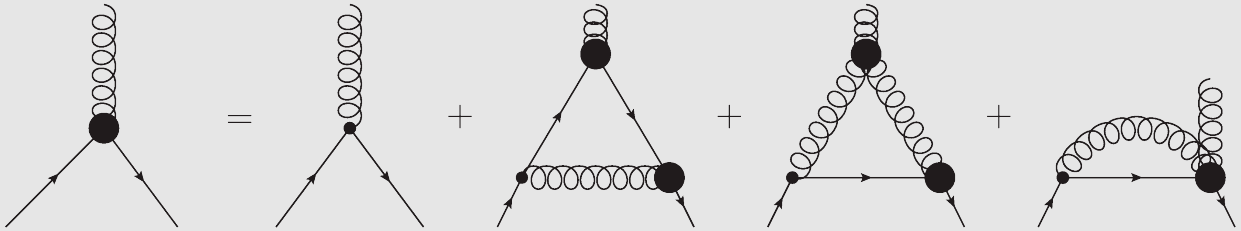
Widely used truncation: Rainbow + ladder + variant of Maris-Tandy interaction

→ Talks by Eichmann, Sanchis Alepuz



How to reduce model dependence

- Improve kernel K
- Use explicit gluon propagator + quark-gluon vertex



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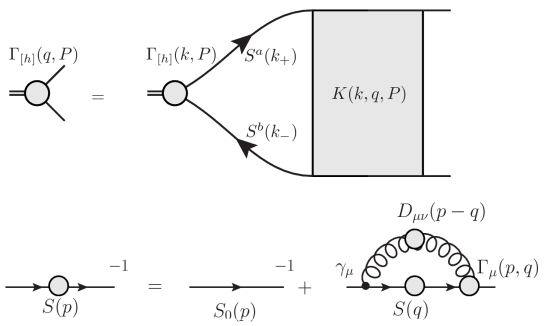

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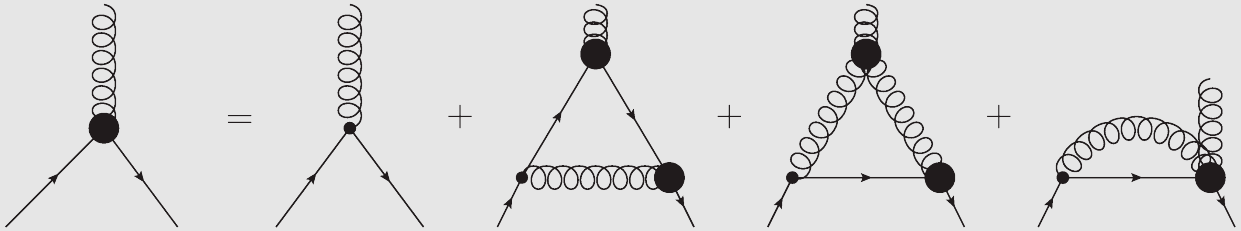
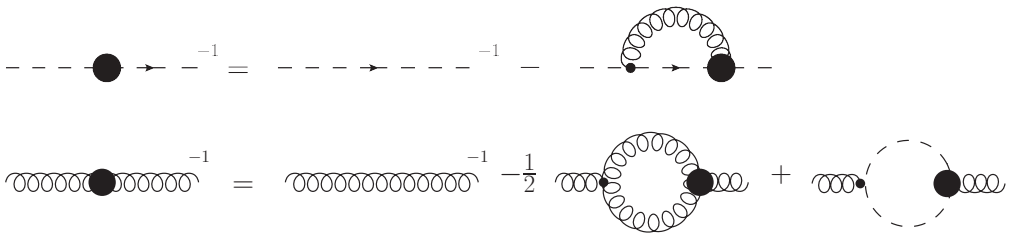


⟶ Full control over the gluonic sector is needed for self-contained calculations.

- Gluon propagator
- Three-gluon vertex
- . . . ?
- Glueballs

Progress in n-point functions: 2, 3, 4, . . .

Status 2010:
Truncated propagator equations,
modeled vertices



Beyond that (with continuum approaches):

- Three-point functions
- Four-point functions
- Two-loop diagrams
- Other functional equations
  $\rightarrow$ Talks by Mitter, Pawlowski

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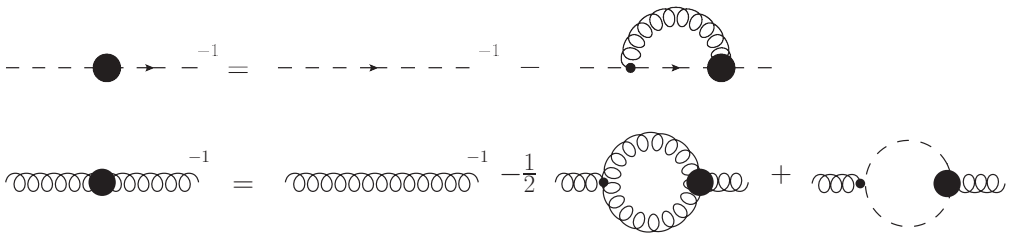

Beyond that (with continuum approaches):

- Three-point functions, e.g., [Schleifenbaum et al. '04; Boucaud et al. '11; Dudal et al. '12; MQH et al. '12; Aguilar et al. '13; Blum et al. 14; Eichmann et al. '14; MQH '17]
- Four-point functions, e.g., [Cyrol et al. '15; MQH '17]
- Two-loop diagrams, e.g., [Mader, Alkofer '13; Hopfer '14; Meyers, Swanson '14; MQH '17]
- Other functional equations: FRG [fQCD collaboration], n-PI effective action
  $\rightarrow$ Talks by Mitter, Pawlowski

Progress in n-point functions: 2, 3, 4, ...

Early models *optimized* to achieve/improve results: Tuning possible. f

Replacement by dynamic quantities:

- Some freedom is lost, system is more constrained.
- $\rightarrow$ New problems emerge, e.g., realization of perturbative resummation.

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Testing truncations:

- Hierarchy of diagrams
- Extensions of truncations:

$\rightarrow$ Two-loop terms

$\rightarrow$ Non-primitively divergent correlation functions

Landau gauge QCD

$$\mathcal{L} = \bar{\boldsymbol{q}}(-\not{D} + m)\boldsymbol{q} + \frac{1}{2}F^2 + \mathcal{L}_{gf} + \mathcal{L}_{gh}$$

$$F_{\mu\nu} = \partial_\mu \boldsymbol{A}_\nu - \partial_\nu \boldsymbol{A}_\mu + i\,g\,[\boldsymbol{A}_\mu, \boldsymbol{A}_\nu]$$

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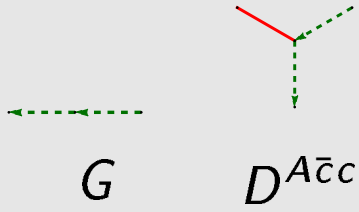
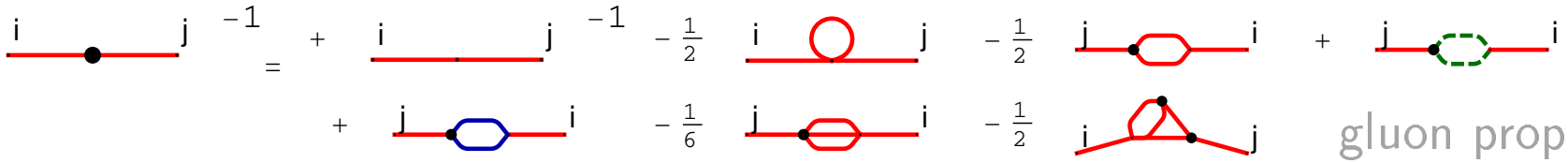
Landau gauge

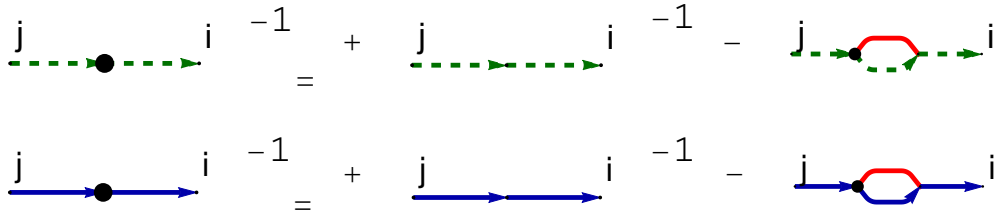
- simplest one for functional equations
- $\partial_\mu \boldsymbol{A}_\mu = 0$: $\quad \mathcal{L}_{gf} = \frac{1}{2\xi}(\partial_\mu \boldsymbol{A}_\mu)^2, \quad \xi \to 0$
- requires ghost fields: $\quad \mathcal{L}_{gh} = \bar{\boldsymbol{c}}\,(-\Box + g\,\boldsymbol{A}\times)\,\boldsymbol{c}$



The tower of DSEs



gluon propagator



ghost propagator

quark propagator

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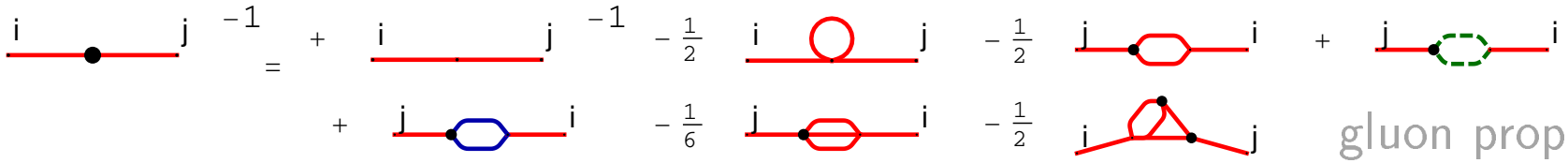

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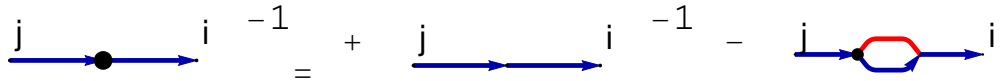

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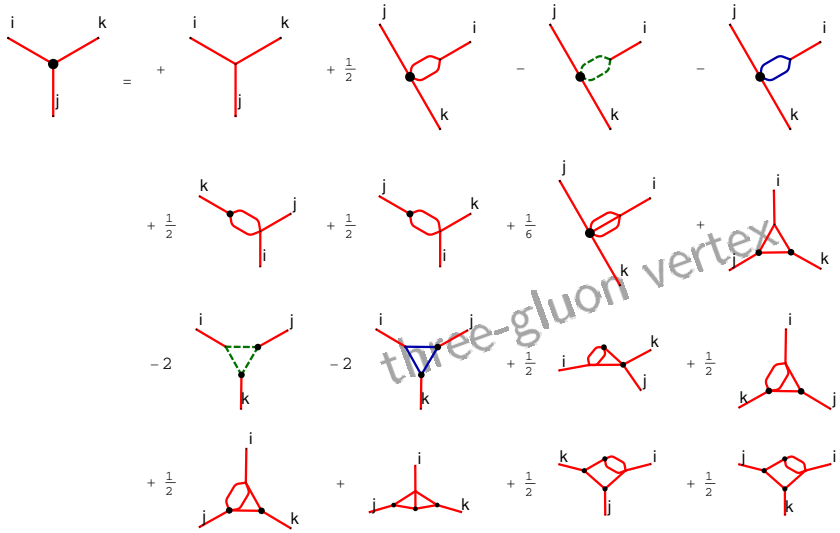

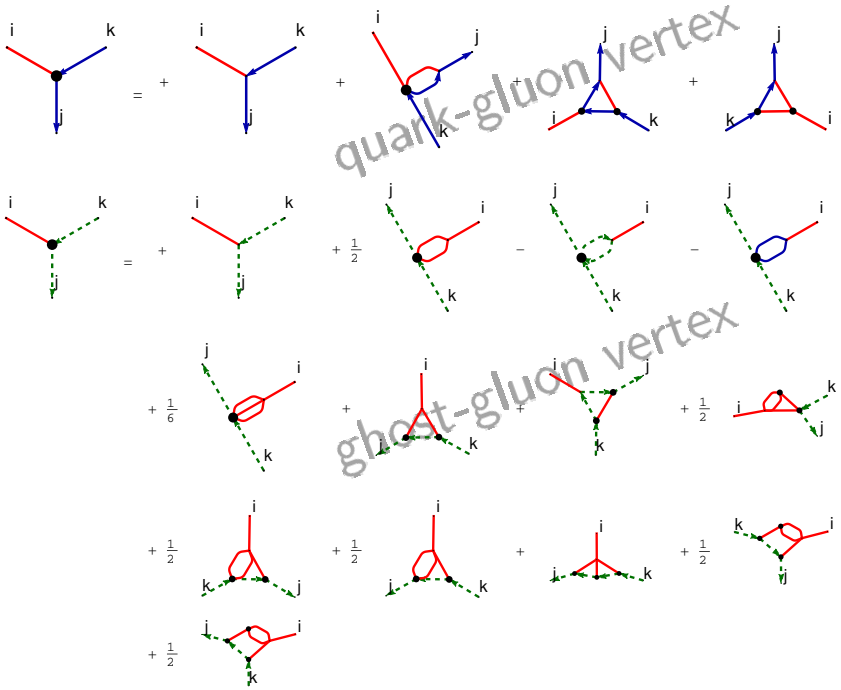

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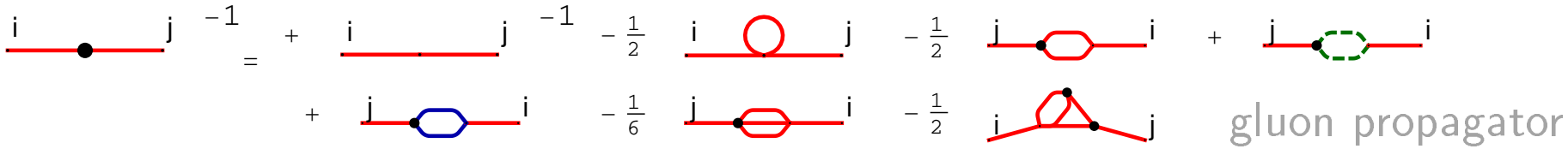
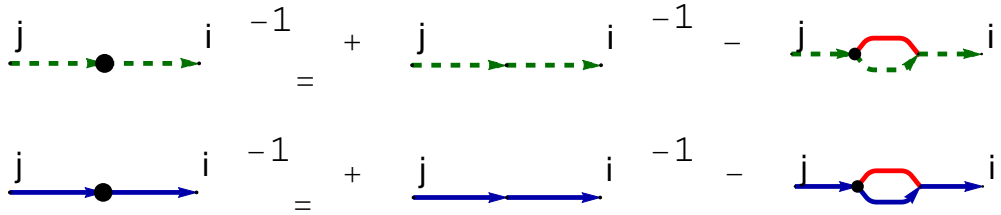
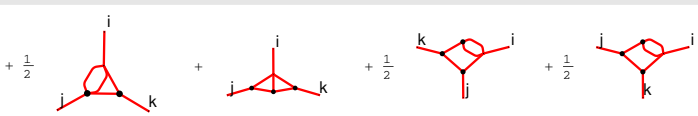
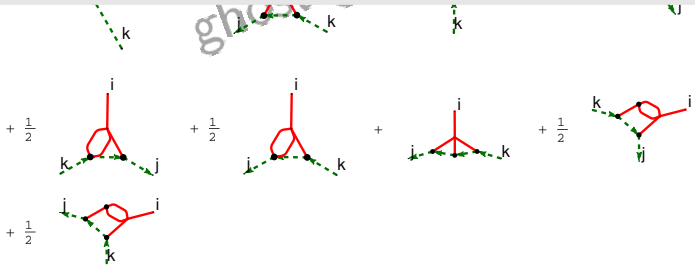




Infinitely many equations. In QCD, every n-point function depends on $(n+1)$- and possibly $(n+2)$-point functions.





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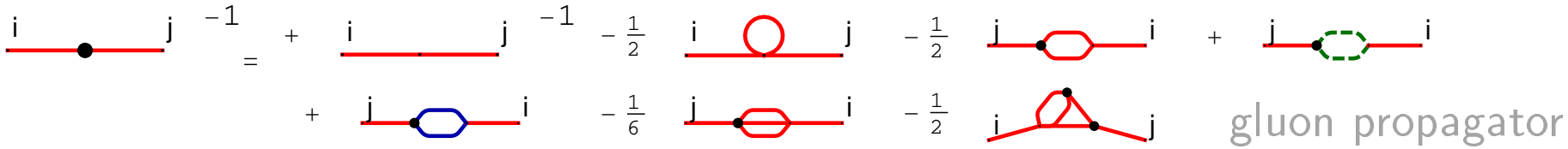

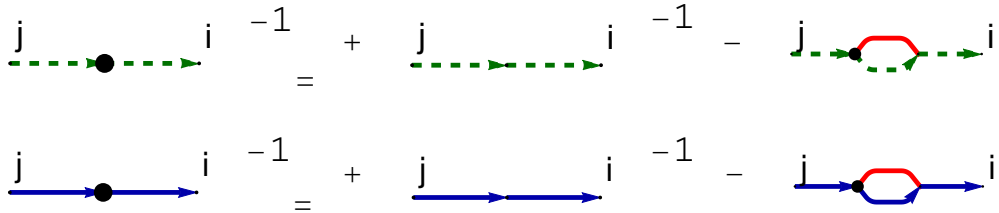

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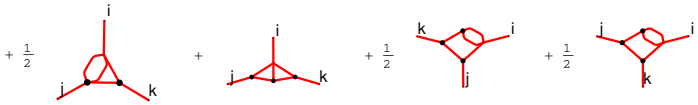
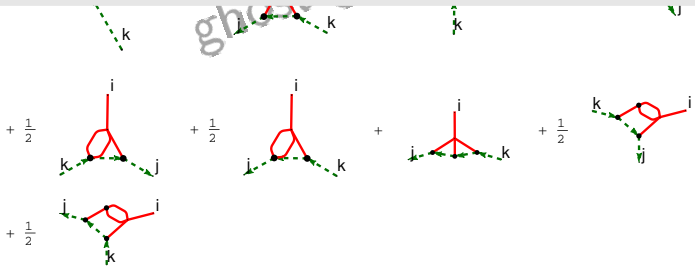
Is it possible to find and solve a truncation with all relevant contributions?





Questions about truncations

- Influence of higher correlation functions?

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- Model dependence $\leftrightarrow$ Self-contained truncation?
- How to realize resummation?
- Equivalence between different functional methods?

UV behavior of the gluon propagator

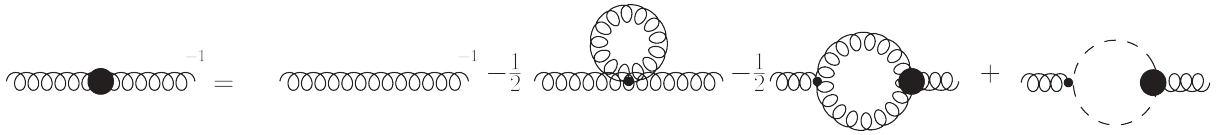
Resummed one-loop order: anomalous dimension $\gamma = -13/22$

$$\left(1 + \frac{\alpha(s) 11 N_c}{12\pi} \ln \frac{p^2}{s}\right)^{\gamma}$$

One-loop anomalous dimension

Origin in resummation of higher order diagrams.

However, one-loop truncation discards some terms.



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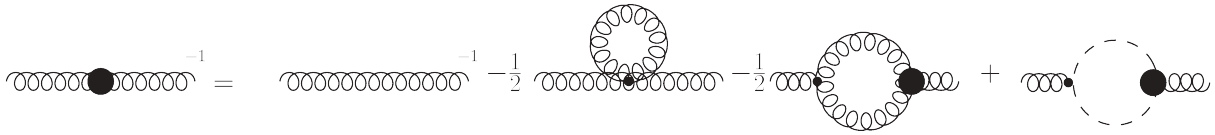

$\rightarrow$ Puts constraints on UV behavior of vertices [von Smekal, Hauck, Alkofer '97]: Include in models.

Resummed behavior

Minimal requirements to obtain one-loop resummed behavior:

- Squint diagram
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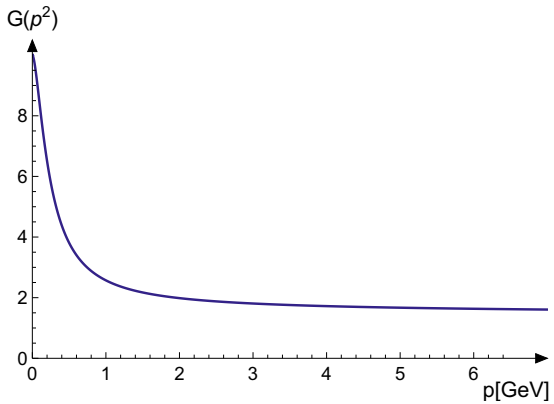
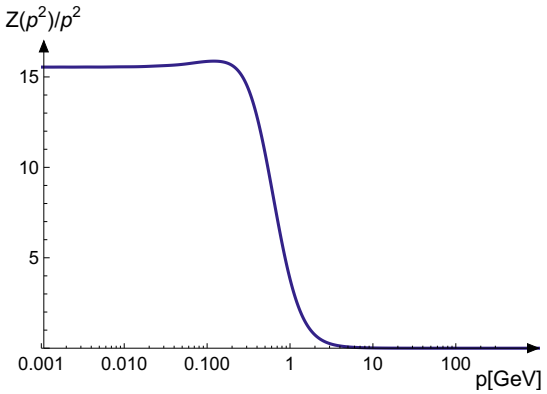
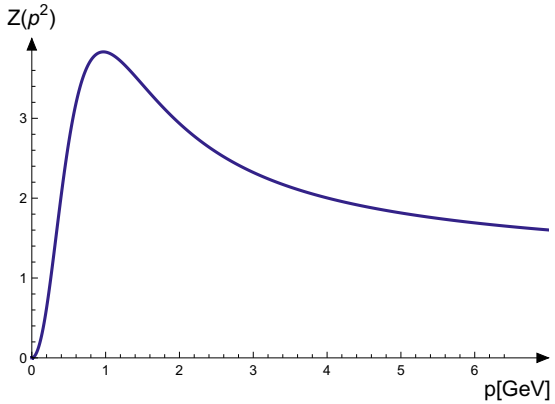
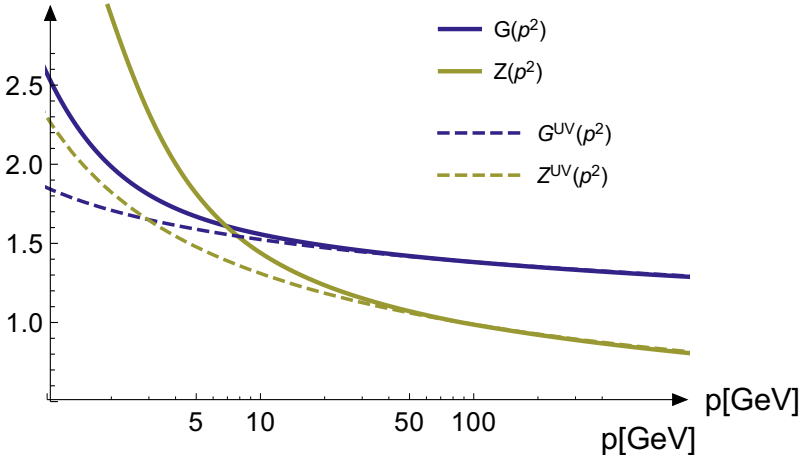






[propagator+ghost-gluon eqs. full, 3-gluon vertex model, bare 4-gluon vertex]

- Resummed behavior is recovered [MQH '17].



Testing truncations in $d = 3$:
Vary equations and systems of equations.

$d = 3$ Yang-Mills theory as testing ground

Advantages:

- UV finite: no renormalization, no anomalous running
- Spurious divergences easier to handle
- UV behavior 'easier': $\propto \frac{g^2}{p}$ instead of resummed logarithm

$\rightarrow$ Many complications from $d = 4$ absent.
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Historically interesting because cheaper on the lattice $\rightarrow$ easier to reach the IR.

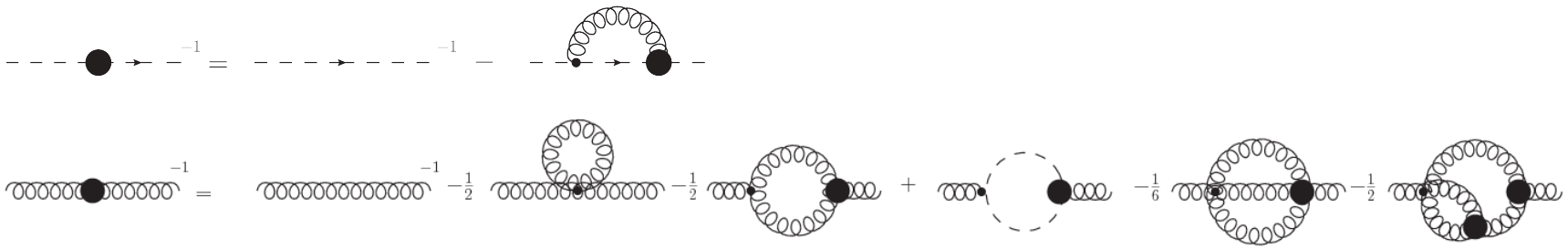
Numerically not cheaper for functional equations of 2- and 3-point functions.

Continuum results:

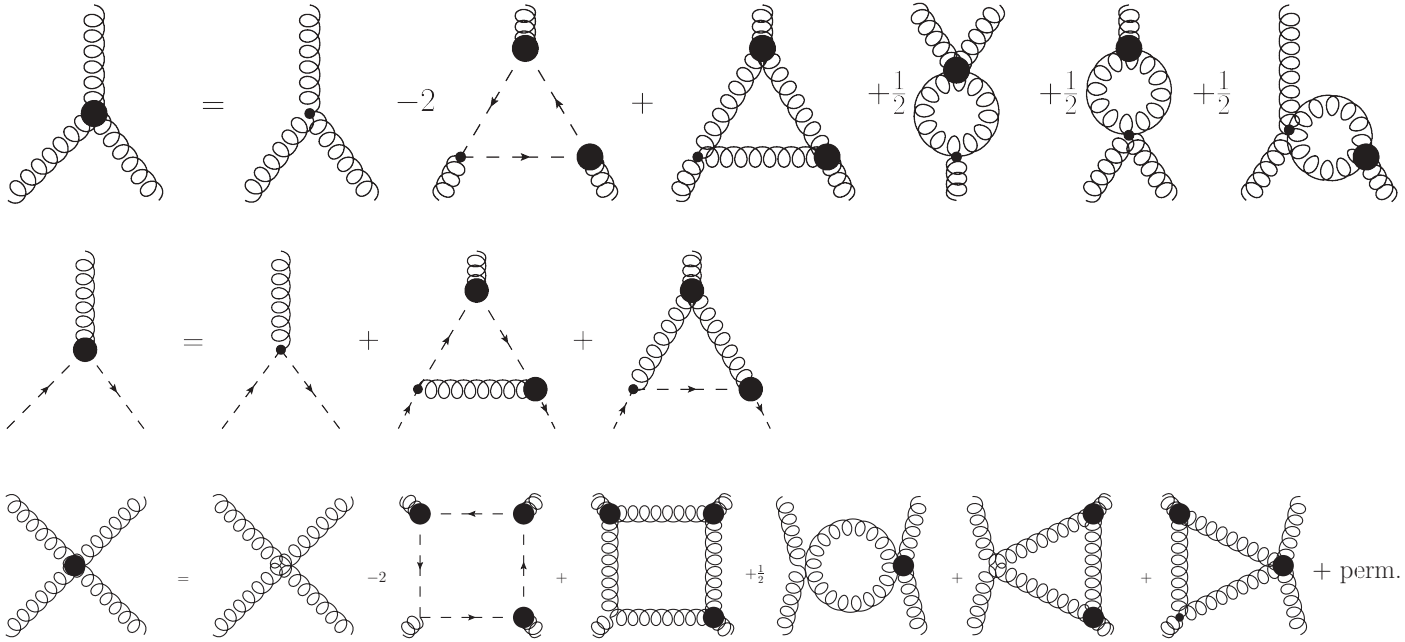
- Coupled propagator DSEs: [Maas, Wambach, Grüter, Alkofer '04]
- (R)GZ: [Dudal, Gracey, Sorella, Vandersickel, Verschelde '08]
- DSEs of PT-BFM: [Aguilar, Binosi, Papavassiliou '10]
- YM + mass term: [Tissier, Wschebor '10, '11]

Dyson-Schwinger equations: Truncation

Untruncated propagators:



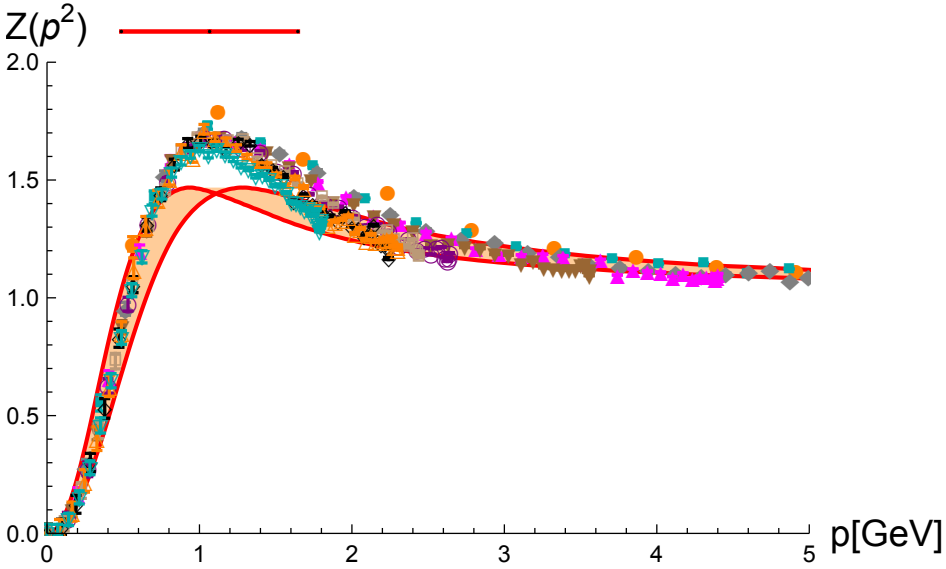
Truncated vertices:

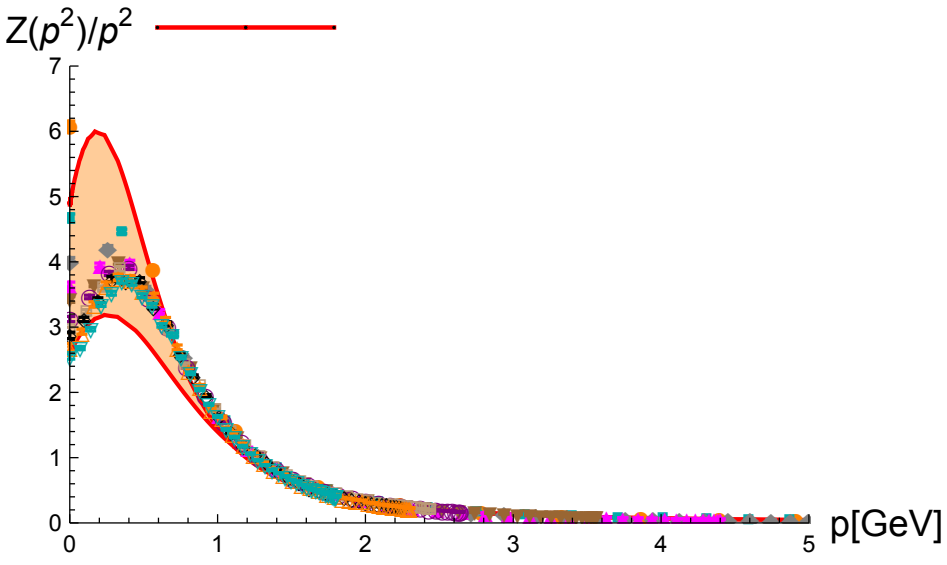


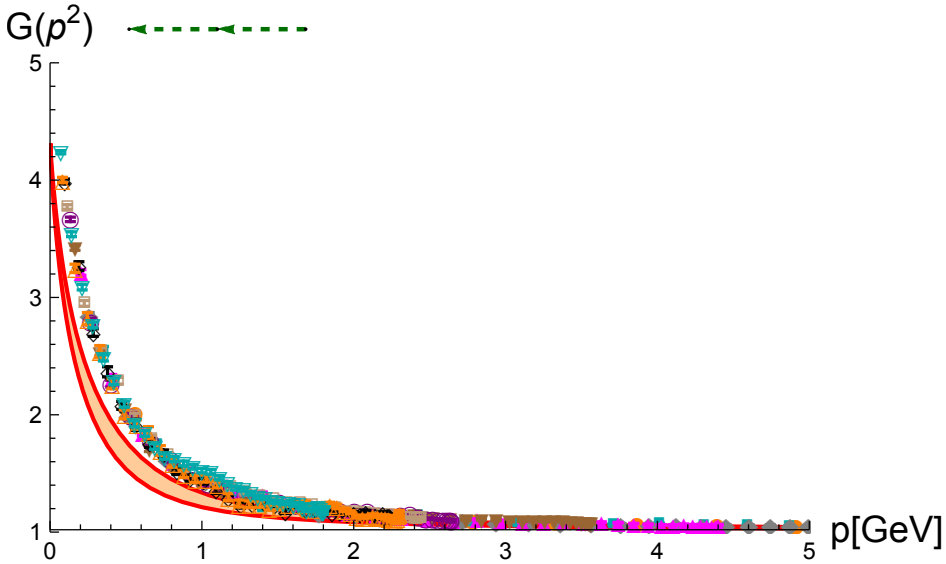
Tests in three dimensions

1. Hierarchy of diagrams.
2. Hierarchy of equations.
3. Different functional equations.

Results: Propagators







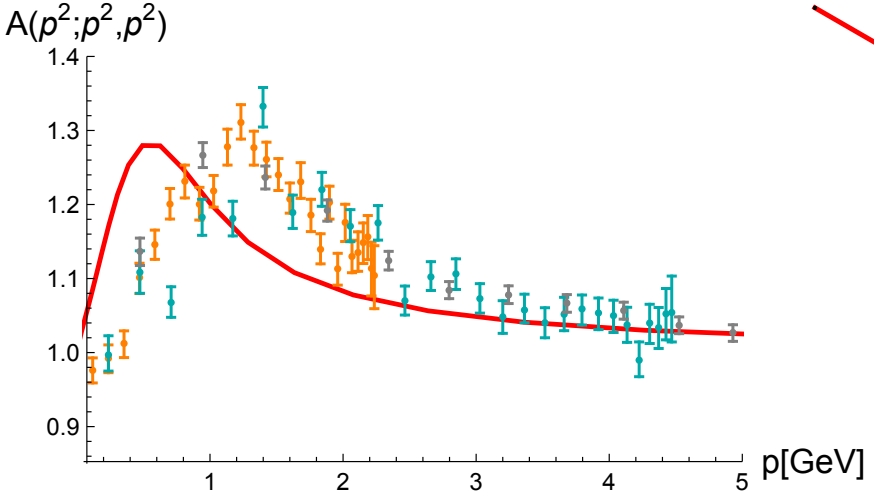
Bands from uncertainty in setting the physical scale.

[MQH '16; lattice: Maas '14]

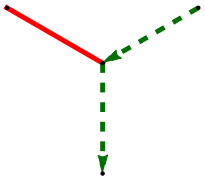
Similar results with FRG [Cyrol '17]

Comparison of three-point functions with lattice results

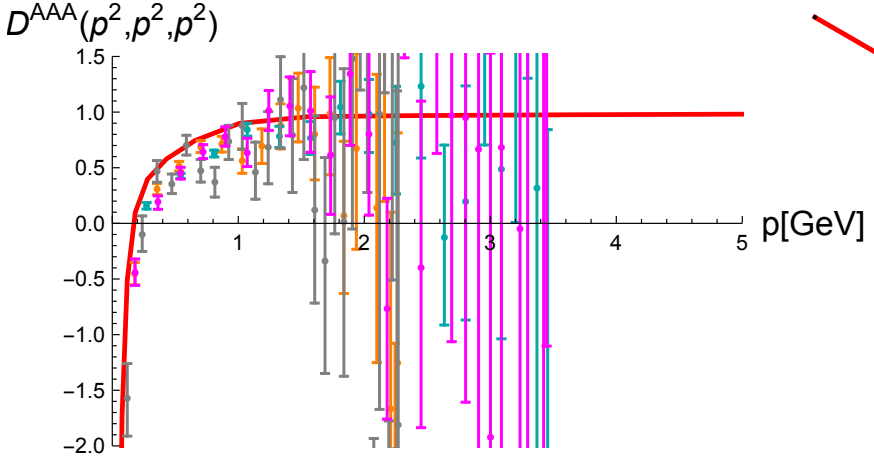
Dressings:



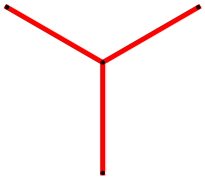
[MQH '16; lattice: Maas, unpublished]



- Deviation from tree-level 'small'
- Position of maximum shifted (as observed with other continuum methods for $SU(2)$, e.g., [Pelaez, Matthieu, Wschebor '13; Cyrol, Fister, Mitter, Pawlowski, Strodthoff '15; Cyrol '17]).



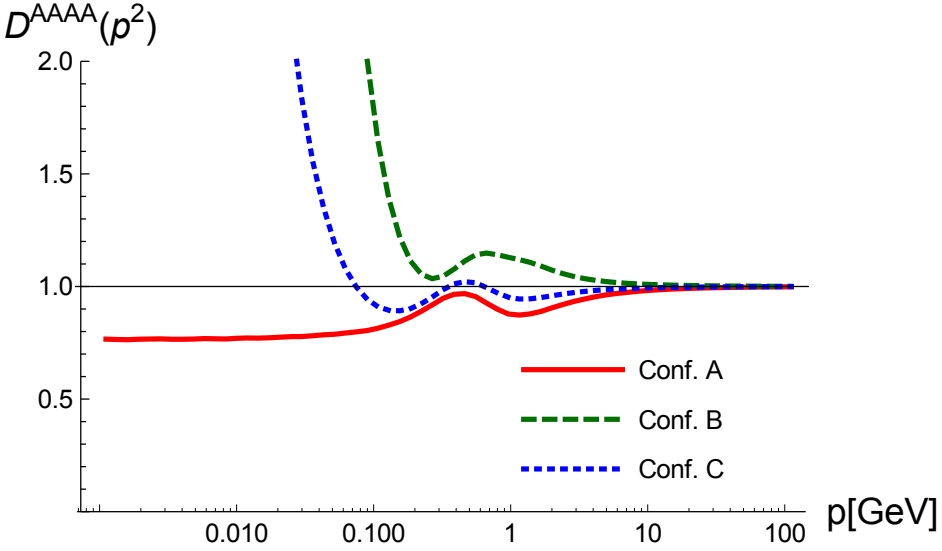
[MQH '16; lattice: Cucchieri, Maas, Mendes '08]



- Close to tree-level above 1 GeV
- Good agreement with lattice data.
- Linear IR divergence [Pelaez, Tissier, Wschebor '13; Aguilar et al. '13]

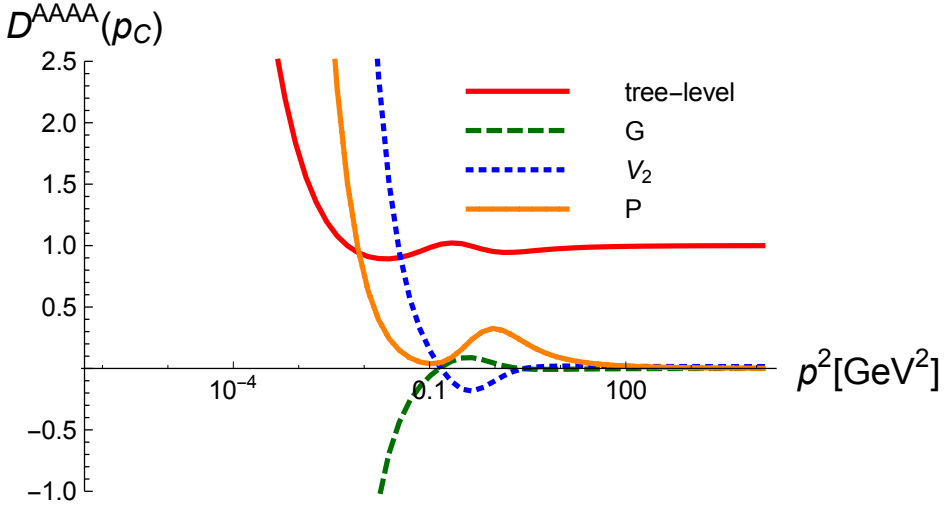
Four-gluon vertex

Different momentum configurations:



[MQH '16]

Different projections:

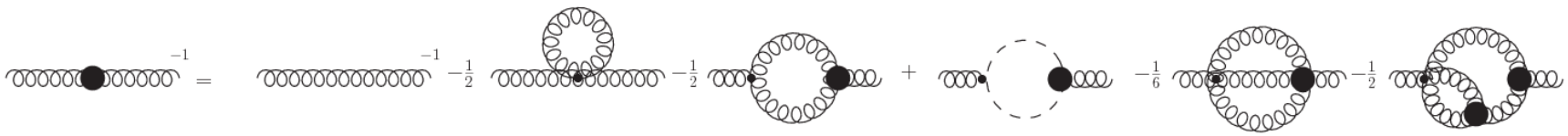


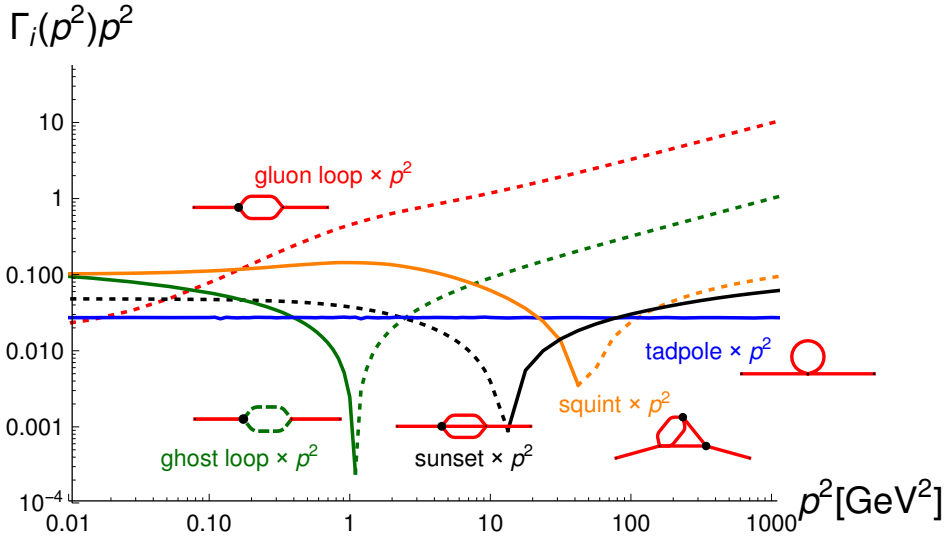
Four-gluon vertex:

- Close to tree-level down to 1 GeV

→ Corrections small individually?

Gluon propagator: Single diagrams





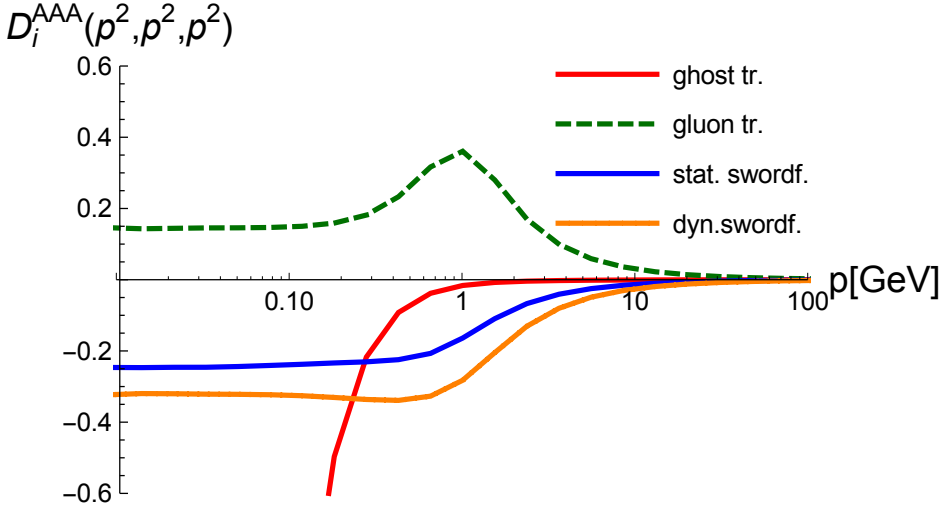
[MQH '16]

→ Clear hierarchies identified.

- UV: as expected perturbatively
- non-perturbative: squint important, sunset small (d=4: [Mader, Alkofer '13; Meyers, Swanson '14])

Cancellations in gluonic vertices

Three-gluon vertex:



[MQH '16]

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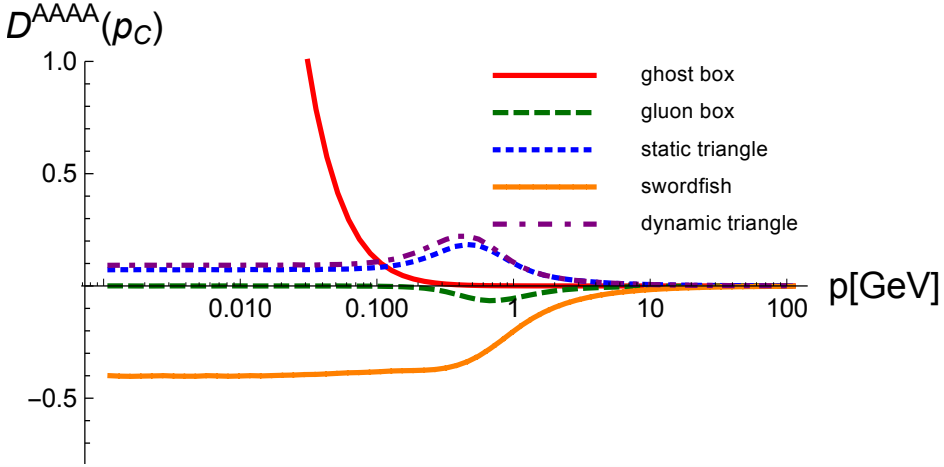
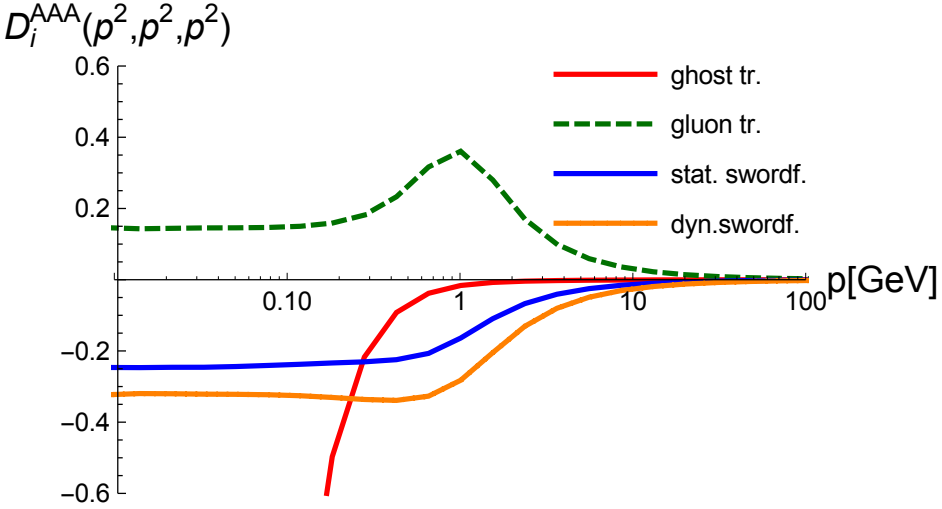


- Individual contributions large.
- Sum is small!

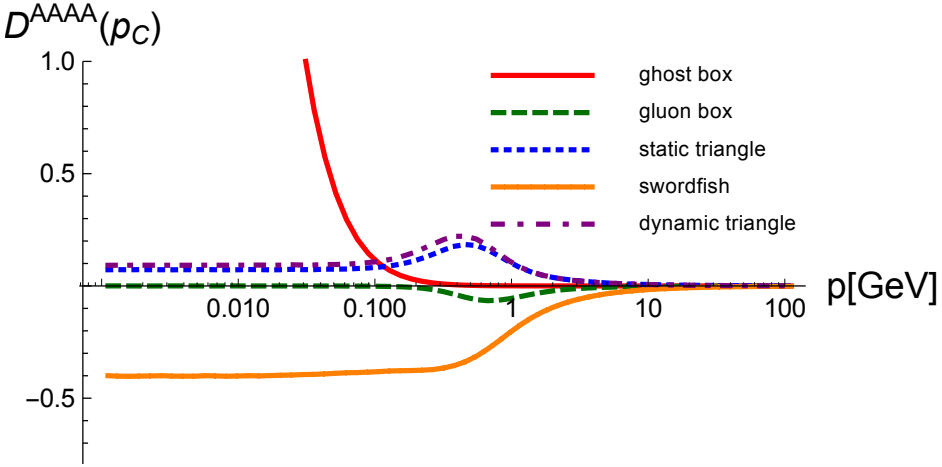
Cancellations in gluonic vertices

Three-gluon vertex:



[MQH '16]

Four-gluon vertex:



- Individual contributions large.
- Sum is small!

$\Downarrow$

Higher contributions:

- Higher vertices close to 'tree-level'? $\rightarrow$ Small.
- If pattern changes (higher vertices large): cancellations required.

Solution from the 3PI effective action

Different set of functional equations:
Equations of motion from 3PI effective action (at three-loop level)

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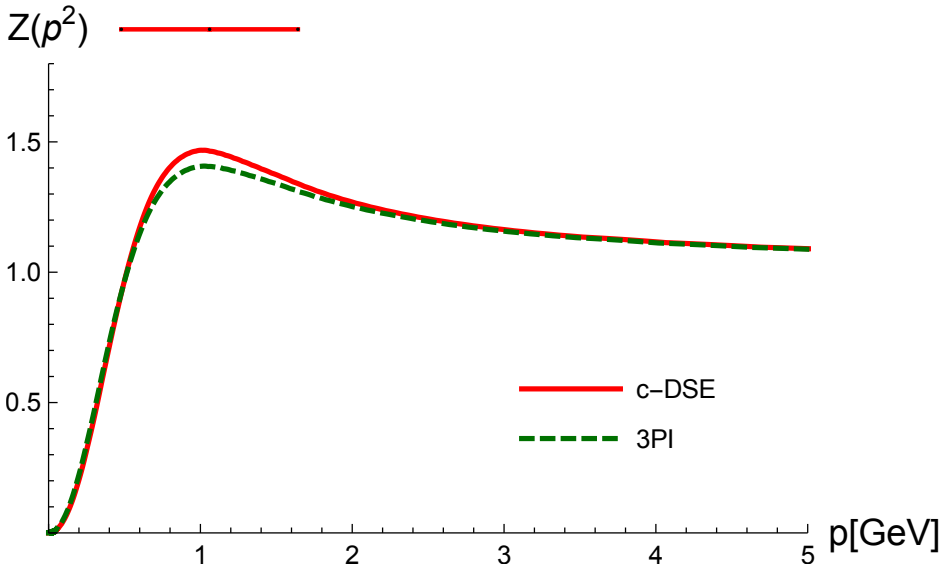
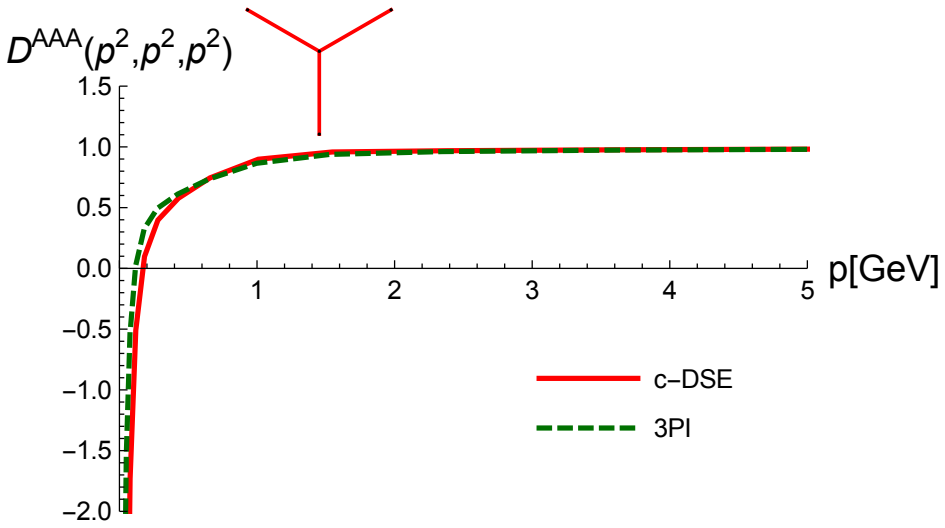




→ Very similar results. [MQH '16]

FRG: Similar results with similar truncation [Cyrol '17, thesis]

Summary: Three dimensions

- Hierarchy of correlation functions and diagrams
- Cancellations lead to small deviations from the perturbative behavior above 2 GeV.
- Some degree of stability (but no complete list of checks done) when
  - varying *system* of equations.
  - varying *equations* of system.
- Discrepancies with lattice results:
  - Nonperturbative gauge fixing?
  - Lattice systematics?
  - Missing diagrams for vertices?
  - Incomplete tensor bases for some vertices?

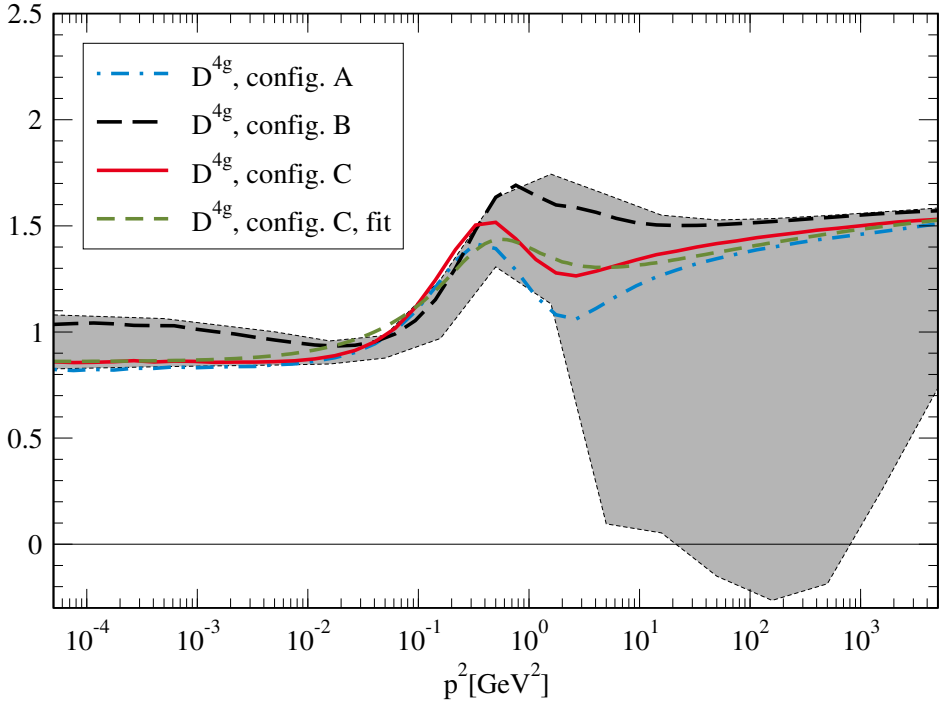
Extending truncations in four dimensions:
Include four-point functions.

Four-gluon vertex

Full calculation with fixed input:

[Cyrol, MQH, von Smekal '14]

Computationally expensive!



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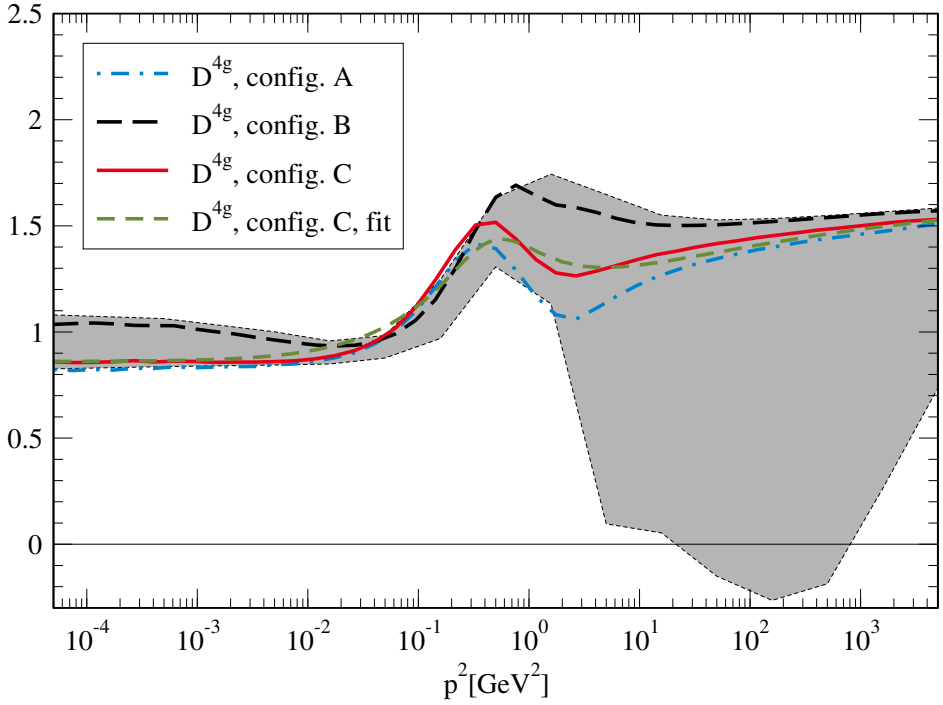
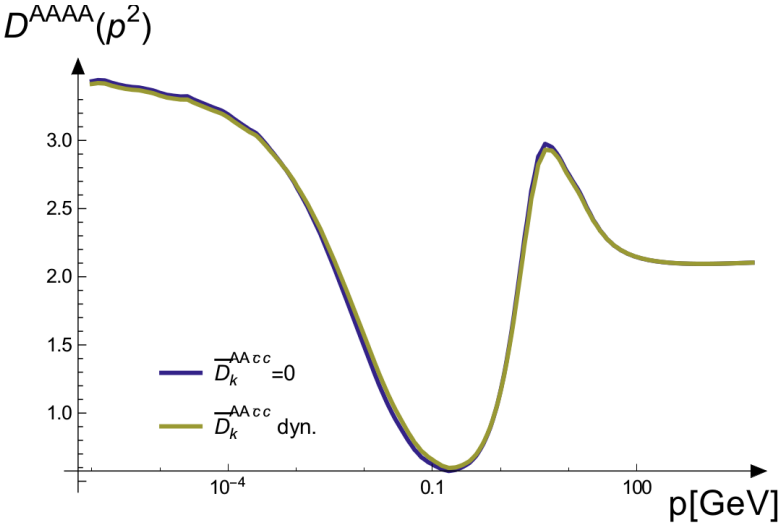


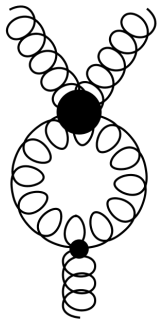


Three-gluon vertex has small angle dependence.

→ For dynamic inclusion: Resort to a one-momentum approximation (symmetric point); see also FRG calculations by fQCD collaboration.

Effect of four-gluon vertex

- In three-gluon vertex DSE:
  Important for convergence within current truncations in $d = 4$
  [Blum, MQH, Mitter, von Smekal '14;
  Eichmann, Williams, Alkofer, Vujinovic '14; MQH '17]
  $\rightarrow$ Related to renormalization and missing two-loop diagrams.



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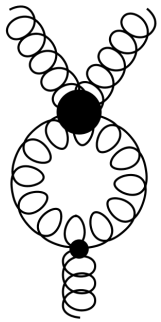
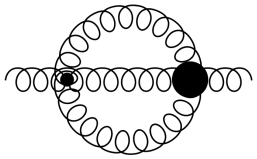
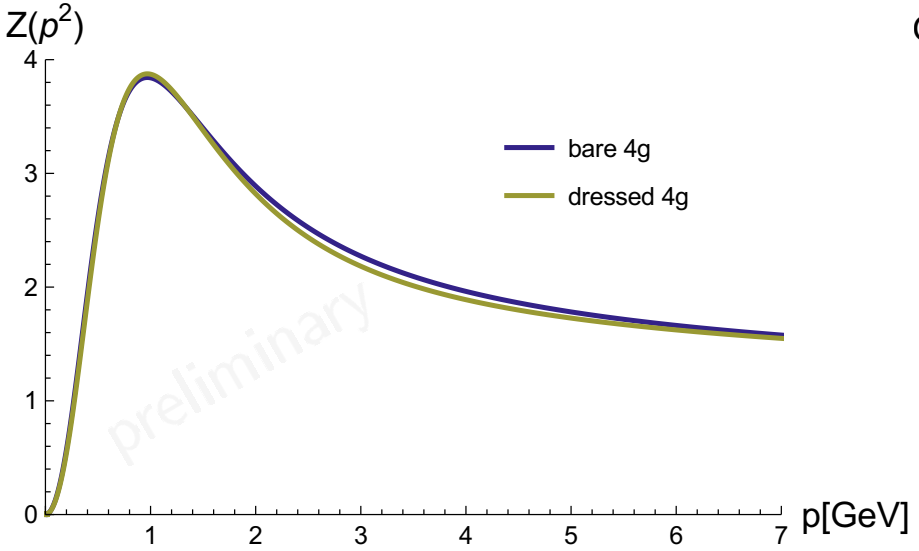
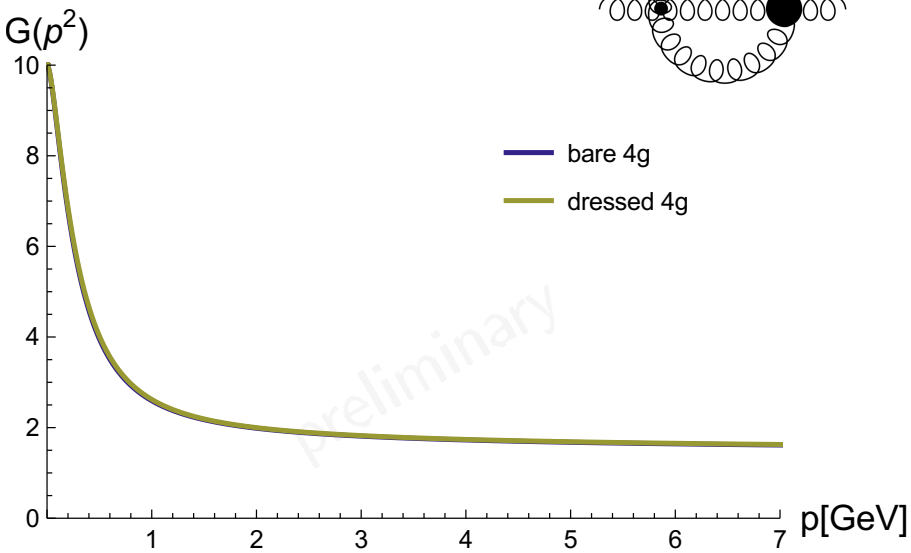


- In gluon propagator: Via sunset diagram, small contribution of tree-level dressing; model studies: [Mader, Alkofer '13; Meyers, Swanson '14]







[propagator eqs. full, 3-gluon model, ghost-gluon fixed, 4-gluon fixed]

Extending truncations of three-point functions

Extend truncations of equations of three-point functions by adding the two-ghost-two-gluon vertex:

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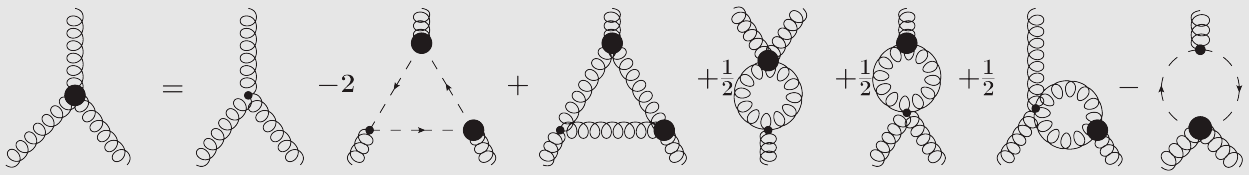

→ One-loop complete equation.

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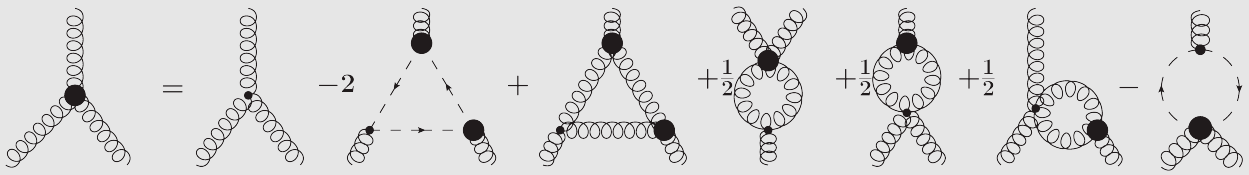

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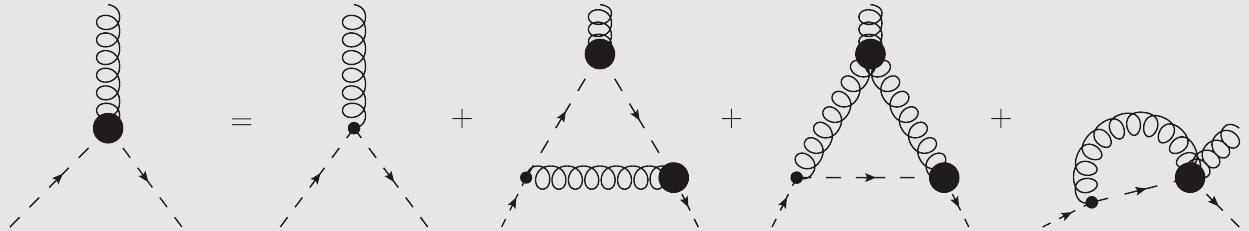
Ghost-gluon vertex



→ Complete equation.

Four-ghost vertex:

In alternative ghost-gluon vertex DSE and in four-point functions.

Four-point functions: Color space

15 possibilities:

$\delta\,\delta$: 3 combinations

$f\,f$: 3 combinations

$d\,d$: 3 combinations

$d\,f$: 6 combinations

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9/8/3 linearly independent in $SU(N/3/2)$, $N > 3$

[Pascual, Tarrach '80].

$SU(3)$: $\{\sigma_1, \ldots, \sigma_8\}$ chosen with these symmetries:

	σ_1	σ_2	σ_3	σ_4	σ_5	σ_6	σ_7	σ_8
$a \leftrightarrow b$	+	+	+	-	-	-	-	+
$c \leftrightarrow d$	+	+	+	-	-	+	-	-

$\{\sigma_1, \ldots, \sigma_5\}$ orthogonal to $\{\sigma_6, \sigma_7, \sigma_8\}$. $\rightarrow$ $\{\sigma_6, \sigma_7, \sigma_8\}$ decouple.

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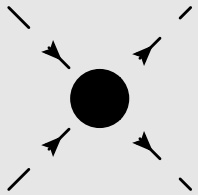

$$\Gamma^{\bar{c}\bar{c}cc,abcd}(p,q,r,s) = g^4 \sum_{k=1}^{8} \sigma^{k,abcd} E_k^{\bar{c}\bar{c}cc}(p,q,r,s).$$

The two-ghost-two-gluon vertex: Lorentz space

Non-primitively divergent correlation function $\rightarrow$ No guide from tree-level tensor. $\rightarrow$ Use full basis.

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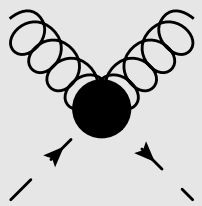
Two-ghost-two-gluon vertex



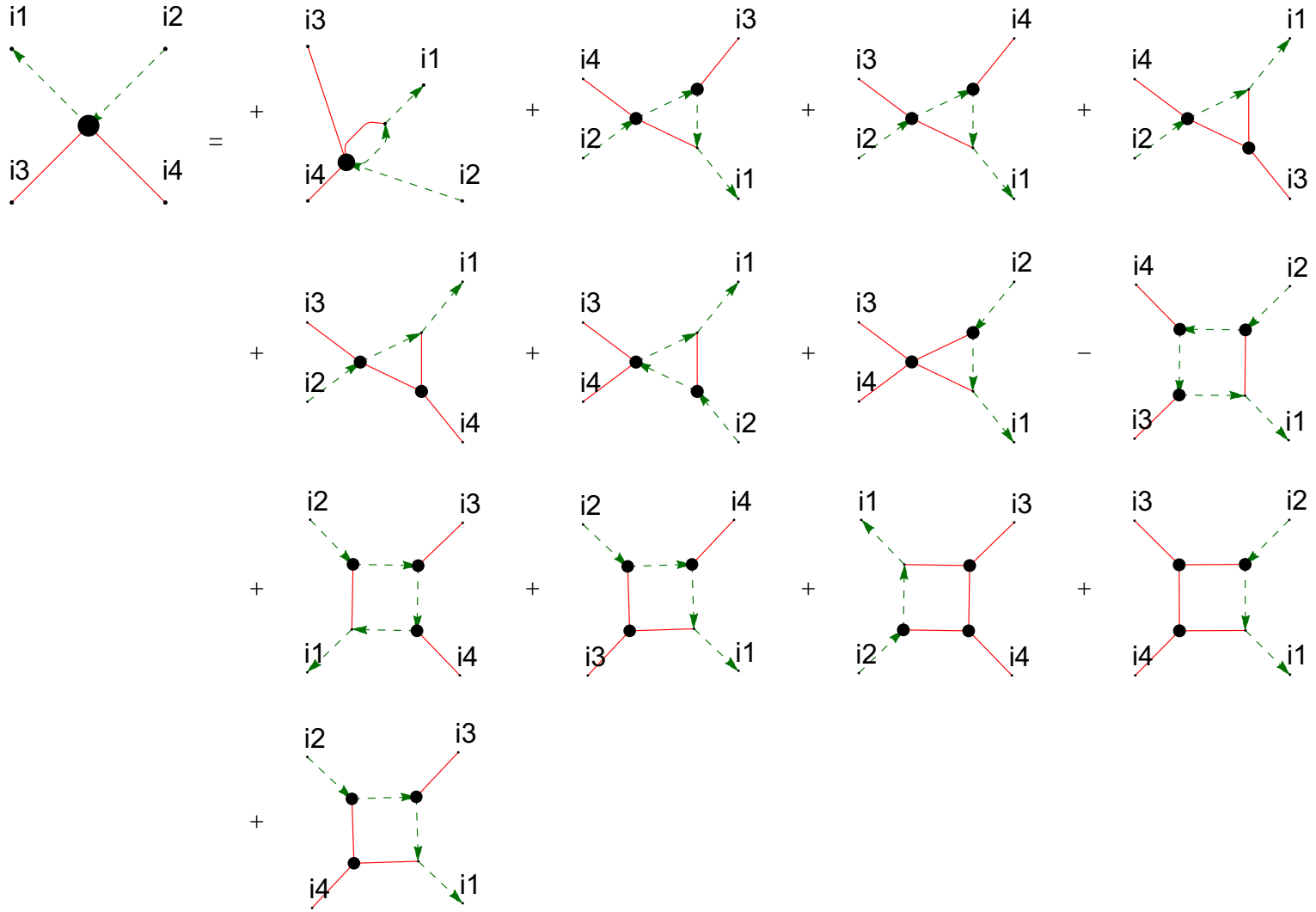
$$\Gamma^{AA\bar{c}c,abcd}_{\mu\nu}(p,q;r,s) = g^4 \sum_{k=1}^{40} \rho^{k,abcd}_{\mu\nu} D^{AA\bar{c}c}_{k(i,j)}(p,q;r,s)$$

with

$$\rho^{k,abcd}_{\mu\nu} = \sigma^{abcd}_i \tau^j_{\mu\nu}, \qquad k = k(i,j) = 5(i-1)+j$$

The two-ghost-two-gluon vertex DSE

2 DSEs, choose the one with the ghost leg attached to the bare vertex
$\rightarrow$ Truncation discards only one diagram.



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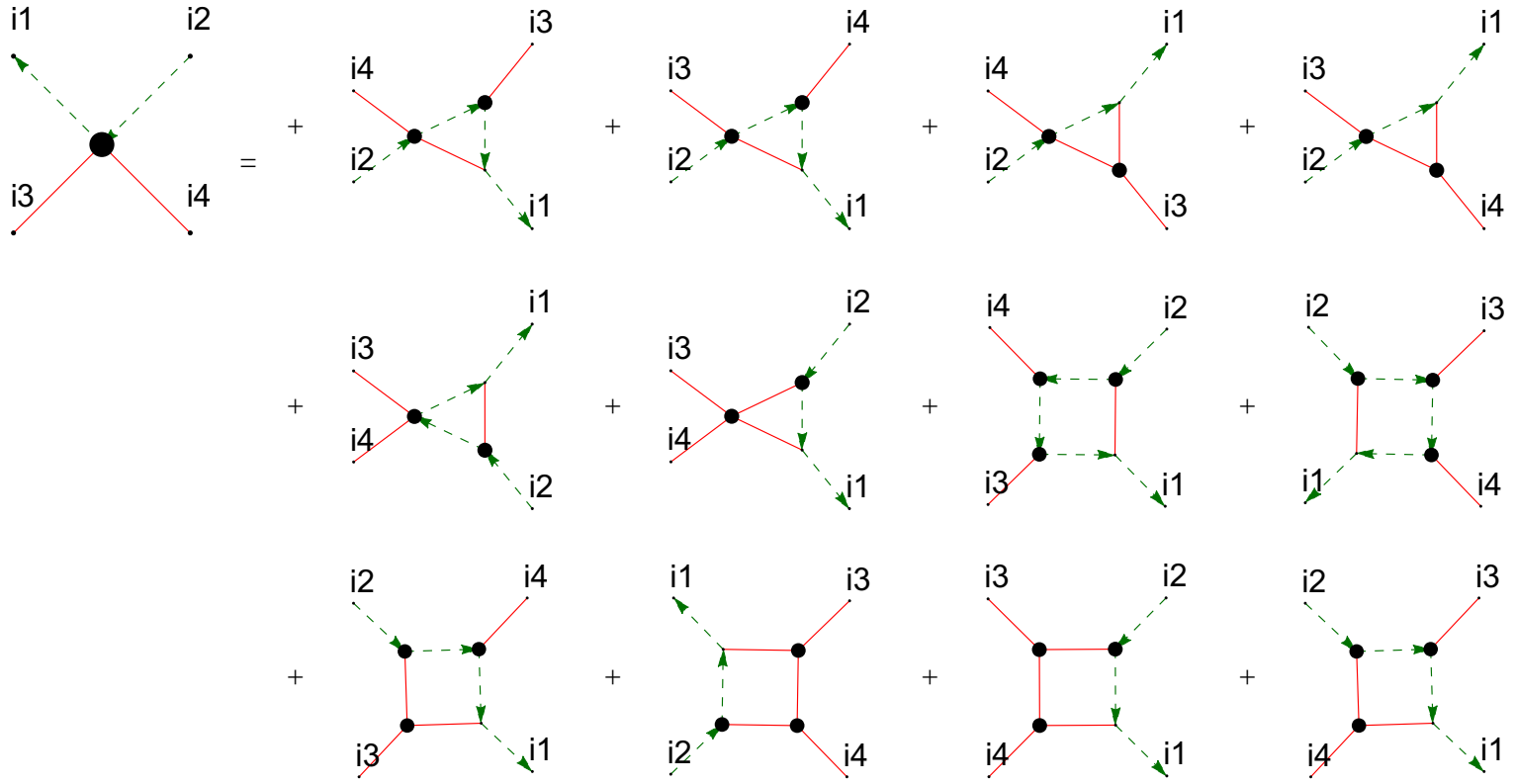
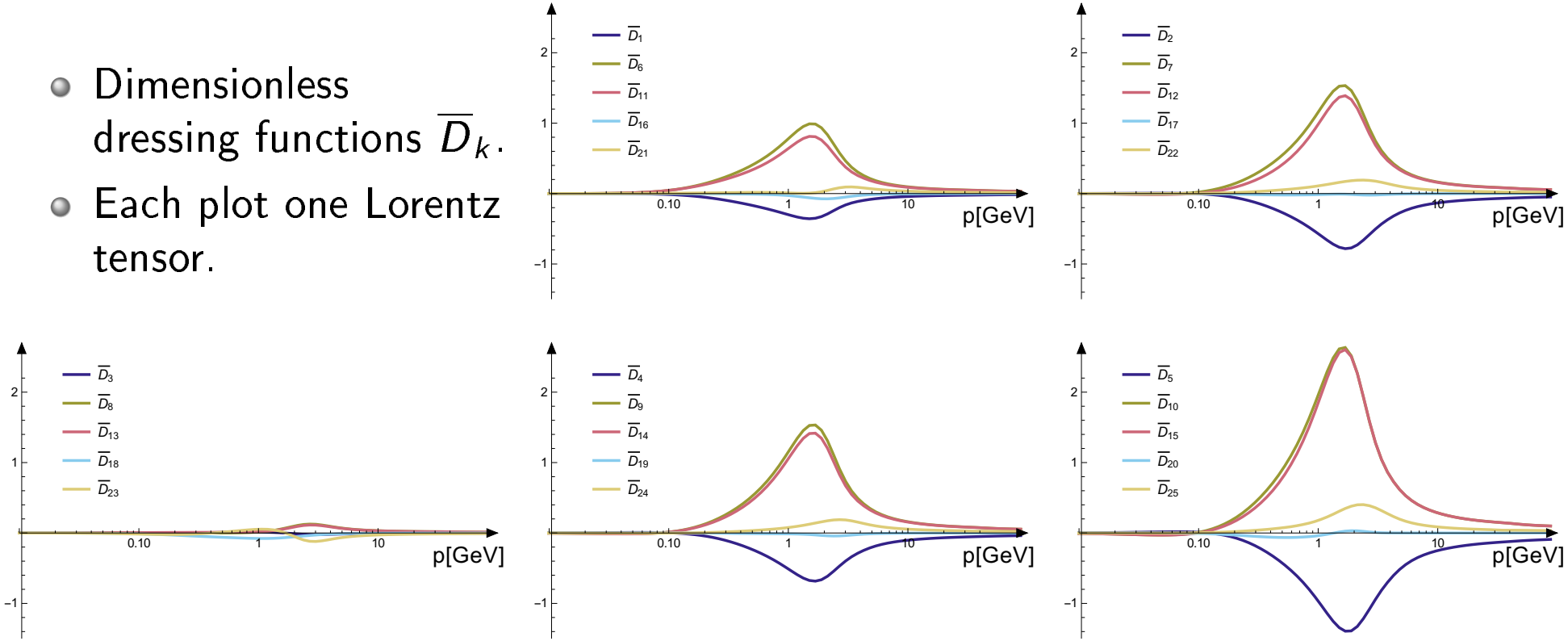


Results for the two-ghost-two-gluon vertex

Kinematic approximation: one-momentum configuration

- Dimensionless dressing functions $\overline{D}_k$.
- Each plot one Lorentz tensor.

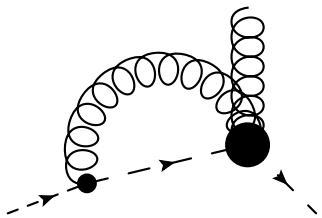


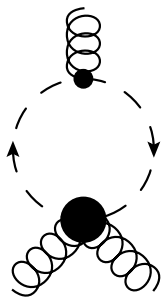
→ Two classes of dressings: 13 very small, 12 not small

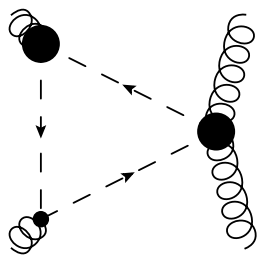
→ No nonzero solution for $\{\sigma_6, \sigma_7, \sigma_8\}$ found. [MQH '17]

Influence of two-ghost-two-gluon vertex

Coupled system of ghost-gluon, three-gluon and four-gluon vertices with and without two-ghost-two-gluon vertex:







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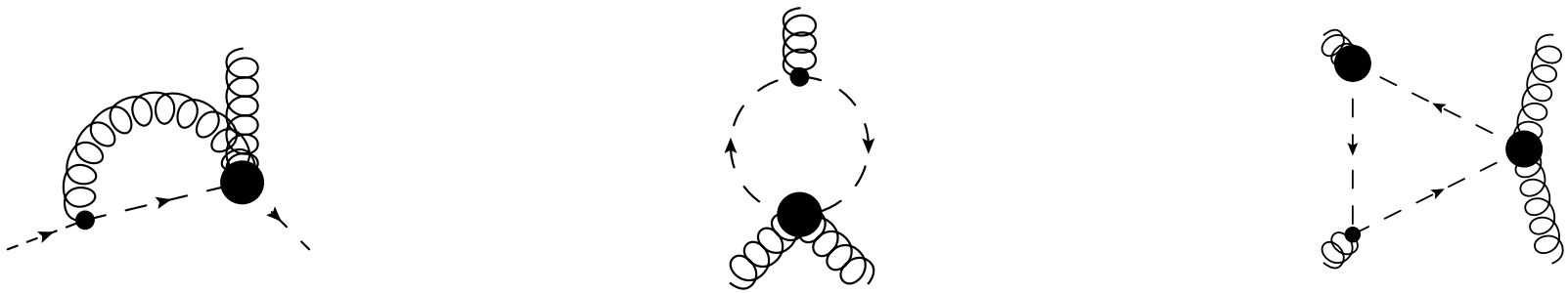
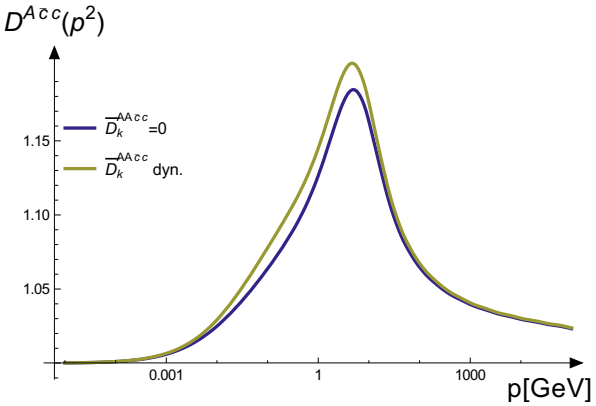
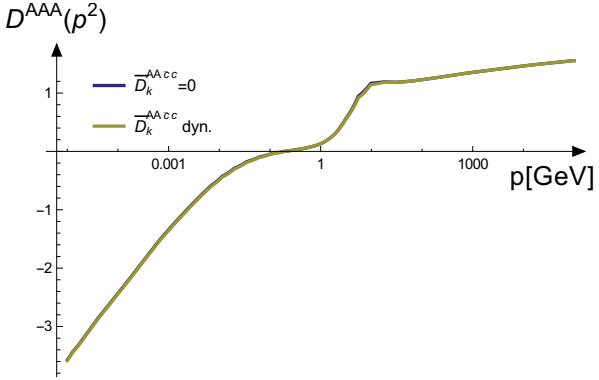
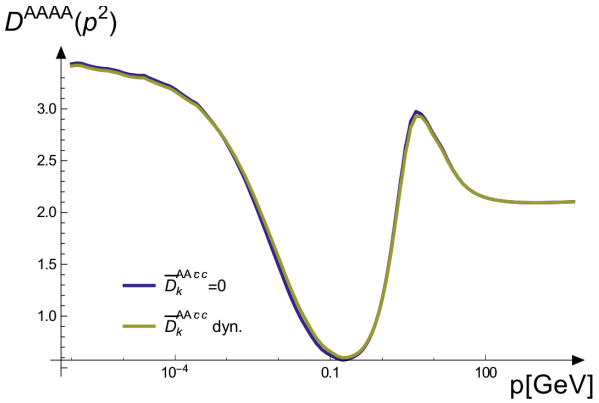


Color space: $\{\sigma_6, \sigma_7, \sigma_8\}$ do not appear here!







- Small influence on ghost-gluon vertex ($< 1.7\%$)
- Negligible influence on three- and four-gluon vertices.

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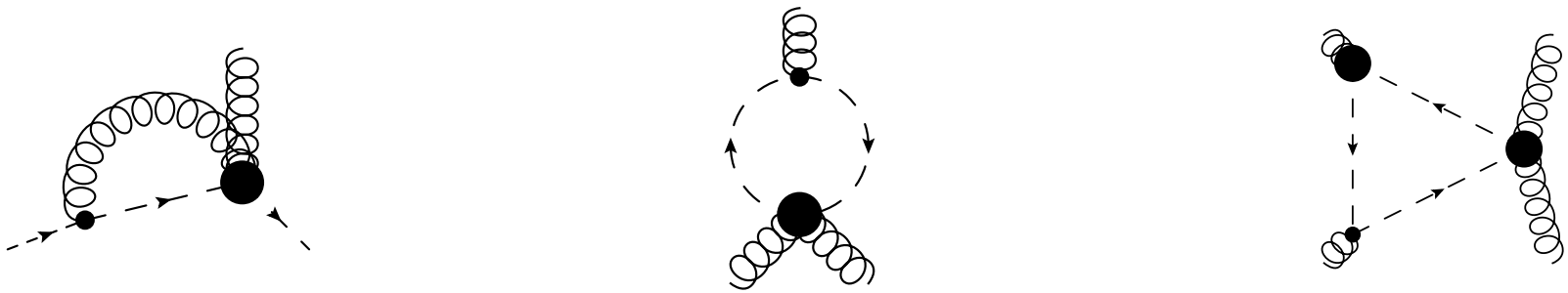
Coupled system of ghost-gluon, three-gluon and four-gluon vertices with and without two-ghost-two-gluon vertex:



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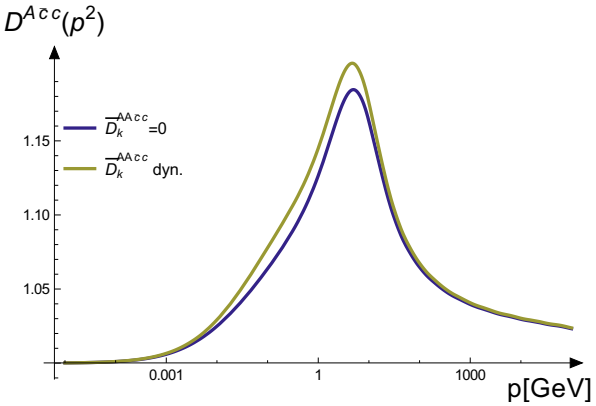

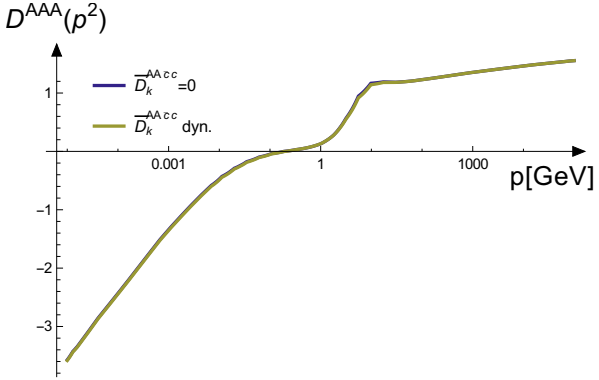

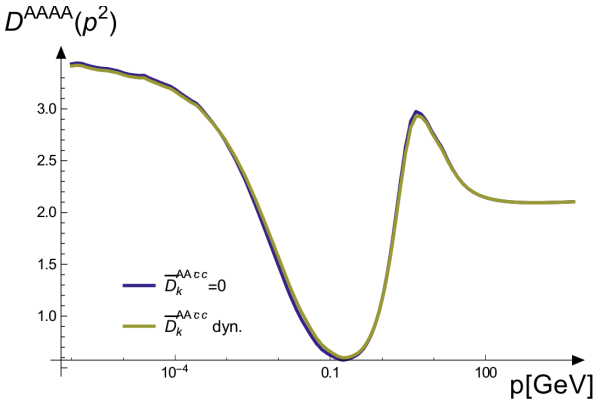

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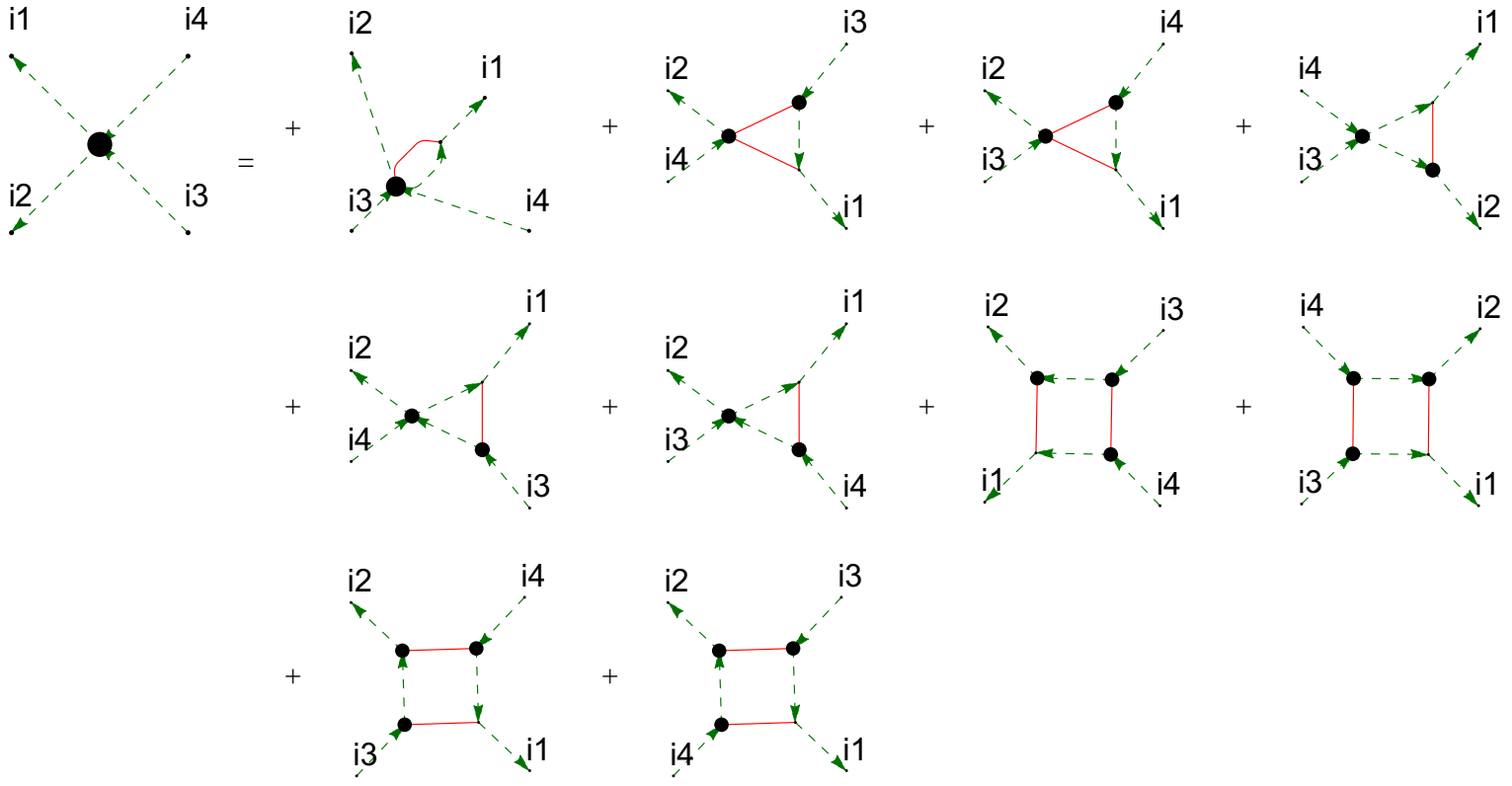
  Color space: Only small dressings couple to three-gluon vertex!

[MQH '17]

The four-ghost vertex DSE

→ Truncation discards only one diagram.



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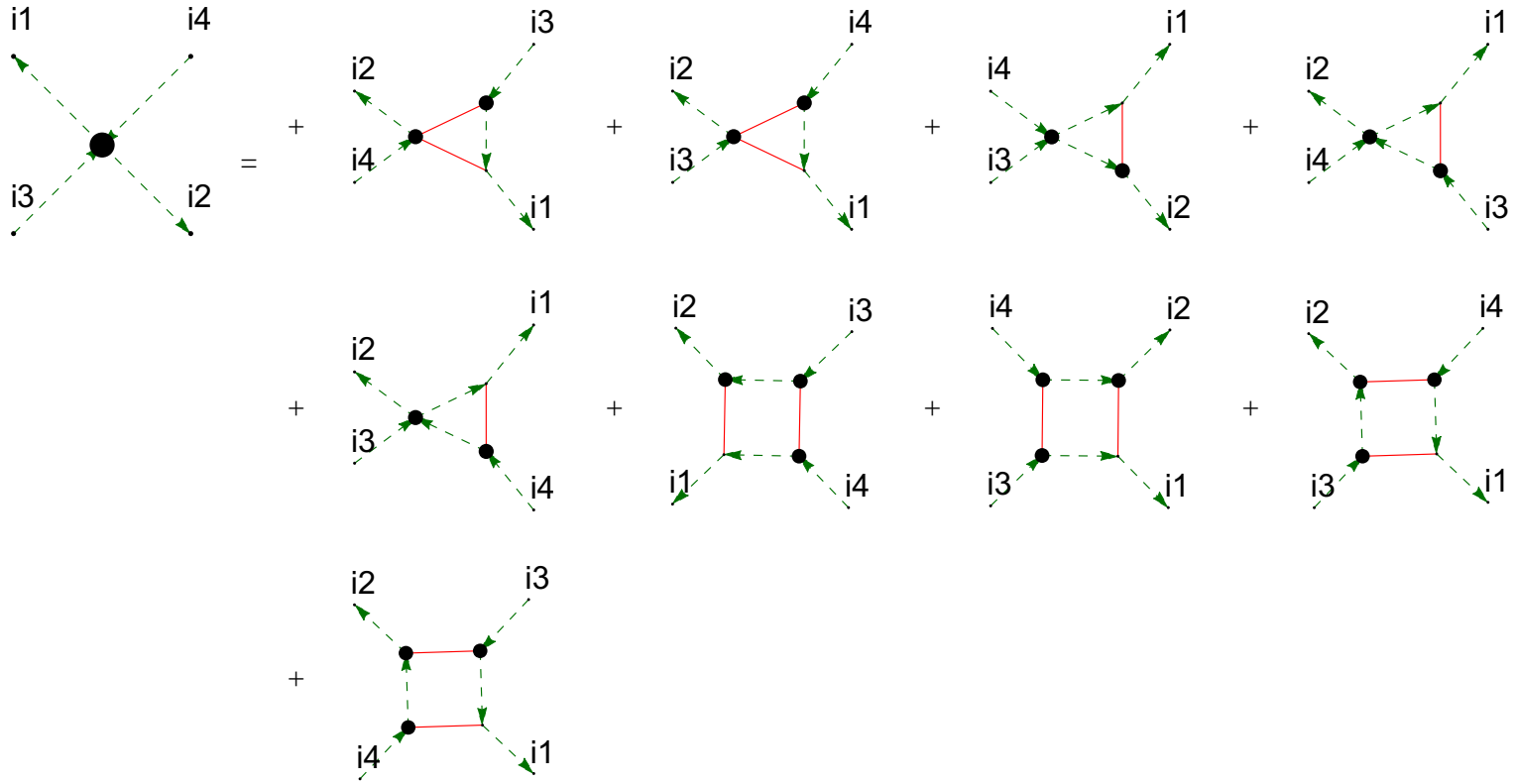
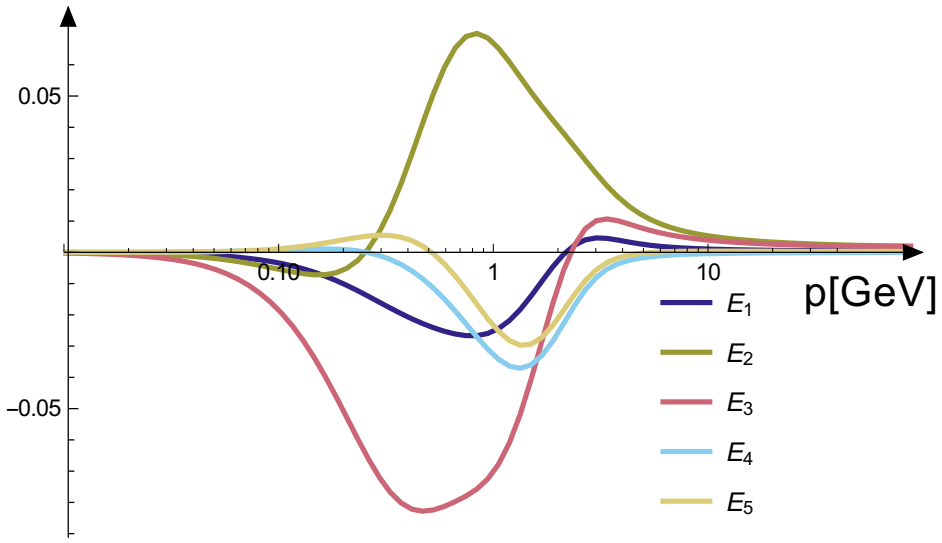


Results for the four-ghost vertex

Kinematic approximation: one-momentum configuration



→ All dressings very small.

[MQH '17]

E_6, E_7, E_8 ($\{\sigma_6, \sigma_7, \sigma_8\}$)

Decouple into a homogeneous, linear equation. → Trivial solution always exists.
Nontrivial one? → None found.
(Same applies to two-ghost-two-gluon vertex.)

Summary and conclusions

Based on

- tests in $d = 3$ including comparison with 3PI calculations
- analysis of one-loop resummation
- testing non-primitively divergent correlation functions

a non-perturbative hierarchy of correlations functions and diagrams can be identified.

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1. Cancellations between diagrams
2. Negligible diagrams

Two-loop diagrams in propagators:

Required quantitatively and for self-consistency.

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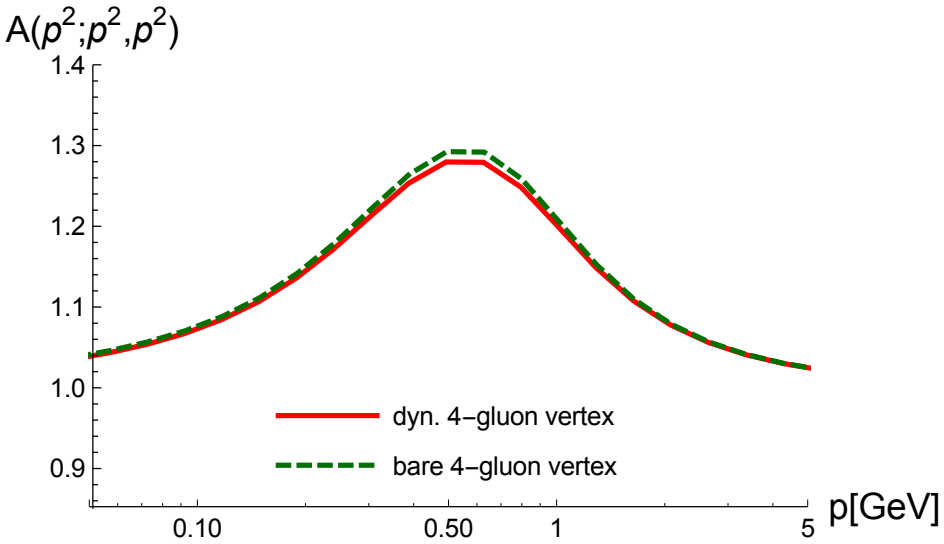
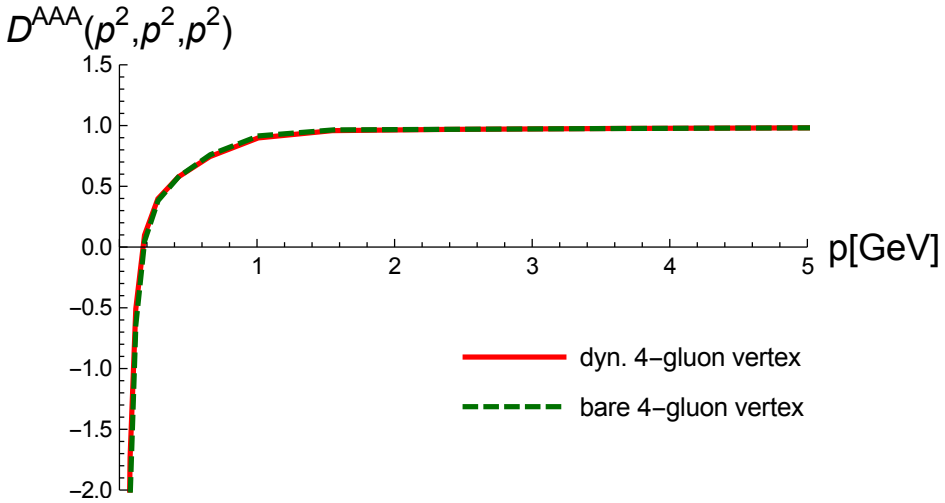
Thank you for your attention!

Influence of four-gluon vertex on three-point functions





[MQH '16]

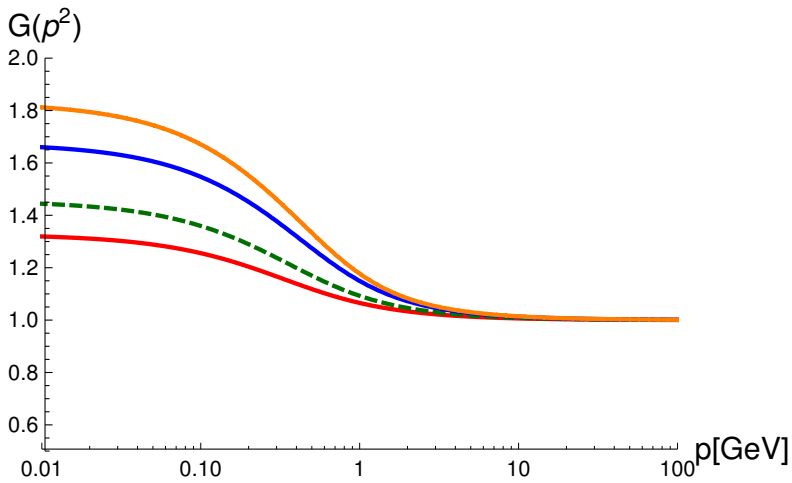
- Influence of four-gluon vertex small.

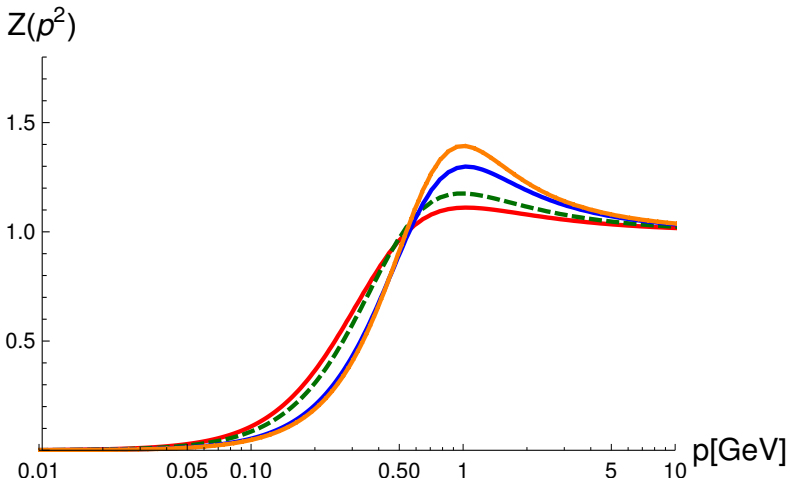
Family of solutions in three dimensions

Cf. FRG results: Bare mass parameter from modified STIs [Cyrol, Fister, Mitter, Pawlowski, Strodthoff '15].

DSEs: Enforce family of solutions by fixing the gluon propagator at $p^2 = 0$.

Simple toy system with bare vertices [MQH, 1606.02068]:





$\Rightarrow$ Possibility of family of solutions.

NB: Effect overestimated here since vertices are fixed.