Recent developments in the calculation of correlation functions of Yang-Mills theory

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Graz, Austria

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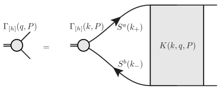
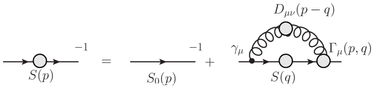
Hadronic bound states

Bound state equations:



Ingredients:

- Interaction kernel K
- Quark propagator S



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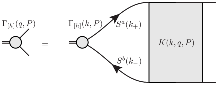

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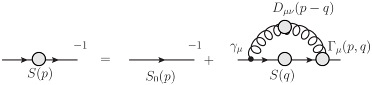

Approaches:

- Phenomenological:
  Model interactions
- From first principles:
  Piecing together the pieces

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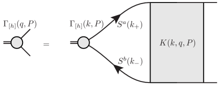

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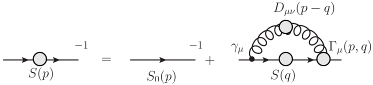

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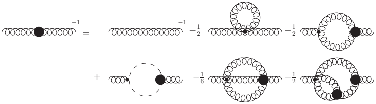
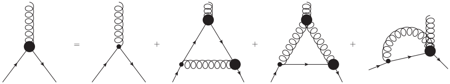
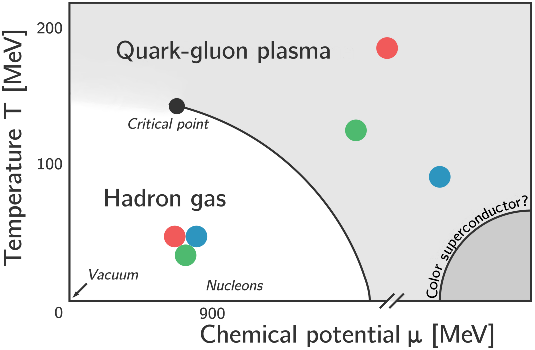




$\rightarrow$ Couples to infinity of equations.

QCD phase diagram



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Questions:

- Phases and transitions between them, critical point
- Experimental signatures

Theoretical challenges:

- Model description
- Mathematical, e.g., complex action for lattice QCD
- Complexity, e.g., truncations of function eqs.
- ...

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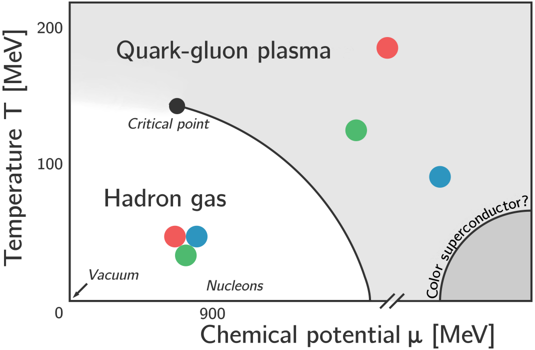

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- How to realize resummation?
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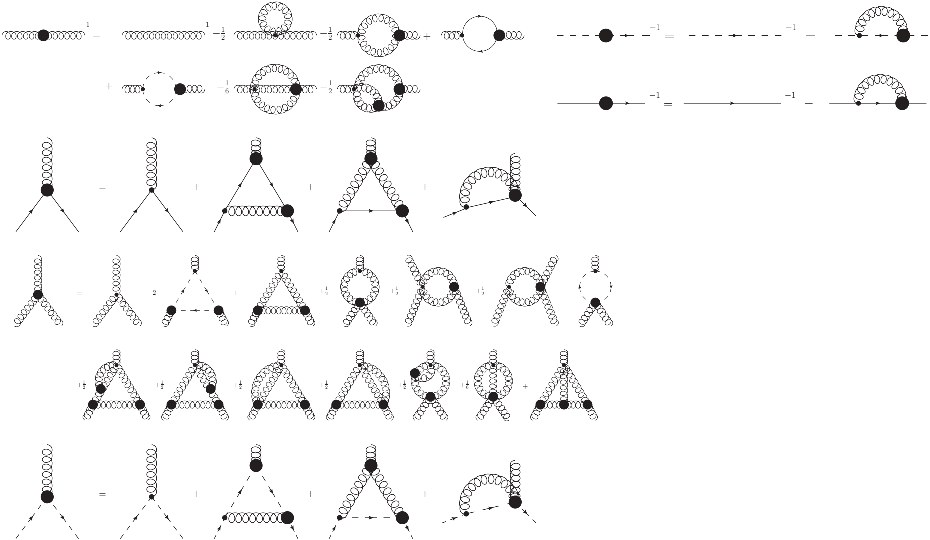
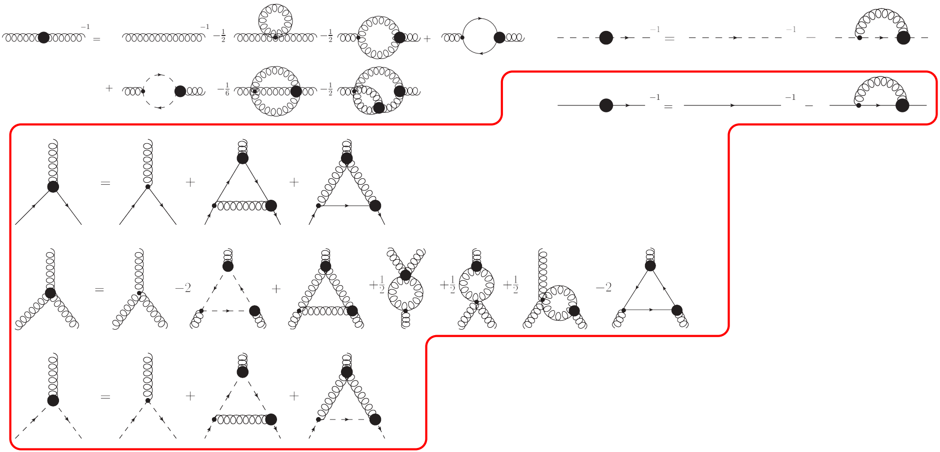
- Equivalence between different functional methods?
  FRG, DSEs, nPI, Hamiltonian approach

Dyson-Schwinger equations



Coupled systems of Dyson-Schwinger equations



quark propagator + 3-point functions: [Williams, Fischer, Heupel '15] → application to bound states

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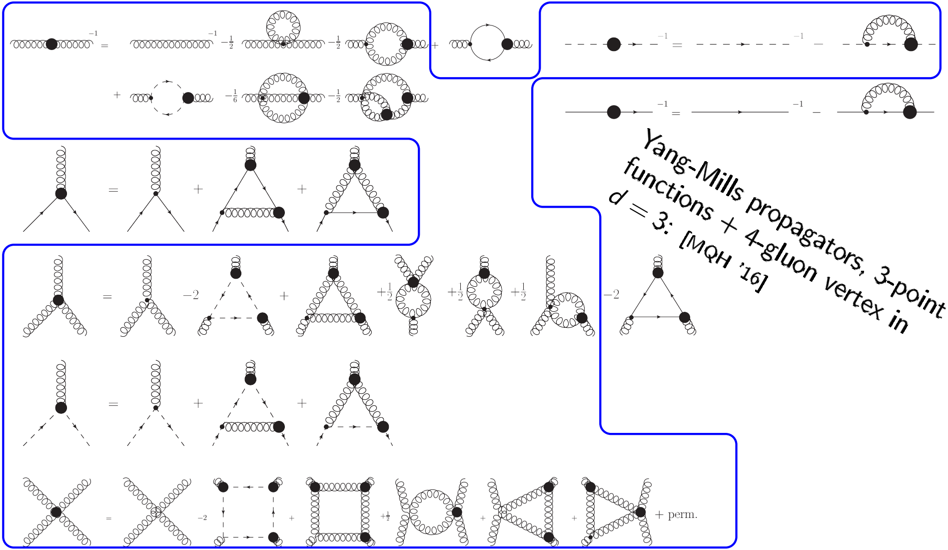
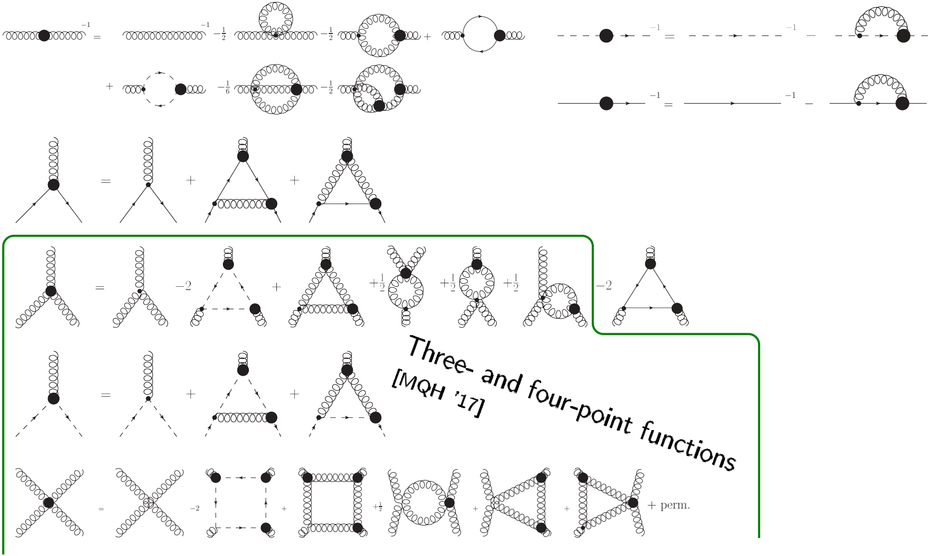


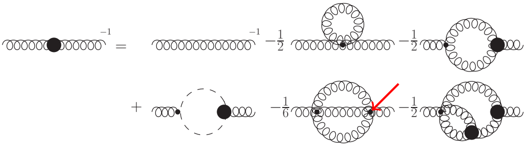
Yang-Mills propagators, 3-point functions + 4-gluon vertex in $d = 3$: [MQH '16]

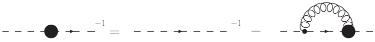
Coupled systems of Dyson-Schwinger equations

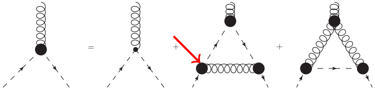


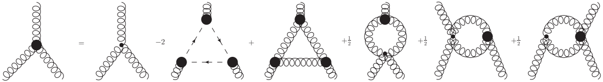
3PI system of equations

Three-loop expansion of PI effective action [Berges '04]:



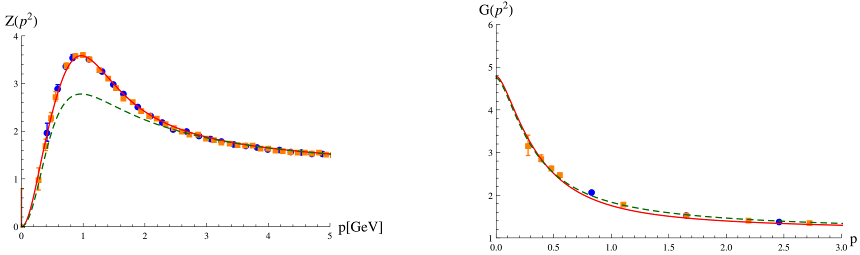






Propagators and ghost-gluon vertex with three-gluon vertex model

One-loop truncation of gluon propagator with an optimized effective model (contains zero crossing) [MQH, von Smekal '13]:



Good quantitative agreement for ghost and gluon dressings.

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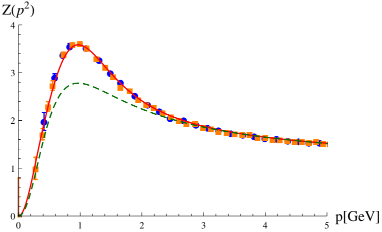
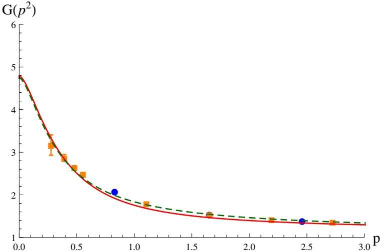




Good quantitative agreement for ghost and gluon dressings.

QCD is only this:

$$\boxed{\mathcal{L} = -\frac{1}{2}\mathrm{Tr}\left(F_{\mu\nu}F^{\mu\nu}\right) + \sum_j \bar{\varphi}_j [i\gamma^\mu D_\mu - m_j]\varphi_j}$$

WOBEI $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + ig[A_\mu, A_\nu]$

UND $D_\mu = \partial_\mu + igA_\mu$

Can we do with only that?

UV behavior of the gluon propagator

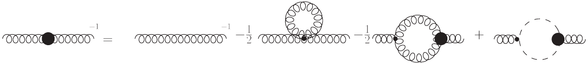
Resummed one-loop order: anomalous dimension $\gamma = -13/22$

$$\left(1 + \frac{\alpha(s) 11 N_c}{12\pi} \ln \frac{p^2}{s}\right)^{\gamma}$$

One-loop anomalous dimension

Origin in resummation of higher order diagrams.

However, one-loop truncation discards some terms.



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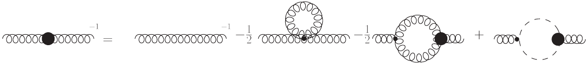

$\rightarrow$ Puts constraints on UV behavior of vertices [von Smekal, Hauck, Alkofer '97].
Way out: Include in models (for now).

Resummed behavior

Minimal requirements to obtain one-loop resummed behavior:

- Squint diagram
- Correct anomalous dimensions of three-point functions
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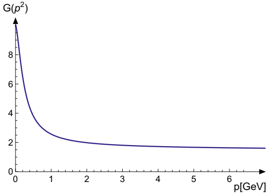
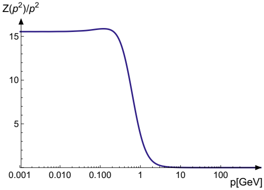
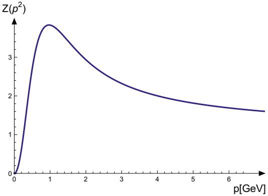
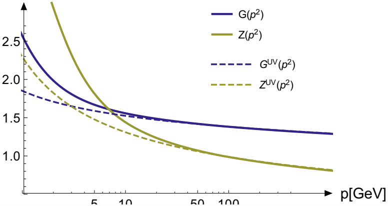






[propagator+ghost-gluon eqs. full, 3-gluon vertex model, bare 4-gluon vertex]

- Resummed behavior is recovered [MQH '17].



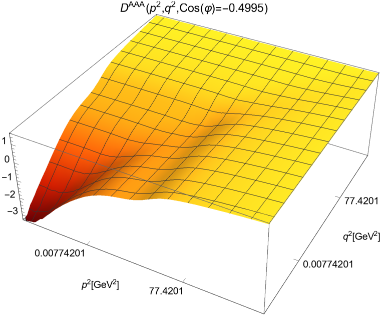
Extending truncations

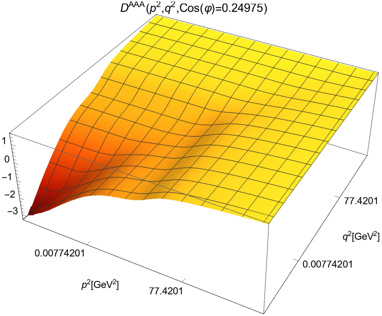
Various ways to extend truncations:

- Vertex tensors beyond tree-level
- Neglected diagrams
- Neglected correlation functions

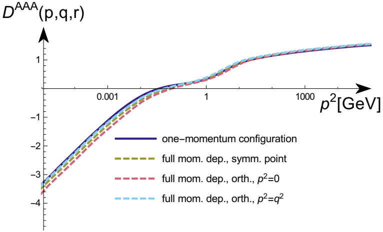
Extensions also test the previous truncations!

Three-gluon vertex: Kinematic dependence



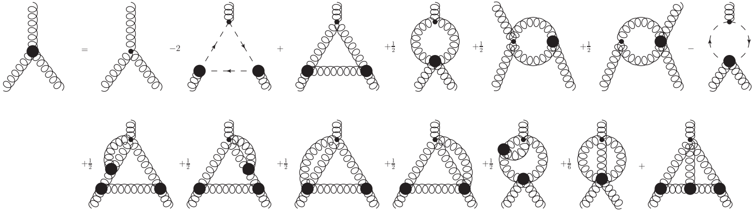


- Kinematic dependence weak.
- In the following: One-momentum approximation



Three-gluon vertex DSE

Full DSE:



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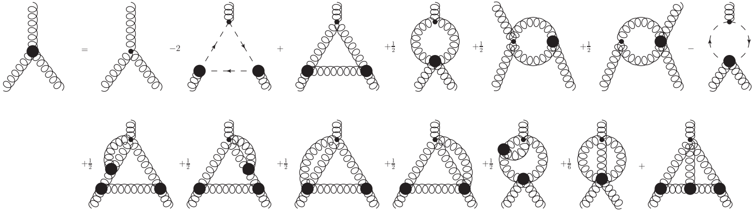
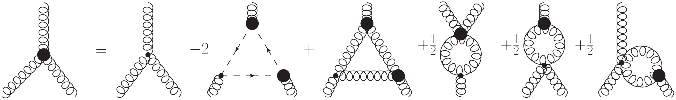


Perturbative one-loop truncation [Blum, MQH, Mitter von Smekal '14; Eichmann, Alkofer, Vujinovic '14]:



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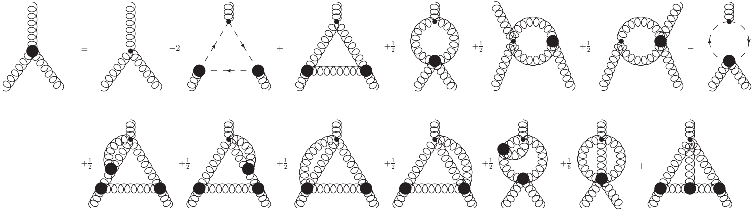

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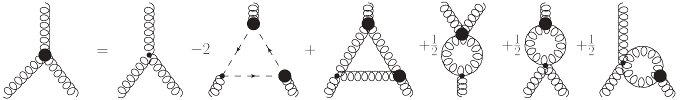
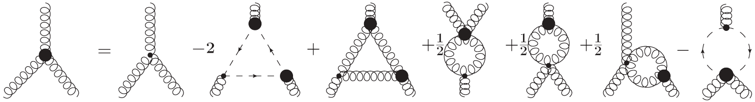
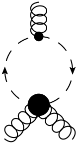


Non-perturbative one-loop truncation [MQH '17]:



Influence of two-ghost-two-gluon vertex



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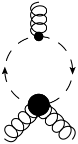
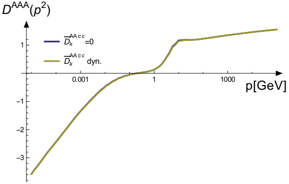


Coupled system of ghost-gluon, three-gluon and four-gluon vertices with and without two-ghost-two-gluon vertex [MQH '17]:



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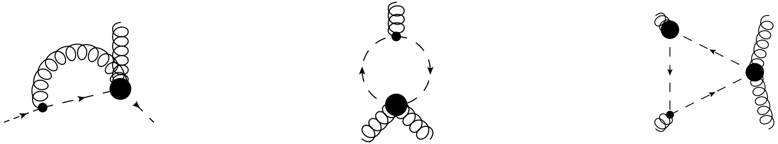

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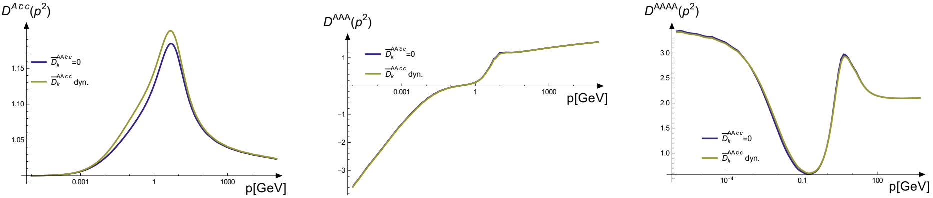
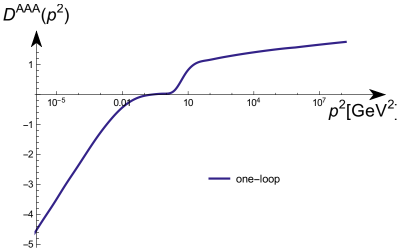


- Small influence on ghost-gluon vertex ($< 1.7\%$)
- Negligible influence on three- and four-gluon vertices.

Three-gluon vertex results



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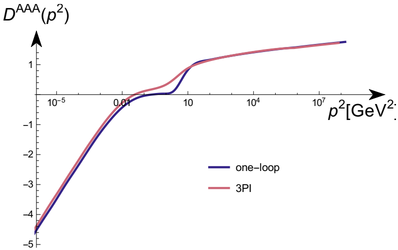

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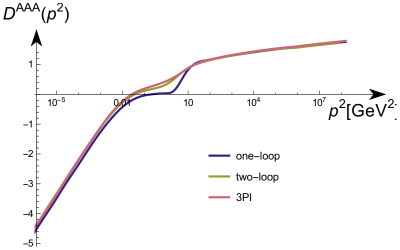

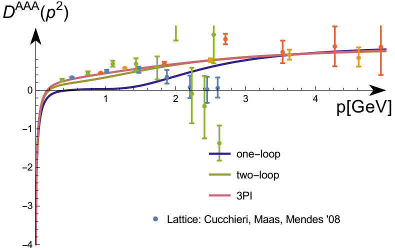

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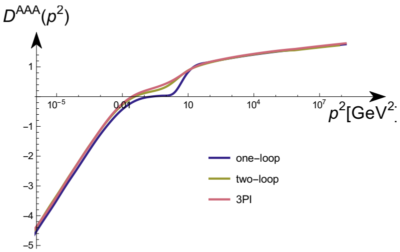
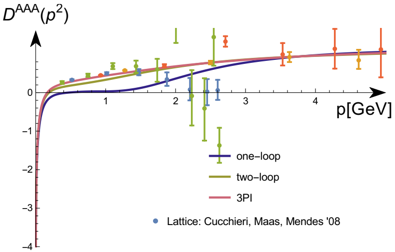
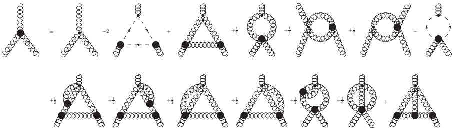




- Two-loop truncation: All diagrams except the one with a five-point function.



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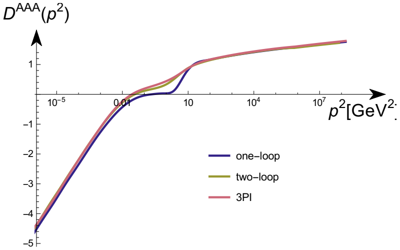
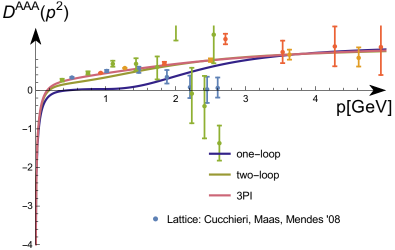
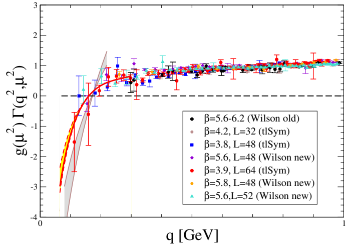




- Difference between two-loop DSE and 3PI smaller than lattice error.
- Resolves ambiguity in zero crossing due to RG improvement [Blum et al. '14; Eichmann et al. '14; Williams et al. '16]
- Zero crossing in agreement with other approaches, e.g., [Pelaez et al. '13; Aguilar et al. '13; Athenodorou et al. '16; Duarte et al. '16; Sternbeck et al. '17]



[Athenodorou et al. '16, '18]

The two-ghost-two-gluon vertex

Non-primitively divergent correlation function $\rightarrow$ No guide from tree-level tensor. $\rightarrow$ Use full basis.

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Lorentz basis transverse wrt gluon legs → 5 tensors $\tau^i_{\mu\nu}(p,q;r,s)$, (anti-)symmetric under exchange of gluon legs.
Color basis: 8 tensors (results show that only 5 required).

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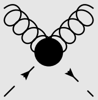


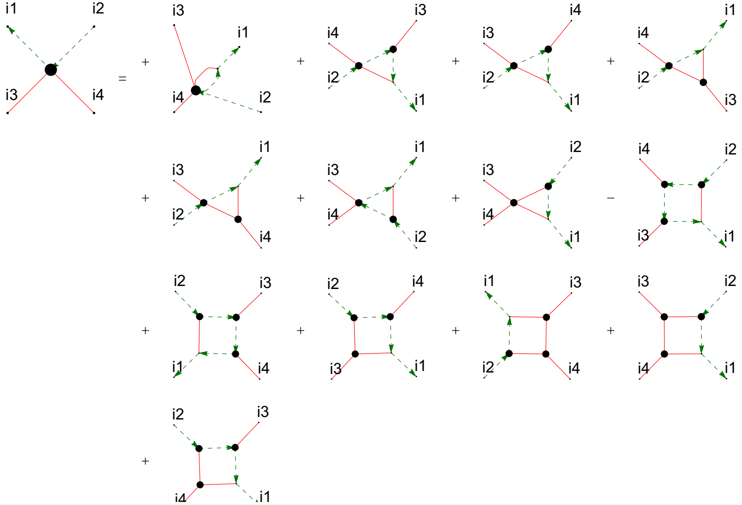
$$\Gamma^{AA\bar{c}c,abcd}_{\mu\nu}(p,q;r,s) = g^4 \sum_{k=1}^{40} \rho^{k,abcd}_{\mu\nu} D^{AA\bar{c}c}_{k(i,j)}(p,q;r,s)$$

with

$$\rho^{k,abcd}_{\mu\nu} = \sigma^{abcd}_i \tau^j_{\mu\nu}, \qquad k = k(i,j) = 5(i-1)+j$$

The two-ghost-two-gluon vertex DSE

2 DSEs, choose the one with the ghost leg attached to the bare vertex
$\rightarrow$ Truncation discards only one diagram.



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2 DSEs, choose the one with the ghost leg attached to the bare vertex
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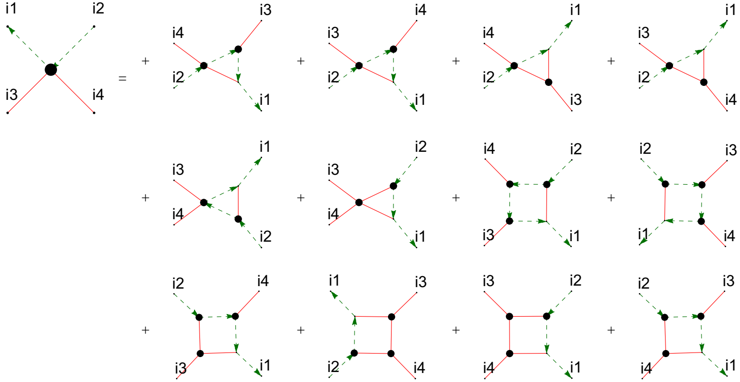
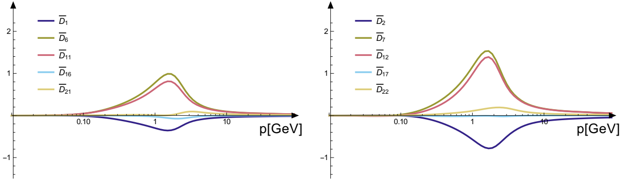
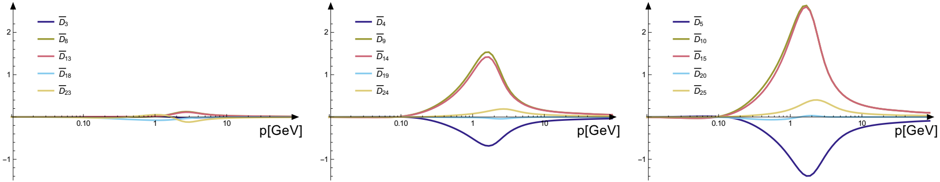


Results for the two-ghost-two-gluon vertex

Kinematic approximation: one-momentum configuration

- Dimensionless dressing functions $\overline{D}_k$.
- Each plot one Lorentz tensor.

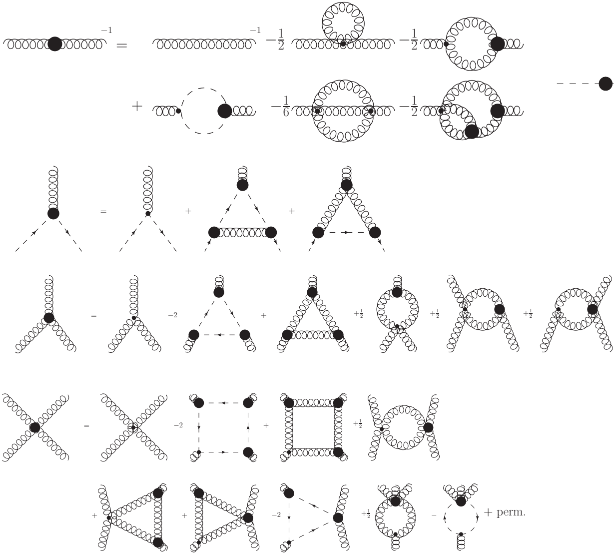




→ Two classes of dressings: 13 very small, 12 not small

→ No nonzero solution for $\{\sigma_6, \sigma_7, \sigma_8\}$ found. [MQH '17]

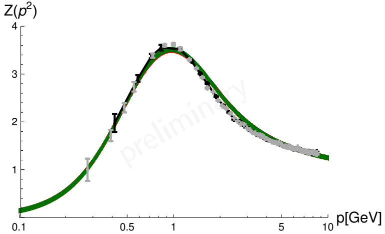
3PI system of primitively divergent correlation functions



Four-gluon vertex included to calculate Z_4.

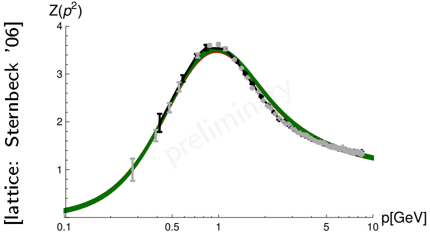
Results for 3PI system

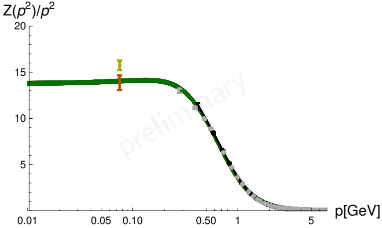
[lattice: Sternbeck '06]



Note: Two solutions with different renormalization parameter $D(0)$ on top of each other.

Results for 3PI system





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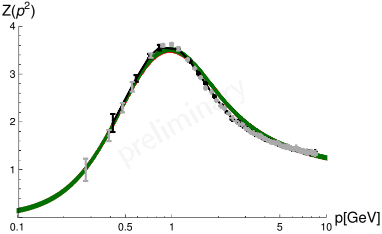
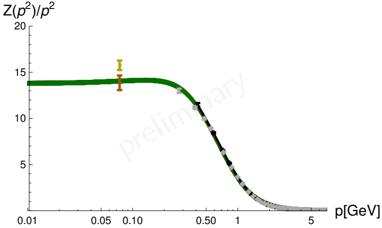
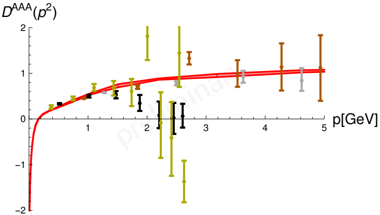




Note: Two solutions with different renormalization parameter $D(0)$ on top of each other.

- Details of renormalization crucial!
- Other details also important.



[lattice: Cucchieri, Maas, Mendes '08]

Summary and conclusions

Towards a systematic understanding of truncations of functional equations to establish them as a first principles method.

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- Add quarks
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- Bound states
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Thank you for your attention!