Green functions of Landau gauge Yang-Mills theory from Dyson-Schwinger equations

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April 17, 2013

Seminar Quantum Many-body Dynamics and Nonequilibrium Physics
Heidelberg

Unterstützt von / Supported by
Alexander von Humboldt
Stiftung/Foundation

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Our view of the world in terms of particles

The standard model:

Quarks	d	s	b	g
	u	c	t	γ
Leptons	e	μ	τ	Z
	ν_e	ν_μ	ν_τ	$W^\pm$

Bosons

Higgs?

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Higgs? → 2012 → Which Higgs?

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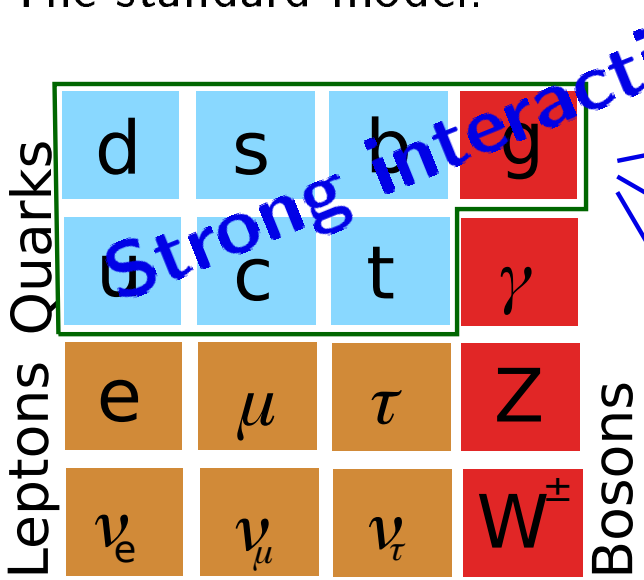
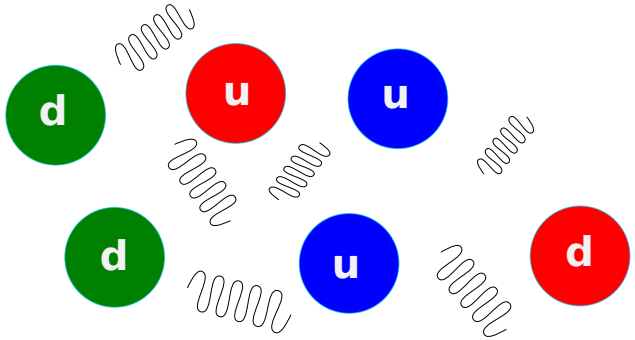


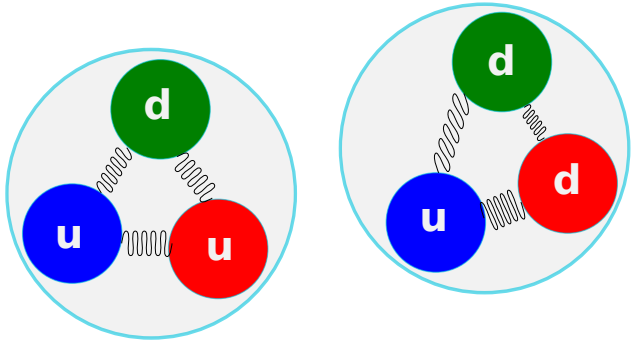
Higgs? 2012 Which Higgs?

fundamental fields: quarks and gluons

u, d quarks are light $\sim$ MeV



physical degrees of freedom: hadrons



Hadrons are bound states of quarks,
but they cannot be split into their building blocks.

Landau gauge Green functions from functional methods

Landau gauge Green functions:

- Information about confinement
- Input for phenomenological calculations, e.g., bound states, QCD phase diagram

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QCD phase diagram with functional equations:

+ no sign problem at non-zero chemical potential
+ varying quark masses easy (physical masses, chiral limit, quenched)
– infinitely large system $\rightarrow$ influence of higher Green functions?
  $\Rightarrow$ tests of truncations:
  analytically, comparison with other methods,
  improve truncation, . . . ?

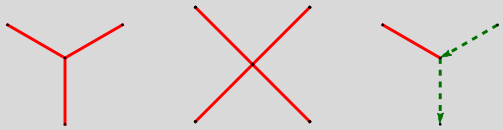
Landau Gauge Yang-Mills theory

Gluonic sector of quantum chromodynamics: Yang-Mills theory

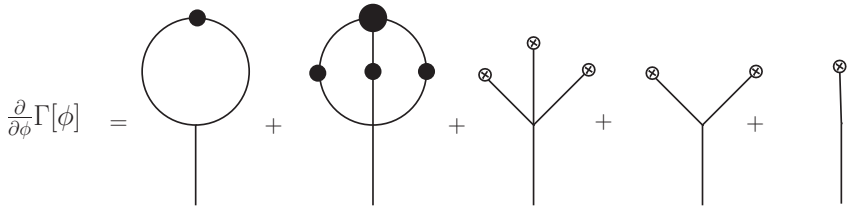
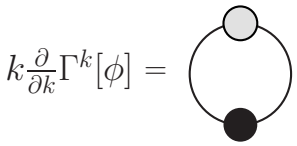
$$\mathcal{L} = \frac{1}{2} F^2 + \mathcal{L}_{gf} + \mathcal{L}_{gh}$$
$$F_{\mu\nu} = \partial_\mu \boldsymbol{A}_\nu - \partial_\nu \boldsymbol{A}_\mu + i\, g\, [\boldsymbol{A}_\mu, \boldsymbol{A}_\nu]$$

Propagators and vertices are gauge dependent
$\longrightarrow$ choose any gauge, ideally one that is convenient.

Landau gauge

- simplest one for functional equations
- $\partial_\mu \boldsymbol{A}_\mu = 0$: $\quad \mathcal{L}_{gf} = \frac{1}{2\xi} (\partial_\mu \boldsymbol{A}_\mu)^2, \quad \xi \to 0$
- requires ghost fields: $\quad \mathcal{L}_{gh} = \bar{\boldsymbol{c}} \left(-\Box + g\, \boldsymbol{A} \times \right) \boldsymbol{c}$
- 2 fields: 3 vertices: 

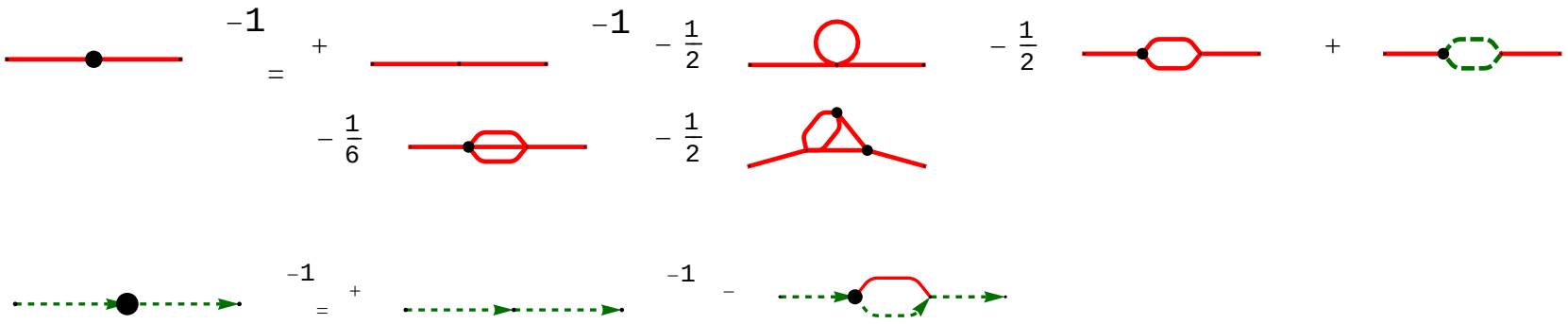
Comparison: DSEs and flow equations

DSEs	FRGEs
effective action $\Gamma[\phi]$	effective average action $\Gamma^k[\phi]$
–	regulator
n-loop structure (n *const.*)	1-loop structure
exactly only bare vertex per diagram	no bare vertices
	

Both systems of equations are exact,
but same truncations may have different (quantitative) effects.

Dyson-Schwinger equations: Propagators

Dyson-Schwinger equations (DSEs) of gluon and ghost propagators:



- Equations of motion of correlation functions:
  Describe how fields propagate and interact non-perturbatively!
- Infinite tower of coupled integral equations.
- Derivation straightforward, but tedious
  $\longrightarrow$ automated derivation with *DoFun* [MQH, Braun, CPC183 (2012)].
- Contain three-point and four-point functions:
  ghost-gluon vertex , three-gluon vertex , four-gluon vertex

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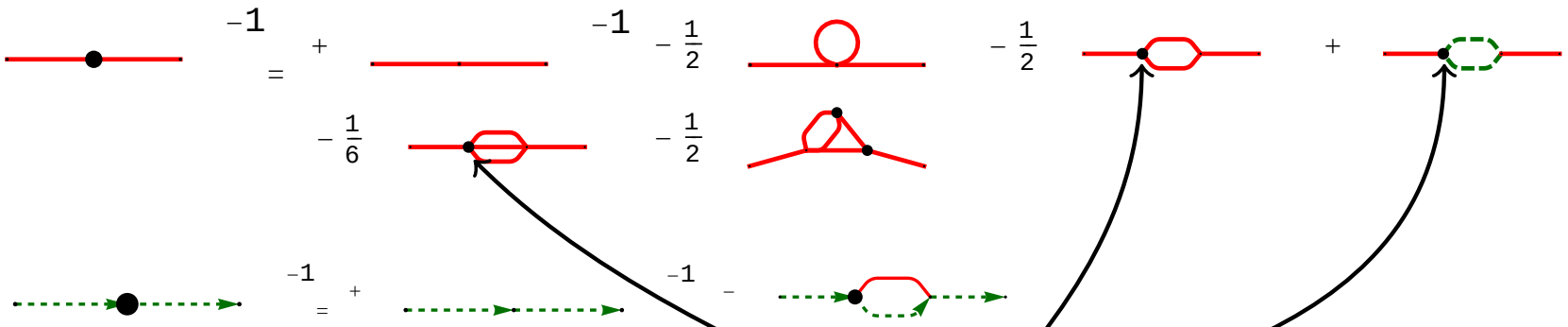

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Truncated propagator Dyson-Schwinger equations

Standard truncation:

- No four-point interactions
- models for ghost-gluon and three-gluon vertices

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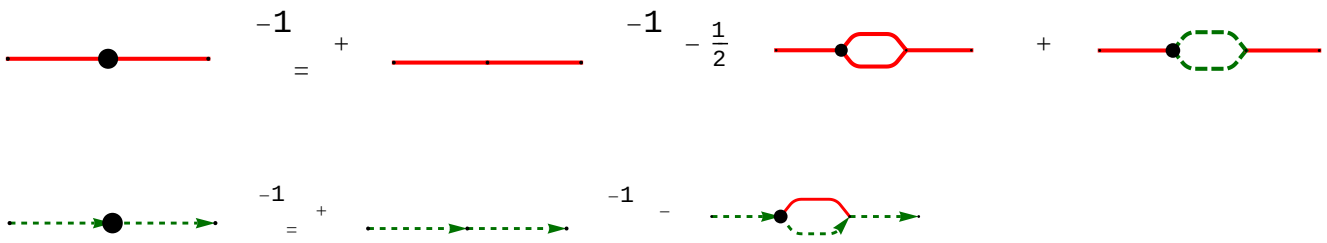

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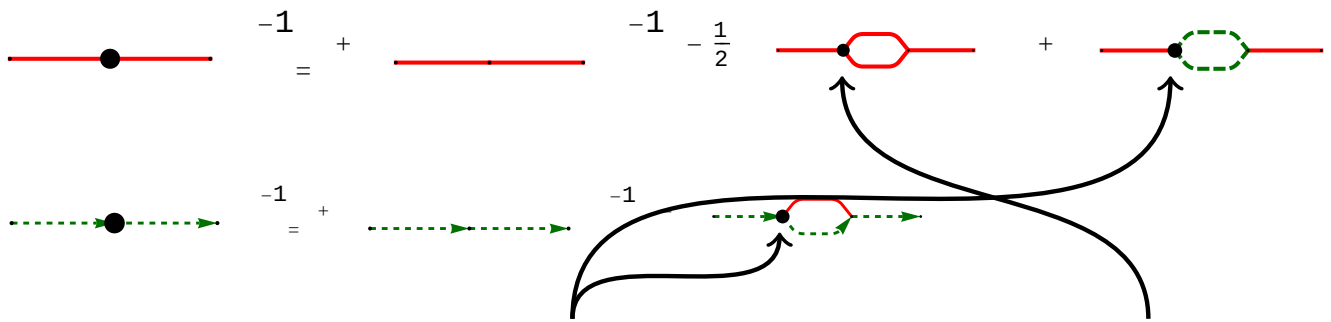

Standard: bare ghost-gluon vertex and three-gluon vertex model

Influence of three-point functions?

$$D^{ab}_{gl,\mu\nu}(p) = \left(g_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}\right) \frac{Z(p^2)}{p^2} \delta^{ab}$$

$$D^{ab}_{gh}(p) = -\frac{G(p^2)}{p^2} \delta^{ab}$$

Truncating Dyson-Schwinger equations

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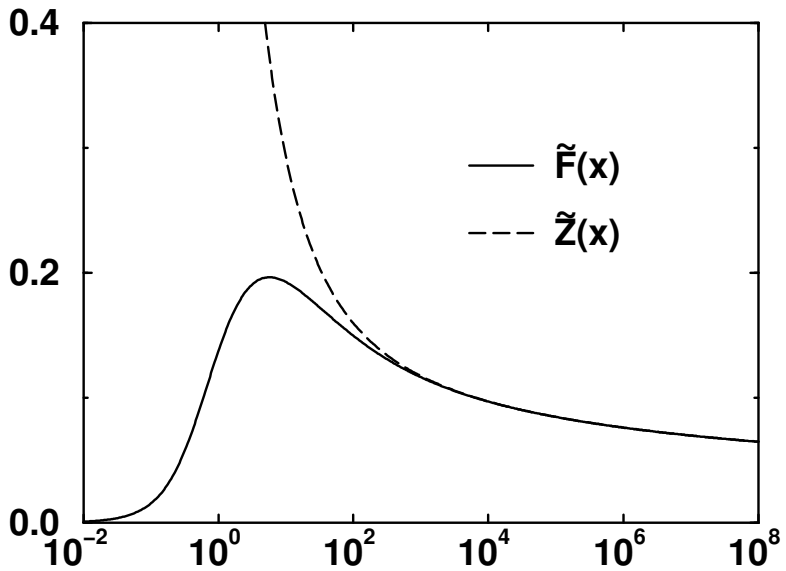

[Hauck, von Smekal, Alkofer, CPC 112 (1998)]

$$D_{gl,\mu\nu}(p) = \left(g_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}\right) \frac{\tilde{Z}(p^2)}{p^2}$$

- gluon dressing $\tilde{Z}(p^2)$ IR divergent $\rightarrow$ IR slavery

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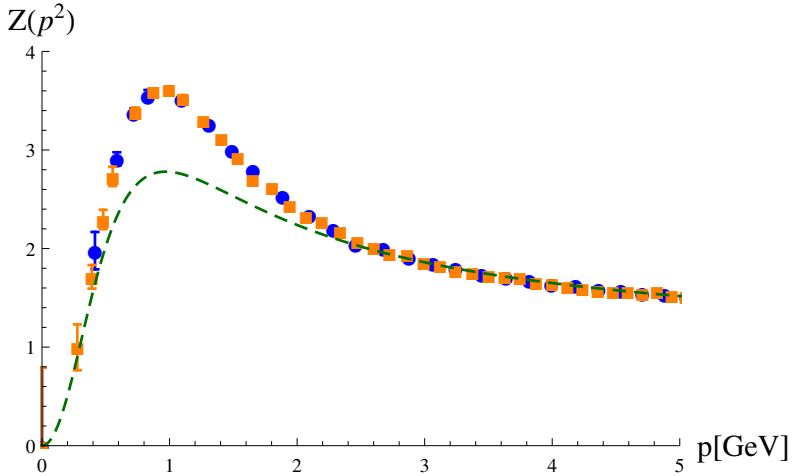

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- gluon dressing $Z(p^2)$ IR vanishing
- deviations from lattice results in mid-momentum regime

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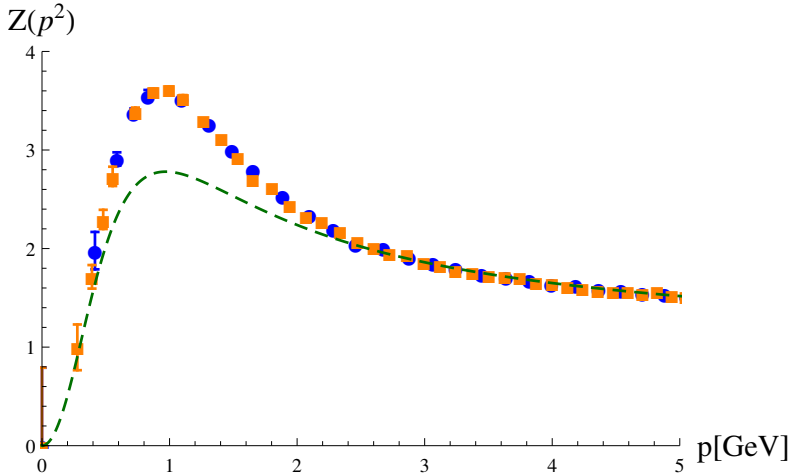

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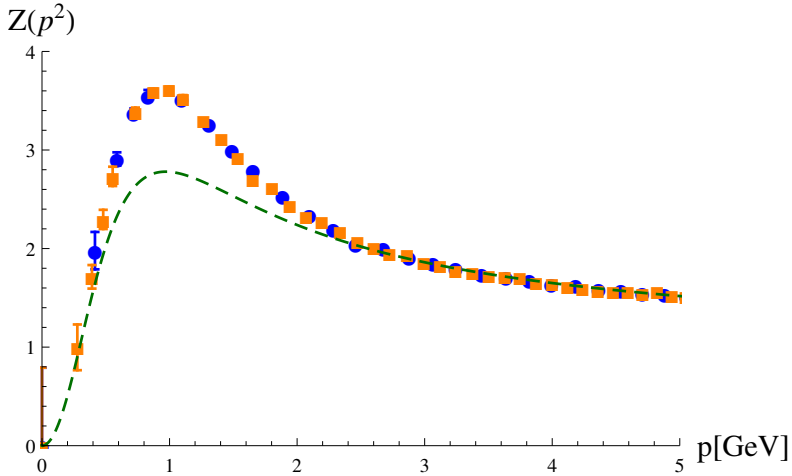

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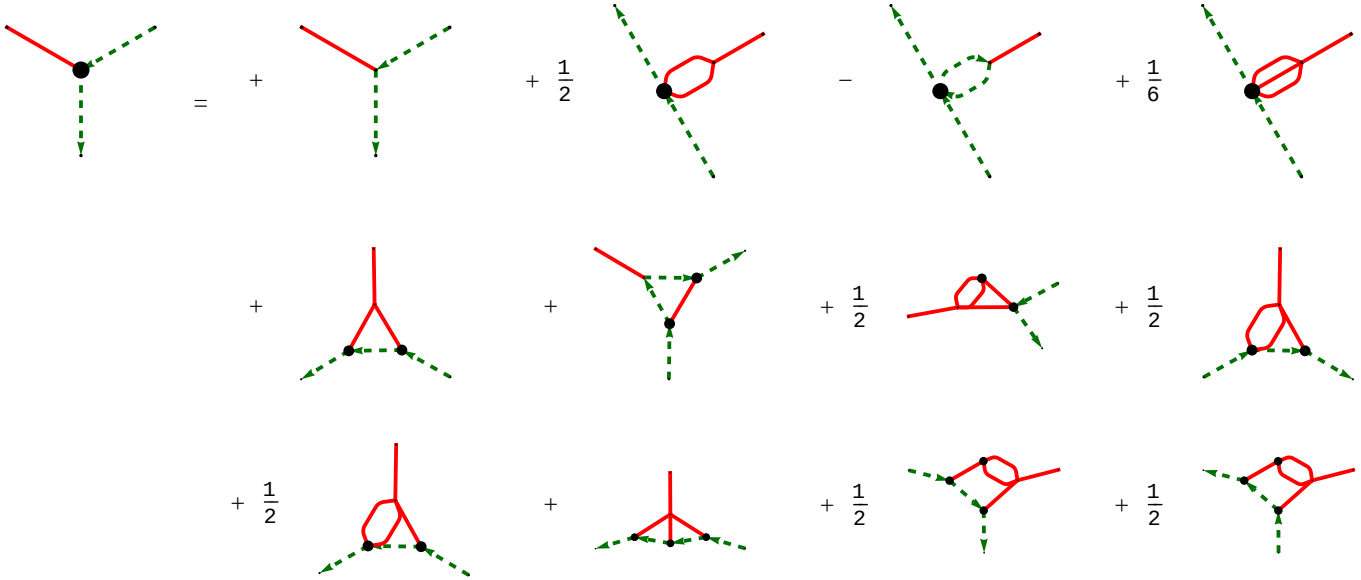
Ghost-gluon vertex DSE

Full DSE:



- Lattice results [Cucchieri, Maas, Mendes, PRD77 (2008); Ilgenfritz et al., BJP37 (2007)]
- OPE analysis [Boucaud et al., JHEP 1112 (2011)]
- Modeling via ghost DSE [Dudal, Oliveira, Rodriguez-Quintero, PRD86 (2012)]
- Semi-perturbative DSE analysis [Schleifenbaum et al., PRD72 (2005)]
- FRG [Fister, Pawlowski, 1112.5440]

Ghost-gluon vertex

$$\Gamma_\mu^{A\bar{c}c,abc}(k;p,q) := i\,g\,f^{abc}\left(p_\mu \boldsymbol{A(k;p,q)} + k_\mu B(k;p,q)\right)$$

Note:

$B(k;p,q)$ is irrelevant in Landau gauge (but it is not the pure longitudinal part).

Taylor argument applies only to longitudinal part (it's an STI).

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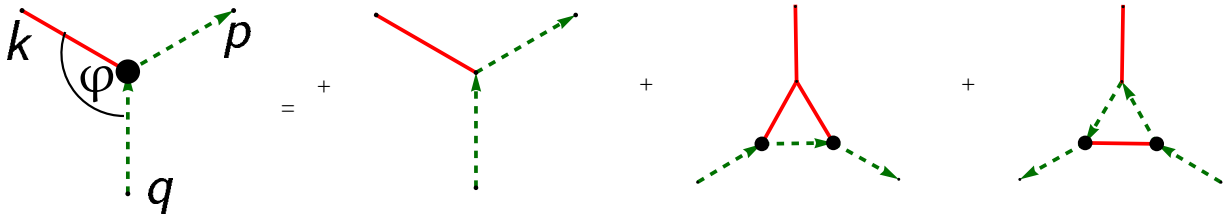
<u>IR and UV consistent truncation:</u>



System of eqs. to solve:
gluon and ghost propagators + ghost-gluon vertex

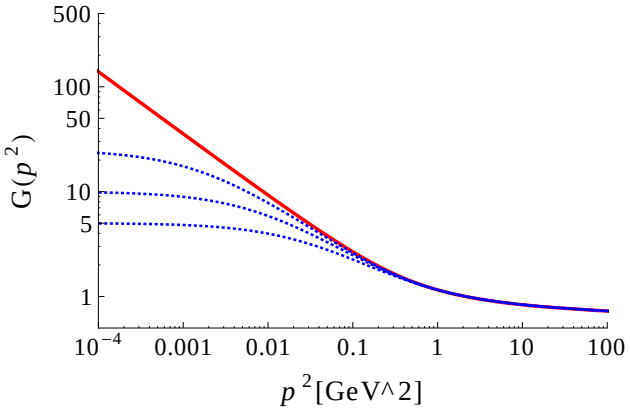
Only unfixed quantity in present truncation: three-gluon vertex.

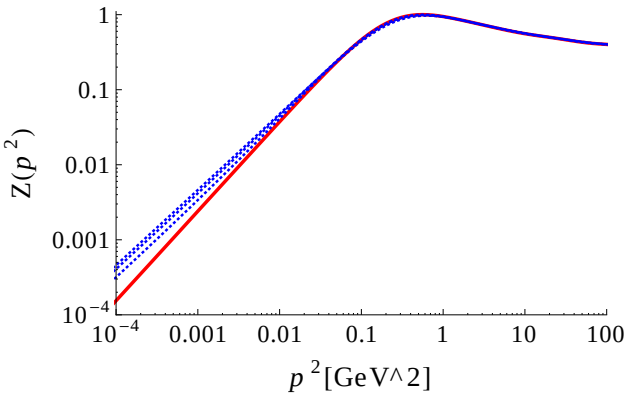
Solutions of functional equations: Decoupling and scaling

- Two types of solutions with functional methods that differ only in deep IR [Boucaud et al., JHEP 0806, 012; Fischer, Maas, Pawlowski, AP 324 (2009)]:
  scaling [von Smekal, Alkofer, Hauck PRL97],
  decoupling [Aguilar, Binosi, Papavassiliou PRD78; Fischer, Maas, Pawlowski, AP 324 (2009)]
- Lattice calculations find only decoupling type solution for $d = 3, 4$ and scaling for $d = 2$
- Decoupling emerges also from Refined Gribov-Zwanziger framework [Dudal, Sorella, Vandersickel, Verschelde, PRD77]

Decoupling and scaling solutions

DSEs: Vary ghost boundary condition [Fischer, Maas, Pawlowski, AP 324 (2009)]





- Dependence of propagators on Gribov copies, e.g., [Bogolubsky, Burgio, Müller-Preussker, Mitrjushkin, PRD 74 (2006); Maas, PR 524 (2013)]
- Ideas:
  - [Sternbeck, Müller-Preussker, 1211.3057]: choose Gribov copies by lowest eigenvalue of the Faddeev-Popov operator
    $\longrightarrow$ modification of both dressings
  - [Maas, PLB689 (2010)]: choose Gribov copies by value of ghost propagator

$d = 2$: Analytic and numerical arguments from DSEs for scaling only [Cucchieri, Dudal, Vandersickel, PRD85 (2012); MQH, Maas, von Smekal, JHEP11 (2012)] as well as from analysis of Gribov region [Zwanziger, 1209.1974].

Why are two dimensions interesting?

- Ambiguity of solutions?
- larger lattices $\longrightarrow$ lower momenta
  lower dimensions require (much) less computer power, e.g.:
  $d = 4$: $128^4 \approx 3 \cdot 10^8$ ($L \approx 27\,fm$)
  [Cucchieri, Mendes, Pos LAT2007 297 (2007)],
  $d = 2$: $2560^2 \approx 7 \cdot 10^6$ ($L \approx 460\,fm$)
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- $\longrightarrow$ good lattice results exist even for three-point functions
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Yang-Mills theory for $d = 2$

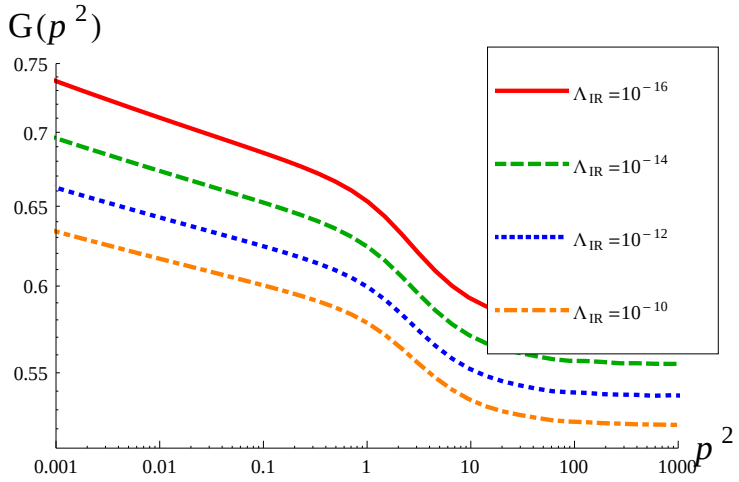
- Perturbation theory does not work because of IR divergences.
  $\longrightarrow$ low- and mid-momentum behavior important for UV
- Gluons have no transverse polarization $\longrightarrow$ no physical degrees of freedom, but we can investigate correlation functions, the Gribov problem, ...

Existence of decoupling solution

- Analytical:
  For $d = 2, 3, 4$ two possible scaling solutions, of which one is unphysical.
  Specific to $d = 2$: One can show analytically which one is unphysical. Coincides with decoupling type.
- Numerical:
  Ghost equation contains IR singularities for decoupling type.

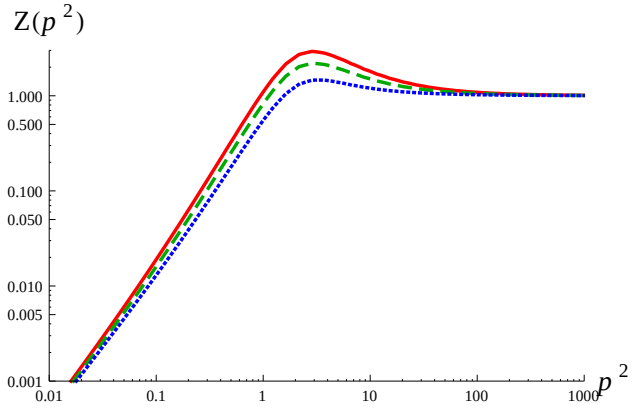
$\Rightarrow$ No decoupling type solution in two dimensions.

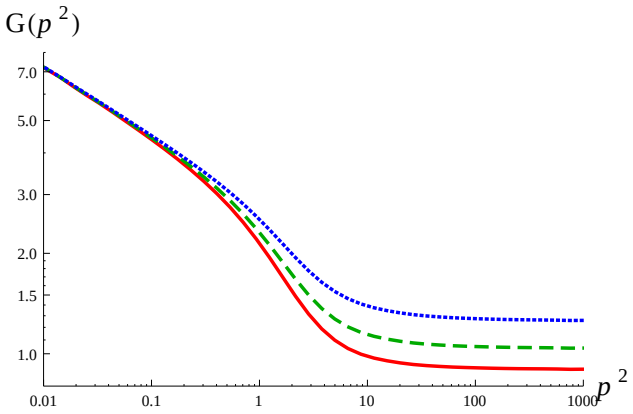
In agreement with [Cucchieri, Dudal, Vandersickel, PRD85 (2012); Zwanziger, 1209.1974].



Aspects of $d = 2$ Dyson-Schwinger equations

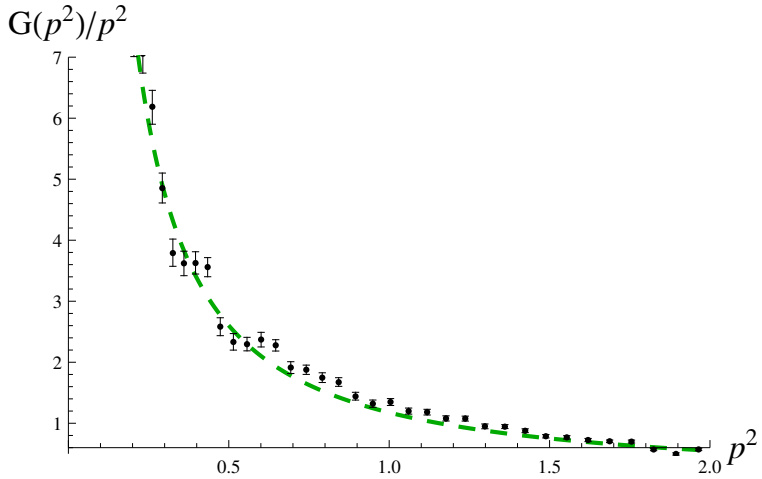
- Different momentum regimes mix,
  e.g., mid-momentum influences UV.

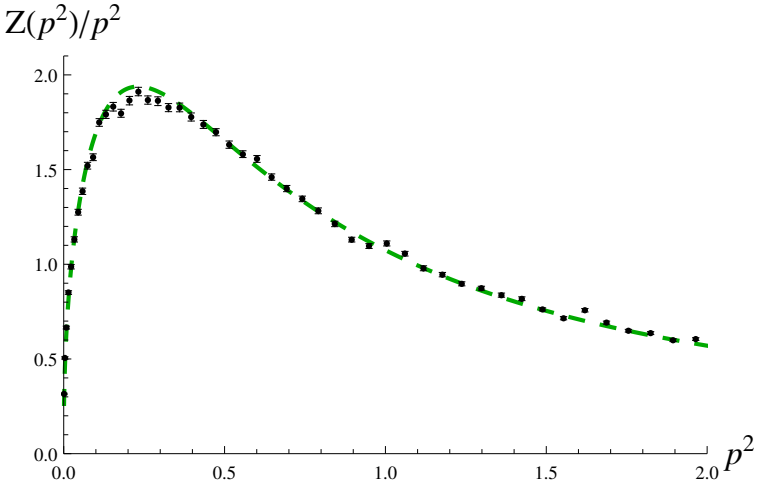




- Ghost dressing must approach 1 in the UV,
  but difficult to achieve due to mixing.
  $\longrightarrow$ Increased vertex dependence.
- Remaining logarithmic divergences:
  Different subtraction methods available.

Propagator results





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- lattice results [Maas, PR 524 (2013)]

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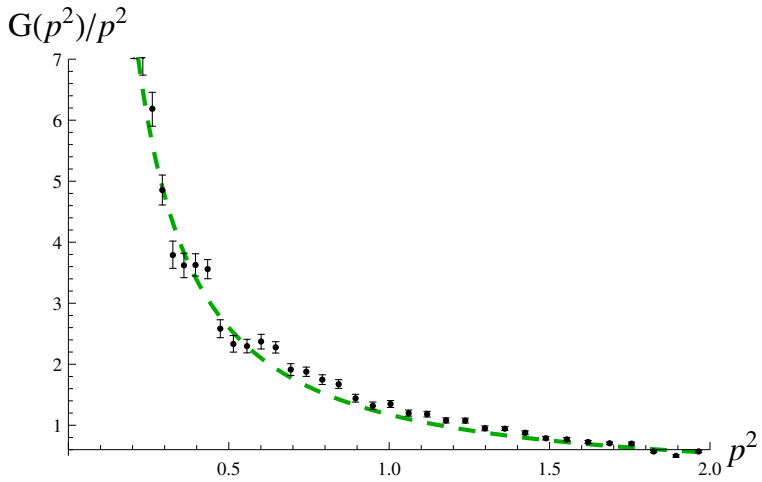

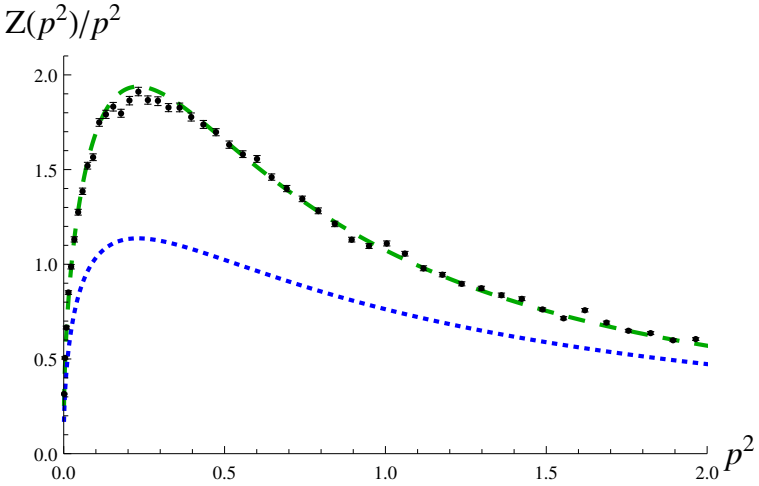

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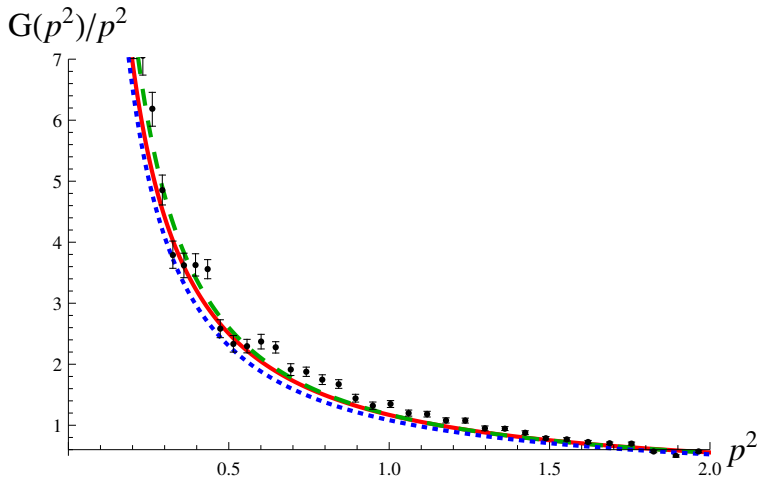
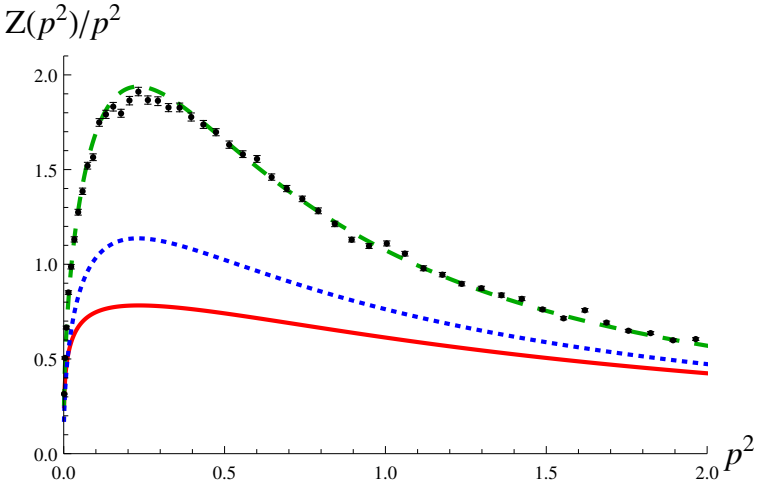


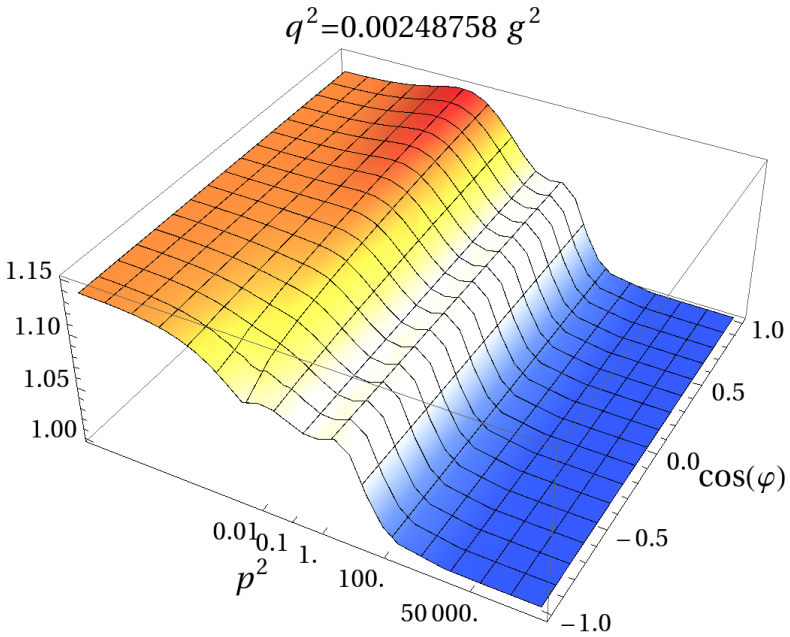


- bare ghost-gluon vertex, three-gluon vertex ansatz (1 parameter),
- lattice results [Maas, PR 524 (2013)]
- lattice inspired models for both vertices,
- dynamic ghost-gluon vertex, lattice inspired three-gluon vertex

$\Rightarrow$ Good agreement with lattice can be obtained,
but strong dependence on vertices.

Ghost-gluon vertex results: Selected configurations

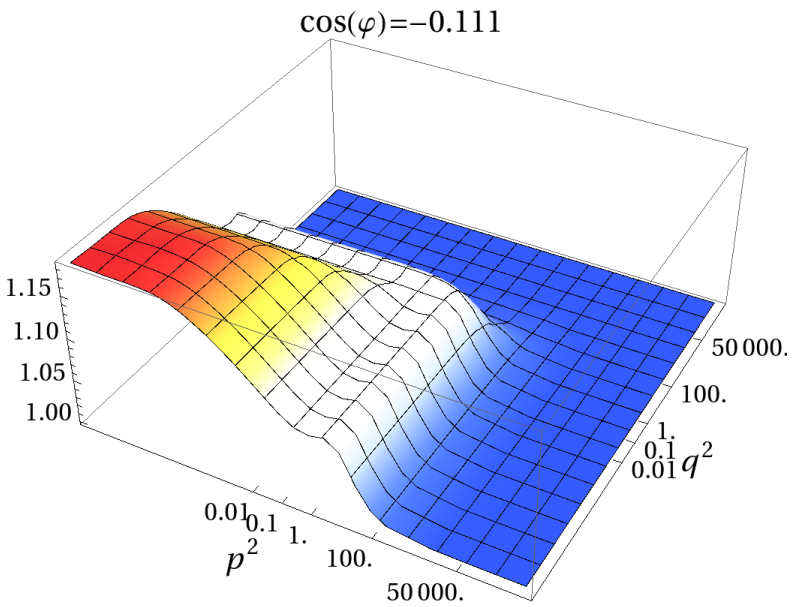
Fixed ghost momentum:



→ 1 in the UV

→ IR constant

Fixed angle:

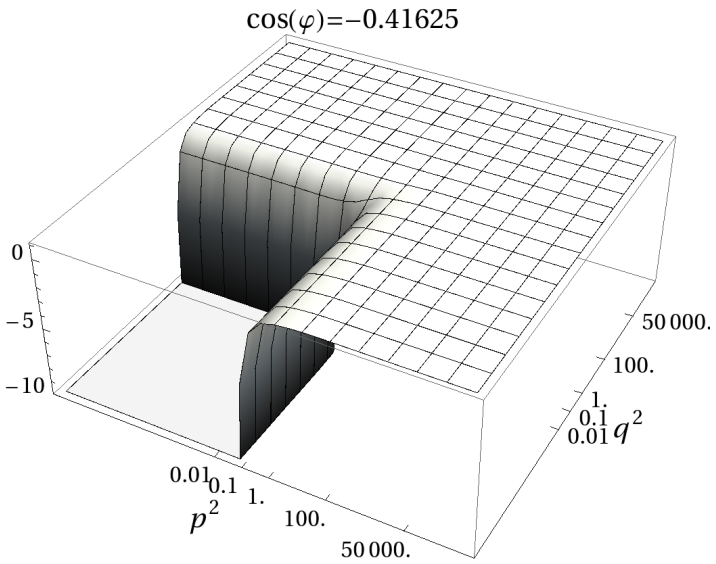


→ Almost no dependence on angle

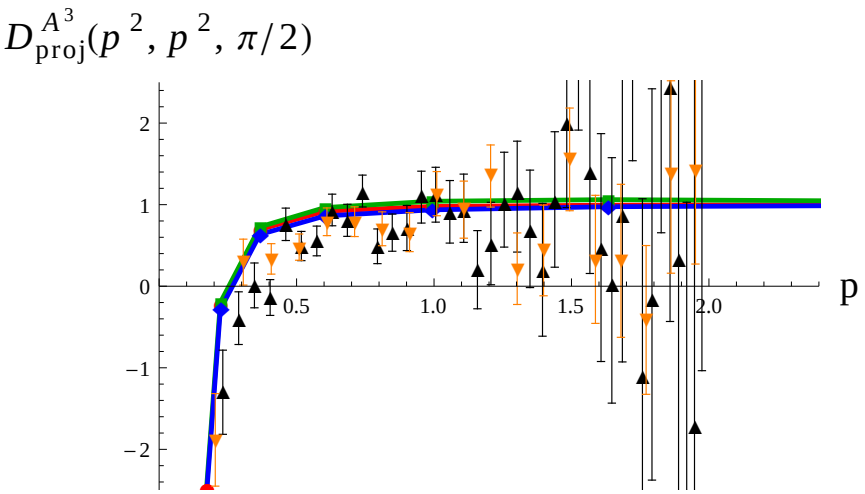
Three-gluon vertex

Three-gluon vertex from propagators and ghost-gluon vertex:

Fixed angle:



Orthogonal configuration:



red, green, blue: DSE calculation with different truncations
black (orange): lattice with $L = 21(12)fm^{-1}$ [Maas, PRD75]

$\Rightarrow$ Leading diagrams reproduce lattice results.

Three-gluon vertex: Ultraviolet

Bose symmetric version:

$$D^{A^3,UV}(x,y,z) = G\left(\frac{x+y+z}{2}\right)^{\alpha} Z\left(\frac{x+y+z}{2}\right)^{\beta}$$

Fix α and β:

1. UV behavior of three-gluon vertex
2. IR behavior of three-gluon vertex?

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2. IR behavior of three-gluon vertex $\rightarrow$ yes, but ...

Three-gluon vertex: Infrared

Three-gluon vertex might have a zero crossing.

$d = 2, 3$: seen on lattice

[Cucchieri, Maas, Mendes, PRD77 (2008); Maas, PRD75 (2007)],

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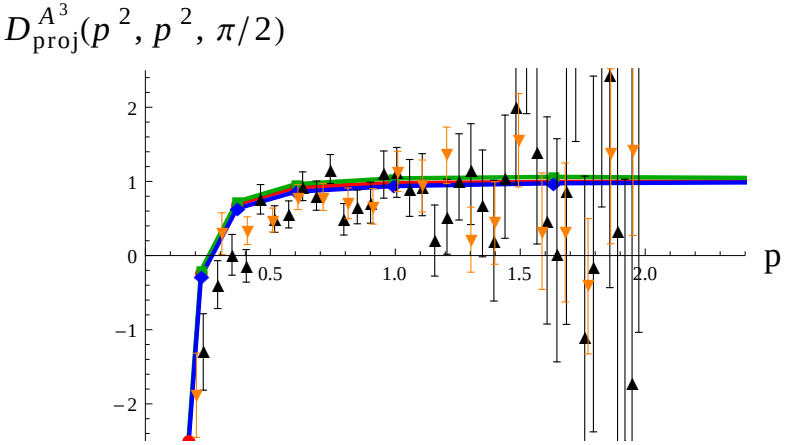

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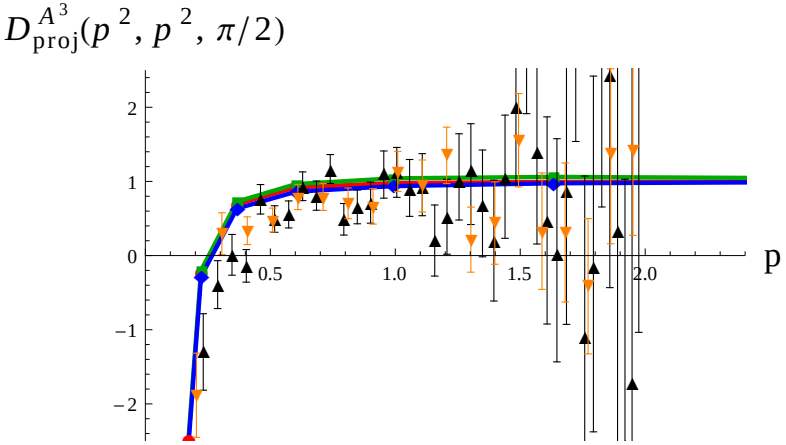

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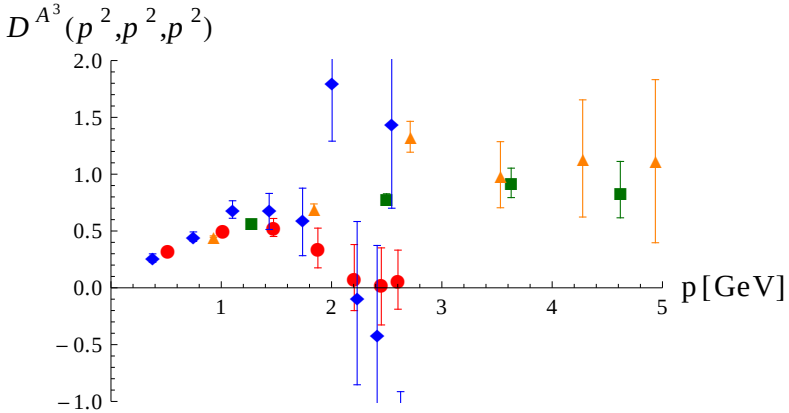

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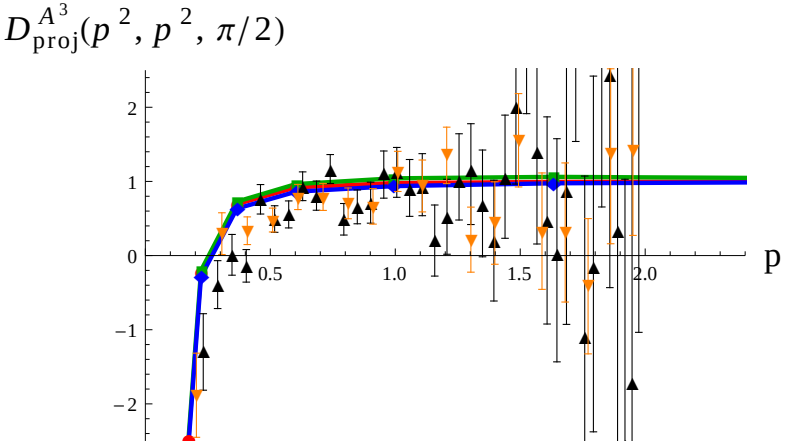

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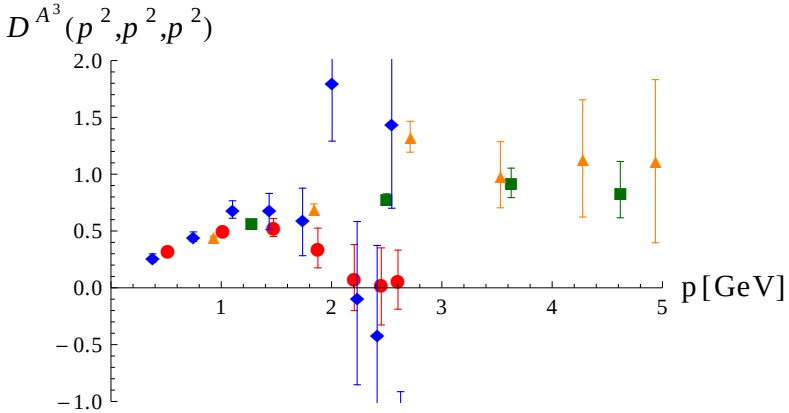
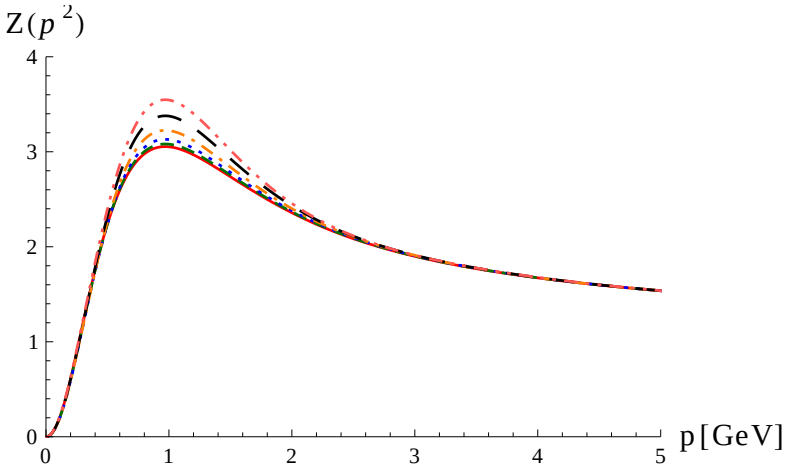
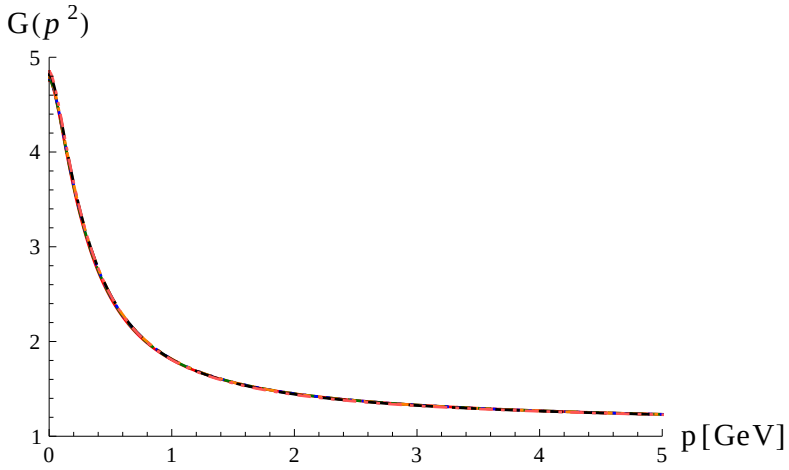


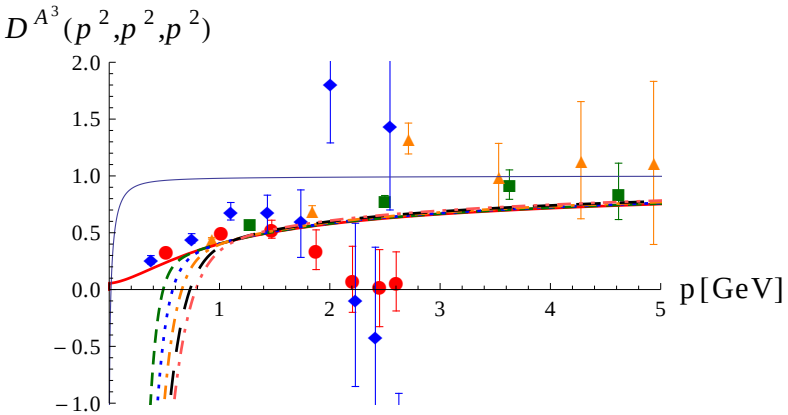
$$D^{A^3,IR}(x,y,z) = h_{IR}\, G(x+y+z)^3 (f^{3g}(x) f^{3g}(y) f^{3g}(z))^4$$

IR damping function $f^{3g}(x) := \dfrac{\Lambda_{3g}^2}{\Lambda_{3g}^2 + x}$

Influence of the three-gluon vertex







- Vary $\Lambda_{3g} \longrightarrow$ vary mid-momentum strength
- Ghost almost unaffected
- Thin line: Leading IR order <u>DSE calculation for three-gluon vertex</u> $\Rightarrow$ zero crossing

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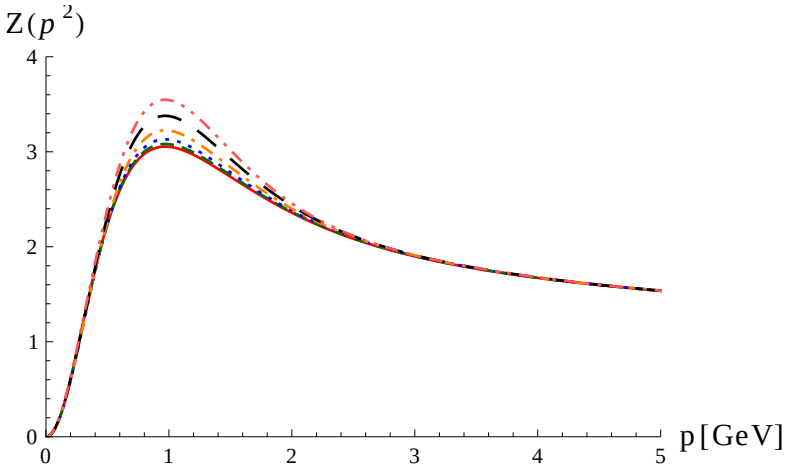

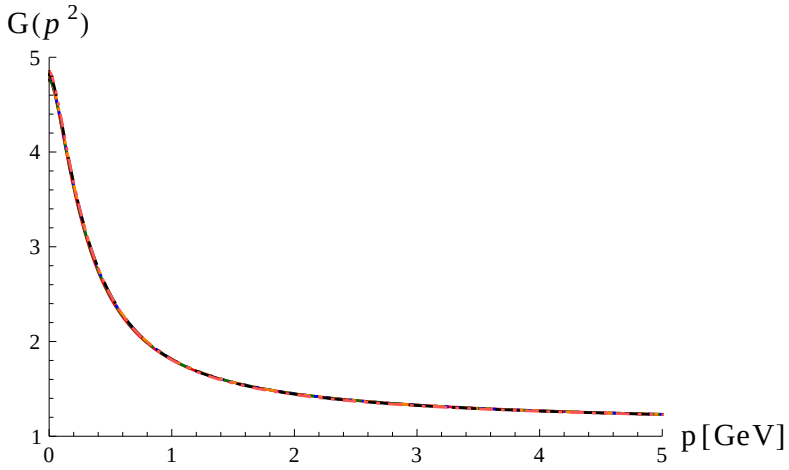

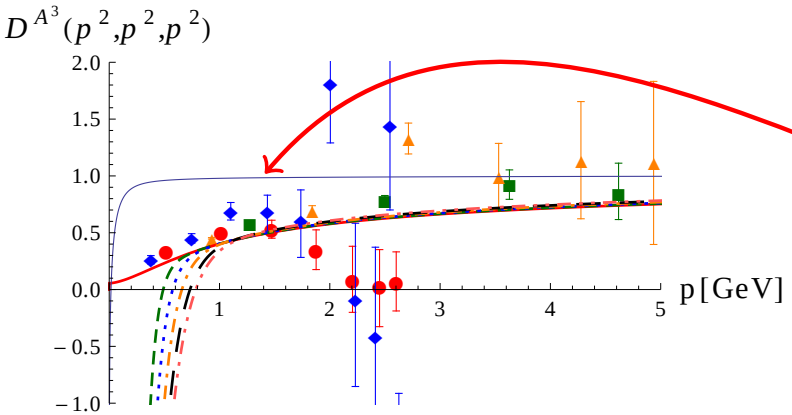

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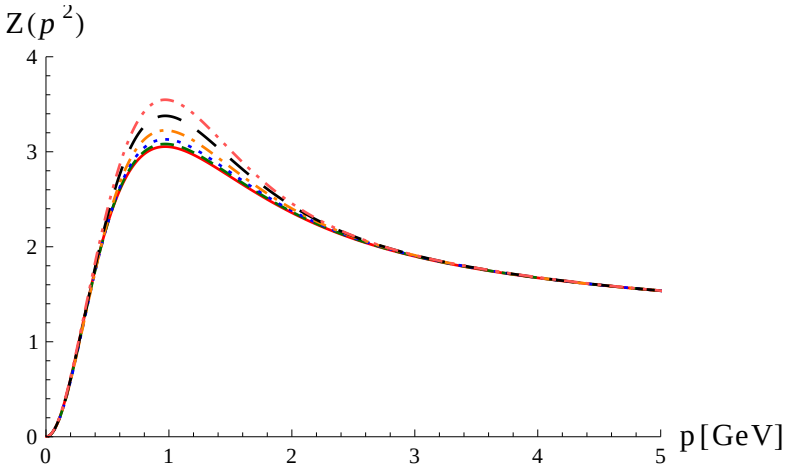

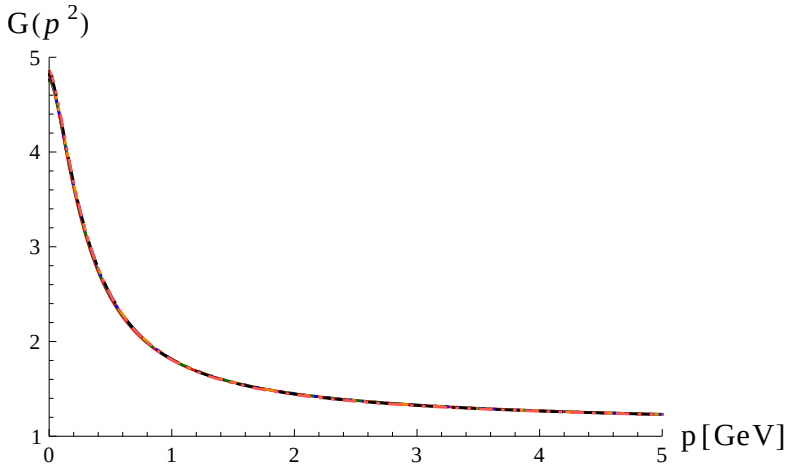

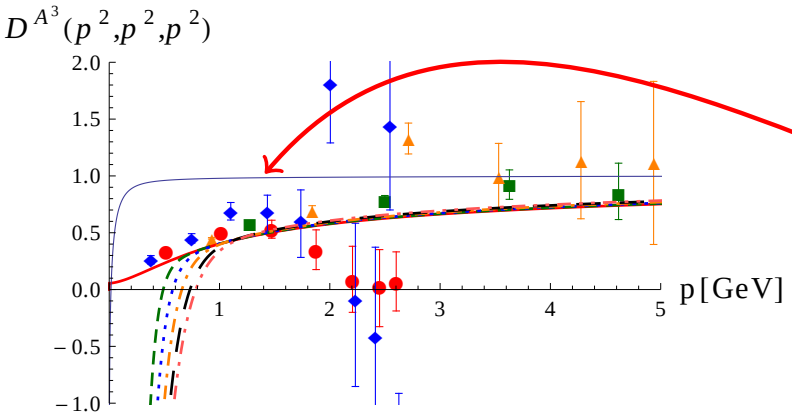

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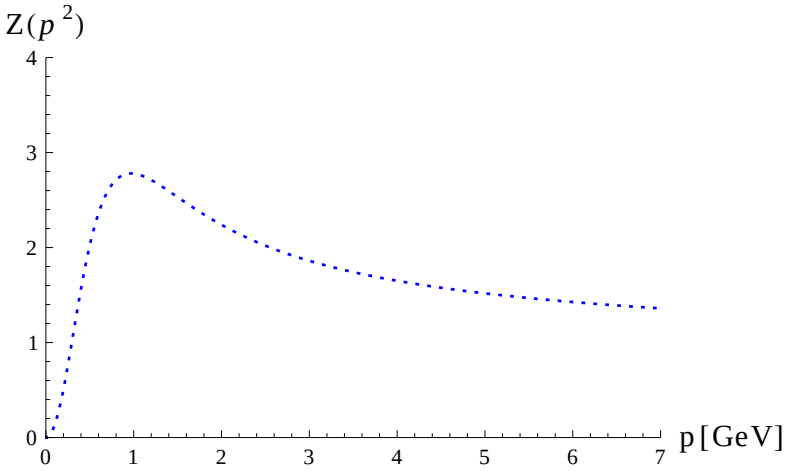
Optimized effective three-gluon vertex:
Choose Λ_{3g} where gluon dressing has best agreement with lattice results.
[MQH, von Smekal, JHEP (2013)]

Influence of the three-gluon vertex

ghost-gluon vertex: bare



original three-gluon vertex

Influence of the three-gluon vertex

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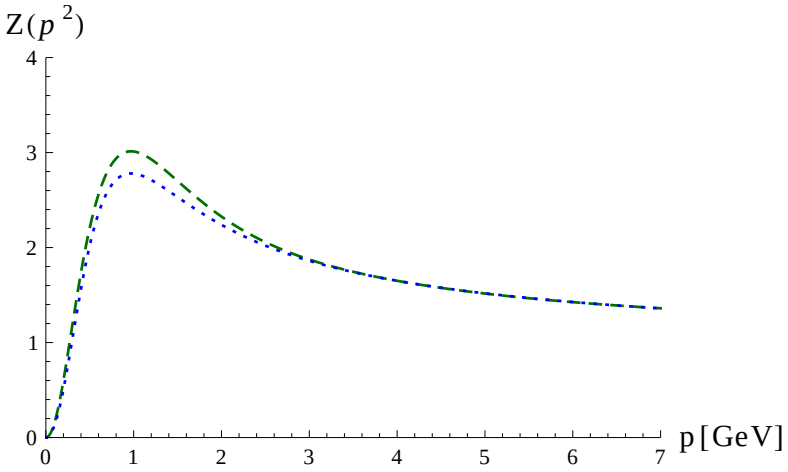

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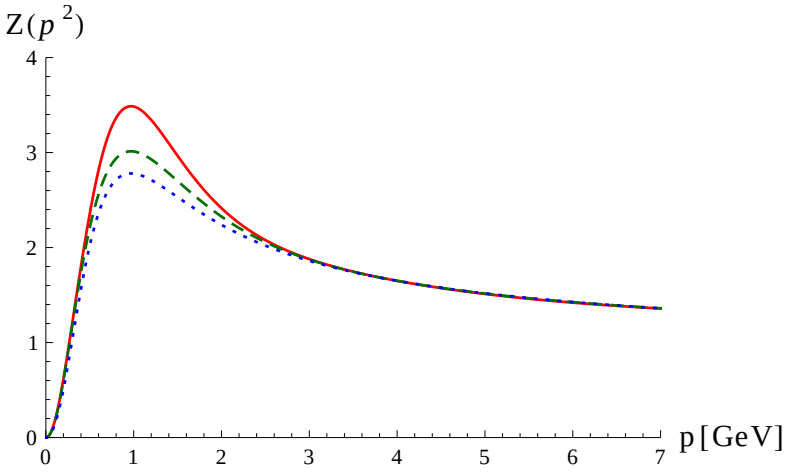
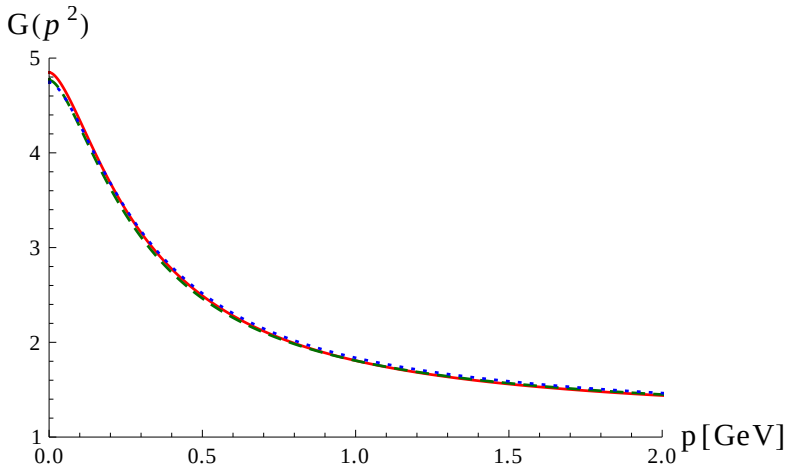


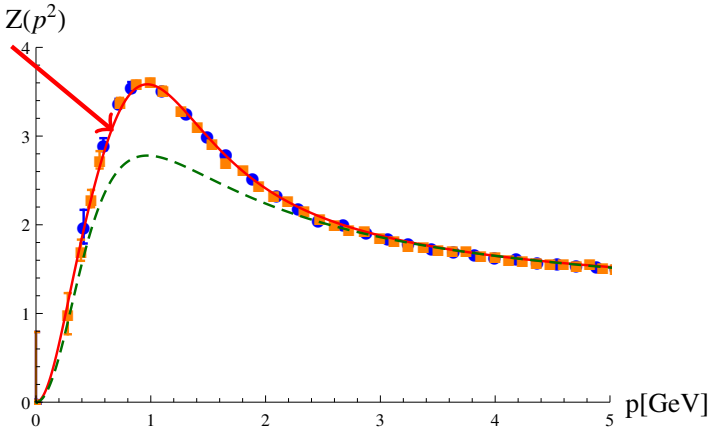


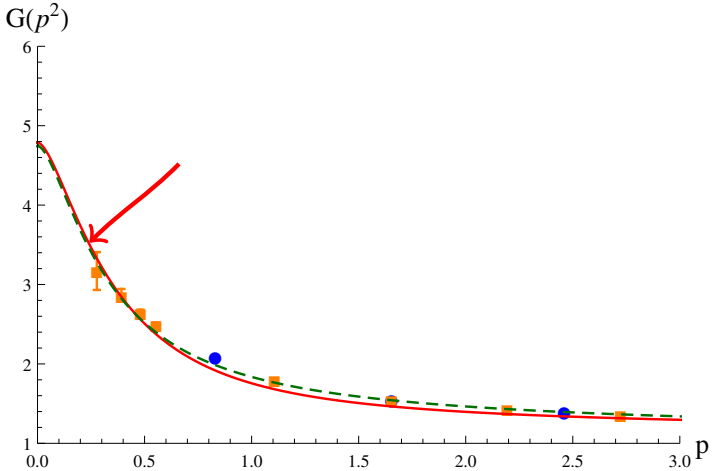
original three-gluon vertex
Bose symmetric three-gluon vertex
Bose symmetric three-gluon vertex with IR part

$\Rightarrow$ Improved three-gluon vertex adds additional strength in the mid-momentum regime.

Dynamic ghost-gluon vertex: Propagator results

Dynamic ghost-gluon vertex, opt. eff.
three-gluon vertex [MQH, von Smekal, JHEP (2013)]





Good quantitative agreement for ghost and gluon dressings.

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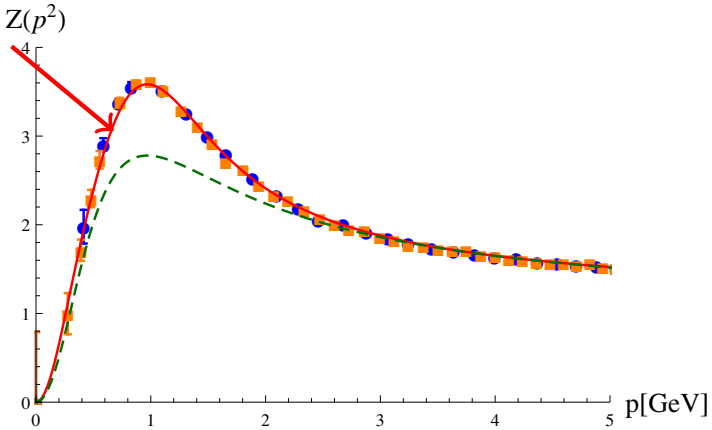
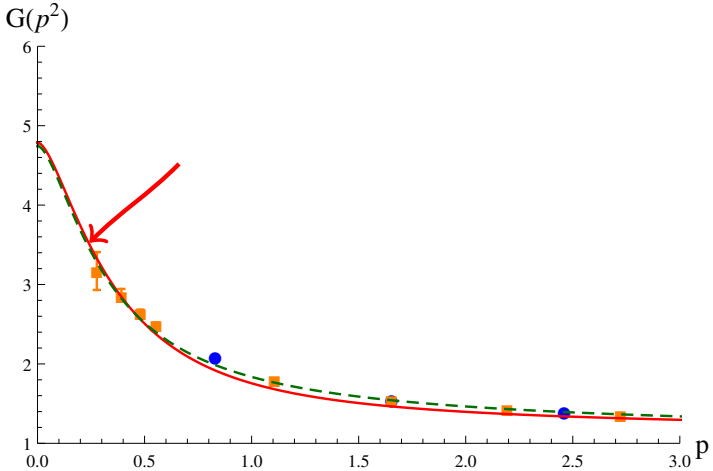
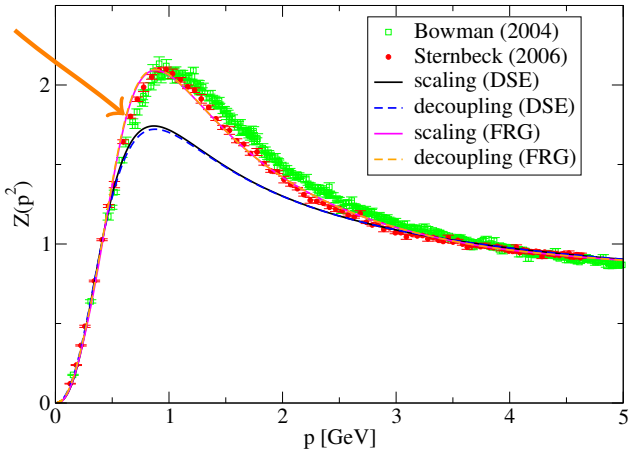
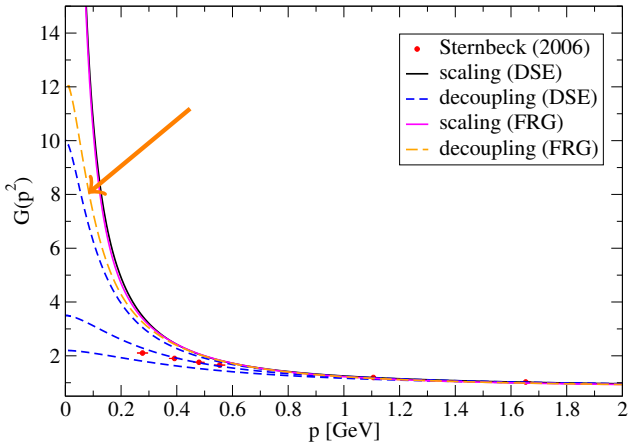




FRG results

[Fischer, Maas, Pawlowski, AP324 (2009)]



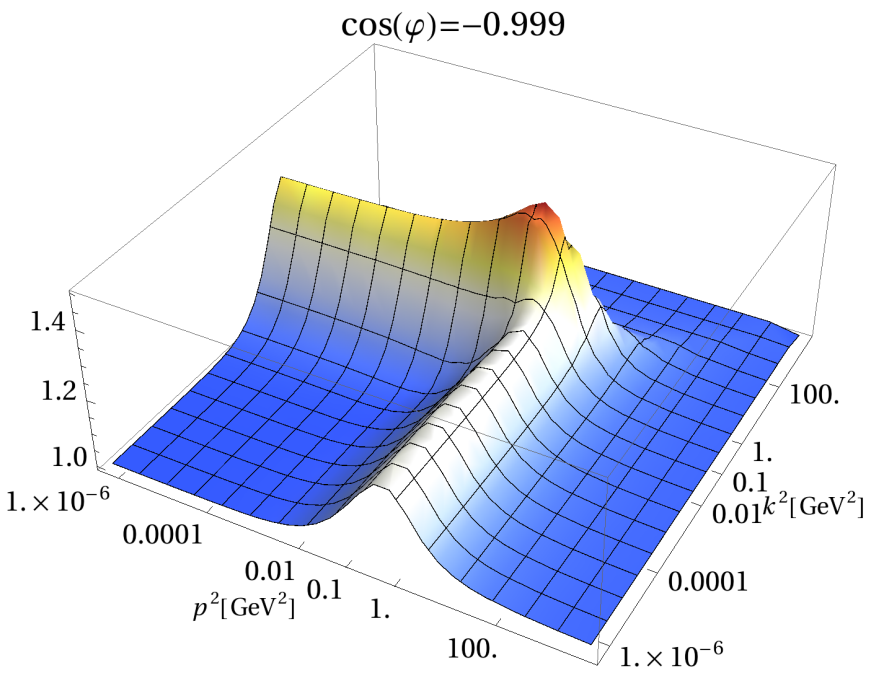


Good quantitative agreement for ghost and gluon dressings.

Ghost-gluon vertex: Selected configurations (decoupling)

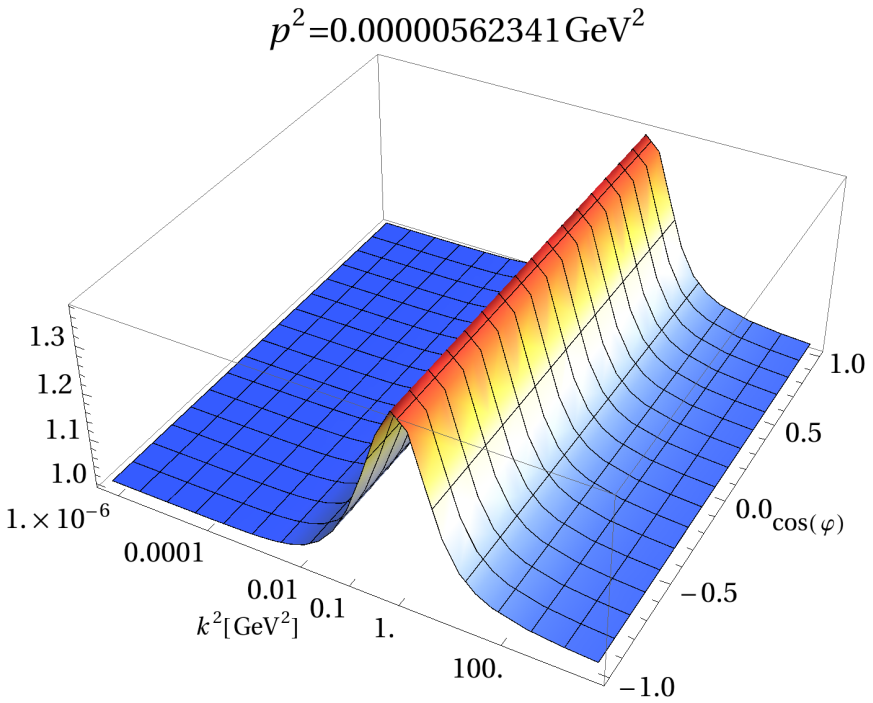
$$\Gamma_\mu^{A\bar{c}c,abc}(k;p,q) := i\,g\,f^{abc}\left(p_\mu A(k;p,q) + k_\mu B(k;p,q)\right)$$

Fixed angle:



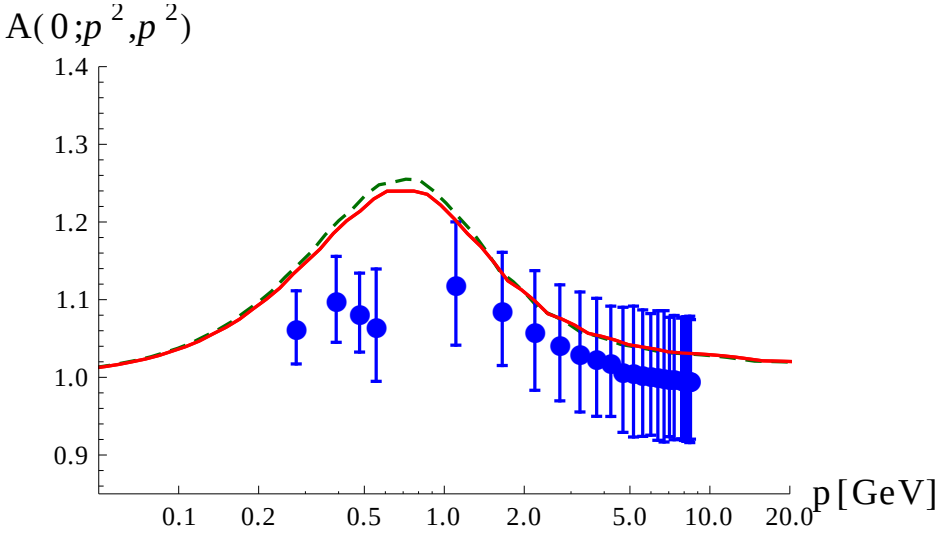
[MQH, von Smekal, JHEP (2013)]

Fixed anti-ghost momentum:



Ghost-gluon vertex: Comparison with lattice data

Orthogonal configuration $k^2 = 0$, $q^2 = p^2$:



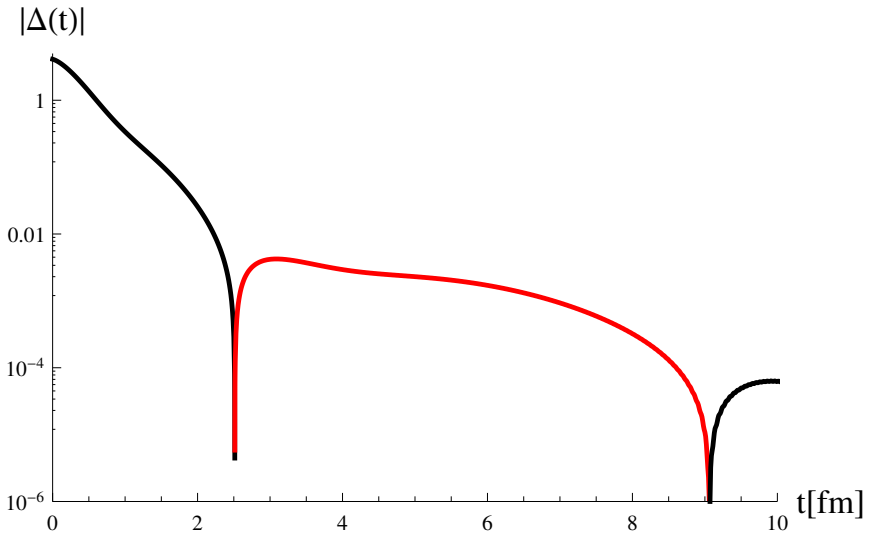
- constant in the IR
- relatively insensitive to changes of the three-gluon vertex (red/green lines: different three-gluon vertex models)

DSE calculation: [MQH, von Smekal, JHEP (2013)]
lattice data: [Sternbeck, hep-lat/0609016]

Schwinger function

Schwinger function $\Delta(t)$:

$$\Delta(t) = \frac{1}{\pi} \int dq \, \cos(q\, t) \frac{Z(q^2)}{q^2}$$



[MQH, von Smekal, PoS CONFX 062 (2013)]

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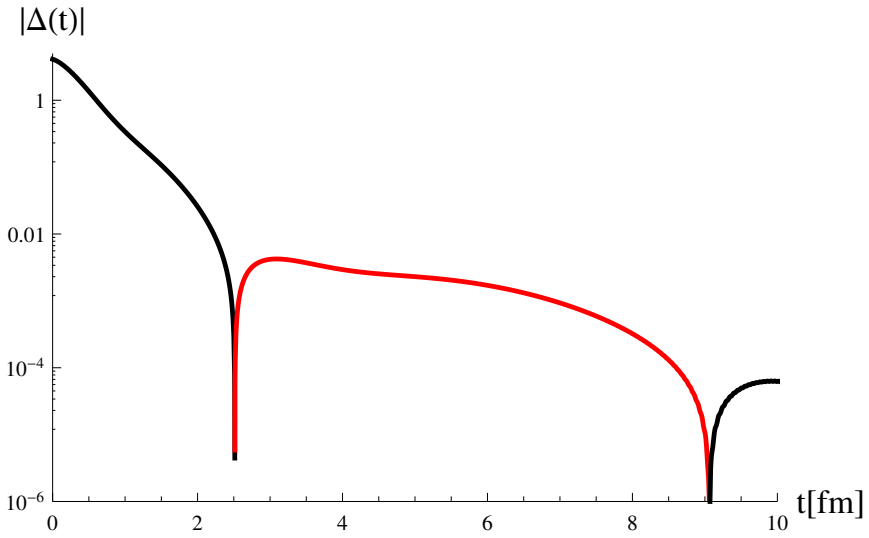


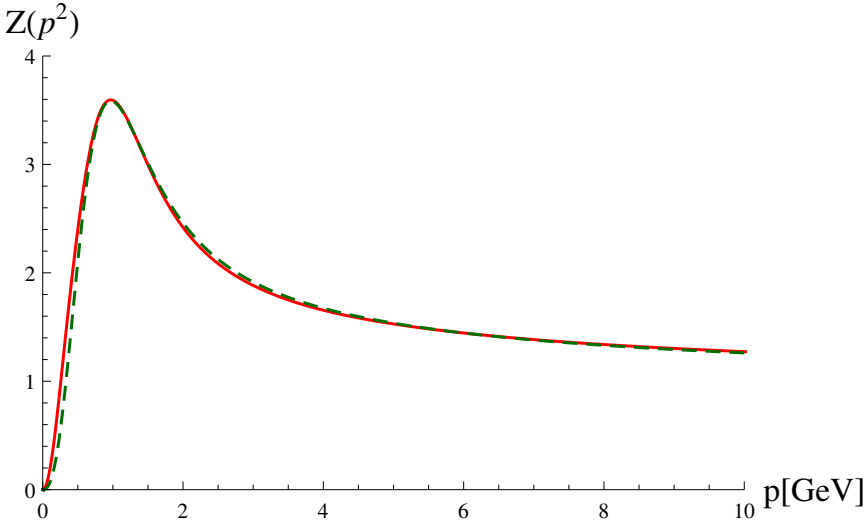
[MQH, von Smekal, PoS CONFX 062 (2013)]

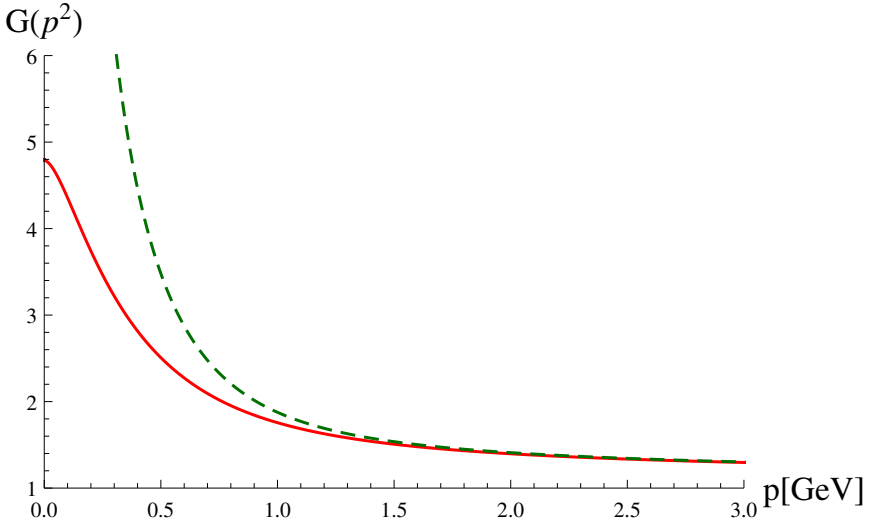
$$\Delta(t) = \int_0^\infty d\nu \, \rho(\nu^2) e^{-\nu p_0} = \mathcal{L}(\rho)$$

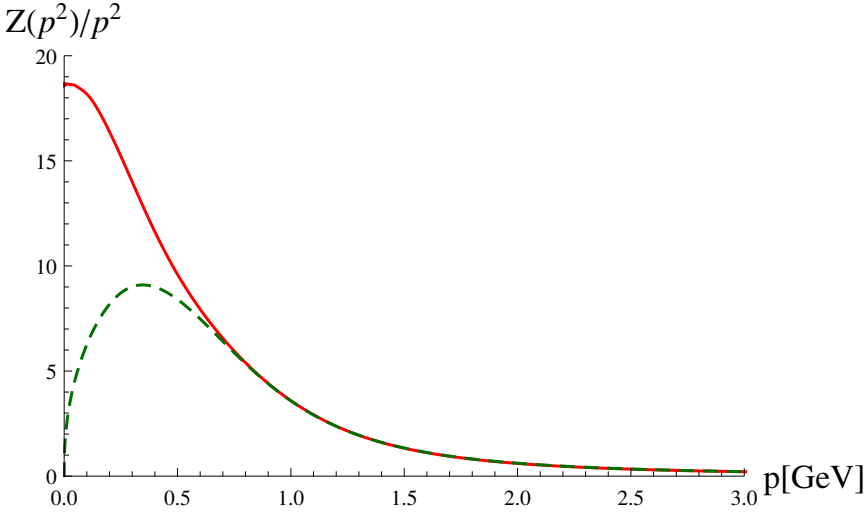
ρ: spectral density, must be positive for physical particles

Positivity violation of propagators $\rightarrow$ confinement.

Scaling solution: Propagators



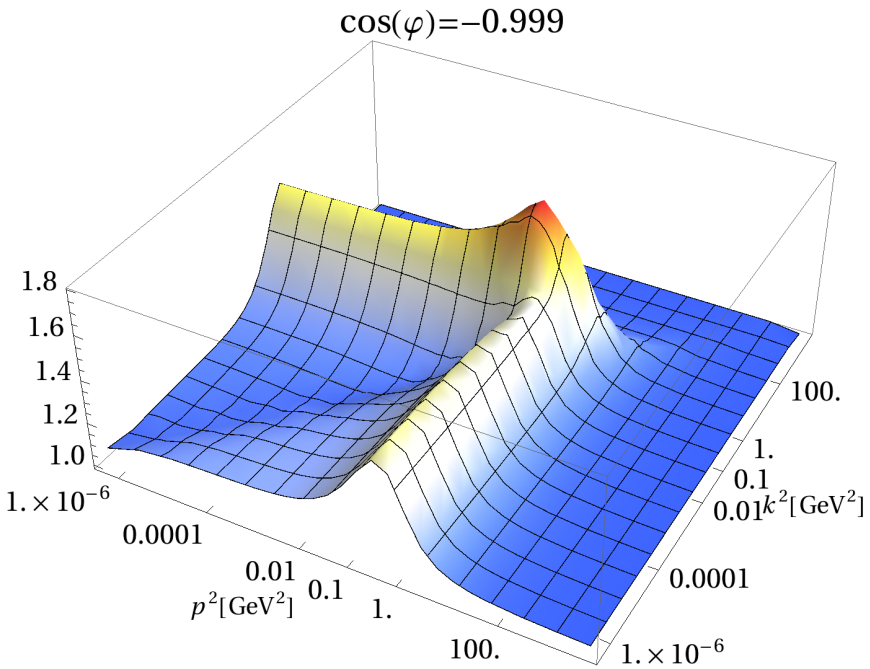




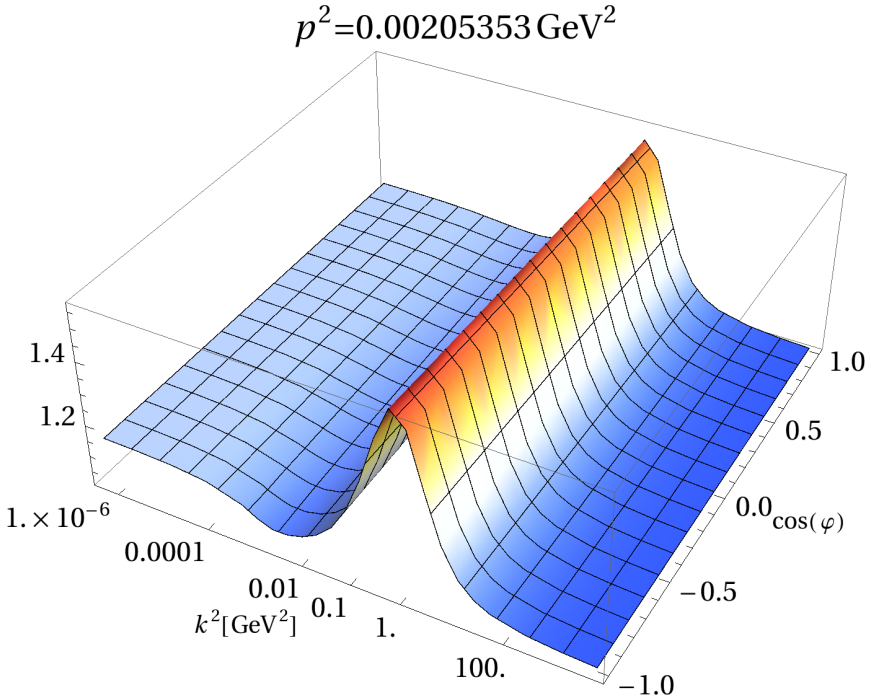
- Scaling solution
- Decoupling solution
- Differences only at low momenta.

Scaling solution: Ghost-gluon vertex

Fixed angle:



Fixed momentum:



- Dressing not 1 in the IR ← Contributions from loop corrections (for decoupling they are suppressed)
- Scaling/decoupling also seen in ghost-gluon vertex

Towards the phase diagram of QCD with DSEs

- Lattice results helpful
  - as input

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Towards the phase diagram of QCD with DSEs

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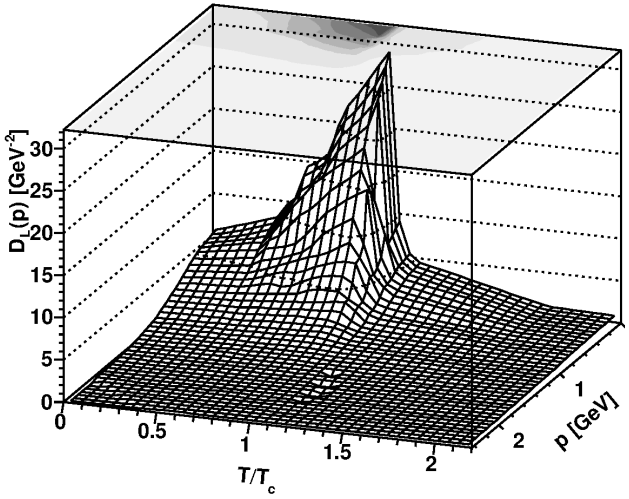
    non-zero chemical potential ↯ $\longrightarrow$ use theories without sign problem, e.g., $SU(2)$, G_2
- Self-consistent functional RG calculation of correlation functions:
  [Fister, Pawlowski, 1112.5440; Fister, PhD thesis, 2012]
- DSE calculations:
  - Yang-Mills propagators: [Maas, Wambach, Alkofer, EPJC42 (2005); Cucchieri, Maas, Mendes, PRD75 (2007)]
  - Phase transitions: [Fischer et al. (2009-2013)]

Ghost propagator

First steps towards full system: Take some lattice input.

Gluon propagator: lattice based fits [Fischer, Maas, Müller, EPJC68 (2010)]



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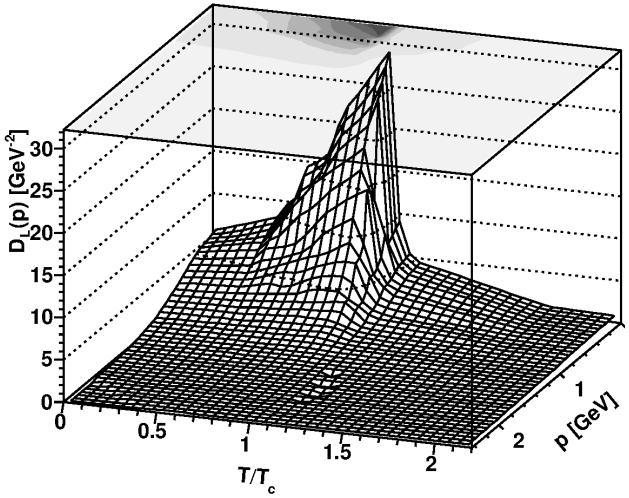
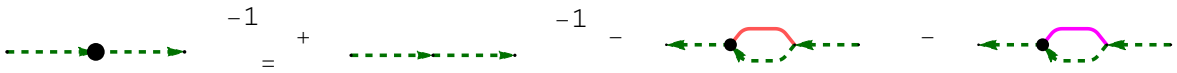


Ghost propagator [preliminary]:



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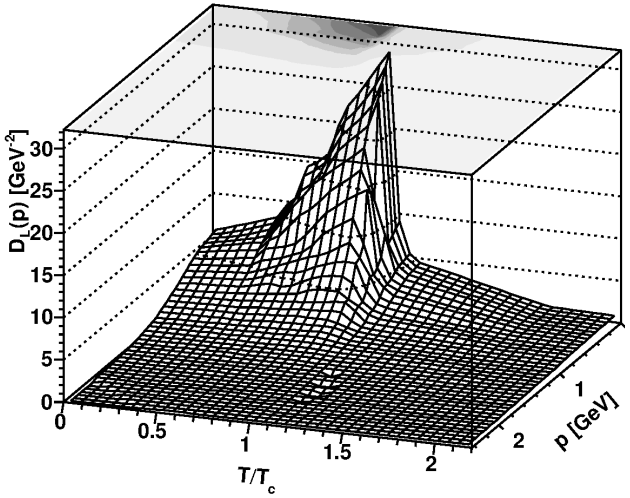

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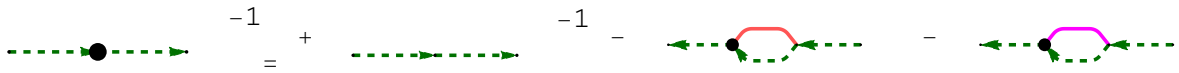
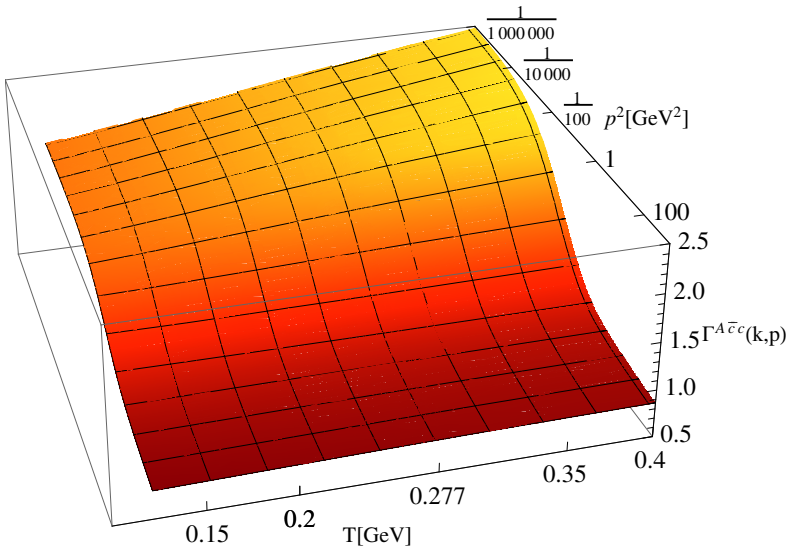


Diagram with long. gluon: For zeroth Matsubara no contribution from (dominant) zeroth Matsubara summand.

$\Rightarrow$ Ghost does not react to phase transition.

Ghost propagator

Ghost propagator at various T:



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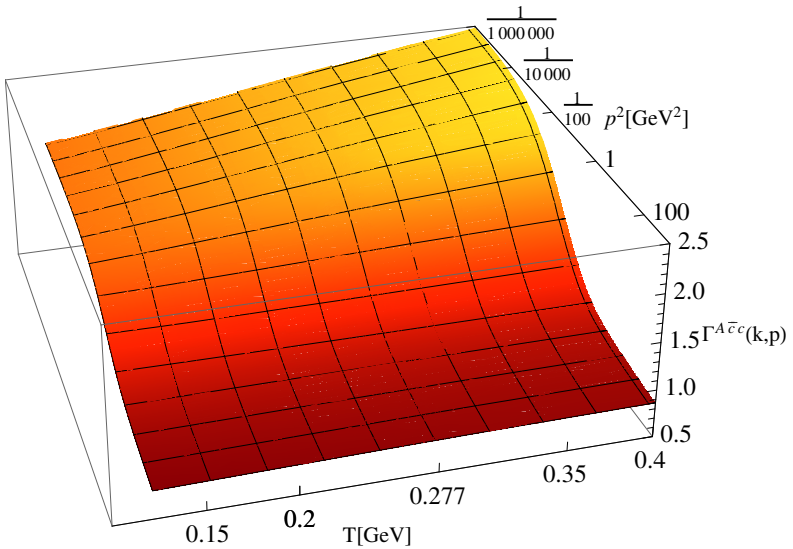

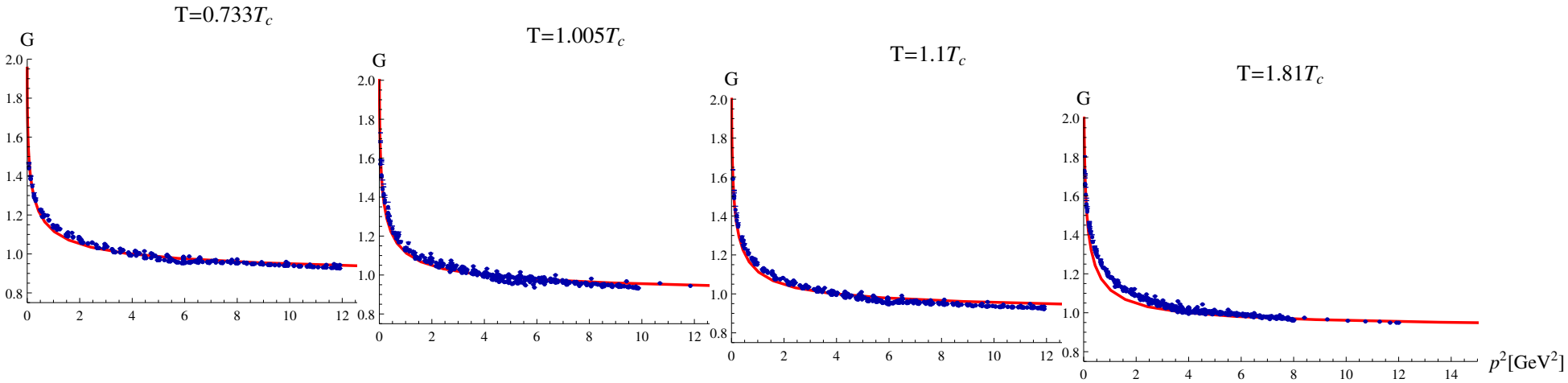

Lattice: [Maas, Pawlowski, Spielmann, von Smekal, PRD85 (2012)]

Ghost-gluon vertex

Remember: Relevance of ghost-gluon vertex for non-zero temperature known from functional RG [Fister, Pawlowski, 1112.5440]!

Simple approximation:

$$\left.\begin{array}{l}\text{Fully iterated ghost propagator}\\ \text{Gluon propagator from the lattice}\\ \text{[Fischer, Maas, Müller, EPJC68 (2010)]}\end{array}\right\}\ \begin{array}{l}\text{Ghost-gluon vertex semi-perturbatively}\\ \text{at symmetric point } (p^2 = q^2 = k^2)\end{array}$$

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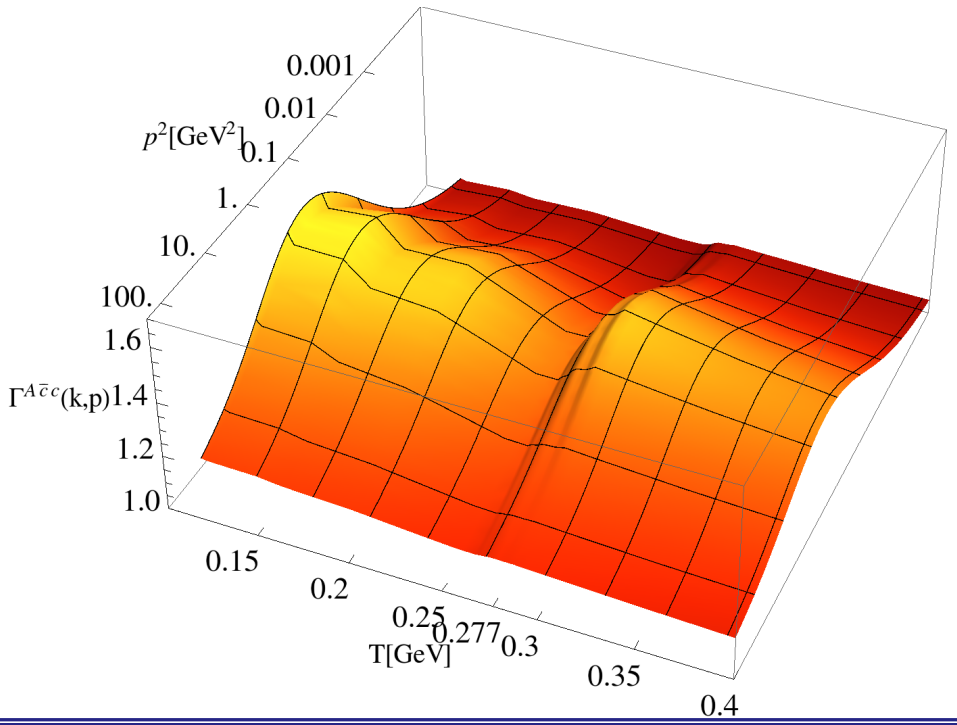

[preliminary]

Summary

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Thank you for your attention.