

# Infrared analysis of Yang-Mills theory in the maximally Abelian gauge and the Landau gauge

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in collaboration with:

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SIC!QFT



# Contents of the talk

- Infrared of Yang-Mills theory: What can we learn from it?
- Maximally Abelian gauge: Why do we need this complicated gauge, anyway? And what is its infrared behavior?
- Landau gauge: Does (partly) solving the Gribov problem change the infrared behavior?
- Non-perturbative tool: Dyson-Schwinger equations; is there an easy way to derive them?

# Confinement of quarks and gluons

- **Confinement** is a long-range  $\leftrightarrow$  IR phenomenon: We do not see individual  $\sim$  infinitely separated quarks or gluons.  
What's the mechanism behind it?
- One expects that the property of being **confined** is encoded in the particles' propagators.
- Different confinement criteria for the propagators:
  - Positivity violations: negative norm contributions  $\rightarrow$  not a particle of the physical state space
  - Kugo-Ojima: quartet mechanism, e. g. Gupta-Bleuler formalism in QED: time-like and longitudinal photon cancel each other.  
**Landau gauge Yang-Mills**,  $p^2 \rightarrow 0$ :  $D_{\text{gluon}} \rightarrow 0$ ,  $p^2 D_{\text{ghost}} \rightarrow \infty$
  - Gribov-Zwanziger (Landau gauge, Coulomb gauge): IR suppression of the gluon propagator due to Gribov horizon  $\rightarrow$  no long-distance propagation.  
Already manifest at perturbative level with Gribov-Zwanziger Lagrangian!

# Dyson-Schwinger equations (DSEs) for investigating QCD

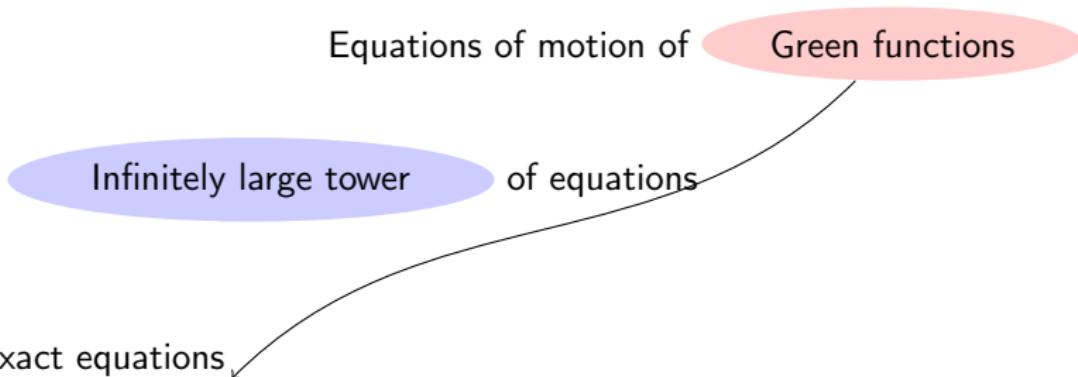
Equations of motion of

Green functions

Infinitely large tower

of equations

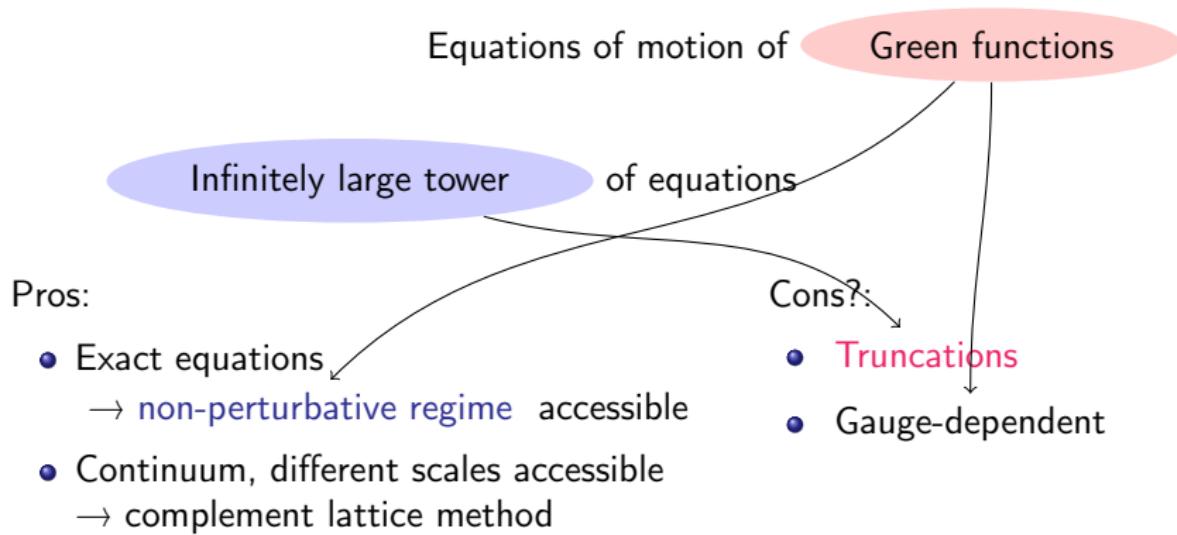
# Dyson-Schwinger equations (DSEs) for investigating QCD



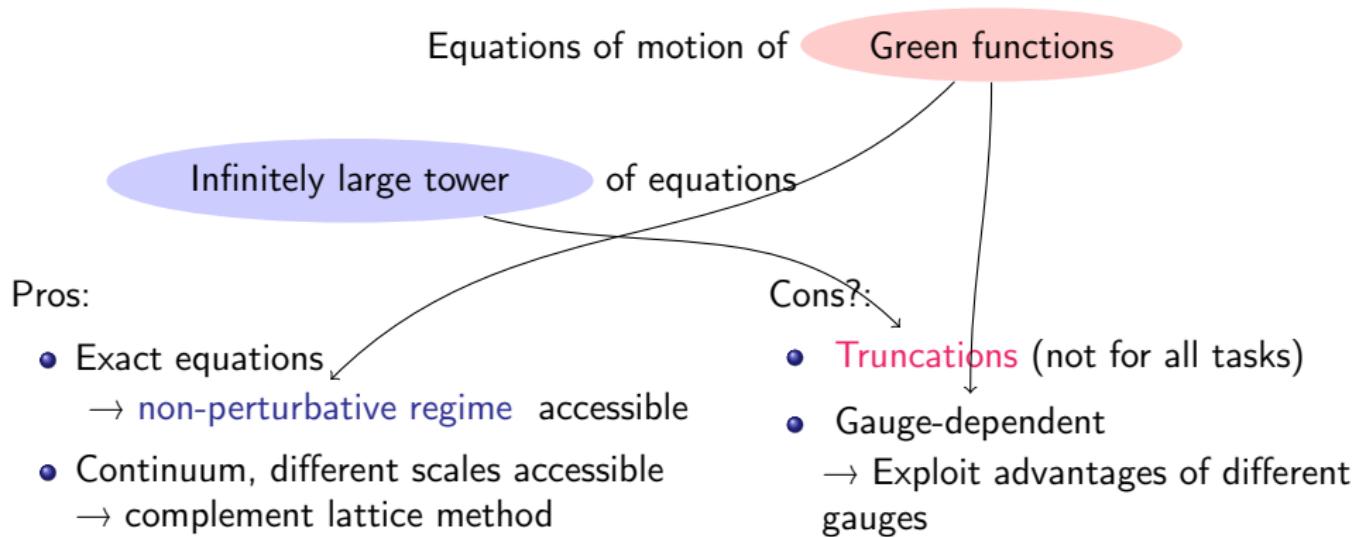
Pros:

- Exact equations  
→ non-perturbative regime accessible
- Continuum, different scales accessible  
→ complement lattice method

# Dyson-Schwinger equations (DSEs) for investigating QCD



# Dyson-Schwinger equations (DSEs) for investigating QCD



# Infrared regime of Yang-Mills theory in Landau gauge I

## Scaling solution [Alkofer, Fischer, Gies, Maas, Pawłowski, von Smekal, ...]

- Dressing functions obey power laws. → Qualitative information provided by **IR exponents**.
- Qualitative IR solution of ALL correlation functions is known.
- Picture of confinement: IR vanishing gluon (→ gluon confinement) and **IR enhanced ghost propagator** (→ long-range force to confine quarks).
- Horizon condition/Kugo-Ojima  $\leftrightarrow$  IR enhanced ghost.
- Gluon propagator violates positivity.
- Confining Polyakov loop potential [Braun, Gies, Pawłowski, arXiv:0708.2413].
- Method transferable to some other gauges (→ MAG, lin. covariant gauge  $\xi \neq 0$ ?).

# Infrared regime of Yang-Mills theory in Landau gauge II

## Decoupling solution

- Gluon massive, ghost tree-level like.
- Seen in most lattice calculations [Bogolubsky, Bornyakov, Cucchieri, Ilgenfritz, Maas, Mendes, Müller-Preussker, Pawłowski, Spielmann, Sternbeck, von Smekal, ...].  
→ Proof of unique solution?
- Adding condensates to the Gribov-Zwanziger action → refined Gribov-Zwanziger scenario [Dudal et al.]
- DSEs, FRGEs [Boucaud et al., Aguilar et al., Fischer et al.]
- Different renormalization of the **ghost propagator** ⇒ tree-level like.  
↔ **boundary condition for DSEs** [Fischer et al., Ann. Phys. 324; Maas, 0907.5185]
- Gluon propagator violates positivity.
- Confining Polyakov loop potential [Braun, Gies, Pawłowski, arXiv:0708.2413].
- What is the mechanism for confinement?

# Hypothesis of Abelian dominance

## Dual superconductor picture of confinement [Mandelstam, 't Hooft]

- Picture a conventional superconductor, where the electric charges condense and force the magnetic flux into vortices.
- Change "electric" and "magnetic" components and you get a dual superconductor, where **condensed magnetic monopoles** squeeze the electric flux into flux tubes.
- QCD: No free chromoelectric charges. Are they confined by condensed magnetic monopoles?

Hypothesis of Abelian dominance [Ezawa, Iwazaki, PRD 25 (1981)]:

Magnetic monopoles live in Abelian part of the theory. → **Abelian part dominates in the IR?**

# Lattice results on Abelian IR dominance

Quenched QCD, linear rising potential between two quarks:

$$V(r) \sim \sigma r.$$

- $\sigma_{Abel}$  (calculated from the Abelian part) is almost the same as  $\sigma$ .
- Suzuki et al. [PRD 80]: Without gauge fixing  $\sigma_{Abel}$  was extracted and agreed exactly with  $\sigma$ .  
Maybe MAG is a simple way to get monopoles?
- Available lattice results of MAG [Cucchieri, Mendes, Mihara, 2008]: all propagators massive, Abelian field has lowest mass  
 $\Rightarrow$  other fields decouple. Realization of Abelian dominance.

# Definition of the maximally Abelian gauge

Look for dominance of Abelian part. What is the **Abelian part**?

Gauge field components:

$$A_\mu = \mathbf{A}_\mu^i T^i + \mathbf{B}_\mu^a T^a, \quad i = 1, \dots, N-1, \quad a = N, \dots, N^2 - 1$$

Abelian subalgebra:  $[T^i, T^j] = 0$ , can be written as diagonal matrices

Abelian  $\leftrightarrow$  diagonal fields **A**,  
non-Abelian  $\leftrightarrow$  off-diagonal fields **B**.

E.g.  $T^1 = \frac{1}{2}\lambda^3$ ,  $T^2 = \frac{1}{2}\lambda^8$  for  $SU(3)$ .

Which interactions are possible ( $[T^r, T^s] = i f^{rst} T^t$ )?

	$SU(2)$	$SU(N > 2)$
$f^{ijk}$	0	0
$f^{ija}$	0	0
$f^{iab}$	✓	✓
$f^{abc}$	0	✓

⇒ 2 off-diagonal and 1 diagonal field  
can interact; 3 off-diagonal fields can  
only interact in  $SU(N > 2)$   
→  $SU(2)$  and  $SU(3)$  different?

# Gauge fixing condition

Stress role of diagonal fields  $\Rightarrow$  minimize norm of off-diagonal field  $B$ :

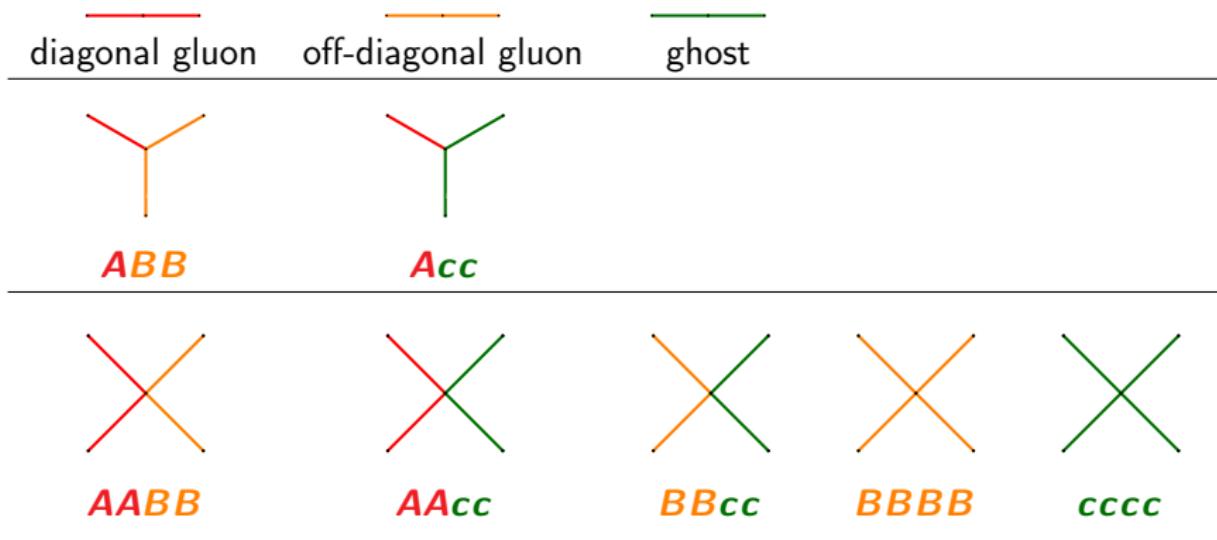
$$\|B_U\| = \int dx B_U^a B_U^a \rightarrow \text{minimize wrt. gauge transformations } U$$

$$D_\mu^{ab} B_\mu^b = (\delta_{ab} \partial_\mu - g f^{abi} A_\mu^i) B_\mu^b = 0 \quad \text{non-linear gauge fixing condition!}$$

Remaining symmetry of diagonal part:  $U(1)^{N-1}$

Fix gauge of diag. gluon field  $A$  by Landau gauge condition:  $\partial_\mu A_\mu = 0$   
 $\Rightarrow$  diagonal ghosts decouple (like in QED).

# Lagrangian for the MAG



$SU(N > 2)$

$BBB \quad Bcc \quad ABBB \quad ABcc$

# Deriving Dyson-Schwinger equations

Translation invariance of the path integral:

$$\int [D\phi] \frac{\delta}{\delta\phi} e^{-S+J\Phi} = \int [D\phi] \left( J - \frac{\delta S}{\delta\phi} \right) e^{-S+J\Phi} = 0.$$

⇒ DSEs for **all Green functions** (full, connected, 1PI) by further differentiations.

Doing it by hand?

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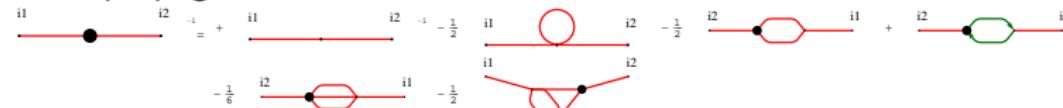
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Doing it by hand?

Example: Landau gauge, only 2 propagators (**AA**, **cc**), 3 interactions  
(**Acc**, **AAA**, **AAAA**)

# Landau Gauge: Propagators

Gluon propagator:

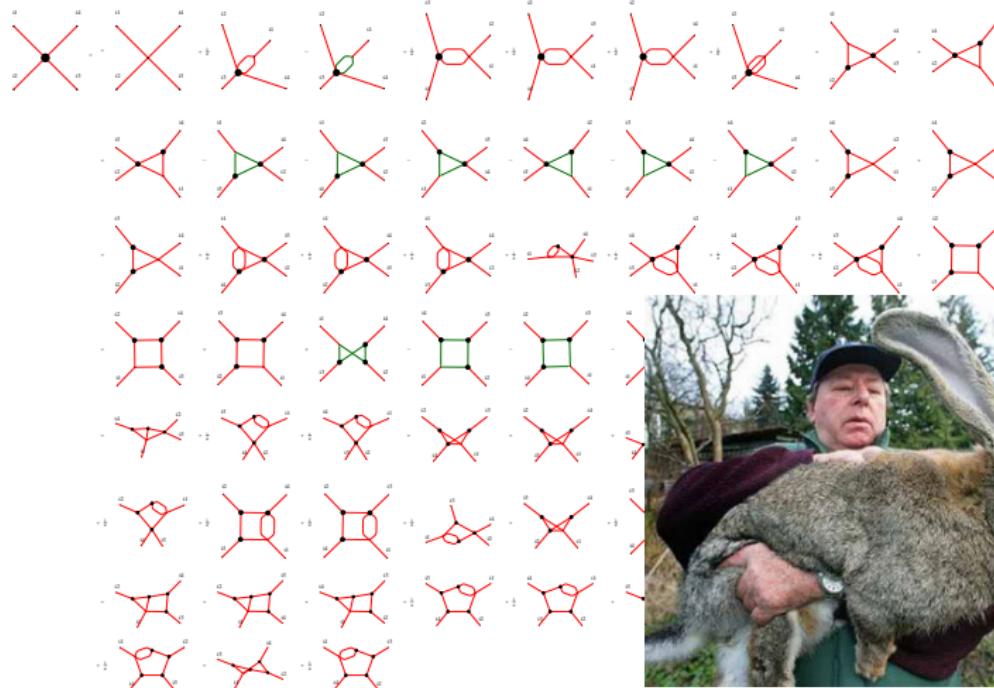


Ghost propagator:



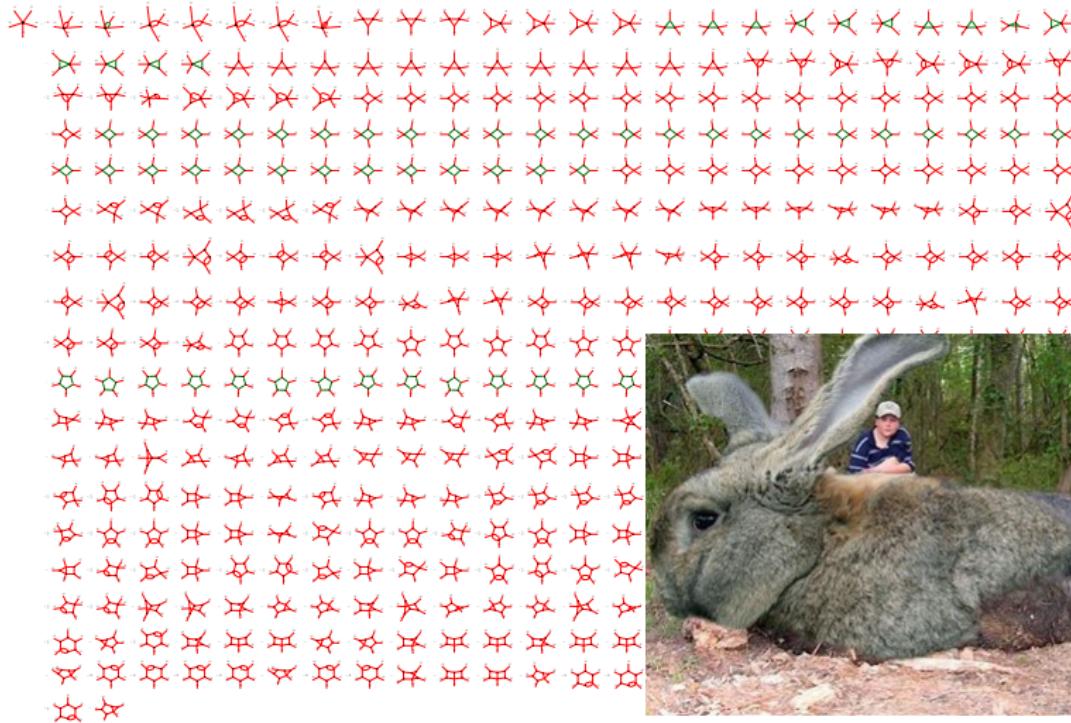
# Landau Gauge: Four-Gluon Vertex

66 terms



# Landau Gauge: Five-Gluon Vertex

434 terms



# DoDSE

⇒ *DoDSE* [Alkofer, M.Q.H., Schwenzer, CPC 180 (2009)]

Given a structure of interactions, the DSEs are derived symbolically using *Mathematica*.

Example (Landau gauge):

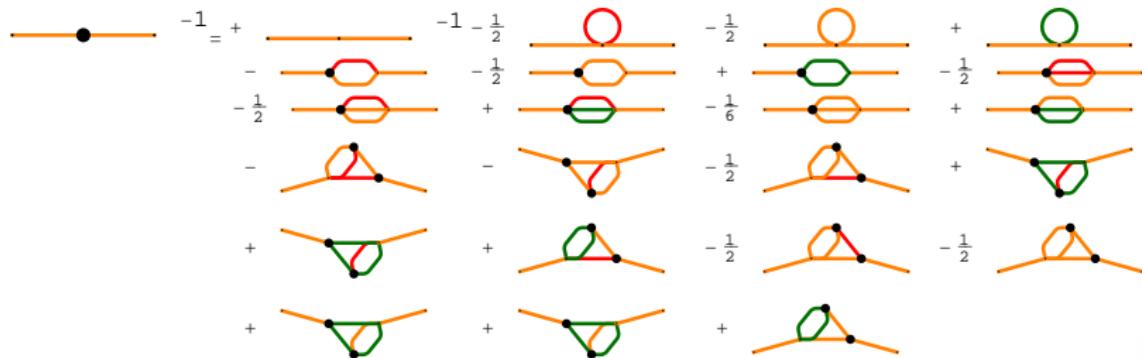
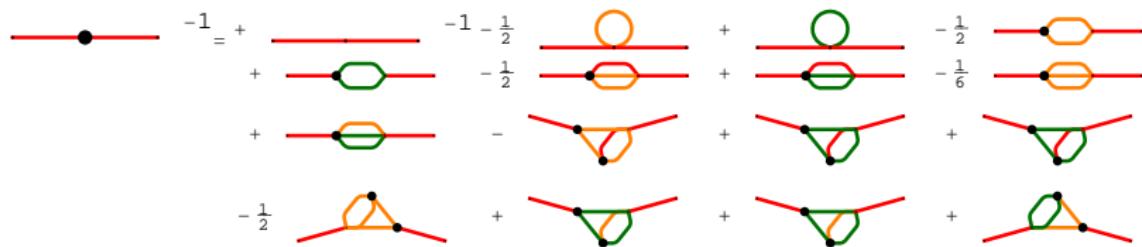
- only input: interactions in Lagrangian (AA, AAA, AAAA, cc, Acc)
  - Which DSE do I want?
- 
- Step-by-step calculations possible.
  - Can handle mixed propagators (then there are really many diagrams; e. g. in Gribov-Zwanziger action).

Upgrade: *Symb2Alg*

Provide Feynman rules and get complete algebraic expressions.

→ E. g. calculate color algebra with *FORM* and integrals with *C*.

# DSEs of the MAG



# Infrared power counting

## Generic propagator

$$T_{(ij)} \cdot \frac{D(p^2)}{p^2},$$

IR exponent

assume power law behavior at low  $p^2$

$$D^{IR}(p^2) = A \cdot (p^2)^\delta$$

- Vertices also assume power law behavior [Alkofer, Fischer, Llanes-Estrada, PLB 611 (2005) (skeleton expansion)].
- Limit of all momenta approaching zero simultaneously.
- Upon integration all momenta converted into powers of external momenta.  $\Rightarrow$  Counting of IR exponents

# System of inequalities

- IR exponent for every diagram
- Lhs is dominated by at least one diagram on rhs and rhs cannot be more divergent than Lhs.  $\rightarrow \delta_{\text{lhs}} \leq \delta_{\text{rhs, any diagram}}$ .
- Not knowing which diagram is leading on the rhs, we can write inequalities from all diagrams.

$$\text{wavy line}^{-1} = \text{wavy line}^{-1} - \frac{1}{2} \text{star diagram} - \frac{1}{2} \text{star diagram} + \text{circle diagram} - \frac{1}{6} \text{star diagram} - \frac{1}{2} \text{star diagram}$$

$$-\delta_{gl} \leq 2\delta_{gl} + \delta_{3g}, \quad -\delta_{gl} \leq 2\delta_{gh} + \delta_{gg}, \quad \dots$$

That's the basic idea.

Still, for a large system a lot of work.

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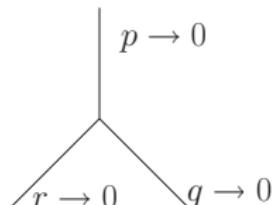
$$-\delta_{gl} \leq 2\delta_{gl} + \delta_{3g}, \quad -\delta_{gl} \leq 2\delta_{gh} + \delta_{gg}, \quad \dots$$

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All inequalities relevant?

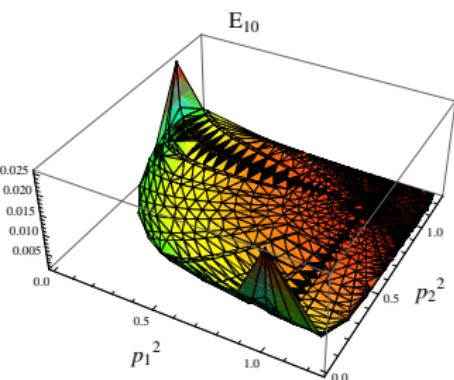
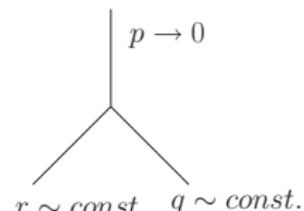
Still, for a large system a lot of work.

# More IR exponents?



Uniform/global scaling: All external momenta go to zero simultaneously.

Kinematic scaling: Some external momenta go to zero simultaneously. → Non-uniform dependence on momenta [Alkofer, M.Q.H., Schwenzer, EPJC 62 (2009)]



Additional singularities only in longitudinal parts!  
[Alkofer, M.Q.H., Schwenzer, EPJC 62 (2009); Fischer, Pawłowski, PRD 80 (2009)]

# Relevant inequalities

A closed form for all relevant inequalities can be derived from DSEs and FRGEs [Huber, Schwenzer, Alkofer, 0904.1873].

type		derived from	#
dressed vertices	$C_1 := \delta_{\text{vertex}} + \frac{1}{2} \sum_{\substack{\text{legs } j \text{ of} \\ \text{vertex}}} \delta_j \geq 0$	FRGEs	infinite
prim. div. vertices	$C_2 := \frac{1}{2} \sum_{\substack{\text{legs } j \text{ of} \\ \text{prim. div.} \\ \text{vertex}}} \delta_j \geq 0$	DSEs+FRGEs	finite

Some inequalities are contained within others.

E. g. in MAG:  $\delta_B \geq 0$  and  $\delta_c \geq 0$  render  $\delta_B + \delta_c \geq 0$  useless.

# Scaling relations

## General analysis of propagator DSEs

[M.Q.H., Schwenzer, Alkofer, arXiv:0804.1873]

- At least one inequality from a prim. divergent vertex has to be saturated, i. e.  $C_2^i = 0$  for at least one  $i$ .
- Necessary condition for a scaling solution.
- Related to bare vertices in DSEs: Fischer-Pawlowski consistency condition DSEs  $\leftrightarrow$  FRGEs [Fischer, Pawlowski, PRD 75 (2007)].

⇒ One primitively divergent vertex is not IR enhanced.

The non-enhancement of at least one primitively divergent vertex  
is now established for all scaling type solutions.

# How to obtain a scaling relation: Landau gauge

- ➊ Look at all inequalities for primitively divergent vertices, i. e. at  $C_2^i$ .
- ➋ Try all possibilities of  $C_2^i = 0$ .
- ➌ Choose the non-trivial solutions.

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Application to Landau gauge:

➊  $\delta_{gl} \geq 0, \delta_{gl} + 2\delta_{gh} \geq 0$

➋ a  $\delta_{gl} = 0$

b  $\delta_{gl} + 2\delta_{gh} = 0$

➌ a  $\underline{\delta_{gl}} = \underline{\delta_{gh}} = 0$

b  $\delta_{gl} + 2\delta_{gh} = 0$

Scaling relation of the Landau gauge:

$$\frac{1}{2}\delta_{gl} = -\delta_{gh} = \kappa_{LG}$$

# How to obtain a scaling relation: MAG

Many interactions  $\Rightarrow$  many inequalities, but some of them are contained within others  $\Rightarrow$  reduces number of possibilities.

- ① Look at all inequalities for primitively divergent vertices, i. e. at  $C_2^i$ .
- ② Try all possibilities of  $C_2^i = 0$ .
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# How to obtain a scaling relation: MAG

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- ② Try all possibilities of  $C_2^j = 0$ .
- ③ Choose the non-trivial solutions.

Application to the MAG:

- ①  $\delta_B \geq 0, \delta_c \geq 0, \delta_A + \delta_B \geq 0, \delta_A + \delta_c \geq 0$
- ②
  - a  $\delta_B = 0$
  - b  $\delta_c = 0$
  - c  $\delta_A + \delta_B = 0$
  - d  $\delta_A + \delta_c = 0$
- ③
  - a  $\delta_A = \delta_B = \delta_c = 0$
  - b  $\delta_A = \delta_B = \delta_c = 0$
  - c  $\delta_A + \delta_B = 0$
  - d  $\delta_A + \delta_c = 0$

Scaling relation of the MAG:  $\boxed{\delta_B = \delta_c = -\delta_A = \kappa_{MAG} \geq 0}$

# IR scaling solution of the MAG

$$\delta_B = \delta_c = -\delta_A = \kappa_{MAG} \geq 0$$

- The Abelian fields are IR enhanced. → Realization of Abelian dominance?
- Off-diagonal fields are IR suppressed.
- $SU(2)$  and  $SU(N > 2)$  have the same solution.
- Qualitative solutions for tower of all Green functions.

Two-loop diagrams are IR leading (sunset, squint). → UV/IR preserving truncation?

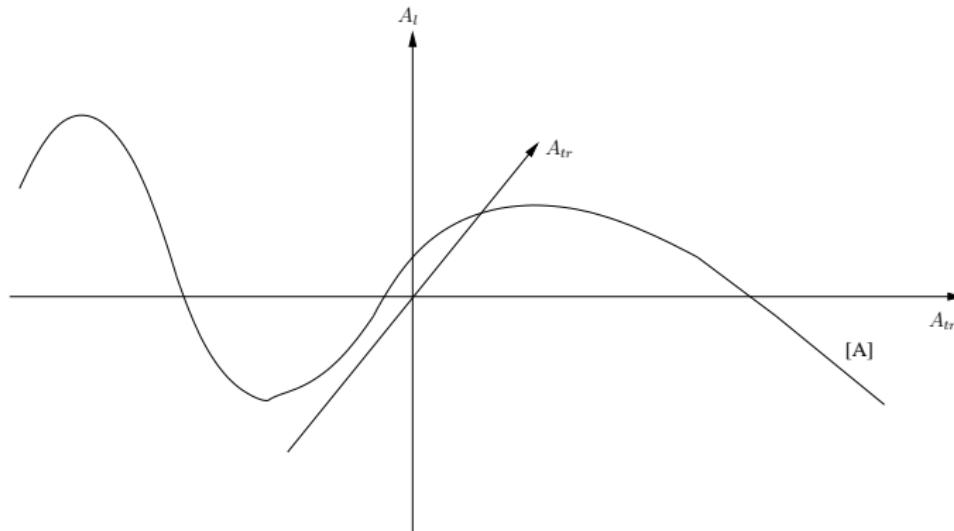
# Relation Landau gauge & MAG

Landau gauge	maximally Abelian gauge
ghost dominance	Abelian (gluon) dominance
Gribov region bounded	Gribov region unbounded in diagonal direction [Capri et al., PRD79]

Greensite, Olejnik, Zwanziger, PRD78:

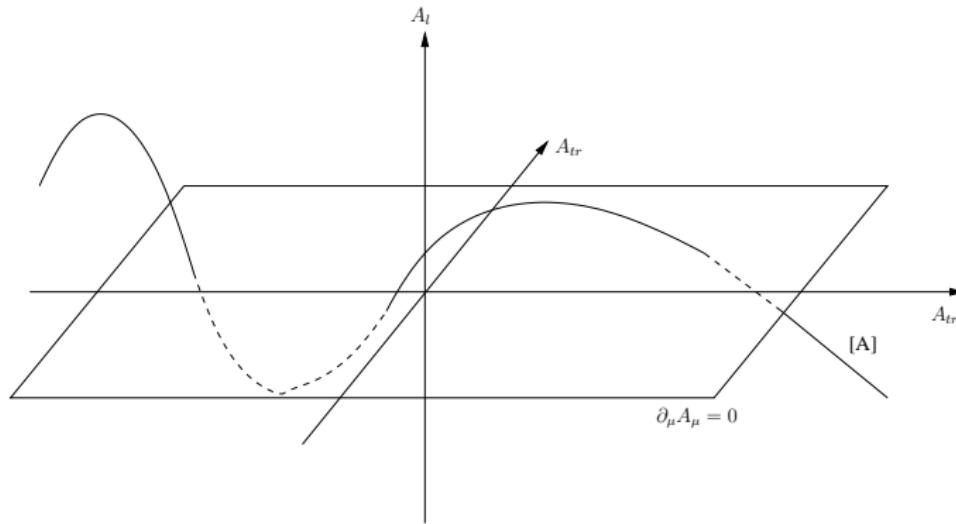
Abelian configurations  $\xrightarrow{\text{Landau gauge}}$  on Gribov horizon

# Gauge orbits and Gribov copies



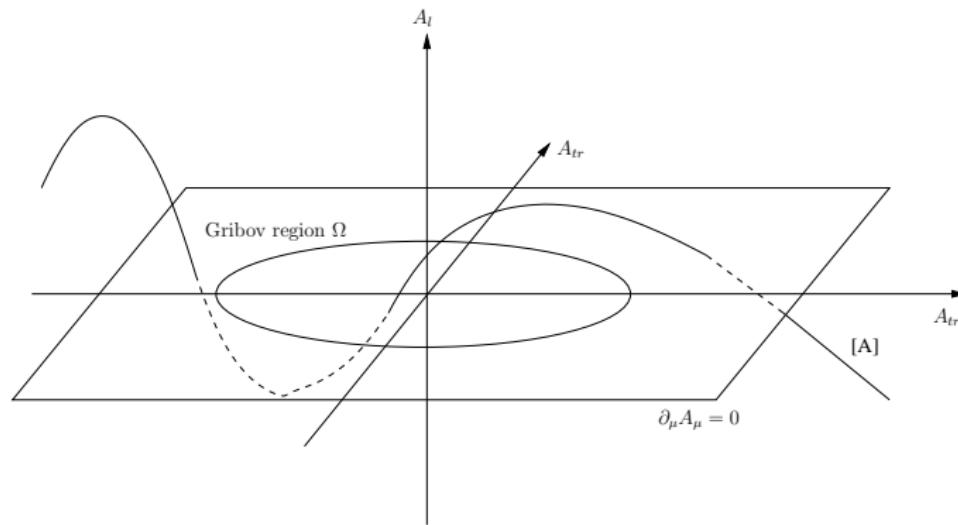
Gauge equivalent configurations (gauge orbit  $[A]$ )  $\Rightarrow$  integration in path integral is overcomplete.

# Gauge orbits and Gribov copies



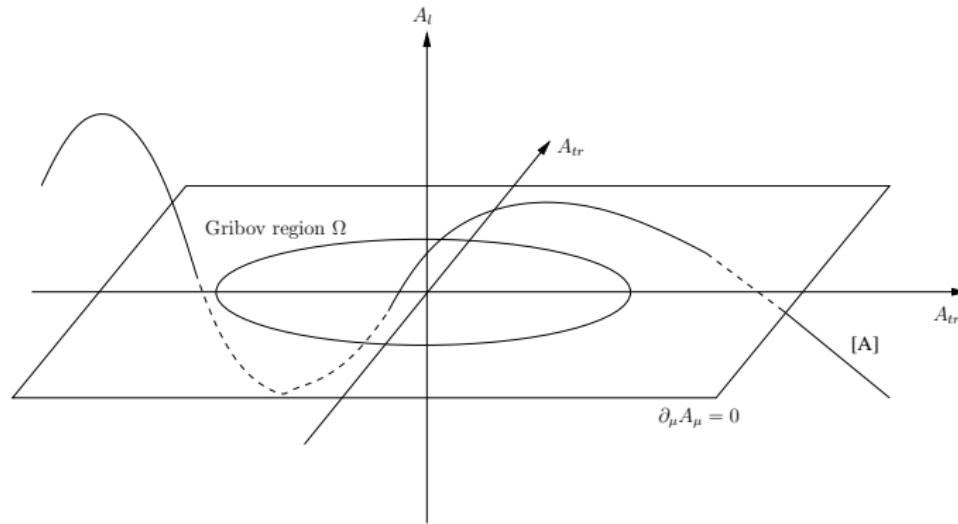
Faddeev and Popov: Restriction of integration to single representative of each gauge orbit possible? Gauge symmetry replaced by BRST symmetry!

# Gauge orbits and Gribov copies



Restriction to Gribov horizon: almost unique gauge fixing.

# Gauge orbits and Gribov copies



Restriction to Gribov horizon: almost unique gauge fixing.

Restriction to Gribov region is done via adding a non-local term to the Lagrangian. → New parameter  $\gamma$ , determined by horizon condition.

# How do DSEs usually deal with this?

Integral of a total derivative vanishes:

$$\int [D\phi] \frac{\delta}{\delta\phi} e^{-S+J\Phi} = \int [D\phi] \left( J - \frac{\delta S}{\delta\phi} \right) e^{-S+J\Phi} = 0.$$

⇒ DSEs for **all Green functions** (full, connected, 1PI) by further differentiations.

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Integral of a total derivative vanishes [Zwanziger, PRD65]:

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$$\int_{\Omega} [D\phi] \left( J - \frac{\delta S}{\delta \phi} \right) \delta(\partial \cdot A) \det(M) e^{-S_{YM}+J\Phi} = 0.$$

$$\det(M) \Big|_{\Omega} = 0$$

# Local renormalizable action

Non-local term can be localized with auxiliary fields  
 $(\bar{\varphi}_\mu^{ab}, \varphi_\mu^{ab}, \bar{\omega}_\mu^{ab}, \omega_\mu^{ab}) \rightarrow$  local Gribov-Zwanziger action:

$$\mathcal{L}_{GZ} = \bar{\varphi}_\mu^{ac} M^{ab} \varphi_\mu^{bc} - \bar{\omega}_\mu^{ac} M^{ab} \omega_\mu^{bc} + \gamma^2 g f^{abc} A_\mu^a (\varphi_\mu^{bc} - \bar{\varphi}_\mu^{bc}) - \gamma^4 d(N^2 - 1)$$

Horizon condition in local form:

$$\langle g f^{abc} A_\mu^a (\varphi_\mu^{bc} - \bar{\varphi}_\mu^{bc}) \rangle = 2\gamma^2 d(N^2 - 1),$$

- Restriction breaks BRST invariance.
- Mixing at the level of two-point functions, e. g.  $\langle A_\mu^a \varphi_\nu^{bc} \rangle$ .  
 $\Rightarrow$  (3x3)-matrix relation between propagators and two-point functions:

$$D^{\phi\phi} = (\Gamma^{\phi\phi})^{-1}, \quad \phi \in \{A, \varphi, \bar{\varphi}\}$$

$\Rightarrow$  non-trivial relation between IR exponents of propagators and two-point functions

# More fields . . .

Simplify to (2x2)-matrix relation by splitting into real and imaginary part  
 [Zwanziger, 0904.2380]:

$$\varphi = \frac{1}{\sqrt{2}} (U + i V), \quad \bar{\varphi} = \frac{1}{\sqrt{2}} (U - i V).$$

$$\begin{aligned}\mathcal{L}'_{GZ} &= \mathcal{L}_U + \mathcal{L}_V + \mathcal{L}_{UV} - \bar{\omega}_\mu^{ac} M^{ab} \omega_\mu^{bc}, \\ \mathcal{L}_U &= \frac{1}{2} U_\mu^{ac} M^{ab} U_\mu^{bc}, \\ \mathcal{L}_V &= \frac{1}{2} V_\mu^{ac} M^{ab} V_\mu^{bc} + \textcolor{red}{ig \gamma^2 \sqrt{2} f^{abc} A_\mu^a V_\mu^{bc}}, \\ \mathcal{L}_{UV} &= \frac{1}{2} i g f^{abc} U_\mu^{ad} V_\mu^{bd} \partial_\nu A_\nu^c \stackrel{LG}{=} 0,\end{aligned}$$

Simplify even further:

$$c, \bar{c}, U, \omega, \bar{\omega} \longrightarrow \eta, \bar{\eta}$$

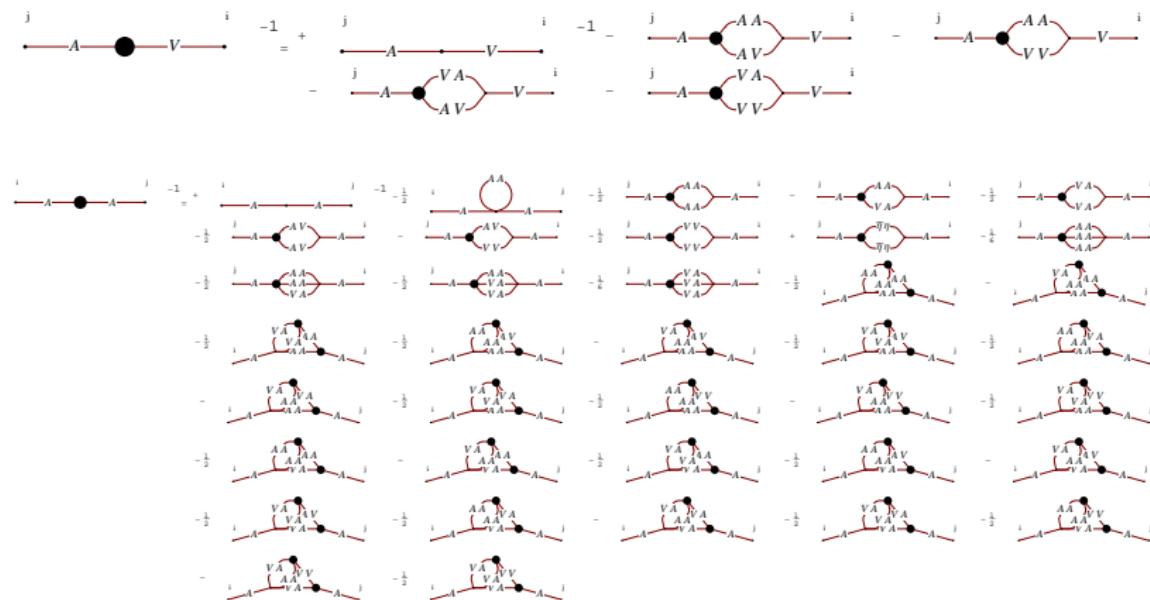
# DSEs of Gribov-Zwanziger action

Just to give an impression:



# DSEs of Gribov-Zwanziger action

Just to give an impression:



Complete analysis of all diagrams!

# Truncation of tensors

Propagators  $\langle A_\mu^a V_\nu^{bc} \rangle$  and  $\langle V_\mu^{ab} V_\nu^{cd} \rangle$  can have many tensors with different dressing functions,  
e. g. color space:  $f^{abc}$ ;  $\delta^{ab}\delta^{cd}$ ,  $\delta^{ac}\delta^{bd}$ ,  $\delta^{ad}\delta^{bc}$ ,  $f^{abe}f^{cde}$ ,  $f^{ace}f^{bde}$ .

Truncation: Take only tree-level tensors of two-point functions.

$$\Gamma^{\Phi\Phi} = \begin{pmatrix} \Gamma^{AA} & \Gamma^{AV} \\ \Gamma^{VA} & \Gamma^{VV} \end{pmatrix},$$

$$\Gamma_{\mu\nu}^{AA,ac} = \delta^{ac} p^2 \mathbf{c}_A^\perp(\mathbf{p}^2) P_{\mu\nu} + \delta^{ac} \frac{1}{\xi} \mathbf{c}_A^{\parallel}(\mathbf{p}^2) p_\mu p_\nu,$$

$$\Gamma_{\mu\nu}^{VV,abcd} = \delta^{ac}\delta^{bd} p^2 \mathbf{c}_V(\mathbf{p}^2) g_{\mu\nu},$$

$$\Gamma_{\mu\nu}^{AV,cab} = f^{cab} i p^2 \mathbf{c}_{AV}(\mathbf{p}^2) g_{\mu\nu},$$

$\mathbf{c}_{ij}(\mathbf{p}^2)$  are dressing functions.

# Propagators of the GZ action

$$D_{cd}^{\eta\bar{\eta}, ab} = (\Gamma_{cd}^{\eta\bar{\eta}, ab})^{-1} = -\delta^{ab}\delta^{cd} \frac{c_\eta(p^2)}{p^2}$$

$D^{VV}$  has two tensors  $\rightarrow$  non-trivial truncation:

$$\begin{aligned} D_{\mu\nu}^{AA, ab} &= \delta^{ab} \frac{1}{p^2} P_{\mu\nu} \frac{c_V(p^2)}{c_A^\perp(p^2) c_V(p^2) + 2N c_{AV}^2(p^2)}, \\ D_{\mu\nu}^{VV, abcd} &= \frac{1}{p^2} \frac{1}{c_V(p^2)} \delta^{ac} \delta^{bd} g_{\mu\nu} - \\ &\quad - f^{abe} f^{cde} \frac{1}{p^2} P_{\mu\nu} \frac{2c_{AV}^2(p^2)}{c_A^\perp(p^2) c_V^2(p^2) + 2N c_{AV}^2(p^2) c_V(p^2)}, \\ D^{AV, abc} &= -i f^{abc} \frac{1}{p^2} P_{\mu\nu} \frac{\sqrt{2} c_{AV}(p^2)}{c_A^\perp(p^2) c_V(p^2) + 2N c_{AV}^2(p^2)} \end{aligned}$$

Appearance of the determinant  $c_A^\perp(p^2) c_V(p^2) + 2N c_{AV}^2(p^2)$

# The four possibilities

Which part of the determinant  $c_A^\perp(p^2)c_V(p^2) + 2N c_{AV}^2(p^2)$  dominates in the IR?

$$c_{ij}(p^2) = d_{ij} \cdot (p^2)^{\kappa_{ij}}$$

- I:  $c_{AV}^2 > c_A c_V \leftrightarrow \kappa_A + \kappa_V > 2\kappa_{AV}$
- II:  $c_A c_V > c_{AV}^2 \leftrightarrow 2\kappa_{AV} > \kappa_A + \kappa_V$
- III:  $c_{AV}^2 \sim c_A c_V \leftrightarrow \kappa_A + \kappa_V = 2\kappa_{AV}$ , no cancelations
- IV:  $c_{AV}^2 \sim c_A c_V \leftrightarrow \kappa_A + \kappa_V = 2\kappa_{AV}$ , cancelations

Cancelations: Leading contributions cancel and some less dominant term takes over.

# The four possibilities

Which part of the determinant  $c_A^\perp(p^2)c_V(p^2) + 2N c_{AV}^2(p^2)$  dominates in the IR?

$$c_{ij}(p^2) = d_{ij} \cdot (p^2)^{\kappa_{ij}}$$

- I:  $\cancel{c_{AV}^2 > c_{ACV}} \leftrightarrow \kappa_A + \kappa_V > 2\kappa_{AV}$
- II:  $\cancel{c_{ACV} > c_{AV}^2} \leftrightarrow 2\kappa_{AV} > \kappa_A + \kappa_V$
- III:  $c_{AV}^2 \sim c_{ACV} \leftrightarrow \kappa_A + \kappa_V = 2\kappa_{AV}$ , no cancelations
- IV:  $\cancel{c_{AV}^2 \sim c_{ACV}} \leftrightarrow \kappa_A + \kappa_V = 2\kappa_{AV}$ , cancelations

Cancelations: Leading contributions cancel and some less dominant term takes over.

Two solutions lead to inconsistencies [M.Q.H., R. Alkofer, S. P. Sorella, 0910.5604].

## Case II: Recovery of standard Landau gauge solution

$$c_A c_V > c_{AV}^2 \leftrightarrow \kappa_A + \kappa_V < 2\kappa_{AV}$$

- The  $VV$ -propagator becomes

$$\frac{1}{p^2} \frac{1}{c_V(p^2)} \delta^{ac} \delta^{bd} g_{\mu\nu} \quad \longrightarrow \quad \kappa_V = \kappa_\eta$$

$\Rightarrow VV$ -propagator could be integrated out in the IR and the FP theory is recovered exactly.

- All contributions containing an  $AV$ -propagator are suppressed.  $\Rightarrow$  DSEs reduce in the IR to the same system as in FP theory.
- Formula for IR exponent of arbitrary n-point functions is obtained.
- IR exponent of  $AV$ -2-point function is not fixed by scaling relation; calculated numerically.  
Several solutions: 0.0668776, 0.981386 and higher.

In the IR this is completely the same as FP theory!

## Case III: The "strict" scaling solution

All IR exponents are connected by the scaling relations ( $\kappa := \kappa_V = \kappa_\eta$ ):

$$\kappa_A + 2\kappa = \kappa + 2\kappa_{AV} = 0$$

Mixed propagator:  $\delta_{AV} = \kappa/2 \Rightarrow$  Less pronounced IR suppression than in case II ( $\delta_{AV} > \kappa/2$ ).

The determinant remains as it is.  $\Rightarrow$  Non-linear relations between the coefficients of the dressing functions.

# Summary Gribov-Zwanziger action

[M.Q.H., Alkofer, Sorella, 0910.5604, to appear in PRD]

Explicitly restricted integration to Gribov region by using the Gribov-Zwanziger action.

Mixed propagators complicate the analysis. Two candidates remain:

- All solutions have the same qualitative behavior.
- Mixed propagator IR suppressed.
- Scaling relation between FP ghost and gluon unaltered:  
 $\kappa_A + 2\kappa_c = 0$ .
- Input for numerical solution of the equations.

Both cases reproduce the qualitative behavior of the Gribov-Zwanziger and Kugo-Ojima scenarios.

# Summary maximally Abelian gauge [M.Q.H., Schwenzer, Alkofer, 0904.1873]

- Existence and form of scaling solutions can easily be obtained directly from the interactions.
- Fischer-Pawlowski consistency condition: one vertex remains bare in the IR.
- Scaling solution may exist in MAG:
  - Abelian gluon propagator is IR enhanced. → Support of hypothesis of Abelian dominance.
  - Complete numerical solution required. ← Input from asymptotic behavior
  - Two-loop terms are IR leading ↔ UV/IR preserving truncation?
  - Relation to chromomagnetic monopoles?

# The end

Thank you very much for your attention.



# IR Scaling solutions for other gauges

The analysis can be used also for other gauges. Beware: This corresponds to a naive application!

## Linear covariant gauges

scaling solution only, if the **longitudinal part of the gluon propagator** gets dressed, but gauge fixing condition  $\Rightarrow$  longitudinal part bare

## Ghost-antighost symmetric gauges

quartic ghost interaction  $\rightarrow \delta_{gh} \geq 0$   
 $\rightarrow$  with non-negative IREs only the **trivial solution** can be realized

This is valid for all possible dressings and agrees with the results from [Alkofer, Fischer, Reinhardt, v. Smekal, PRD 68 (2003)], where only certain dressings were considered.



- Either the existence of a scaling solution is something special (?) or
- a **more refined analysis** (symmetries  $\leftrightarrow$  cancelations) is needed in these cases.