

# $n$ -point functions in $d$ dimensions

(of Landau gauge Yang-Mills theory)



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Workshop of the Doktoratskolleg  
“Hadrons in Vacuum, Nuclei and Stars”

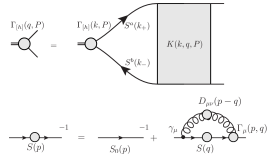
Feb. 26, 2018



# Motivation: Where Yang-Mills theory is important in QCD

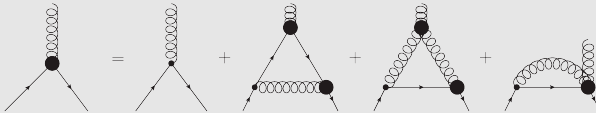
Widely used truncation: Rainbow + ladder + variant of Maris-Tandy interaction

→ Talks by Eichmann, Sanchis Alepuz



## How to reduce model dependence

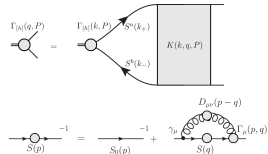
- Improve kernel  $K$
- Use explicit **gluon propagator** + **quark-gluon vertex**



# Motivation: Where Yang-Mills theory is important in QCD

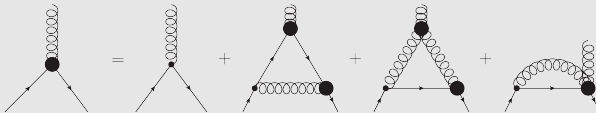
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## How to reduce model dependence

- Improve kernel  $K$
- Use explicit **gluon propagator** + **quark-gluon vertex**



→ Full control over the gluonic sector is needed for self-contained calculations.

- Gluon propagator
- Three-gluon vertex
- ...?
- **Glueballs**

# Progress in $n$ -point functions: 2, 3, 4, ...

Status 2010:

Truncated propagator equations,  
modeled vertices

The image shows two equations for truncated propagators. The top equation is for a fermion propagator, represented by a dashed line with an arrow. It states that the inverse of the truncated propagator (a dashed line with a black dot) is equal to the inverse of the bare propagator (a simple dashed line) plus a term representing a self-energy loop (a dashed line with a fermion loop). The bottom equation is for a gluon propagator, represented by a wavy line. It states that the inverse of the truncated gluon propagator (a wavy line with a black dot) is equal to the inverse of the bare gluon propagator (a simple wavy line) minus half the inverse of the bare gluon propagator multiplied by a term representing a self-energy loop (a wavy line with a gluon loop), plus a term representing a ghost loop (a dashed line with a gluon loop).

Beyond that (with continuum approaches):

- Three-point functions
- Four-point functions
- Two-loop diagrams
- Other functional equations  
→ Talks by Mitter, Pawłowski

Status 2010:

- **Three-point functions**, e.g., [Schleifenbaum et al. '04; Boucaud et al. '11; Dudal et al. '12; MQH et al. '12; Aguilar et al. '13; Blum et al. '14; Eichmann et al. '14; MQH '17]
- **Four-point functions**, e.g., [Cyrol et al. '15; MQH '17]
- **Two-loop diagrams**, e.g., [Mader, Alkofer '13; Hopfer '14; Meyers, Swanson '14; MQH '17]
- **Other functional equations: FRG [fQCD collaboration],  $n$ -PI effective action**  
→ Talks by Mitter, Pawłowski

# Progress in $n$ -point functions: 2, 3, 4, ...

Early models *optimized* to achieve/improve results: Tuning possible. f

Replacement by dynamic quantities:

- Some freedom is lost, system is more constrained.
- → New problems emerge, e.g., realization of perturbative resummation.

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Testing truncations:

- **Hierarchy** of diagrams
- **Extensions** of truncations:

→ Two-loop terms

→ Non-primitively divergent correlation functions

# Landau gauge QCD

$$\mathcal{L} = \bar{\mathbf{q}}(-\not{D} + m)\mathbf{q} + \frac{1}{2}F^2 + \mathcal{L}_{gf} + \mathcal{L}_{gh}$$

$$F_{\mu\nu} = \partial_\mu \mathbf{A}_\nu - \partial_\nu \mathbf{A}_\mu + ig [\mathbf{A}_\mu, \mathbf{A}_\nu]$$



# Landau gauge QCD

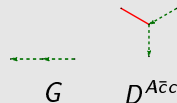
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## Landau gauge

- simplest one for functional equations
- $\partial_\mu \mathbf{A}_\mu = 0$ :  $\mathcal{L}_{gf} = \frac{1}{2\xi}(\partial_\mu \mathbf{A}_\mu)^2$ ,  $\xi \rightarrow 0$
- requires ghost fields:  $\mathcal{L}_{gh} = \bar{\mathbf{c}}(-\square + g \mathbf{A} \times) \mathbf{c}$



# The tower of DSEs

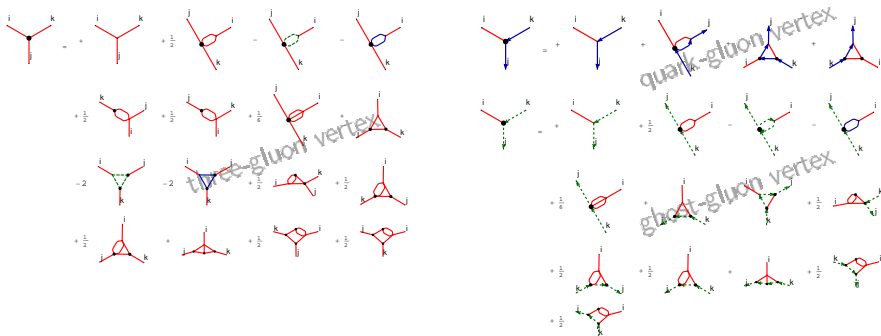
$$\begin{aligned}
 & \text{gluon propagator} \\
 & \text{ghost propagator} \\
 & \text{quark propagator}
 \end{aligned}$$

The diagram shows the Dyson-Schwinger equations (DSEs) for the gluon, ghost, and quark propagators. Each equation is represented by a diagrammatic equation where the full propagator is equal to the sum of the tree-level propagator and various loop corrections.

- gluon propagator:** The full gluon propagator (red line with a black dot) is equal to the sum of the tree-level gluon propagator (red line) and several loop corrections involving gluons and ghosts.
- ghost propagator:** The full ghost propagator (green dashed line with a black dot) is equal to the sum of the tree-level ghost propagator (green dashed line) and a loop correction involving a gluon.
- quark propagator:** The full quark propagator (blue line with a black dot) is equal to the sum of the tree-level quark propagator (blue line) and a loop correction involving a gluon.

# The tower of DSEs

$$\begin{aligned}
 & \text{gluon propagator: } i \text{---} j \text{---}^{-1} = i \text{---} j \text{---}^{-1} - \frac{1}{2} i \text{---} \text{loop} \text{---} j - \frac{1}{2} i \text{---} \text{ghost loop} \text{---} j + \dots \\
 & \text{ghost propagator: } j \text{---} i \text{---}^{-1} = j \text{---} i \text{---}^{-1} - j \text{---} \text{ghost loop} \text{---} i - \dots \\
 & \text{quark propagator: } j \text{---} i \text{---}^{-1} = j \text{---} i \text{---}^{-1} - j \text{---} \text{quark loop} \text{---} i - \dots
 \end{aligned}$$



# The tower of DSEs

$$\begin{aligned}
 & \text{quark propagator} \\
 & i \text{---} \bullet \text{---} j \quad^{-1} = + \text{---} i \text{---} j \quad^{-1} - \frac{1}{2} \text{---} i \text{---} \text{loop} \text{---} j - \frac{1}{2} \text{---} i \text{---} \text{gluon} \text{---} i + \text{---} i \text{---} \text{ghost} \text{---} i \\
 & + \text{---} i \text{---} \text{gluon} \text{---} i - \frac{1}{6} \text{---} i \text{---} \text{gluon} \text{---} i - \frac{1}{2} \text{---} i \text{---} \text{ghost} \text{---} j \quad \text{gluon propagator} \\
 & j \text{---} \bullet \text{---} i \quad^{-1} = + j \text{---} i \quad^{-1} - j \text{---} \text{gluon} \text{---} i \quad \text{ghost propagator} \\
 & j \text{---} \bullet \text{---} i \quad^{-1} = + j \text{---} i \quad^{-1} - i \text{---} \text{gluon} \text{---} i \quad \text{quark propagator}
 \end{aligned}$$

$$\begin{aligned}
 & i \text{---} \bullet \text{---} k + i \text{---} \bullet \text{---} k + \frac{1}{2} i \text{---} \text{gluon} \text{---} i - \frac{1}{2} i \text{---} \text{ghost} \text{---} i - \frac{1}{2} i \text{---} \text{gluon} \text{---} i \\
 & i \text{---} \bullet \text{---} k + i \text{---} \bullet \text{---} k + i \text{---} \text{gluon} \text{---} i + i \text{---} \text{ghost} \text{---} i + i \text{---} \text{gluon} \text{---} i
 \end{aligned}$$

Infinitely many equations. In QCD, every  $n$ -point function depends on  $(n + 1)$ - and possibly  $(n + 2)$ -point functions.

$$\begin{aligned}
 & + \frac{1}{2} i \text{---} \text{gluon} \text{---} i + \frac{1}{2} i \text{---} \text{ghost} \text{---} i + \frac{1}{2} i \text{---} \text{gluon} \text{---} i + \frac{1}{2} i \text{---} \text{ghost} \text{---} i \\
 & + \frac{1}{2} i \text{---} \text{gluon} \text{---} i + \frac{1}{2} i \text{---} \text{ghost} \text{---} i + \frac{1}{2} i \text{---} \text{gluon} \text{---} i + \frac{1}{2} i \text{---} \text{ghost} \text{---} i
 \end{aligned}$$

# The tower of DSEs

$$\begin{aligned}
 i \text{---} \bullet \text{---} j^{-1} &= + i \text{---} j^{-1} - \frac{1}{2} i \text{---} \text{gluon loop} \text{---} j - \frac{1}{2} i \text{---} \text{ghost loop} \text{---} j + i \text{---} \text{ghost loop} \text{---} j \\
 &+ i \text{---} \text{gluon loop} \text{---} i - \frac{1}{6} i \text{---} \text{gluon loop} \text{---} i - \frac{1}{2} i \text{---} \text{gluon loop} \text{---} j \quad \text{gluon propagator}
 \end{aligned}$$

$$j \text{---} \bullet \text{---} i^{-1} = + j \text{---} i^{-1} - j \text{---} \text{gluon loop} \text{---} i \quad \text{ghost propagator}$$

$$j \text{---} \bullet \text{---} i^{-1} = + j \text{---} i^{-1} - i \text{---} \text{gluon loop} \text{---} j \quad \text{quark propagator}$$

$$\begin{aligned}
 i \text{---} \text{gluon vertex} \text{---} k &= + i \text{---} \text{gluon vertex} \text{---} k + \frac{1}{2} i \text{---} \text{gluon loop} \text{---} k - \frac{1}{2} i \text{---} \text{ghost loop} \text{---} k - \frac{1}{2} i \text{---} \text{gluon loop} \text{---} k \\
 &+ i \text{---} \text{gluon vertex} \text{---} k + i \text{---} \text{gluon vertex} \text{---} k + i \text{---} \text{gluon vertex} \text{---} k + i \text{---} \text{gluon vertex} \text{---} k
 \end{aligned}$$

Infinitely many equations. In QCD, every  $n$ -point function depends on  $(n + 1)$ - and possibly  $(n + 2)$ -point functions.

Is it possible to find and solve a truncation with all **relevant contributions**?

$$\begin{aligned}
 &+ \frac{1}{2} i \text{---} \text{gluon loop} \text{---} k + \frac{1}{2} i \text{---} \text{ghost loop} \text{---} k + \frac{1}{2} i \text{---} \text{gluon loop} \text{---} k + \frac{1}{2} i \text{---} \text{ghost loop} \text{---} k \\
 &+ \frac{1}{2} i \text{---} \text{gluon loop} \text{---} k + \frac{1}{2} i \text{---} \text{ghost loop} \text{---} k + \frac{1}{2} i \text{---} \text{gluon loop} \text{---} k + \frac{1}{2} i \text{---} \text{ghost loop} \text{---} k
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# Questions about truncations

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- Influence of **higher correlation functions**?
- Hierarchy of diagrams/correlation functions?
- **Model** dependence  $\leftrightarrow$  **Self-contained** truncation?
- How to realize **resummation**?
- Equivalence between different functional methods?



# UV behavior of the gluon propagator

Resummed one-loop order: anomalous dimension  $\gamma = -13/22$

$$\left(1 + \frac{\alpha(s) 11 N_c}{12\pi} \ln \frac{p^2}{s}\right)^\gamma$$

## One-loop anomalous dimension

Origin in resummation of higher order diagrams.

However, one-loop truncation discards some terms.

The diagram shows the Dyson-Schwinger equation for the gluon propagator at one-loop order. On the left, a gluon line with a black vertex is followed by an equals sign. To the right of the equals sign are four terms: 
 1. A gluon line with a black vertex, preceded by a minus sign and a superscript -1.
 2. A gluon line with a black vertex, preceded by a minus sign and a superscript -1/2.
 3. A gluon line with a black vertex, preceded by a minus sign and a superscript -1/2, with a gluon loop attached to the vertex.
 4. A gluon line with a black vertex, preceded by a plus sign, with a ghost loop attached to the vertex.
 The loops are represented by wavy lines for gluons and dashed lines for ghosts.

→ Puts constraints on UV behavior of vertices [von Smekal, Hauck, Alkofer '97]:  
Include in models.

# Resummed behavior

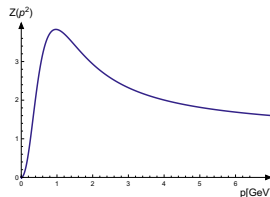
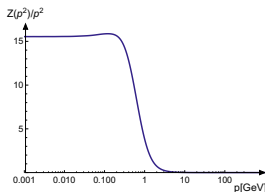
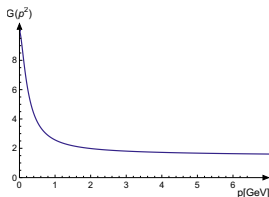
Minimal requirements to obtain one-loop resummed behavior:

- Squint diagram
- Correct anomalous dimensions of three-point functions
- Correct renormalization (constants)

# Resummed behavior

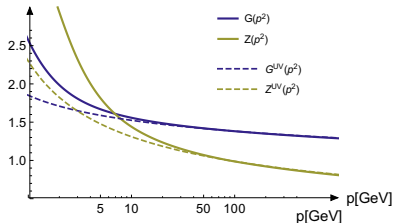
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[propagator+ghost-gluon eqs. full, 3-gluon vertex model, bare 4-gluon vertex]

- Resummed behavior is recovered [MQH '17].



Testing truncations in  $d = 3$ :  
Vary equations and systems of equations.

# $d = 3$ Yang-Mills theory as testing ground

## Advantages:

- UV finite: no renormalization, no anomalous running
- Spurious divergences easier to handle
- UV behavior 'easier':  $\propto \frac{g^2}{p}$  instead of resummed logarithm

→ Many complications from  $d = 4$  absent.

→ Disentanglement of UV easier.

⇒ 'Cleaner' system → Focus on truncation effects.

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⇒ '**Cleaner**' system → Focus on truncation effects.

Historically interesting because cheaper on the lattice → easier to reach the IR.

Numerically not cheaper for functional equations of 2- and 3-point functions.

Continuum results:

- Coupled propagator DSEs: [Maas, Wambach, Grüter, Alkofer '04]
- (R)GZ: [Dudal, Gracey, Sorella, Vandersickel, Verschelde '08]
- DSEs of PT-BFM: [Aguilar, Binosi, Papavassiliou '10]
- YM + mass term: [Tissier, Wschebor '10, '11]

# Dyson-Schwinger equations: Truncation

## Untruncated propagators:

$$\text{---}\bullet\text{---}^{-1} = \text{---}\text{---}^{-1} - \text{---}\bullet\text{---}$$

$$\text{---}\bullet\text{---}^{-1} = \text{---}\text{---}^{-1} - \frac{1}{2} \text{---}\bullet\text{---} - \frac{1}{2} \text{---}\bullet\text{---} + \text{---}\bullet\text{---} - \frac{1}{4} \text{---}\bullet\text{---} - \frac{1}{2} \text{---}\bullet\text{---}$$

## Truncated vertices:

$$\text{---}\bullet\text{---} = \text{---}\text{---} - 2 \text{---}\bullet\text{---} + \text{---}\bullet\text{---} + \frac{1}{2} \text{---}\bullet\text{---} + \frac{1}{2} \text{---}\bullet\text{---} + \frac{1}{2} \text{---}\bullet\text{---}$$

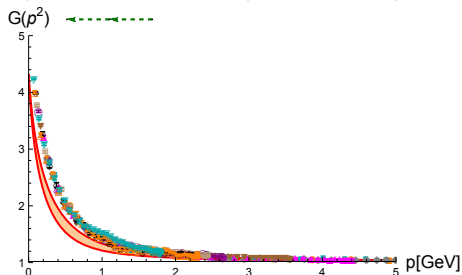
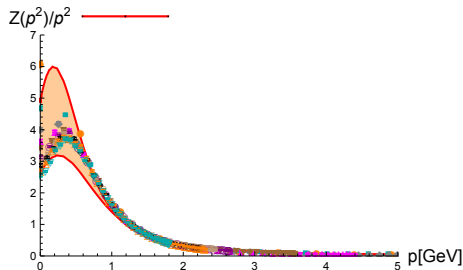
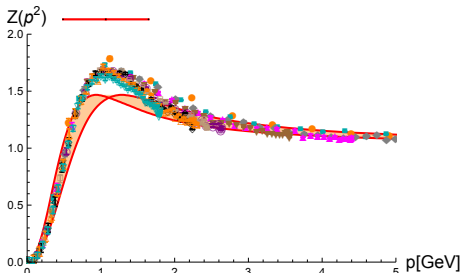
$$\text{---}\bullet\text{---} = \text{---}\text{---} + \text{---}\bullet\text{---} + \text{---}\bullet\text{---}$$

$$\text{---}\bullet\text{---} = \text{---}\text{---} - 2 \text{---}\bullet\text{---} + \text{---}\bullet\text{---} + \text{---}\bullet\text{---} + \text{---}\bullet\text{---} + \text{---}\bullet\text{---} + \text{perm.}$$

# Tests in three dimensions

- ① Hierarchy of diagrams.
- ② Hierarchy of equations.
- ③ Different functional equations.

# Results: Propagators



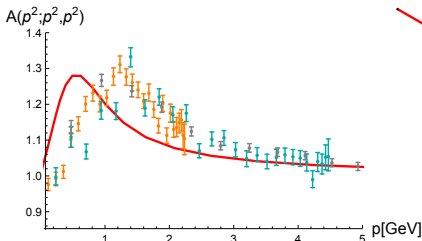
Bands from uncertainty in setting the physical scale.

[MQH '16; lattice: Maas '14]

Similar results with FRG [Cyrol '17]

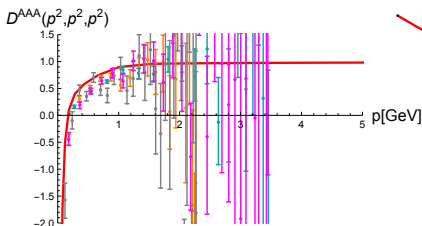
# Comparison of three-point functions with lattice results

## Dressings:



[MQH '16; lattice: Maas, unpublished]

- Deviation from tree-level 'small'
- Position of maximum shifted (as observed with other continuum methods for  $SU(2)$ , e.g., [Pelaez, Matthieu, Wschebor '13; Cyrol, Fister, Mitter, Pawłowski, Strodthoff '15; Cyrol '17]).

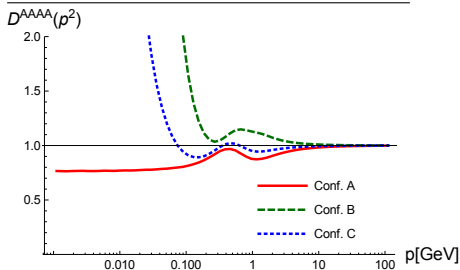


[MQH '16; lattice: Cucchieri, Maas, Mendes '08]

- Close to tree-level above 1 GeV
- Good agreement with lattice data.
- Linear IR divergence [Pelaez, Tissier, Wschebor '13; Aguilar et al. '13]

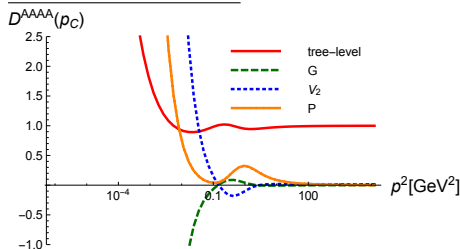
# Four-gluon vertex

Different momentum configurations:



[MQH '16]

Different projections:



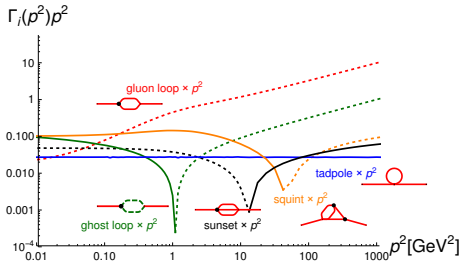
Four-gluon vertex:

- Close to tree-level down to 1 GeV

→ Corrections small individually?

# Gluon propagator: Single diagrams

$$\text{Gluon propagator}^{-1} = \text{Gluon propagator}^{-1} - \frac{1}{2} \left( \text{Gluon loop} \right) \text{Gluon propagator}^{-1} - \frac{1}{2} \left( \text{Gluon tadpole} \right) \text{Gluon propagator}^{-1} + \frac{1}{6} \left( \text{Gluon sunset} \right) \text{Gluon propagator}^{-1} - \frac{1}{2} \left( \text{Gluon squint} \right) \text{Gluon propagator}^{-1}$$



[MQH '16]

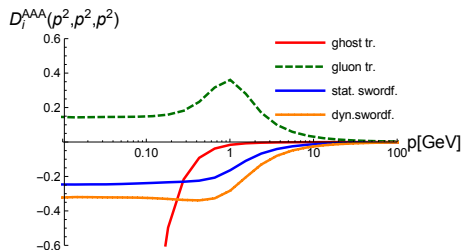
→ Clear **hierarchies** identified.

- UV: as expected perturbatively
  - non-perturbative: squint important, sunset small
- ( $d=4$ :

[Mader, Alkofer '13; Meyers, Swanson '14])

# Cancellations in gluonic vertices

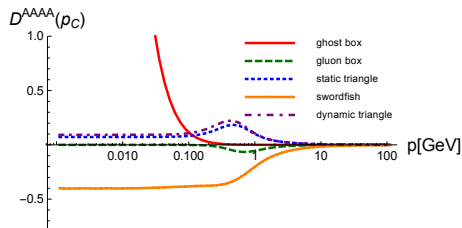
## Three-gluon vertex:



- Individual contributions large.
- Sum is small!

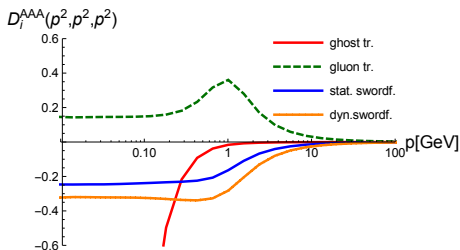
[MQH '16]

## Four-gluon vertex:



# Cancellations in gluonic vertices

## Three-gluon vertex:

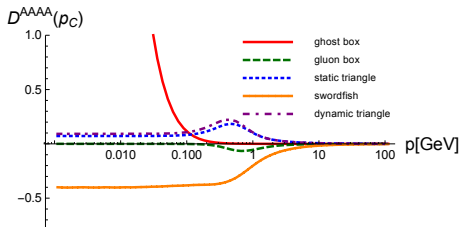


- Individual contributions large.
- **Sum is small!**



[MQH '16]

## Four-gluon vertex:



Higher contributions:

- Higher vertices close to 'tree-level'?  
→ Small.
- If pattern changes (higher vertices large): cancellations required.

# Solution from the 3PI effective action

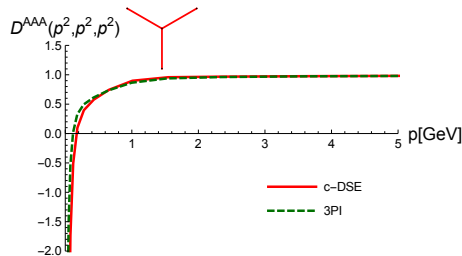
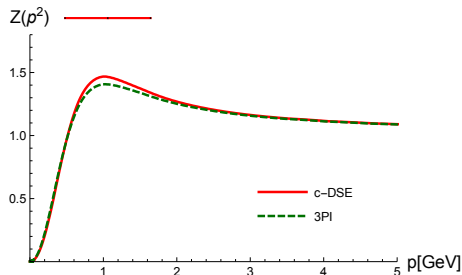
Different set of functional equations:

Equations of motion from 3PI effective action (at three-loop level)

# Solution from the 3PI effective action

Different set of functional equations:

Equations of motion from 3PI effective action (at three-loop level)



→ Very similar results.

[MQH '16]

FRG: Similar results with similar truncation [Cyrol '17, thesis]

# Summary: Three dimensions

- **Hierarchy** of correlation functions and diagrams
- **Cancellations** lead to small deviations from the perturbative behavior above  $2 \text{ GeV}$ .
- Some degree of **stability** (but no complete list of checks done) when
  - varying *system* of equations.
  - varying *equations* of system.
- Discrepancies with lattice results:
  - Nonperturbative gauge fixing?
  - Lattice systematics?
  - Missing diagrams for vertices?
  - Incomplete tensor bases for some vertices?

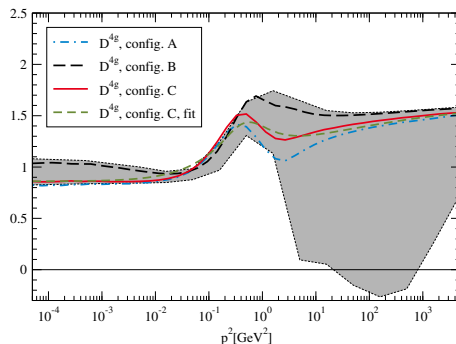
Extending truncations in four dimensions:  
Include four-point functions.

# Four-gluon vertex

Full calculation with fixed input:

[Cyrol, MQH, von Smekal '14]

Computationally expensive!

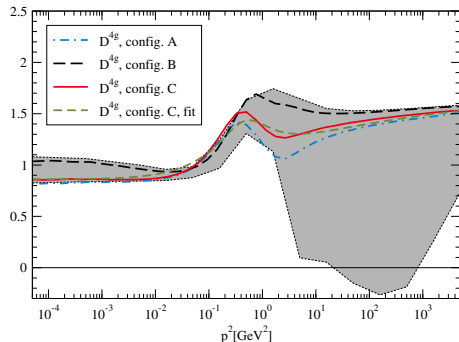
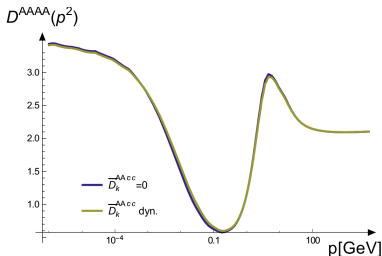


# Four-gluon vertex

Full calculation with fixed input:

[Cyrol, MQH, von Smekal '14]

Computationally expensive!



Three-gluon vertex has small angle dependence.

→ For dynamic inclusion: Resort to a **one-momentum approximation** (symmetric point); see also FRG calculations by fQCD collaboration.

# Effect of four-gluon vertex

- In **three-gluon vertex DSE**:

Important for convergence within current truncations in  $d = 4$

[Blum, MQH, Mitter, von Smekal '14;

Eichmann, Williams, Alkofer, Vujanovic '14; MQH '17]

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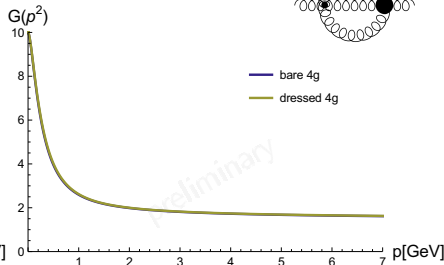
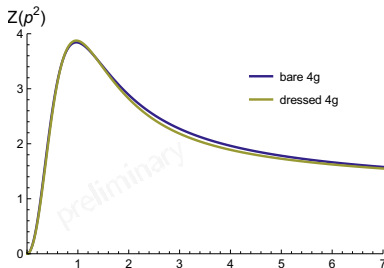
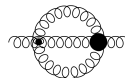
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→ Related to renormalization and missing two-loop diagrams.



- In **gluon propagator**: Via sunset diagram, small contribution of tree-level dressing; model studies: [Mader, Alkofer '13; Meyers, Swanson '14]



[propagator eqs. full, 3-gluon model, ghost-gluon fixed, 4-gluon fixed]

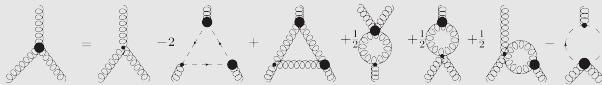
# Extending truncations of three-point functions

Extend truncations of equations of three-point functions by adding the **two-ghost-two-gluon vertex**:

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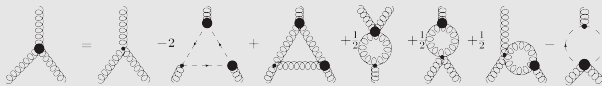


→ One-loop complete equation.

# Extending truncations of three-point functions

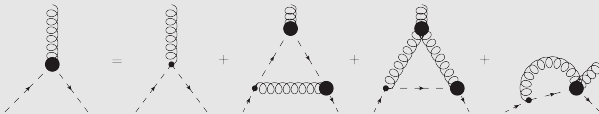
Extend truncations of equations of three-point functions by adding the **two-ghost-two-gluon vertex**:

## Three-gluon vertex



→ One-loop complete equation.

## Ghost-gluon vertex



→ Complete equation.

## Four-ghost vertex:

In alternative ghost-gluon vertex DSE and in four-point functions.

# Four-point functions: Color space

15 possibilities:

$\delta\delta$  : 3 combinations

$ff$  : 3 combinations

$dd$  : 3 combinations

$df$  : 6 combinations

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9/8/3 linearly independent in  $SU(N/3/2)$ ,  $N > 3$   
[Pascual, Tarrach '80].

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$SU(3)$ :  $\{\sigma_1, \dots, \sigma_8\}$  chosen with these symmetries:

	$\sigma_1$	$\sigma_2$	$\sigma_3$	$\sigma_4$	$\sigma_5$	$\sigma_6$	$\sigma_7$	$\sigma_8$
$a \leftrightarrow b$	+	+	+	-	-	-	-	+
$c \leftrightarrow d$	+	+	+	-	-	+	-	-

$\{\sigma_1, \dots, \sigma_5\}$  orthogonal to  $\{\sigma_6, \sigma_7, \sigma_8\}$ .  $\rightarrow \{\sigma_6, \sigma_7, \sigma_8\}$  decouple.

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## Four-ghost vertex



$$\Gamma^{\bar{c}\bar{c}cc,abcd}(p, q, r, s) = \mathbf{g}^4 \sum_{k=1}^8 \sigma^{k,abcd} E_k^{\bar{c}\bar{c}cc}(p, q, r, s).$$

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Non-primitively divergent correlation function  $\rightarrow$  No guide from tree-level tensor.  $\rightarrow$  Use full basis.

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## Two-ghost-two-gluon vertex



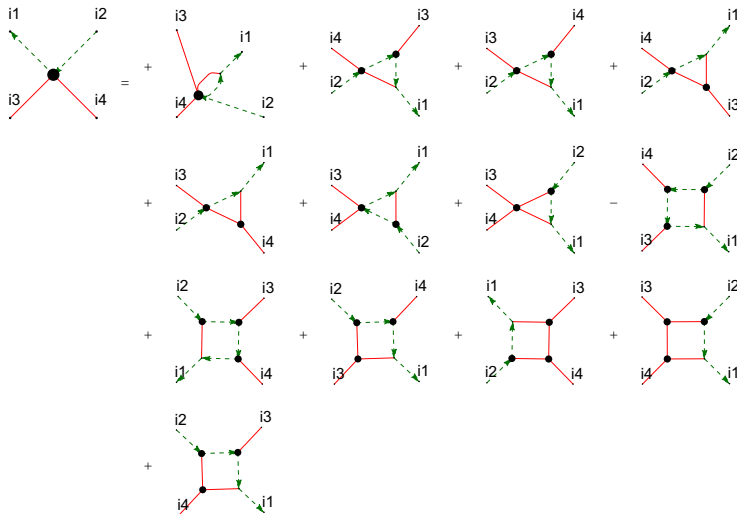
$$\Gamma_{\mu\nu}^{AA\bar{c}c,abcd}(p, q; r, s) = g^4 \sum_{k=1}^{40} \rho_{\mu\nu}^{k,abcd} D_{k(i,j)}^{AA\bar{c}c}(p, q; r, s)$$

with

$$\rho_{\mu\nu}^{k,abcd} = \sigma_i^{abcd} \tau_{\mu\nu}^j, \quad k = k(i, j) = 5(i - 1) + j$$

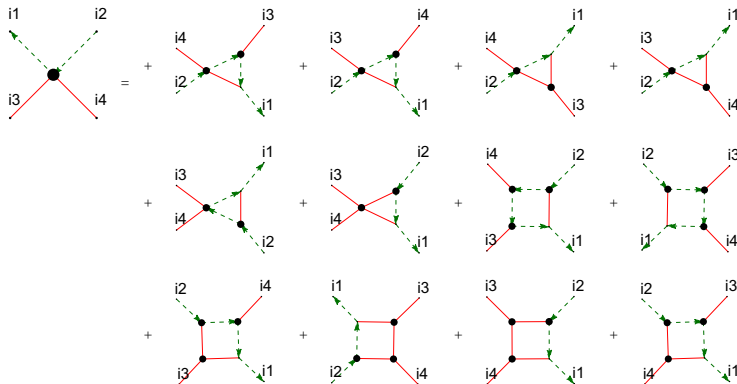
# The two-ghost-two-gluon vertex DSE

2 DSEs, choose the one with the ghost leg attached to the bare vertex  
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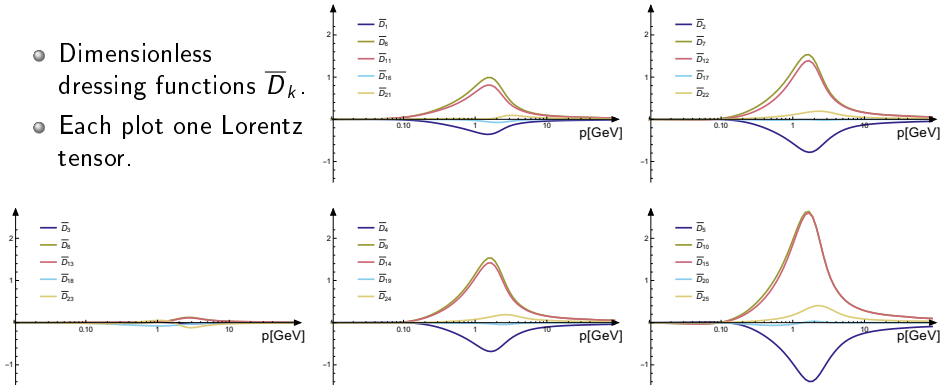
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# Results for the two-ghost-two-gluon vertex

Kinematic approximation: one-momentum configuration

- Dimensionless dressing functions  $\bar{D}_k$ .
- Each plot one Lorentz tensor.



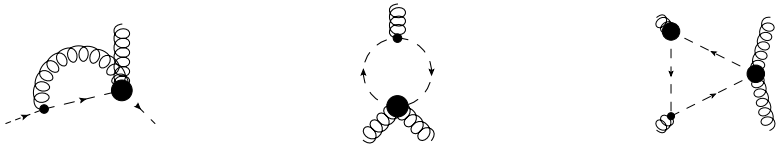
→ Two classes of dressings: 13 very small, 12 not small

→ No nonzero solution for  $\{\sigma_6, \sigma_7, \sigma_8\}$  found.

[MQH '17]

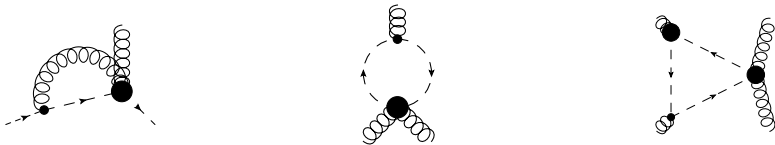
# Influence of two-ghost-two-gluon vertex

Coupled system of ghost-gluon, three-gluon and four-gluon vertices **with and without** two-ghost-two-gluon vertex:

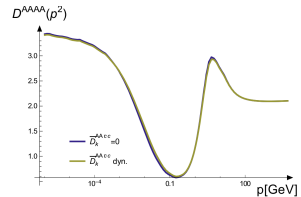
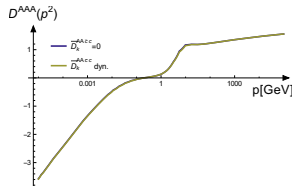
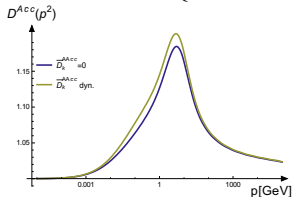


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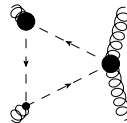
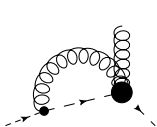
Color space:  $\{\sigma_6, \sigma_7, \sigma_8\}$  do not appear here!



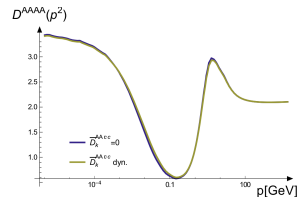
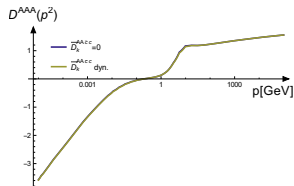
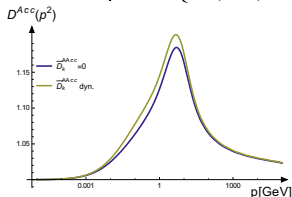
- **Small** influence on ghost-gluon vertex ( $< 1.7\%$ )
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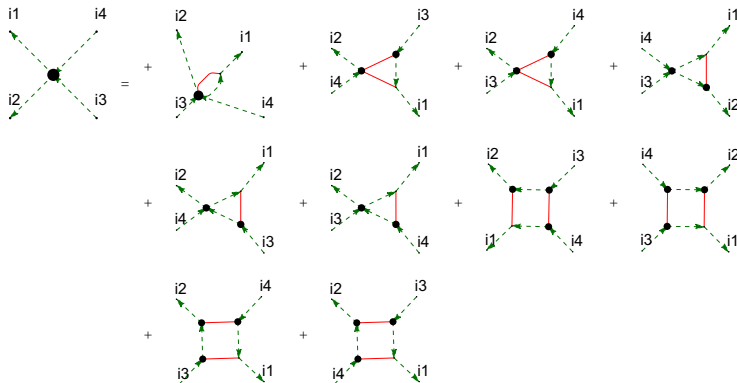
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Color space: Only small dressings couple to three-gluon vertex!

[MQH '17]

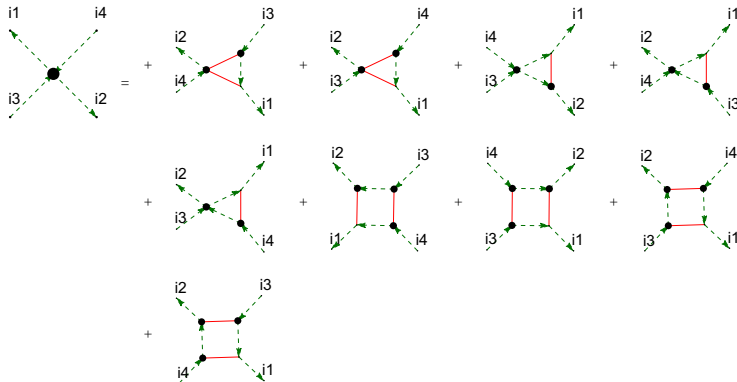
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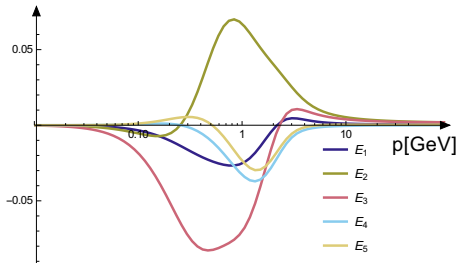
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# Results for the four-ghost vertex

Kinematic approximation: one-momentum configuration



→ All dressings very small.

[MQH '17]

$E_6, E_7, E_8 (\{\sigma_6, \sigma_7, \sigma_8\})$

Decouple into a homogeneous, linear equation. → Trivial solution always exists.  
Nontrivial one? → None found.

(Same applies to two-ghost-two-gluon vertex.)

# Summary and conclusions

Based on

- tests in  $d = 3$  including comparison with 3PI calculations
- analysis of one-loop resummation
- testing non-primitively divergent correlation functions

a **non-perturbative hierarchy** of correlations functions and diagrams can be identified.

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Required quantitatively and for self-consistency.

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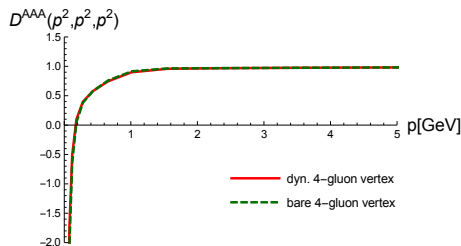
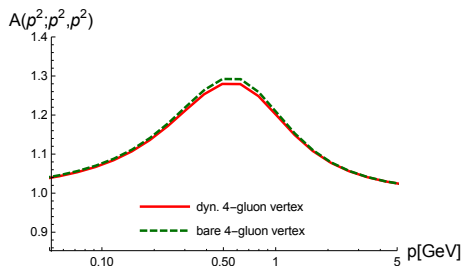
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Thank you for your attention!

# Influence of four-gluon vertex on three-point functions



[MQH '16]

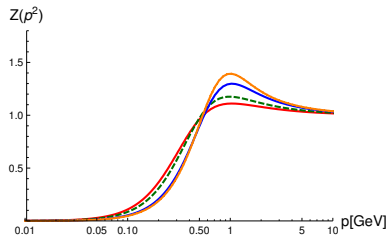
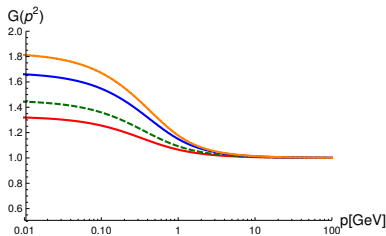
- Influence of four-gluon vertex small.

# Family of solutions in three dimensions

Cf. FRG results: Bare mass parameter from modified STIs [Cyrol, Fister, Mitter, Pawłowski, Strodthoff '15].

DSEs: Enforce family of solutions by fixing the gluon propagator at  $p^2 = 0$ .

Simple toy system with bare vertices [MQH, 1606.02068]:



⇒ Possibility of family of solutions.

NB: Effect overestimated here since vertices are fixed.