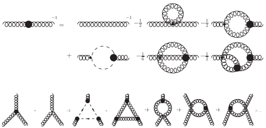
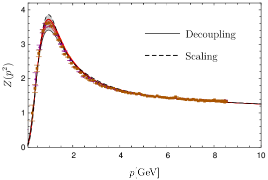
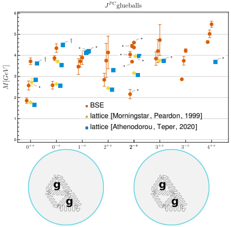
From correlation functions to bound states: A gauge-dependent way to observables







Markus Q. Huber

Institute of Theoretical Physics, Giessen University

Institute of Physics, Graz, Austria

February 24, 2021

FWF Der Wissenschaftsfonds.

JUSTUS-LIEBIG-UNIVERSITÄT GIESSEN

DFG Deutsche Forschungsgemeinschaft

Gauge theories

Gauge theory: Lagrangian with gauge fields (+matter fields)

$\rightarrow$ Defines propagators and vertices of elementary fields.

Gauge theories

Gauge theory: Lagrangian with gauge fields (+matter fields)

$\rightarrow$ Defines propagators and vertices of elementary fields.

Use propagators and vertices:

- Expansion in coupling: Perturbative methods work well if coupling small (despite being a divergent series).
- Nonperturbative calculation with one of various functional methods

Calculations with propagators and vertices require fixing of the gauge to remove physically equivalent configurations.

Gauge dependence must cancel for observables.

Gauge theories

Gauge theory: Lagrangian with gauge fields (+matter fields)

$\rightarrow$ Defines propagators and vertices of elementary fields.

Use propagators and vertices:

- Expansion in coupling: Perturbative methods work well if coupling small (despite being a divergent series).
- Nonperturbative calculation with one of various functional methods

Calculations with propagators and vertices require fixing of the gauge to remove physically equivalent configurations.

Gauge dependence must cancel for observables.

- Alternative: Numerical evaluation of the path integral $\rightarrow$ lattice methods

Quantum chromodynamics

Many facets to study:

- Hadron masses: Origin? (Symmetry) patterns? Numbers?
- Searches for new physics in high- and low-energy regimes

  e.g., background at hadron colliders, anomalous magnetic moment of muon
- Dense system: neutron stars $\rightarrow$ astrophysics
- Hot system: evolution of the universe $\rightarrow$ cosmology
- ...

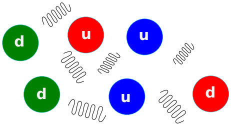
Quantum chromodynamics

Many facets to study:

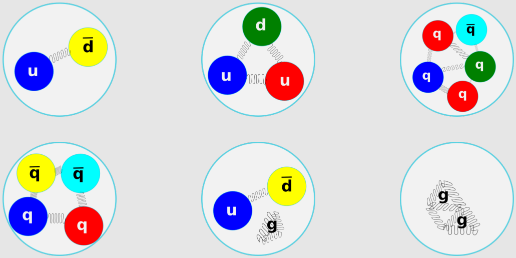
- Hadron masses: Origin? (Symmetry) patterns? Numbers?
- Searches for new physics in high- and low-energy regimes

  e.g., background at hadron colliders, anomalous magnetic moment of muon
- Dense system: neutron stars → astrophysics
- Hot system: evolution of the universe → cosmology
- ...

Quarks and gluons



Meson, baryons, tetraquarks, pentaquarks, hybrids, glueballs



Yang-Mills theory

- Quark models: Descriptive picture of bound states
- Effective theories can capture the relevant degrees of freedom, e.g., chiral perturbation theory, quark-meson model. Underlying dynamics is absorbed in effective parameters.

Yang-Mills theory

- Quark models: Descriptive picture of bound states
- Effective theories can capture the relevant degrees of freedom, e.g., chiral perturbation theory, quark-meson model. Underlying dynamics is absorbed in effective parameters.

So why consider gluons alone (Yang-Mills theory)?

Yang-Mills theory

- Quark models: Descriptive picture of bound states
- Effective theories can capture the relevant degrees of freedom, e.g., chiral perturbation theory, quark-meson model. Underlying dynamics is absorbed in effective parameters.

So why consider gluons alone (Yang-Mills theory)?

- Gluons a challenge (IR behavior) for functional methods.

Warning: Adding quarks not trivial after all.

Yang-Mills theory

- Quark models: Descriptive picture of bound states
- Effective theories can capture the relevant degrees of freedom, e.g., chiral perturbation theory, quark-meson model. Underlying dynamics is absorbed in effective parameters.

So why consider gluons alone (Yang-Mills theory)?

- Gluons a challenge (IR behavior) for functional methods.

  Warning: Adding quarks not trivial after all.
- Important for dynamics, e.g., dynamical chiral symmetry breaking strongly dependent on gauge sector (if calculated and not modeled).

Yang-Mills theory

- Quark models: Descriptive picture of bound states
- Effective theories can capture the relevant degrees of freedom, e.g., chiral perturbation theory, quark-meson model. Underlying dynamics is absorbed in effective parameters.

So why consider gluons alone (Yang-Mills theory)?

- Gluons a challenge (IR behavior) for functional methods.

  Warning: Adding quarks not trivial after all.
- Important for dynamics, e.g., dynamical chiral symmetry breaking strongly dependent on gauge sector (if calculated and not modeled).
- Glueballs.

Yang-Mills theory

- Quark models: Descriptive picture of bound states
- Effective theories can capture the relevant degrees of freedom, e.g., chiral perturbation theory, quark-meson model. Underlying dynamics is absorbed in effective parameters.

So why consider gluons alone (Yang-Mills theory)?

- Gluons a challenge (IR behavior) for functional methods.

  Warning: Adding quarks not trivial after all.
- Important for dynamics, e.g., dynamical chiral symmetry breaking strongly dependent on gauge sector (if calculated and not modeled).
- Glueballs.
- Template for other gauge theories.

Hadrons

Hadron masses from correlation functions of color singlet operators.

Hadrons

Hadron masses from correlation functions of color singlet operators.

Example: For $J^{PC} = 0^{++}$ glueball take $O(x) = F_{\mu\nu}(x)F^{\mu\nu}(x)$:

$$D(x-y) = \langle O(x)O(y)\rangle$$

$\rightarrow$ Lattice: Mass from this correlator by exponential Euclidean time decay.

Hadrons

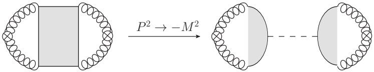
Hadron masses from correlation functions of color singlet operators.

Example: For $J^{PC} = 0^{++}$ glueball take $O(x) = F_{\mu\nu}(x)F^{\mu\nu}(x)$:

$$D(x-y) = \langle O(x)O(y)\rangle$$

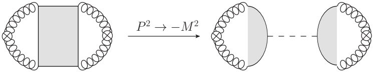
$\rightarrow$ Lattice: Mass from this correlator by exponential Euclidean time decay.

A^4-part of $D(x-y)$, total momentum on-shell:

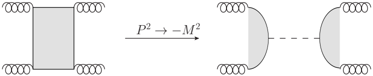


Hadrons

A^4-part of $D(x-y)$, total momentum on-shell:

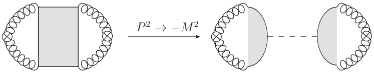


For bound state equations, consider general four-point function:
$\to$ Bethe-Salpeter wave functions

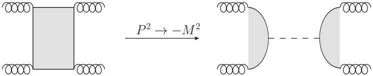


Hadrons

A^4-part of $D(x-y)$, total momentum on-shell:



For bound state equations, consider general four-point function:
$\to$ Bethe-Salpeter wave functions

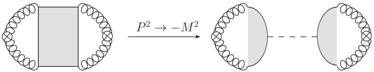


Four-point functions is gauge dependent

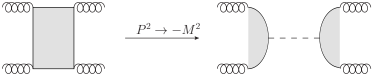
The pole is gauge invariant!

Hadrons

A^4-part of $D(x-y)$, total momentum on-shell:



For bound state equations, consider general four-point function:
$\to$ Bethe-Salpeter wave functions

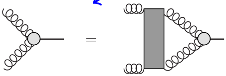


Four-point functions is gauge dependent

The pole is gauge invariant!

$\to$ '2-gluon'-component sufficient to calculate mass.

Glueball bound state equation (BSE) analogous to mesons?



Need  and , solve for . $\to$ Mass

Calculation of correlation functions

Various functional methods

- Dyson-Schwinger equations
- Equations of motion for an *n*PI effective action
- Functional renormalization group
- Nielsen identities
- Bound state equations

Calculation of correlation functions

Various functional methods

- Dyson-Schwinger equations
- Equations of motion for an *n*PI effective action
- Functional renormalization group
- Nielsen identities
- Bound state equations

- Work with fully dressed quantities
- Infinite sets of equations
- Process of making them finite: Truncation

Calculation of correlation functions

Various functional methods

- Dyson-Schwinger equations
- Equations of motion for an *n*PI effective action
- Functional renormalization group
- Nielsen identities
- Bound state equations

- Work with fully dressed quantities
- Infinite sets of equations
- Process of making them finite: Truncation

- Large scale separations
- Fermions technically 'straightforward'
- Kinematic dependences can be resolved
- Higher correlation functions accessible
- (Time-like momenta accessible)

Truncations

Neglecting/Modeling some part

- Guidance from analysis of asymptotic behavior (UV, IR).
- Systematic tests possible by extending a truncation.

Truncations

Neglecting/Modeling some part

- Guidance from analysis of asymptotic behavior (UV, IR).
- Systematic tests possible by extending a truncation.

Even truncated equations can, e.g.,

- elucidate chiral symmetry breaking (dynamical mass creation)
- give hints on the internal structure (tetraquarks)
- identify dominant contributions.

Truncations

Neglecting/Modeling some part

- Guidance from analysis of asymptotic behavior (UV, IR).
- Systematic tests possible by extending a truncation.

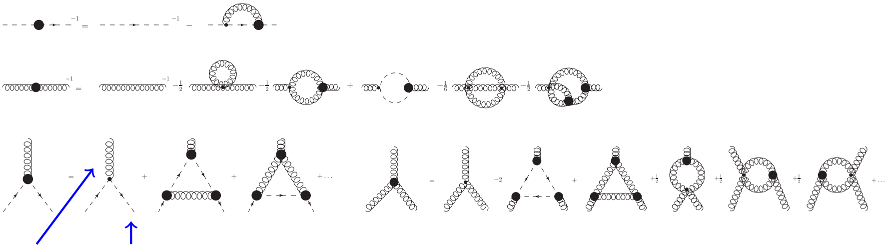
Even truncated equations can, e.g.,

- elucidate chiral symmetry breaking (dynamical mass creation)
- give hints on the internal structure (tetraquarks)
- identify dominant contributions.

Improving truncations

Iterative procedure: Define one, test it, learn from it, find a better one

Equations of motion from 3-loop 3PI effective action

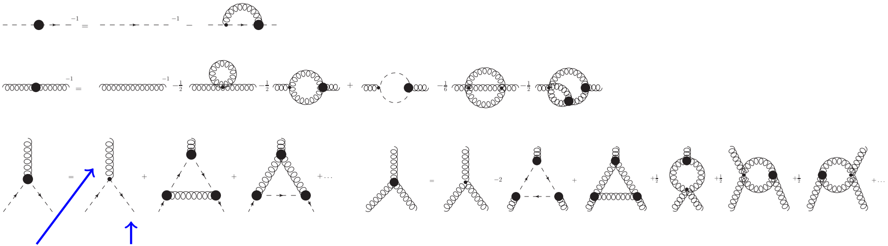


Gluon and ghost fields: Elementary fields of Yang-Mills theory in the Landau gauge

Self-contained system of equations with the scale as the only input.

Truncation?

Equations of motion from 3-loop 3PI effective action

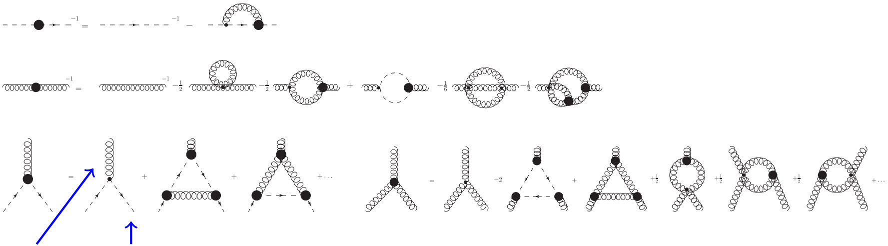


Gluon and ghost fields: Elementary fields of Yang-Mills theory in the Landau gauge

Self-contained system of equations with the scale as the only input.

Truncation? $\rightarrow$ 3-loop expansion of 3PI effective action

Equations of motion from 3-loop 3PI effective action



Gluon and ghost fields: Elementary fields of Yang-Mills theory in the Landau gauge

Self-contained system of equations with the scale as the only input.

Truncation? $\rightarrow$ 3-loop expansion of 3PI effective action

- 4 coupled integral equations with full kinematic dependence.
- Sufficient numerical accuracy required for renormalization.
- One- and two-loop diagrams [Meyers, Swanson '14; MQH '17].

Technical tools

DoFun [Alkofer, MQH, Schwenzer '08; MQH, Braun '11; MQH, Cyrol, Pawlowski '19]

Collection of Mathematica packages for

- Deriving functional equations: Dyson-Schwinger eqs., flow eqs., correlation functions for composite operators
- Automatization of Feynman rules
- `https://github.com/markusqh/DoFun`

CrasyDSE [MQH, Mitter '11]

C++ framework for

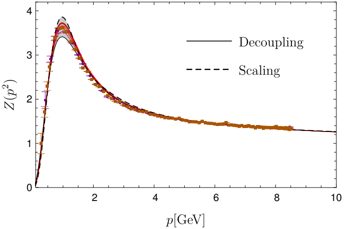
- Interpolation
- Integration
- Kernel code creation from Mathematica

Other tools

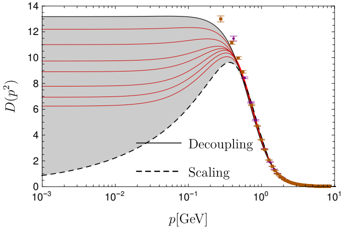
- FORM (code optimization for higher correlation functions) [Ruijl, Ueda, Vermaseren '17]
- Self-made Mathematica packages for color and Lorentz algebra

Landau gauge propagators

Gluon dressing function:

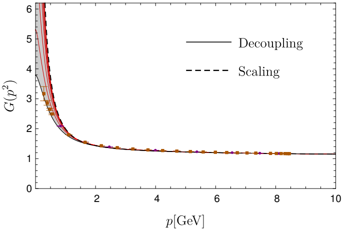


Gluon propagator:



- Family of solutions: Nonperturbative completions of Landau gauge [Maas '10]
- Realized by condition on $G(0)$ [Fischer, Maas, Pawlowski '08; Alkofer, Huber, Schwenzer '08]

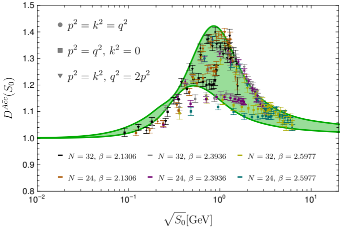
Ghost dressing function:



[Sternbeck '06; MQH '20]

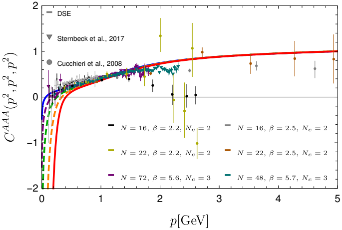
Landau gauge vertices

Ghost-gluon vertex:



[Maas '19; MQH '20]

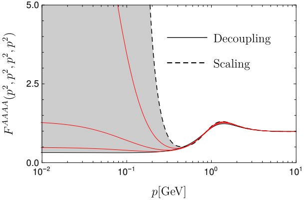
Three-gluon vertex:



[Cucchieri, Maas, Mendes '08; Sternbeck et al. '17; MQH '20]

- Nontrivial kinematic dependence of ghost-gluon vertex
- Simple kinematic dependence of three-gluon vertex
- Four-gluon vertex from solution

Four-gluon vertex:

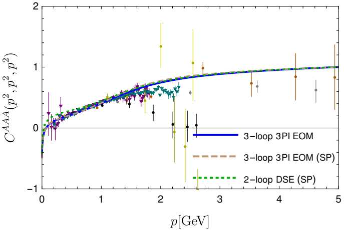


[MQH '20]

Concurrence of functional methods

Exemplified with three-gluon vertex.

3PI vs. 2-loop DSE:

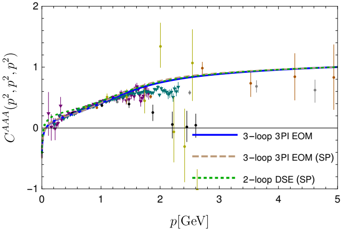


[Cucchieri, Maas, Mendes ’08; Sternbeck et al. ’17; MQH ’20]

Concurrence of functional methods

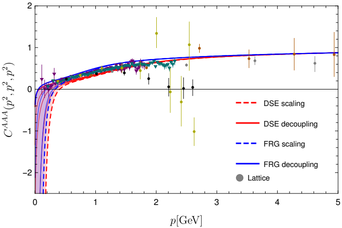
Exemplified with three-gluon vertex.

3PI vs. 2-loop DSE:



[Cucchieri, Maas, Mendes '08; Sternbeck et al. '17; MQH '20]

DSE vs. FRG:

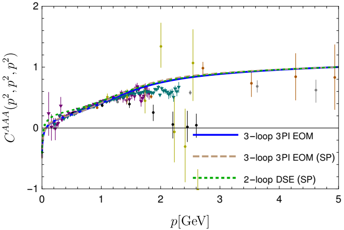


[Cucchieri, Maas, Mendes '08; Sternbeck et al. '17; Cyrol et al. '16; MQH '20]

Concurrence of functional methods

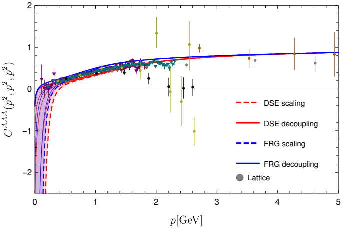
Exemplified with three-gluon vertex.

3PI vs. 2-loop DSE:



[Cucchieri, Maas, Mendes ’08; Sternbeck et al. ’17; MQH ’20]

DSE vs. FRG:

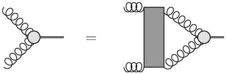


[Cucchieri, Maas, Mendes ’08; Sternbeck et al. ’17; Cyrol et al. ’16; MQH ’20]

Beyond this truncation

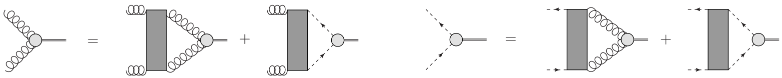
- Further dressings of three-gluon vertex [Eichmann, Williams, Alkofer, Vujinovic ’14]
- Effects of four-point functions [MQH ’16, MQH ’17, MQH ’18]

Glueballs as bound states



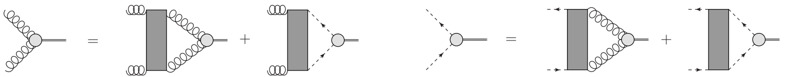
Need  and , solve for . $\rightarrow$ Mass

Glueballs as bound states



Gluons couple to ghosts $\rightarrow$ Include 'ghostball'-part.

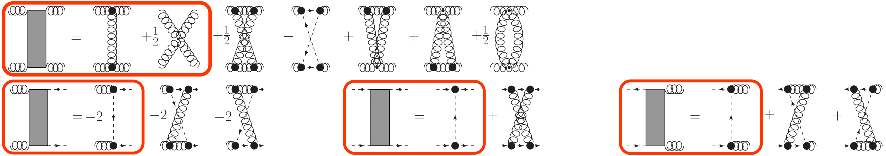
Glueballs as bound states



Gluons couple to ghosts → Include 'ghostball'-part.

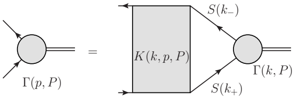
Construction of kernel

3-loop expansion of 3PI effective action [Fukuda '87; McKay, Munczek '89; Sanchis-Alepuz, Williams '15; MQH, Fischer, Sanchis-Alepuz '20]

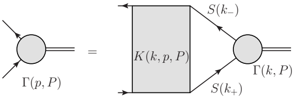


- Some diagrams vanish for certain quantum numbers.
- Full QCD: Same for quarks → Mixing with mesons.

Solving a BSE



Solving a BSE

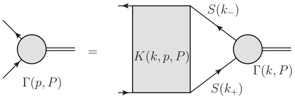


Consider the eigenvalue problem (Γ is the BSE amplitude)

$$\mathcal{K}\cdot\Gamma(P)=\boldsymbol{\lambda}(\boldsymbol{P})\,\Gamma(P).$$

$\lambda(P^2)=1$ is a solution to the BSE $\Rightarrow$ Glueball mass $P^2=-M^2$

Solving a BSE



Consider the eigenvalue problem (Γ is the BSE amplitude)

$$\mathcal{K}\cdot\Gamma(P)=\boldsymbol{\lambda(P)}\,\Gamma(P).$$

$\lambda(P^2)=1$ is a solution to the BSE $\Rightarrow$ Glueball mass $P^2=-M^2$

Calculation requires quantities for

$$k_\pm^2=P^2+k^2\pm 2\sqrt{P^2\,k^2}\cos\theta=-M^2+k^2\pm 2\,\boldsymbol{i}\,M\sqrt{k^2}\,\cos\theta.$$

$\Rightarrow$ Complex momentum arguments.

Landau gauge propagators in the complex plane

Propagators for complex momenta

- Reconstruction from Euclidean results: mathematically ill-defined, bias in solution
- Direct calculation from functional methods possible

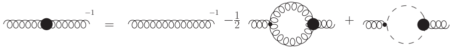
Landau gauge propagators in the complex plane

Propagators for complex momenta

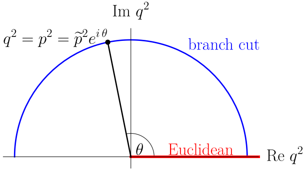
- Reconstruction from Euclidean results: mathematically ill-defined, bias in solution
- Direct calculation from functional methods possible

- Special techniques necessary: Respect analyticity [Fischer, MQH '20]

Simpler truncation:

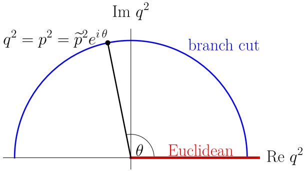


Landau gauge propagators in the complex plane



Appearance of branch cuts for complex momenta forbids integration directly to cutoff.

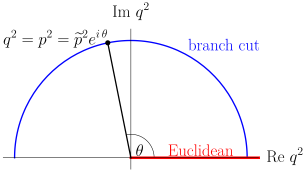
Landau gauge propagators in the complex plane



Appearance of branch cuts for complex momenta forbids integration directly to cutoff.

Deformation of integration contour necessary [Maris '95]. Recent resurgence: [Alkofer et al. '04; Windisch, MQH, Alkofer, '13; Williams '19; Miramontes, Sanchis-Alepuz '19; Eichmann et al. '19], ...

Landau gauge propagators in the complex plane

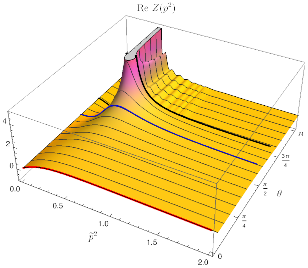


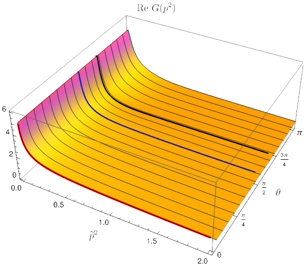
Appearance of branch cuts for complex momenta forbids integration directly to cutoff.

Deformation of integration contour necessary [Maris '95]. Recent resurgence: [Alkofer et al. '04; Windisch, MQH, Alkofer, '13; Williams '19; Miramontes, Sanchis-Alepuz '19; Eichmann et al. '19], . . .

Ray technique for self-consistent solution of a DSE: [Strauss, Fischer, Kellermann; Fischer, MQH '20].

Landau gauge propagators in the complex plane





- Method works,
- but current truncation leads to a pole-like structure in the gluon propagator.
- Analyticity up to 'pole' confirmed by various tests (Cauchy-Riemann, Schlessinger, reconstruction)
- Warning: No proof of existence of complex conjugate poles.

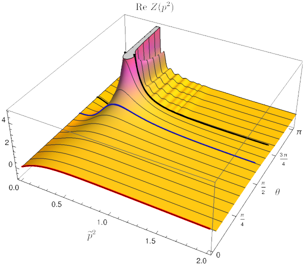
[Fischer, MQH '20]

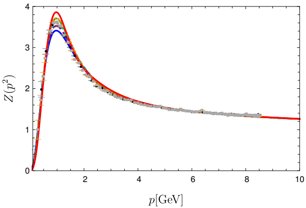
Input for glueballs

Low quality results in complex plane

vs.

Quantitative results for real momenta



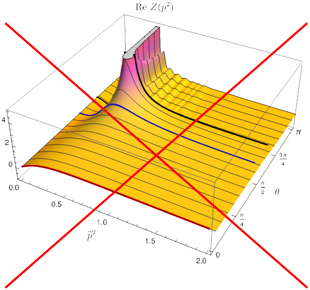


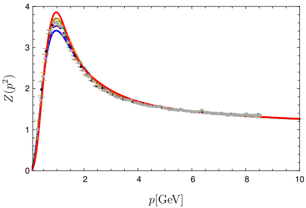
Input for glueballs

Low quality results in complex plane

vs.

Quantitative results for real momenta





$\Rightarrow$ Solve eigenvalue problem for $P^2 > 0$ and extrapolate $\lambda(P^2)$ to glueball mass.

Extrapolation of $\lambda(P^2)$

Extrapolation method

- Extrapolation to time-like P^2 using Schlessinger's continued fraction method (proven superior to default Padé approximants) [Schlessinger '68]
- Average over extrapolations using subsets of points for error estimate

Extrapolation of $\lambda(P^2)$

Extrapolation method

- Extrapolation to time-like P^2 using Schlessinger's continued fraction method (proven superior to default Padé approximants) [Schlessinger '68]
- Average over extrapolations using subsets of points for error estimate

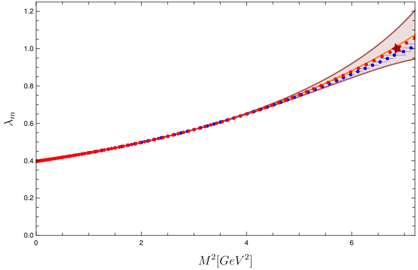
Test extrapolation for solvable system: Heavy meson

Extrapolation of $\lambda(P^2)$

Extrapolation method

- Extrapolation to time-like P^2 using Schlessinger's continued fraction method (proven superior to default Padé approximants) [Schlessinger '68]
- Average over extrapolations using subsets of points for error estimate

Test extrapolation for solvable system: Heavy meson



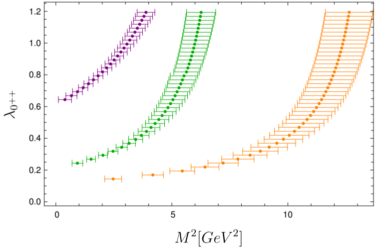
[MQH, Sanchis-Alepuz, Fischer '20]

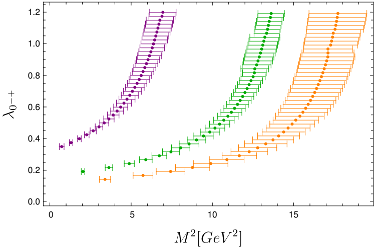
Extrapolation of $\lambda(P^2)$ for glueballs

Higher eigenvalues: Excited states.

Extrapolation of $\lambda(P^2)$ for glueballs

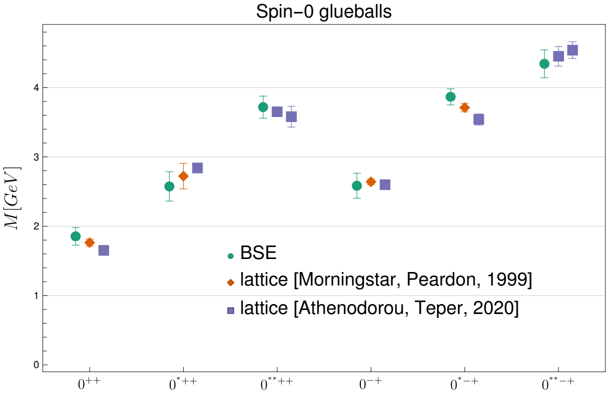
Higher eigenvalues: Excited states.





Physical solutions for $\lambda(P^2) = 1$.

Glueballs with $J = 0^{\pm +}$



Lattice 0^{**++}: Conjectured based on irred. rep. of octahedral group

All results for $r_0 = 1/418(5)$ MeV.

[MQH, Fischer, Sanchis-Alepuz '20]

Extension to $J > 0$

How is spin determined in a BSE?

Extension to $J > 0$

How is spin determined in a BSE?

Encoded in basis tensors for BSE amplitudes.

Extension to $J > 0$

How is spin determined in a BSE?

Encoded in basis tensors for BSE amplitudes.

Example:
$g_{\mu\nu}$ and $p_\mu p_\nu$ correspond to an amplitude with two gluon legs and spin 0, $P = 1$.

Extension to $J > 0$

How is spin determined in a BSE?

Encoded in basis tensors for BSE amplitudes.

Example:
$g_{\mu\nu}$ and $p_\mu p_\nu$ correspond to an amplitude with two gluon legs and spin 0, $P = 1$.

For higher J, construct most general $(J+2)$-dimensional tensor basis and apply spin projector. $\rightarrow$ Traceless, symmetric, transverse in spin indices.

$\rightarrow$ At most 4/5 tensors for $P = \pm 1$ [MQH, Sanchis-Alepuz, Fischer, in prep.].

Extension to $J > 0$

How is spin determined in a BSE?

Encoded in basis tensors for BSE amplitudes.

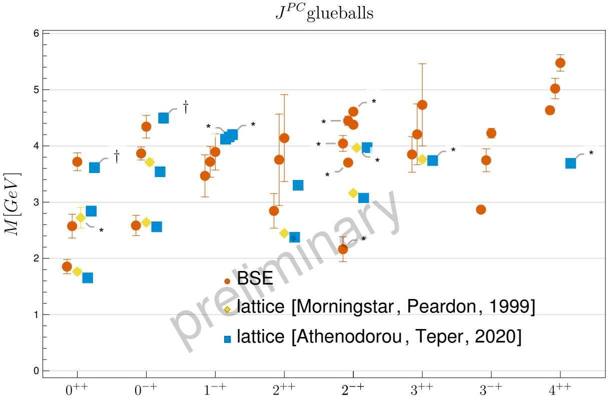
Example:
$g_{\mu\nu}$ and $p_\mu p_\nu$ correspond to an amplitude with two gluon legs and spin 0, $P = 1$.

For higher J, construct most general $(J+2)$-dimensional tensor basis and apply spin projector. $\rightarrow$ Traceless, symmetric, transverse in spin indices.

$\rightarrow$ At most 4/5 tensors for $P = \pm 1$ [MQH, Sanchis-Alepuz, Fischer, in prep.].

Repeat calculations...

Glueball masses



Lattice:
*: identification with some uncertainty
†: conjecture based on irred. rep of octahedral group

[MQH, Fischer, Sanchis-Alepuz, in preparation]

Linear covariant gauges

- The Landau gauge is the endpoint ($\xi = 0$) of linear covariant gauges.
- $\mathcal{L}_{\text{gf}} = \frac{1}{2\xi}(\partial \cdot A)^2 - \bar{c}\, M\, c$
- Gluon propagator: $D(p^2) = \left(g_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}\right)\frac{Z(p^2)}{p^2} + \xi \frac{p_\mu p_\nu}{p^4}$

Nielsen identities

- Describe gauge parameter dependence of correlation functions by a differential equation.
- Traditional use: Show gauge parameter independence of pole masses.

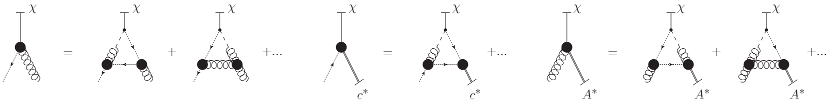
Nielsen identities

- Describe gauge parameter dependence of correlation functions by a differential equation.
- Traditional use: Show gauge parameter independence of pole masses.

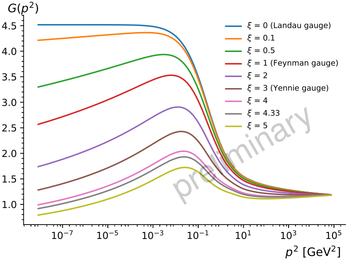
- Here: Solve them for the propagators

$$\partial_\xi Z(p^2;\xi) = K_Z(p^2;\xi) Z(p^2;\xi), \qquad \partial_\xi G(p^2;\xi) = K_G(p^2;\xi) G(p^2;\xi)$$

- Initial condition: Landau gauge ($\xi = 0$)
- K_Z, K_G: nonperturbative one-loop integrals



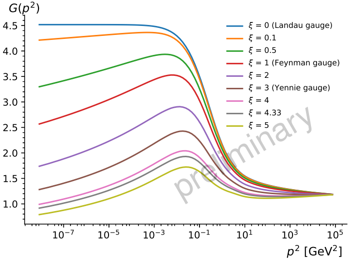
Ghost propagator

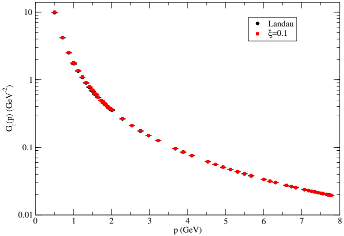


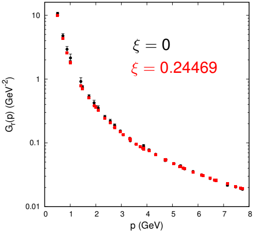
- Logarithmic IR suppression for $\xi > 0$

  [Aguilar, Binosi, Papavassiliou '15; MQH '15]
- Otherwise effects small for low ξ.

Ghost propagator



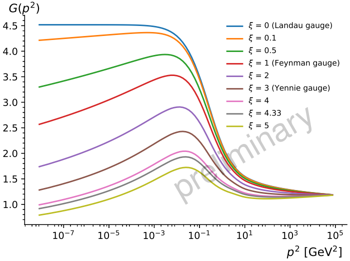


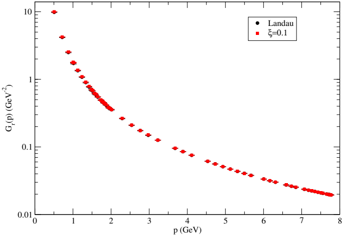


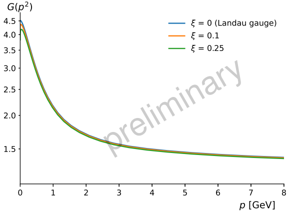
[Napetschnig, Alkofer, MQH, Pawlowski, in prep.]

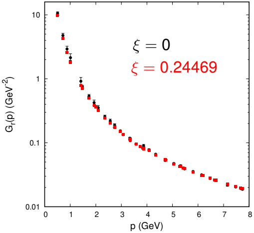
[Cucchieri et al. '18]

Ghost propagator





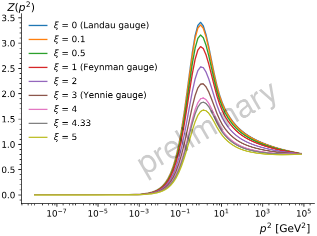


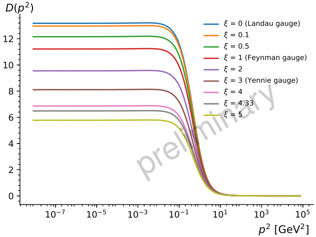


[Napetschnig, Alkofer, MQH, Pawlowski, in prep.]

[Cucchieri et al. '18]

Gluon propagator

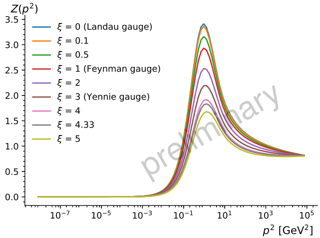


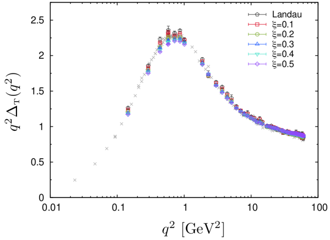


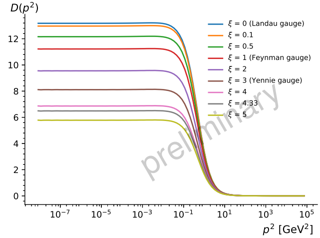
[Napetschnig, Alkofer, MQH, Pawlowski, in prep.]

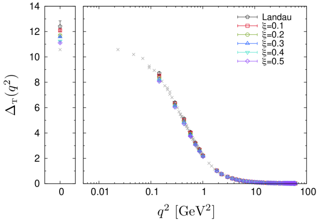
[Bicudo et al. '15]

Gluon propagator





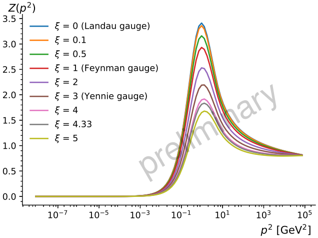


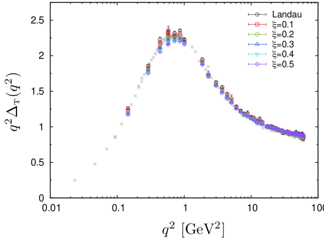


[Napetschnig, Alkofer, MQH, Pawlowski, in prep.]

[Bicudo et al. '15]

Gluon propagator





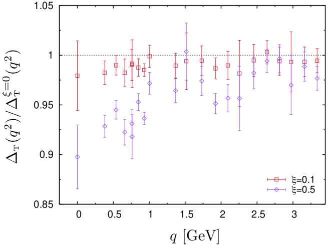
Ratios from Nielsen identities:

$\xi = 0.1$:

- 0 GeV: 0.98
- 1 GeV: 0.98

$\xi = 0.5$:

- 0 GeV: 0.92
- 1 GeV: 0.93



[Napetschnig, Alkofer, MQH, Pawlowski, in prep.]

[Bicudo et al. '15]

Stability of solving Nielsen identities

Nontrivial check: UV behavior

Stability of solving Nielsen identities

Nontrivial check: UV behavior

Exceptional values of ξ where 1-loop anomalous dimensions vanish

$$Z_{\mathrm{UV}}(p^2) = Z(s)\left(1+\omega(s)\ln\frac{p^2}{s}\right)^{-\frac{13-3\xi}{22}}$$

$$G_{\mathrm{UV}}(p^2) = G(s)\left(1+\omega(s)\ln\frac{p^2}{s}\right)^{-\frac{9-3\xi}{44}}$$

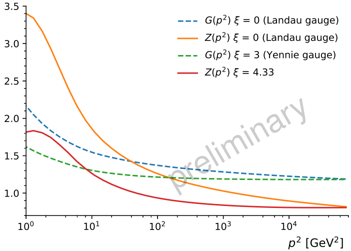
Stability of solving Nielsen identities

Nontrivial check: UV behavior

Exceptional values of ξ where 1-loop anomalous dimensions vanish

$$Z_{\mathrm{UV}}(p^2) = Z(s)\left(1+\omega(s)\ln\frac{p^2}{s}\right)^{-\frac{13-3\xi}{22}}$$

$$G_{\mathrm{UV}}(p^2) = G(s)\left(1+\omega(s)\ln\frac{p^2}{s}\right)^{-\frac{9-3\xi}{44}}$$



[Napetschnig, Alkofer, MQH, Pawlowski, in prep.]

$\rightarrow$ Exceptionally stable process.

Remaining uncertainties: vertices

Summary

Parameter-free determination of glueball masses from functional methods.

Summary

Parameter-free determination of glueball masses from functional methods.

Functional equations:

- Quantitatively reliable correlation functions (Euclidean)

Summary

Parameter-free determination of glueball masses from functional methods.

Functional equations:

- Quantitatively reliable correlation functions (Euclidean)
  - Comparison with lattice results

Summary

Parameter-free determination of glueball masses from functional methods.

Functional equations:

- Quantitatively reliable correlation functions (Euclidean)
  - Comparison with lattice results
  - Concurrence of different functional methods

Summary

Parameter-free determination of glueball masses from functional methods.

Functional equations:

- Quantitatively reliable correlation functions (Euclidean)
  - Comparison with lattice results
  - Concurrence of different functional methods
  - Connection to observables

Summary

Parameter-free determination of glueball masses from functional methods.

Functional equations:

- Quantitatively reliable correlation functions (Euclidean)
  - Comparison with lattice results
  - Concurrence of different functional methods
  - Connection to observables
- Systematic improvements (now) possible

Summary

Parameter-free determination of glueball masses from functional methods.

Functional equations:

- Quantitatively reliable correlation functions (Euclidean)
  - Comparison with lattice results
  - Concurrence of different functional methods
  - Connection to observables
- Systematic improvements (now) possible
- Direct access to analytic structure

Summary

Parameter-free determination of glueball masses from functional methods.

Functional equations:

- Quantitatively reliable correlation functions (Euclidean)
  - Comparison with lattice results
  - Concurrence of different functional methods
  - Connection to observables
- Systematic improvements (now) possible
- Direct access to analytic structure
- Complementarity to lattice methods

Outlook

Going beyond $m = \infty$, $p^2 > 0$, $\mu = T = 0$, and $SU(N)$:

Outlook

Going beyond $m = \infty$, $p^2 > 0$, $\mu = T = 0$, and $SU(N)$:

- Glueballs: Unquenching

Outlook

Going beyond $m = \infty$, $p^2 > 0$, $\mu = T = 0$, and $SU(N)$:

- Glueballs: Unquenching
- Analytic structure: discard extrapolation procedure for glueballs; access to spectral functions; decays; real-time properties

Outlook

Going beyond $m = \infty$, $p^2 > 0$, $\mu = T = 0$, and $SU(N)$:

- Glueballs: Unquenching
- Analytic structure: discard extrapolation procedure for glueballs; access to spectral functions; decays; real-time properties
- QCD phase diagram: From bottom-up to top-down approaches for

Outlook

Going beyond $m = \infty$, $p^2 > 0$, $\mu = T = 0$, and $SU(N)$:

- Glueballs: Unquenching
- Analytic structure: discard extrapolation procedure for glueballs; access to spectral functions; decays; real-time properties
- QCD phase diagram: From bottom-up to top-down approaches for
  - fluctuations of conserved quantities

Outlook

Going beyond $m = \infty$, $p^2 > 0$, $\mu = T = 0$, and $SU(N)$:

- Glueballs: Unquenching
- Analytic structure: discard extrapolation procedure for glueballs; access to spectral functions; decays; real-time properties
- QCD phase diagram: From bottom-up to top-down approaches for
  - fluctuations of conserved quantities
  - transport coefficients

Outlook

Going beyond $m = \infty$, $p^2 > 0$, $\mu = T = 0$, and $SU(N)$:

- Glueballs: Unquenching
- Analytic structure: discard extrapolation procedure for glueballs; access to spectral functions; decays; real-time properties
- QCD phase diagram: From bottom-up to top-down approaches for
  - fluctuations of conserved quantities
  - transport coefficients
  - equation of state

Outlook

Going beyond $m = \infty$, $p^2 > 0$, $\mu = T = 0$, and $SU(N)$:

- Glueballs: Unquenching
- Analytic structure: discard extrapolation procedure for glueballs; access to spectral functions; decays; real-time properties
- QCD phase diagram: From bottom-up to top-down approaches for
  - fluctuations of conserved quantities
  - transport coefficients
  - equation of state
- 'QCD technology' can be applied to other theories: From dark matter to composite Higgs models

Outlook

Going beyond $m = \infty$, $p^2 > 0$, $\mu = T = 0$, and $SU(N)$:

- Glueballs: Unquenching
- Analytic structure: discard extrapolation procedure for glueballs; access to spectral functions; decays; real-time properties
- QCD phase diagram: From bottom-up to top-down approaches for
  - fluctuations of conserved quantities
  - transport coefficients
  - equation of state
- 'QCD technology' can be applied to other theories: From dark matter to composite Higgs models

Outlook

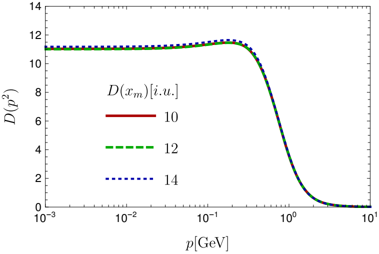
Going beyond $m = \infty$, $p^2 > 0$, $\mu = T = 0$, and $SU(N)$:

- Glueballs: Unquenching
- Analytic structure: discard extrapolation procedure for glueballs; access to spectral functions; decays; real-time properties
- QCD phase diagram: From bottom-up to top-down approaches for
  - fluctuations of conserved quantities
  - transport coefficients
  - equation of state
- 'QCD technology' can be applied to other theories: From dark matter to composite Higgs models

Thank your for your attention.

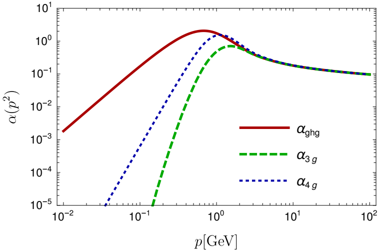
Some properties of the Landau gauge solution

- Renormalization: First parameter-free subtraction of quadratic divergences



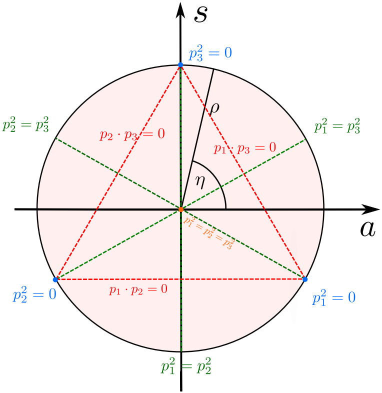
Some properties of the Landau gauge solution

- Renormalization: First parameter-free subtraction of quadratic divergences
- Slavnov-Taylor identities (gauge invariance): Vertex couplings agree down to GeV regime



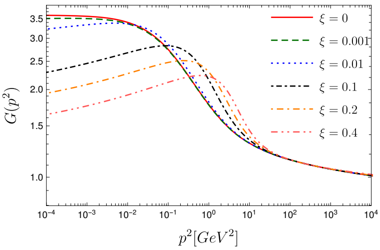
Some properties of the Landau gauge solution

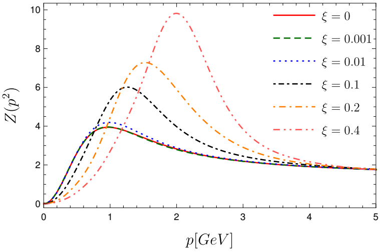
- Renormalization: First parameter-free subtraction of quadratic divergences
- Slavnov-Taylor identities (gauge invariance): Vertex couplings agree down to GeV regime
- Full momentum dependence of vertices:
  Parametrization via permutation group S_3 superior [Eichmann, Williams, Alkofer, Vujinovic '14]



DSEs in linear covariant gauges

DSE results from 2015 [MQH '15]:

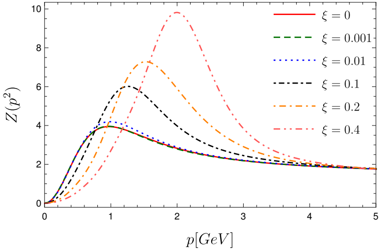




DSEs in linear covariant gauges

DSE results from 2015 [MQH '15]:

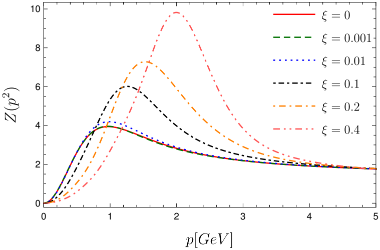
- Much stronger effects for $\xi > 0$



DSEs in linear covariant gauges

DSE results from 2015 [MQH '15]:

- Much stronger effects for $\xi > 0$
- Unnatural effects: Nonperturbative features move too far to perturbative regime.



DSEs in linear covariant gauges

DSE results from 2015 [MQH '15]:

- Much stronger effects for $\xi > 0$
- Unnatural effects: Nonperturbative features move too far to perturbative regime.
- Simple one-loop truncation which is known to be insufficient for $\xi = 0$, so not unexpected.