From three-dimensional Yang-Mills theory to QCD

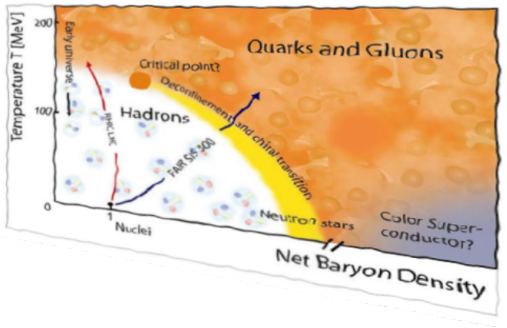
University of Graz, Institute of Physics

Quantum seminar, TPI Friedrich-Schiller-Universität Jena

June 9, 2016

Motivation: QCD phase diagram

$$\frac{1}{4}F^a_{\mu\nu}F^{a,\mu\nu} + \bar{q}(i\not{D} - M)q$$



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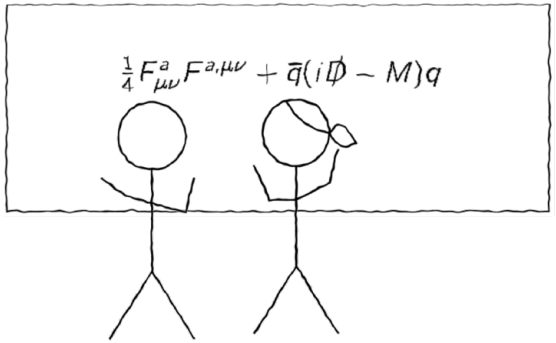
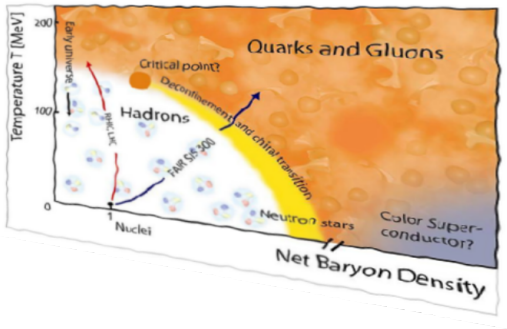
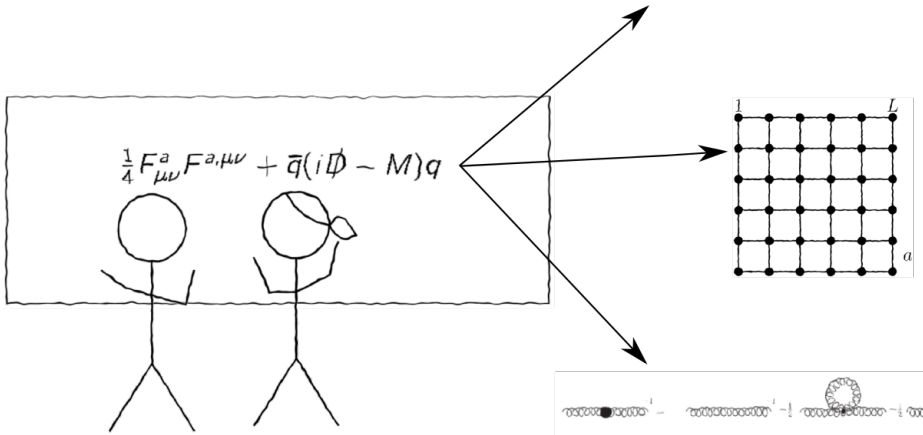
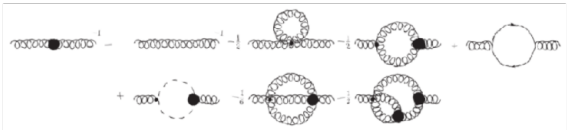


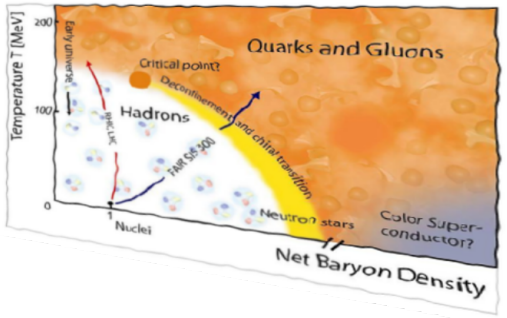


Motivation: QCD phase diagram

$$\mathcal{L}_{\text{eff}} = \frac{1}{2}(\partial^\mu \pi)(\partial_\mu \pi) + \dots$$







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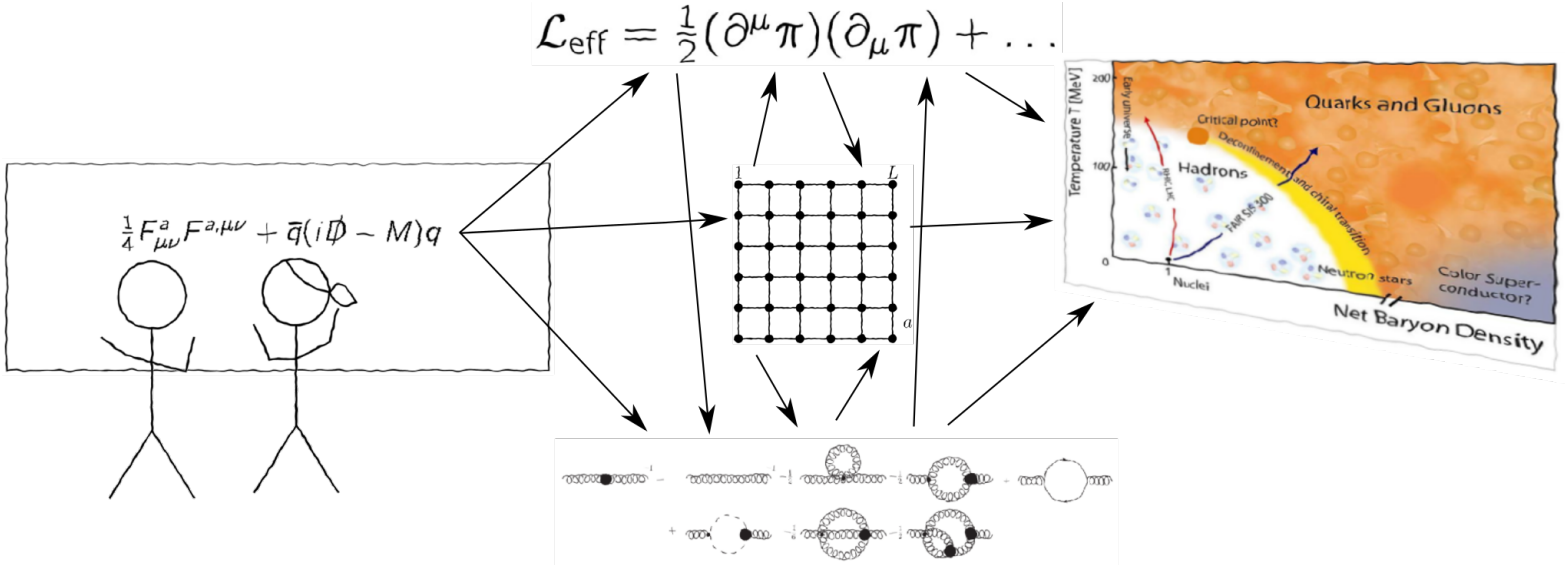

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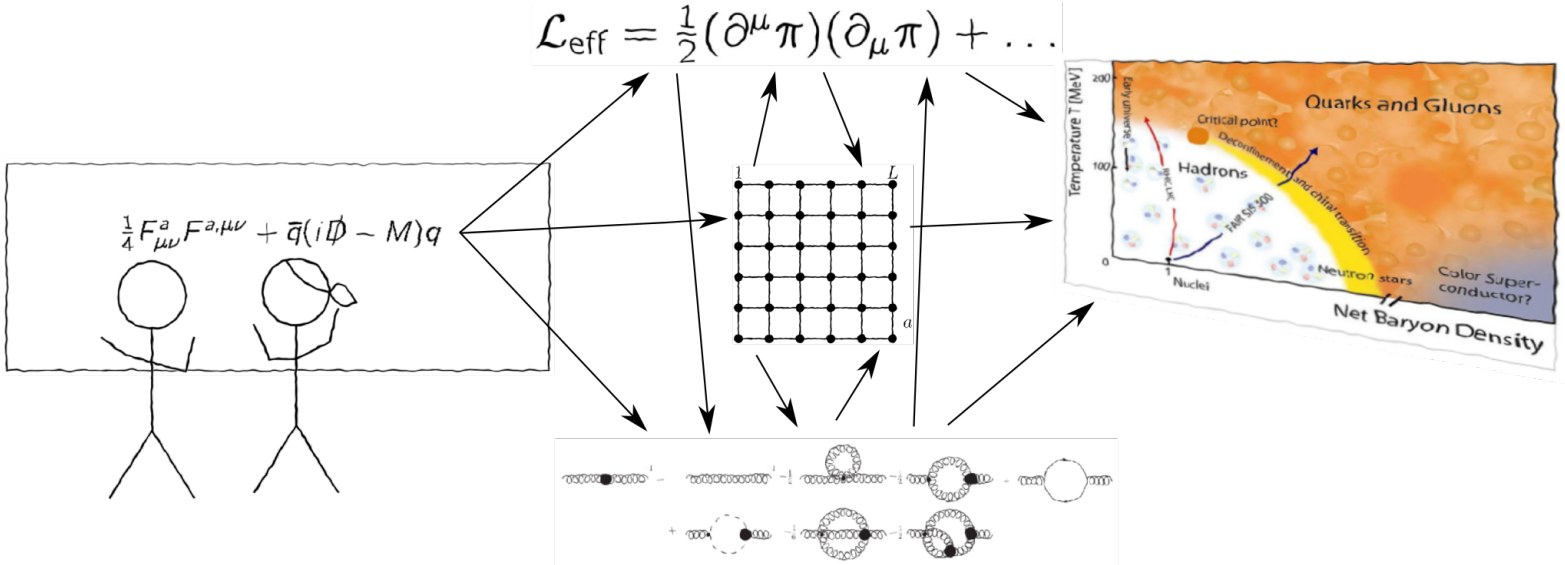

- Region $\mu > T$ unknown: (position) critical endpoint? phases?
- Challenges for all methods, e.g.
  - Lattice QCD: complex action problem
  - Models: parameters
  - Functional methods: reliability of truncations

Functional methods and the QCD phase diagram

Functional equations: Exact equations derived from QCD action.

$$\frac{1}{4}F^a_{\mu\nu}F^{a,\mu\nu} + \bar{q}(i\not{D} - M)q$$

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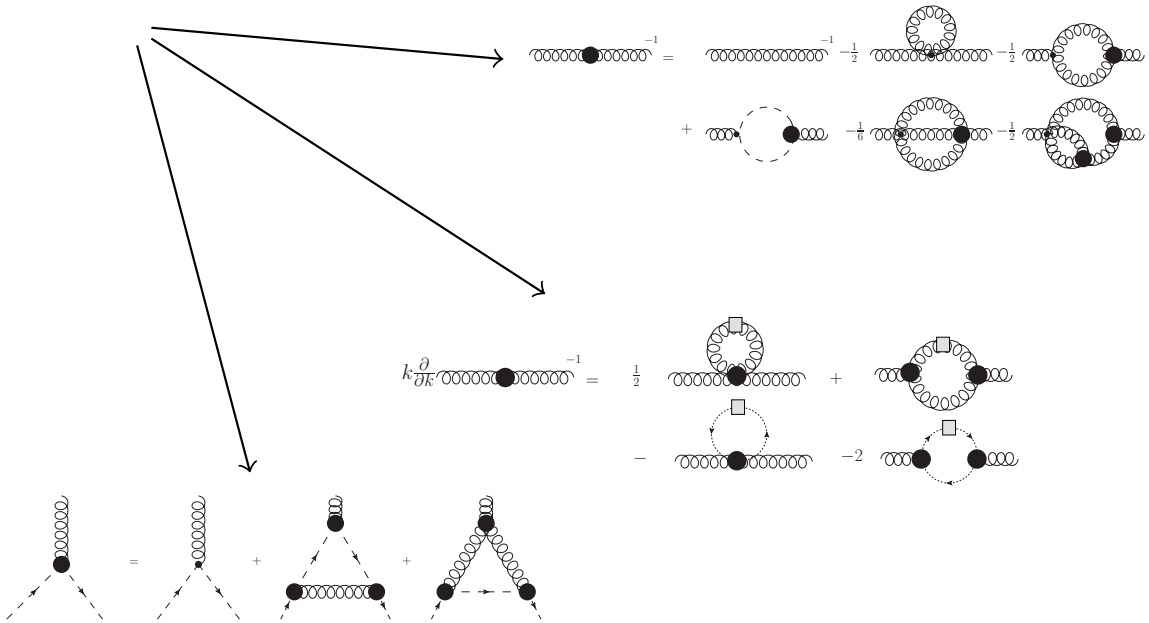

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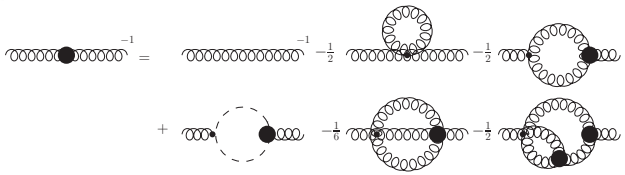

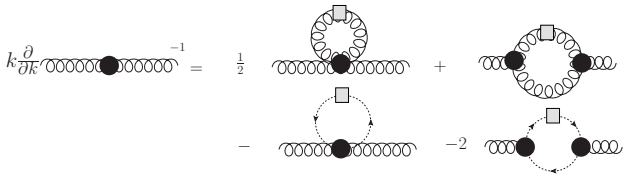

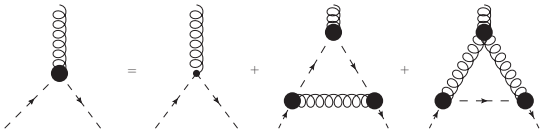

- Chiral limit accessible.
- No sign problem.
- Large scale separations easy.

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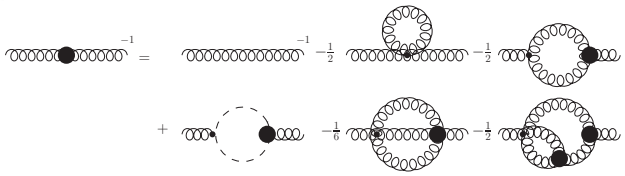

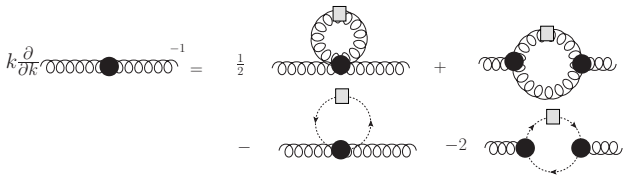

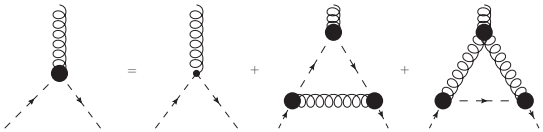

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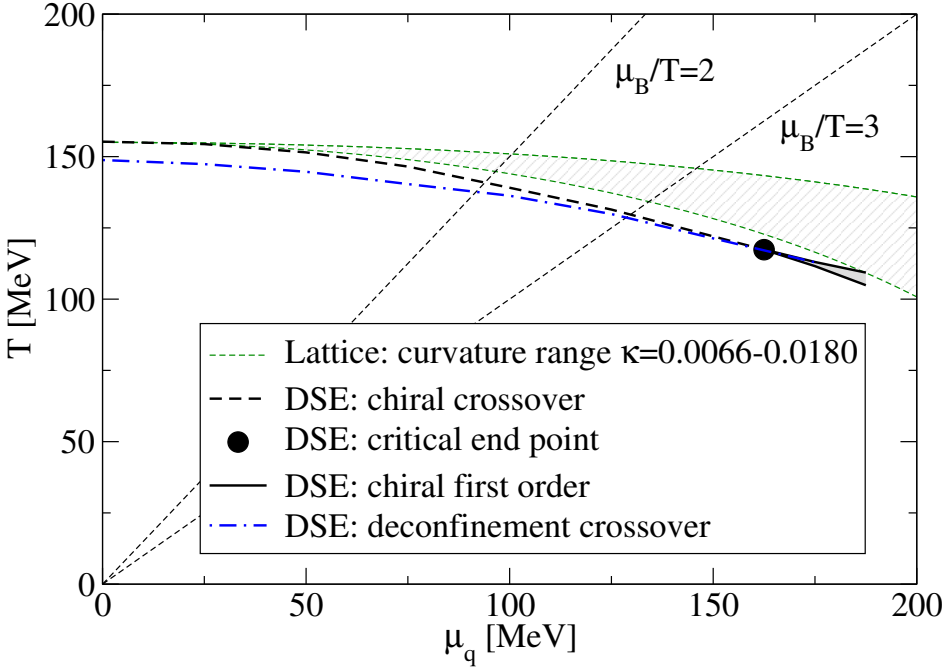
Difficulty

Infinitely large systems of equations without obvious ordering scheme.

QCD phase diagram from functional equations

2+1 flavor QCD from DSEs

[Fischer, Lücker, Welzbacher '14]:



Positions of critical endpoint:

$\sim (168\,\text{MeV}, 115\,\text{MeV})$

lattice gluon from $T = 0$, vertex model

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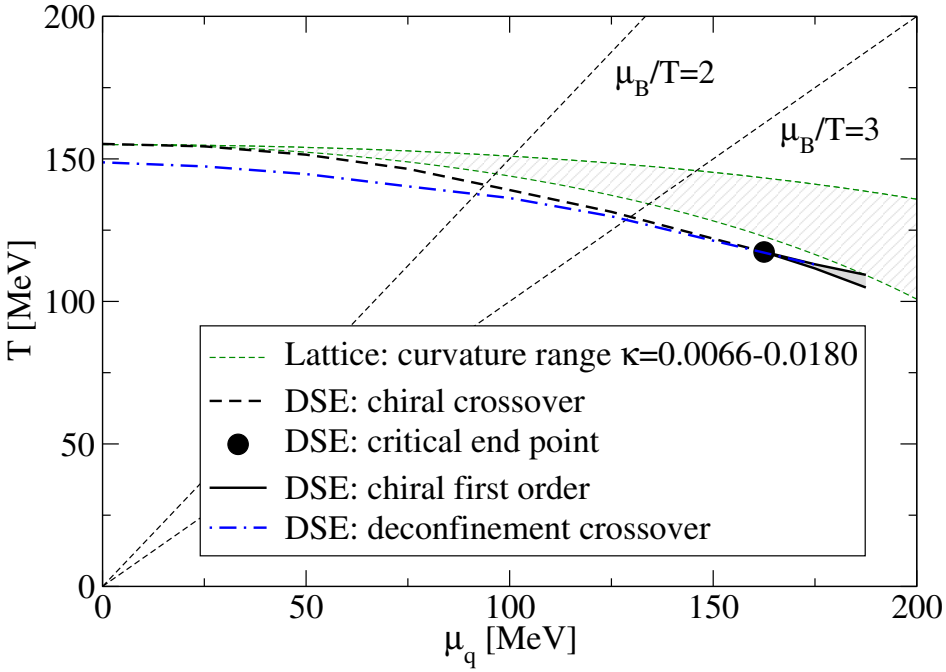
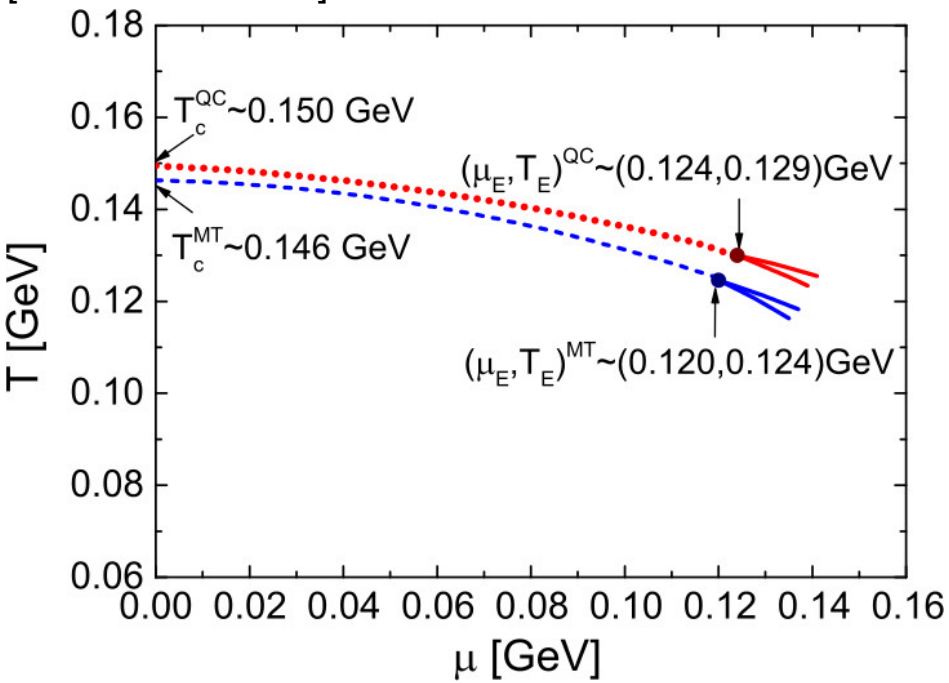


2 flavor QCD from DSEs

[Xin, Qin, Liu '15]:



Positions of critical endpoint:

$\sim (168\,\text{MeV}, 115\,\text{MeV})$

lattice gluon from $T = 0$, vertex model

$\sim (122\,\text{MeV}, 126\,\text{MeV})$

rainbow approximation

QCD phase diagram from functional equations

Input for DSEs:

- model for quark-gluon vertex (parameters fixed at $\mu = 0$)
- fits for gluon propagators at $\mu = 0$ from the lattice

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Possible improvements:

- fully dynamical propagators
- fully dynamical quark-gluon vertex

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Possible improvements:

- fully dynamical propagators $\rightarrow$ require other vertices
- fully dynamical quark-gluon vertex $\rightarrow$ requires propagators & other vertices

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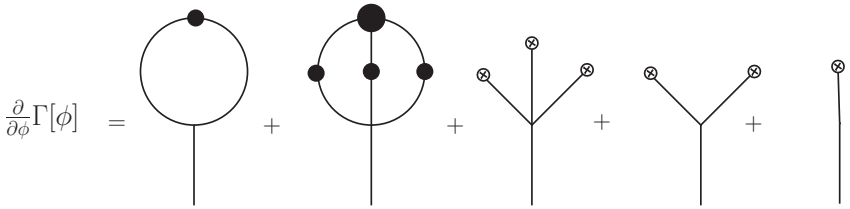
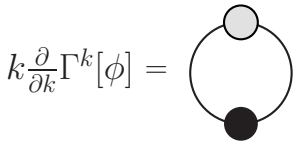
- fully dynamical quark-gluon vertex $\rightarrow$ requires propagators & other vertices

Ultimately, full control over Yang-Mills part required!

Comparison: DSEs and flow equations

Dyson-Schwinger equations (DSEs)	Functional RG equations (FRGEs)
'integrated flow equations'	'differential DSEs'
effective action $\Gamma[\phi]$	effective average action $\Gamma^k[\phi]$
–	regulator
n-loop structure (n *const.*)	1-loop structure
exactly only bare vertex per diagram	no bare vertices
	

- Both systems of equations are exact.
- Both contain infinitely many equations.

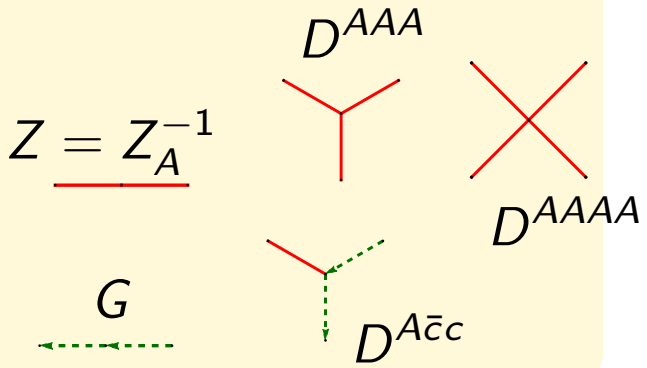
Landau gauge Yang-Mills theory

Gluonic sector of quantum chromodynamics: Yang-Mills theory

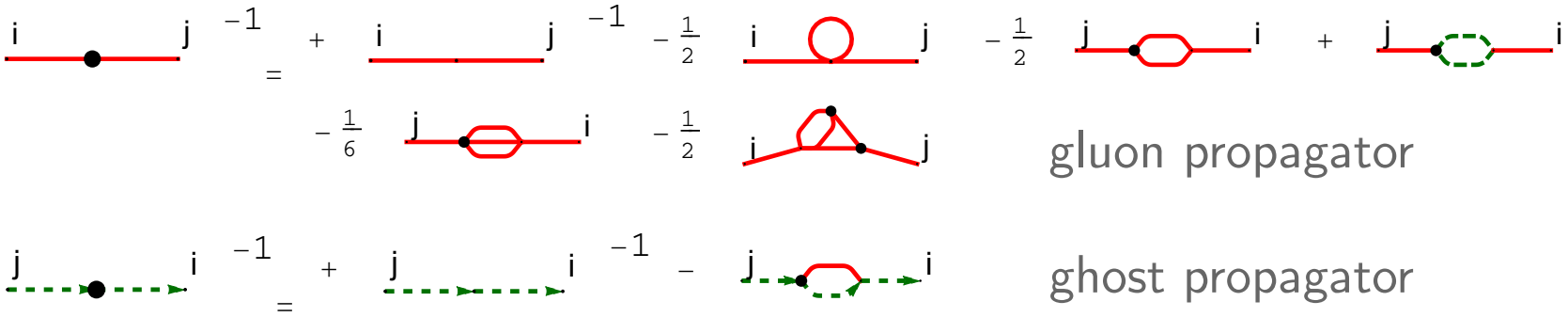
$$\mathcal{L} = \frac{1}{2} F^2 + \mathcal{L}_{gf} + \mathcal{L}_{gh}$$
$$F_{\mu\nu} = \partial_\mu \boldsymbol{A}_\nu - \partial_\nu \boldsymbol{A}_\mu + i\, g\, [\boldsymbol{A}_\mu, \boldsymbol{A}_\nu]$$

Landau gauge

- simplest one for functional equations
- $\partial_\mu \boldsymbol{A}_\mu = 0$: $\quad \mathcal{L}_{gf} = \frac{1}{2\xi} (\partial_\mu \boldsymbol{A}_\mu)^2, \quad \xi \to 0$
- requires ghost fields: $\quad \mathcal{L}_{gh} = \bar{\boldsymbol{c}} \left(-\Box + g\, \boldsymbol{A} \times \right) \boldsymbol{c}$



The tower of DSEs



gluon propagator

ghost propagator

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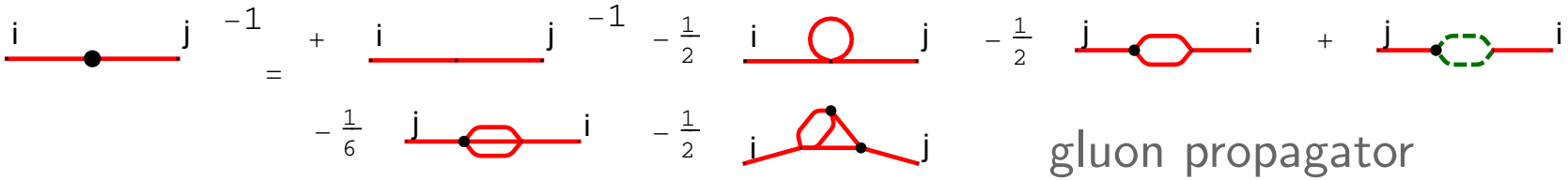

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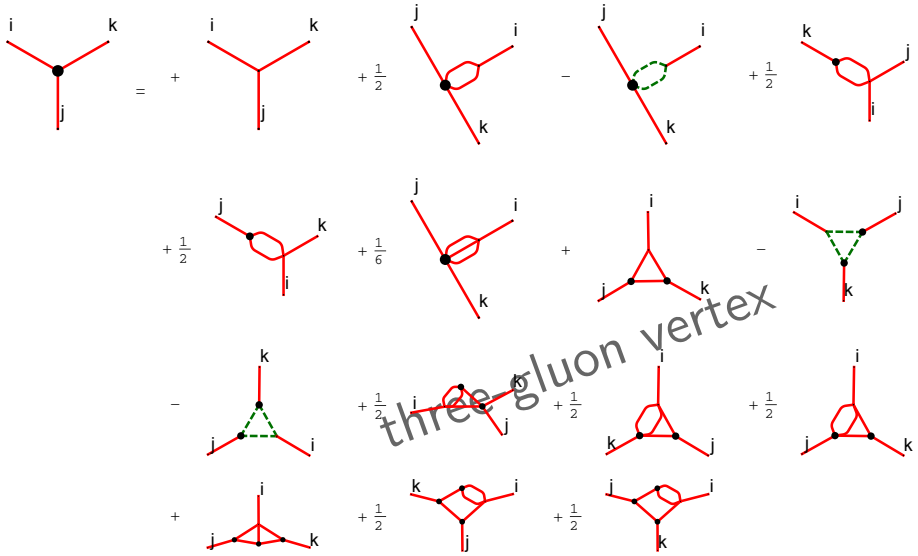

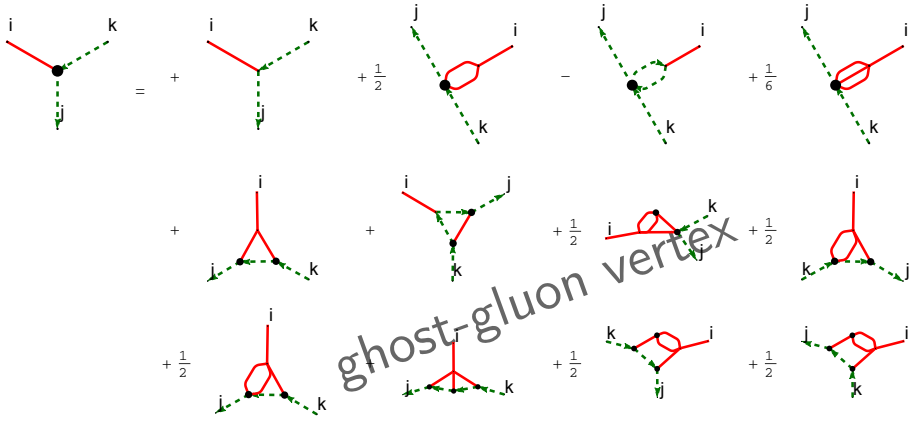

Infinitely many equations. In QCD, every n-point function depends on $(n+1)$- and possibly $(n+2)$-point functions.

Truncating the equations

Guides

- Perturbation theory
- Symmetries
- Lattice
- Analytic results

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Truncation

- Drop quantities (unimportant?)
- Use fits
- Model quantities (good models available? 'true' or 'effective'?)

Ideally: Find a truncation that has (I) no parameters and yields (II) quantitative results.

Truncation of Yang-Mills system

Neglect all non-primitively divergent Green functions. $\rightarrow$ Self-contained.

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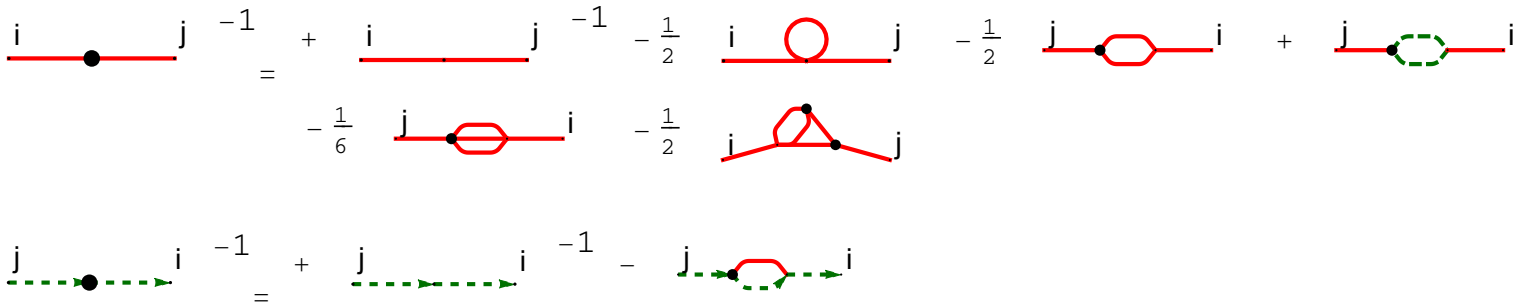
Full propagator equations (two-loop diagrams!):



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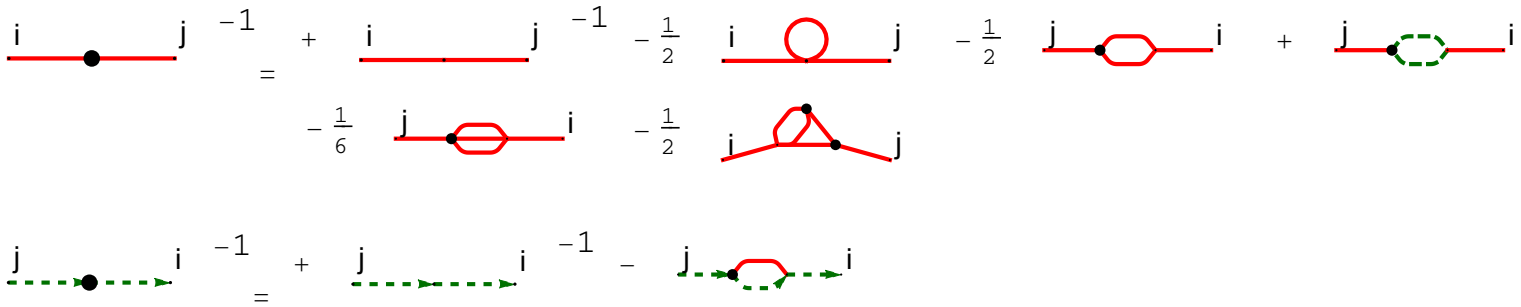
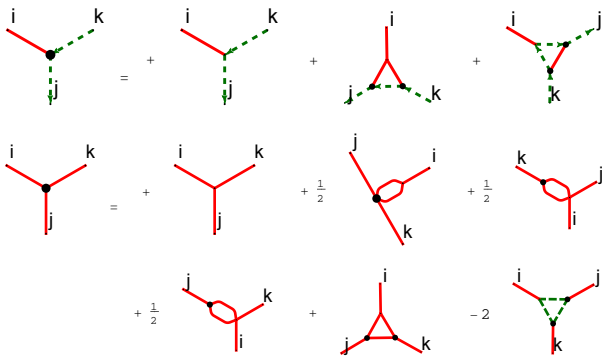
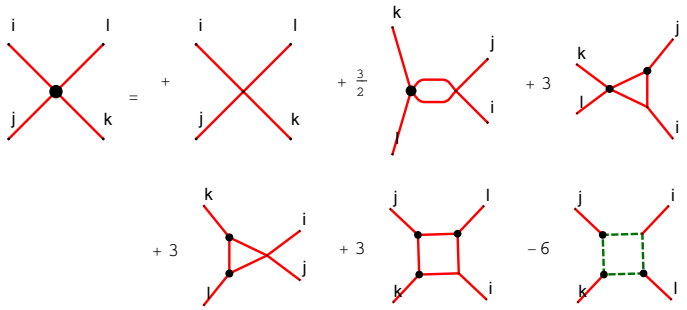


Truncated three-point functions:



Truncated four-gluon vertex:



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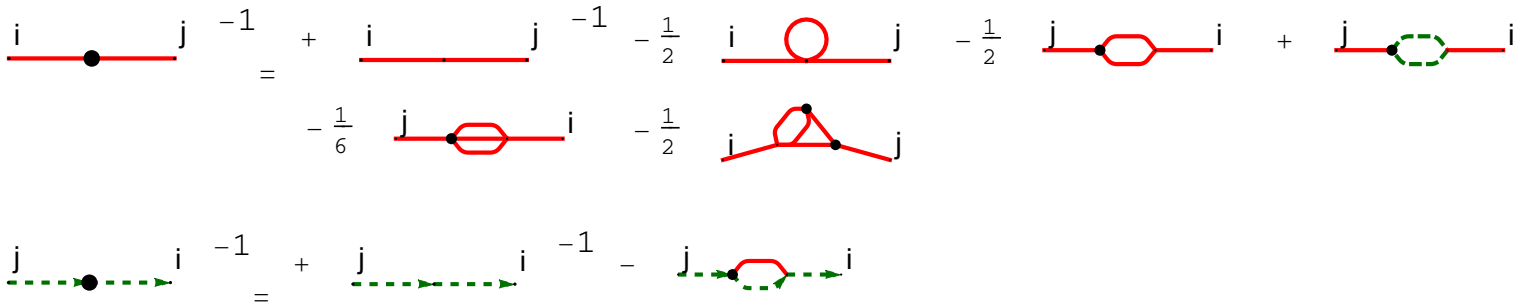

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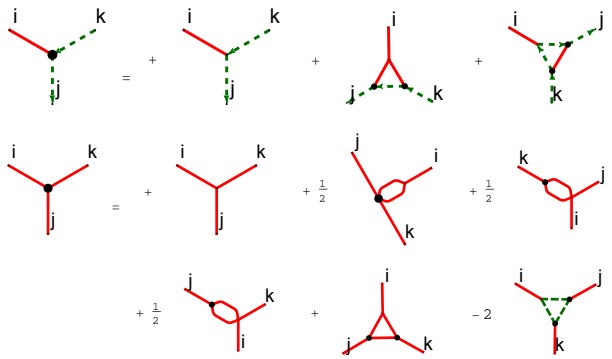

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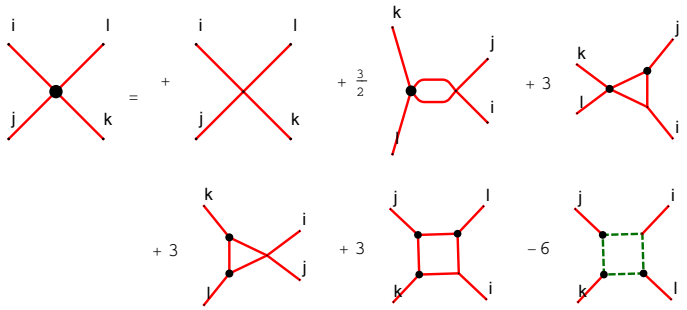

Technical questions: spurious divergences in gluon propagator, RG resummation

Automated derivation

Derivation by hand becomes tedious:

- Large Lagrangians.
- Higher Green functions.
- Larger truncations.
- Error-prone.

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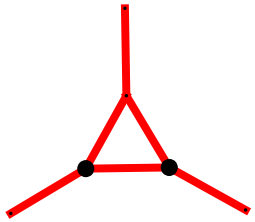

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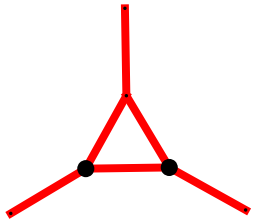

```
(2 g^2 Nc Z1 DAAA[y, qs + y + 2 sp[q, q1], (-y - sp[q, q1])/Sqrt[y (qs + y + 2 sp[q, q1])]]
  DAAA[x2 + y + 2 sp[p, q], qs + x2 - 2 sp[p, q1], (-x2 - sp[p, q] + sp[p, q1] + sp[q, q1])/Sqrt[(x2 + y + 2 sp[p, q]) (qs + x2 - 2 sp[p, q1])]] Dgl[qs] Dgl[qs + x2 - 2 sp[p, q1]] Dgl[
    sp[p, q]^4 (sp[p, q1]^2 sp[q, q1] (y + sp[q, q1]) + qs x2 (y (9 qs + 6 (x2 + y)) + (5 qs + 6 x2 + 10 y) sp[q, q1]) - sp[p, q1] (qs y (5 qs
    sp[p, q]^3 (2 sp[p, q1]^3 (qs y - sp[q, q1]^2) + sp[p, q1] (qs y (10 qs^2 + (-5 x2 - 3 y) y + qs (19 x2 + 3 y)) + (3 qs^3 + 8 qs x2 y + 21 qs^2 (
      qs x2 (y (-9 qs^2 + 3 x2^2 + 7 x2 y + 3 y^2 + 2 qs (x2 + y)) + (-10 qs^2 + qs (-3 x2 - 19 y) + x2 (3 x2 + 5 y)) sp[q, q1] + (-16 qs - 7 x2 - 11 y
      sp[p, q1]^2 (qs (-16 qs - 11 x2 - 7 y) y + (-5 qs^2 + qs (-9 x2 - 19 y) + 2 y (5 x2 + 3 y)) sp[q, q1] + (-5 qs + 12 (x2 + y)) sp[q, q1]^2 +
    sp[p, q]^2 (sp[p, q1]^4 sp[q, q1] (qs + sp[q, q1]) + sp[p, q1]^3 (qs y (7 qs + 11 x2 + 16 y) + (-6 qs^2 + y (9 x2 + 5 y) + qs (-10 x2 + 19 y)
      qs x2 (y (-3 qs^3 - 10 qs^2 (x2 + y) - 6 x2 y (x2 + y) + qs (-3 x2^2 - 19 x2 y - 3 y^2)) + (-6 qs^3 + qs^2 (-21 x2 - 32 y) + qs (-9 x2^2 - 60 x2 y
        (-15 qs^2 - 15 x2^2 + qs (-46 x2 - 41 y) - 41 x2 y - 12 y^2) sp[q, q1]^2 + (-7 qs - 16 x2 - 11 y) sp[q, q1]^3) + sp[p, q1]^2 (qs y (-15 q
        (3 qs^3 + qs^2 (5 x2 - 39 y) + qs (-81 x2 - 39 y) y + y^2 (5 x2 + 3 y)) sp[q, q1] + (12 qs^2 + 12 x2^2 + 3 x2 y + 12 y^2 + 3 qs (x2 + y)) sp[q
      sp[p, q1] (qs y (6 qs^3 + qs^2 (32 x2 + 21 y) + qs (25 x2^2 + 60 x2 y + 9 y^2) + x2 (3 x2^2 + 25 x2 y + 15 y^2)) + (15 qs^3 (x2 + y) + x2 y (-3 x2
        (-3 qs^3 + x2^2 (-3 x2 - 5 y) + qs^2 (39 x2 - 5 y) + qs x2 (39 x2 + 81 y)) sp[q, q1]^2 + (-6 qs^2 + qs (19 x2 - 10 y) + x2 (5 x2 + 9 y)) sp[
    x2 y (-sp[p, q1]^5 (qs + sp[q, q1]) + sp[p, q1]^4 (qs (6 qs + 6 x2 + 9 y) + (10 qs + 6 x2 + 5 y) sp[q, q1]) - qs (qs y - sp[q, q1]^2) (x2 (-
        (6 qs + 9 x2 + 6 y) sp[q, q1]^2 + sp[q, q1]^3) + sp[p, q1]^3 (qs (-3 qs^2 - 3 x2^2 + qs (-7 x2 - 2 y) - 2 x2 y + 9 y^2) + (-3 x2^2 + 3 x2 y +
      sp[p, q1]^2 (qs (-3 qs^2 (2 x2 + y) + qs (-6 x2^2 - 19 x2 y - 10 y^2) + y (-3 x2^2 - 10 x2 y - 3 y^2)) + (-3 qs^3 - 25 qs^2 (x2 + y) + qs (-15 x2
        (-12 qs^2 - 15 x2^2 - 46 x2 y - 15 y^2 - 41 qs (x2 + y)) sp[q, q1]^2 + (-11 qs - 16 x2 - 7 y) sp[q, q1]^3) +
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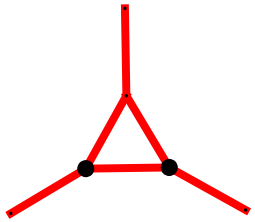
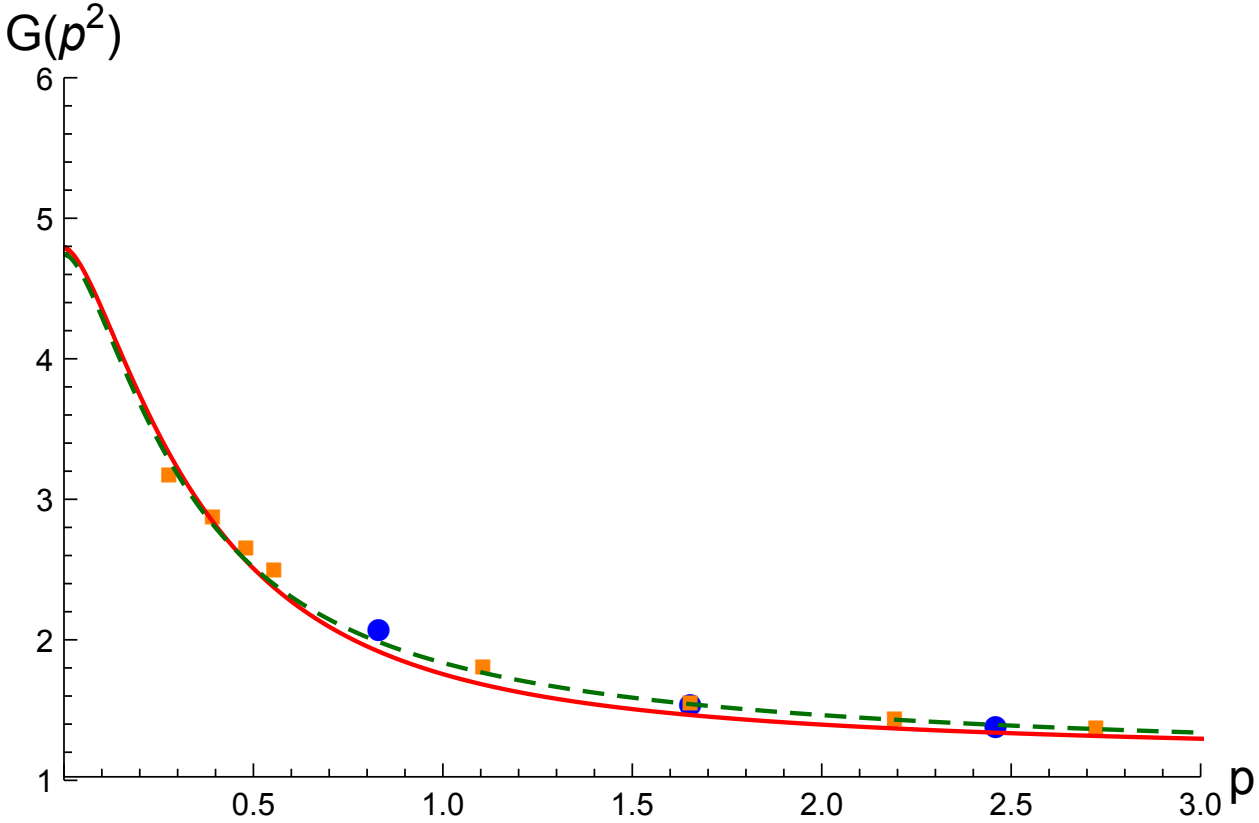


- Automated derivation of DSEs and flow equations:
  Mathematica package *DoFun* [Alkofer, MQH, Schwenzer '08; MQH, Braun '11]
  `http://tinyurl.com/dofun2`
- Framework for numeric handling:
  C++ program *CrasyDSE* [MQH, Mitter '11]
  `http://tinyurl.com/crasydse`

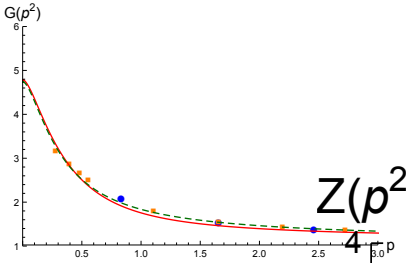
Go ahead and calculate . . .

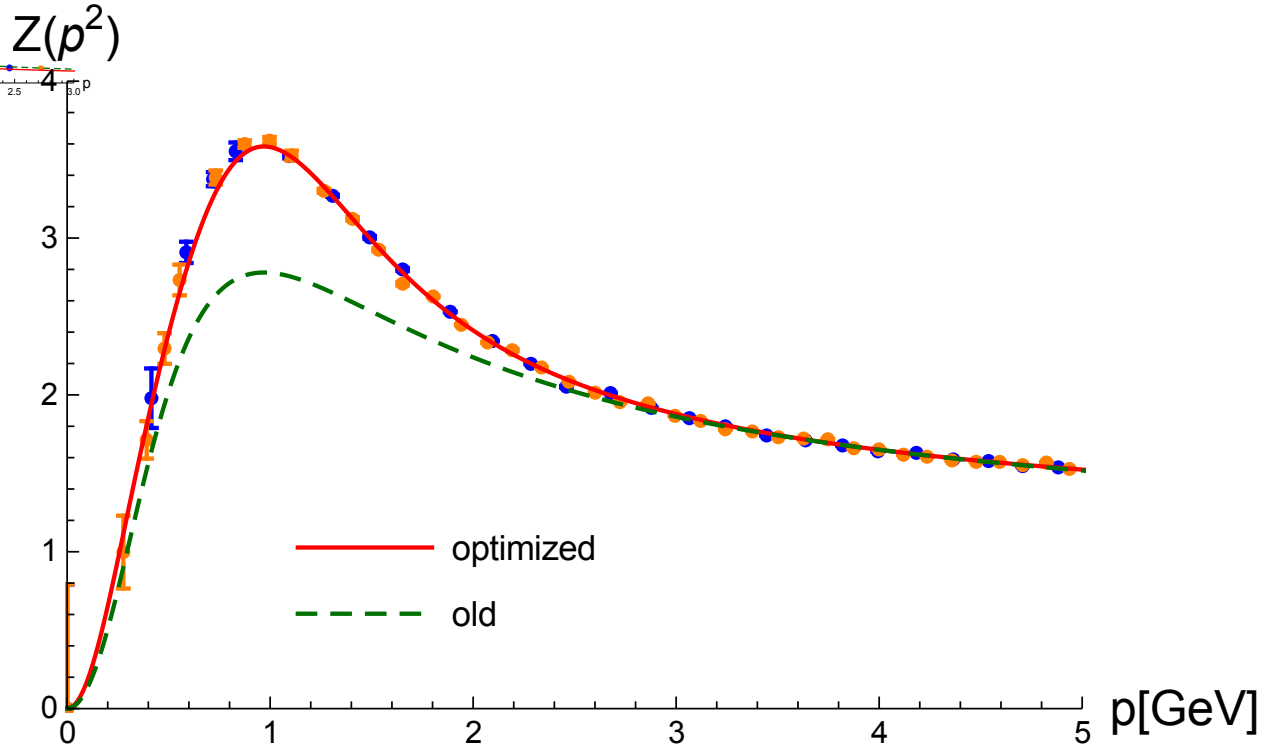


[MQH, von Smekal '12]

Go ahead and calculate . . .

Landau gauge, vacuum:

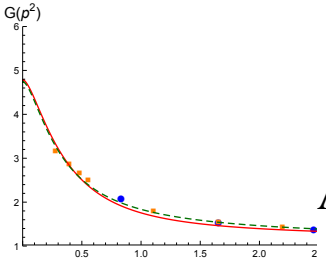


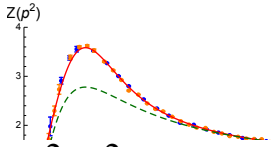


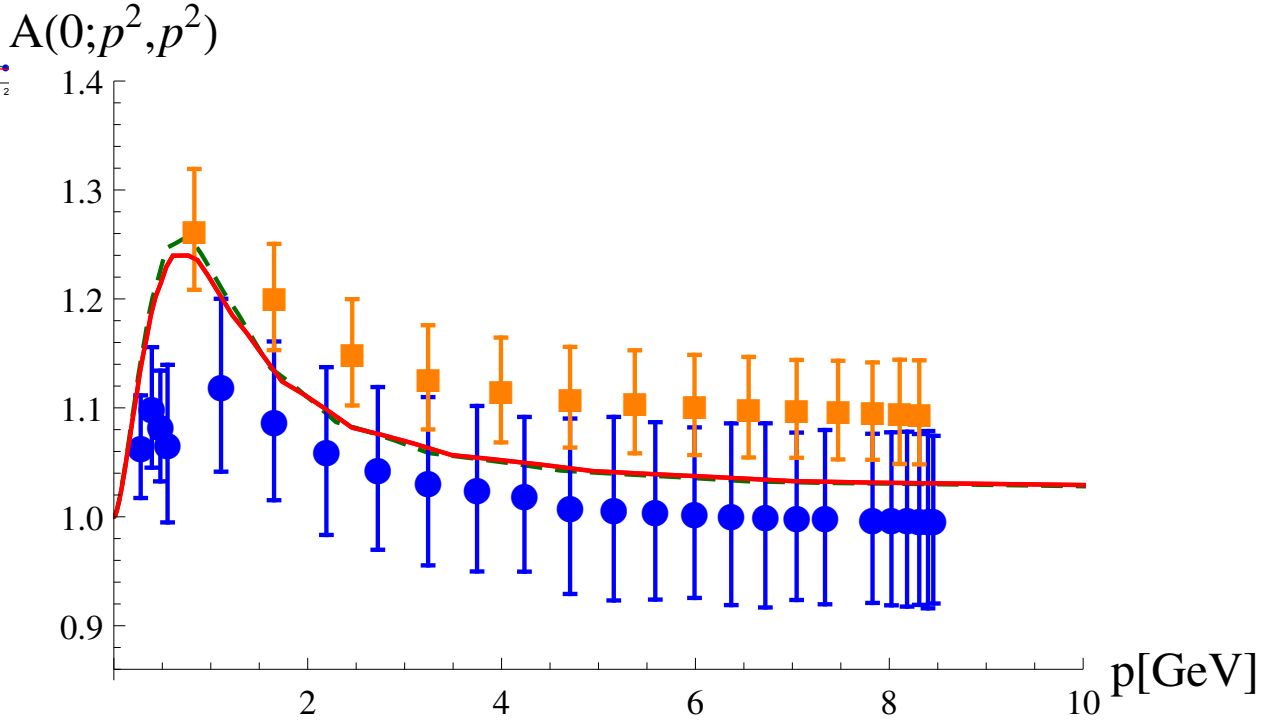
[MQH, von Smekal '12]

Go ahead and calculate . . .

Landau gauge, vacuum:



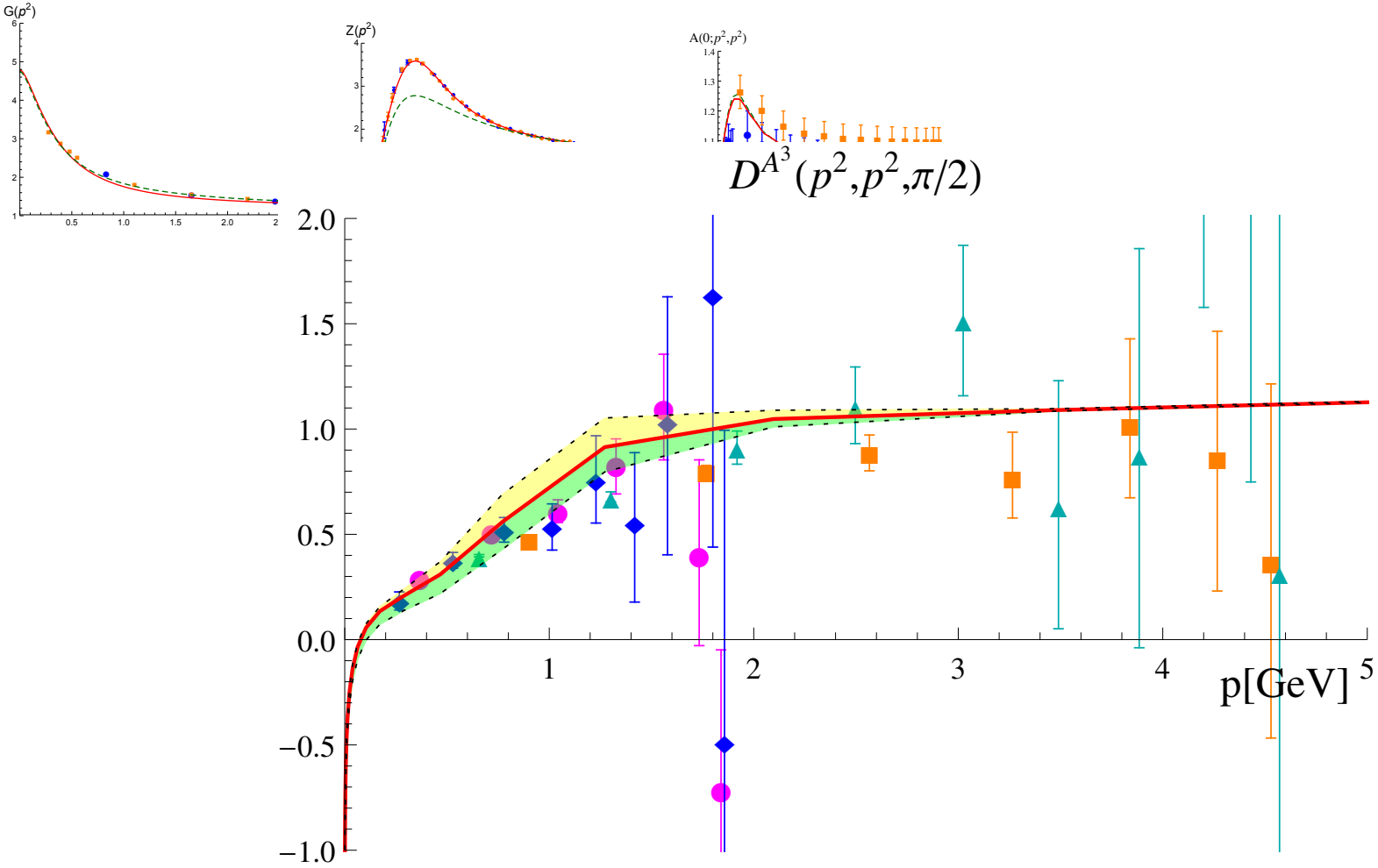




[MQH, von Smekal '12]

Go ahead and calculate . . .

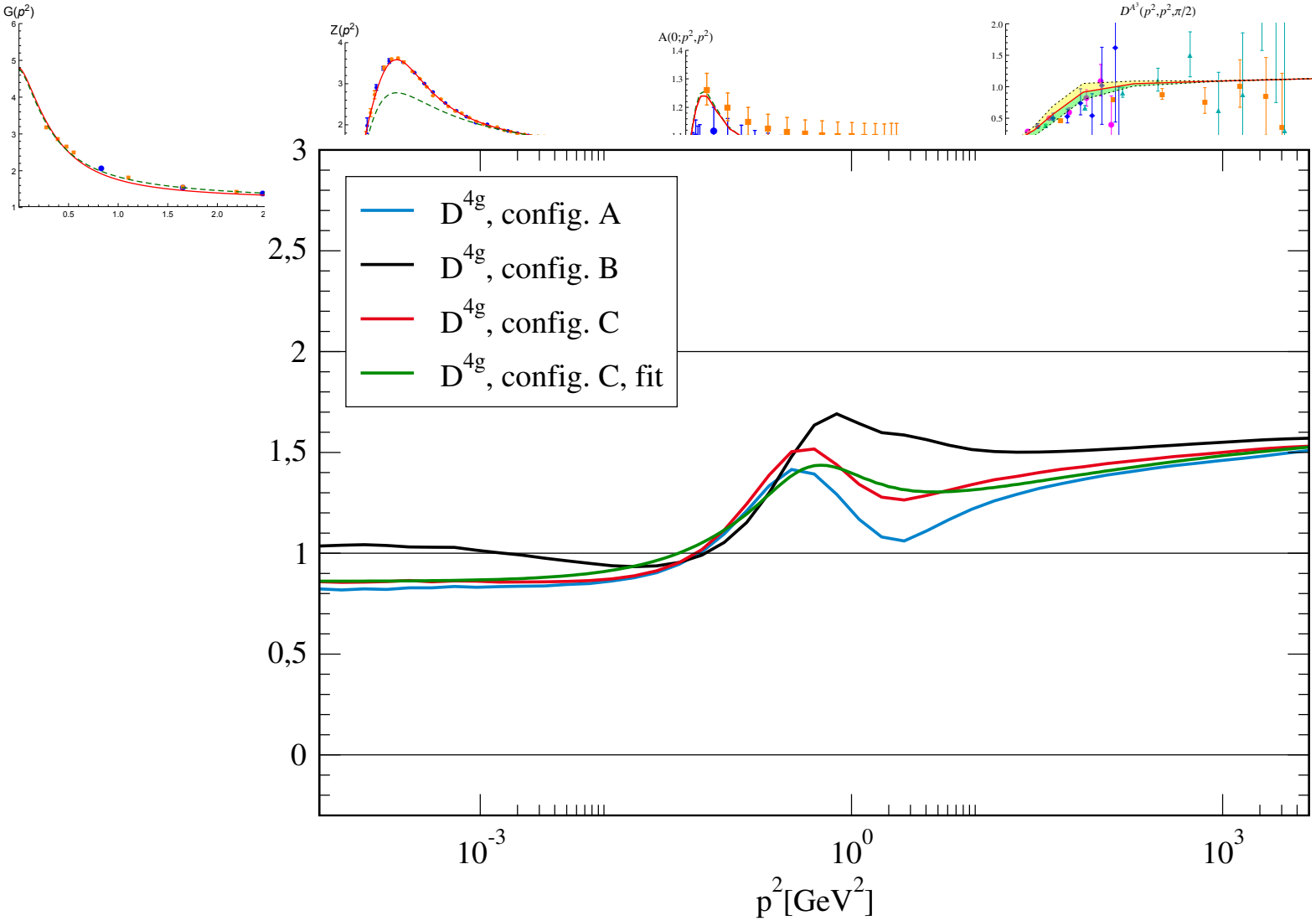
Landau gauge, vacuum:



[Blum, MQH, Mitter, von Smekal '14]

Go ahead and calculate . . .

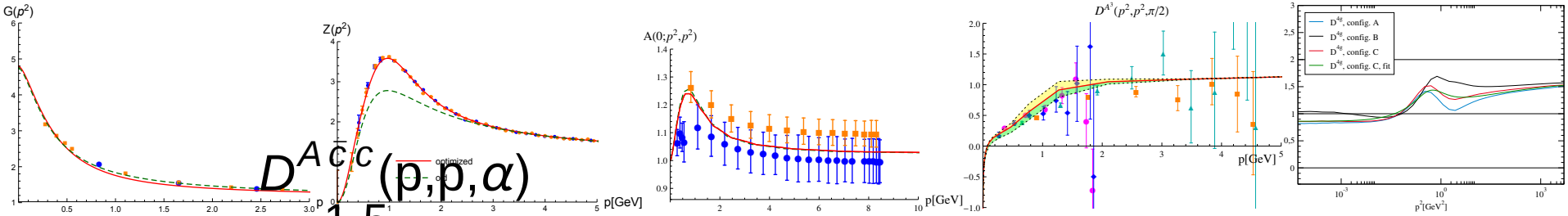
Landau gauge, vacuum:

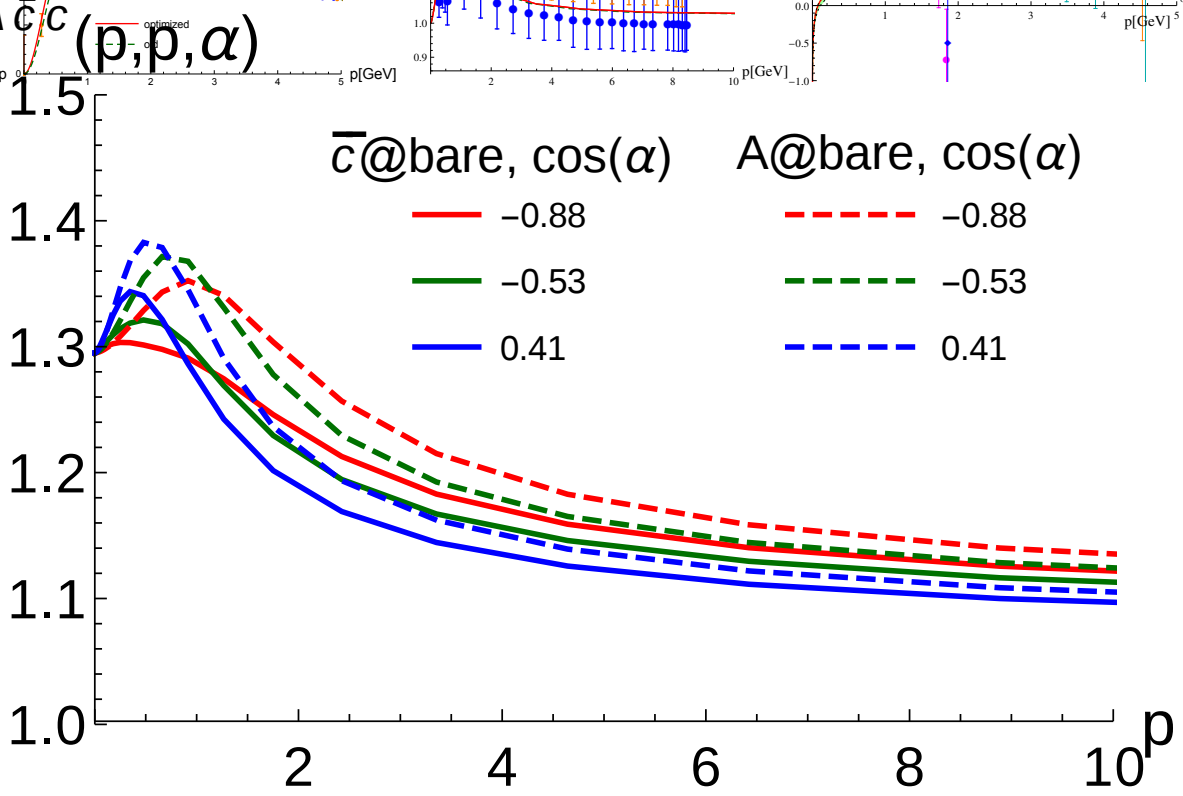


[Cyrol, MQH, von Smekal '14]

Go ahead and calculate . . .

Landau gauge, vacuum:

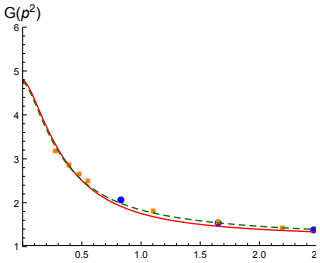


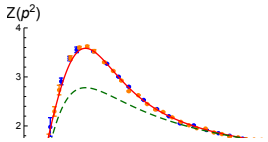


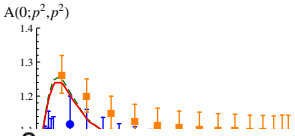
[MQH, Campagnari, Reinhardt '14]

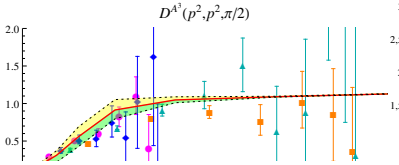
Go ahead and calculate . . .

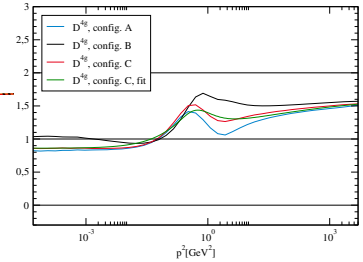
Landau gauge, vacuum:



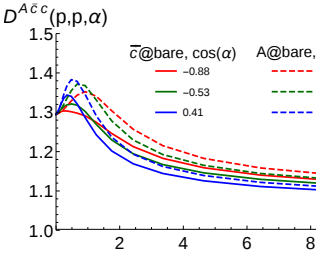


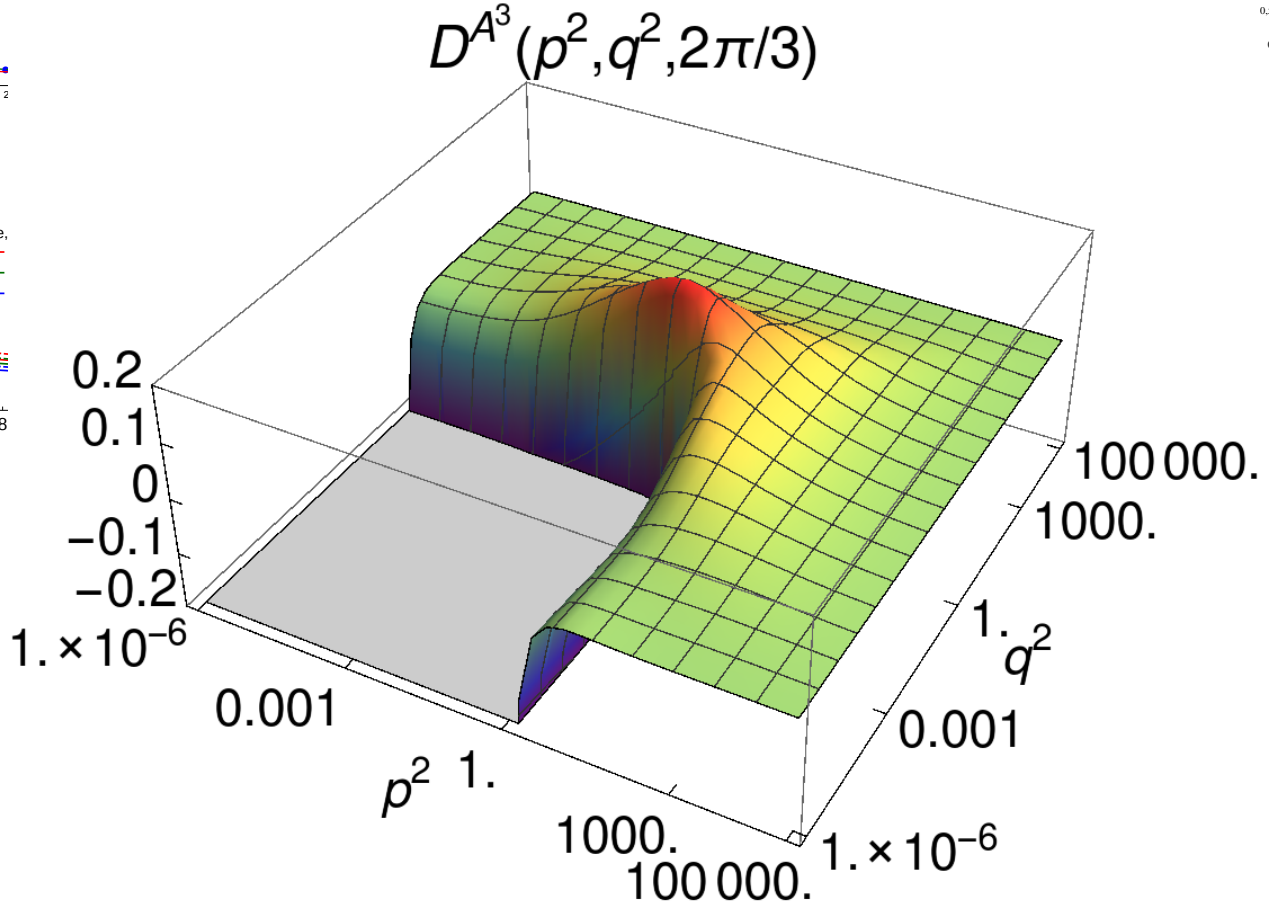






Coulomb and

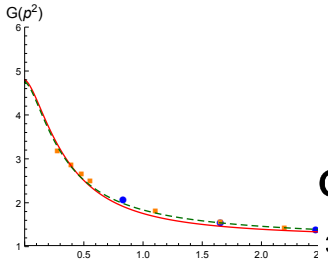


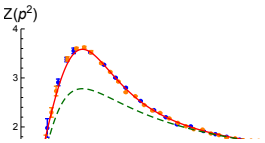


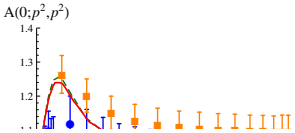
[MQH, Campagnari, Reinhardt '14]

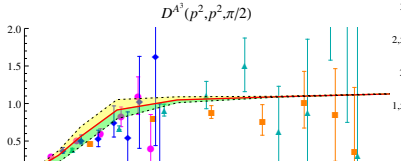
Go ahead and calculate . . .

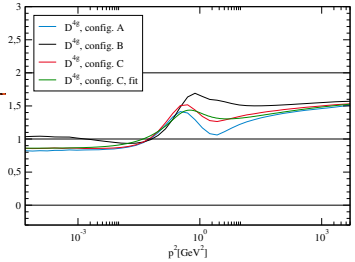
Landau gauge, vacuum:



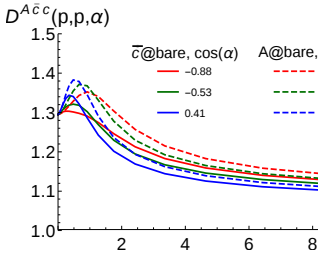


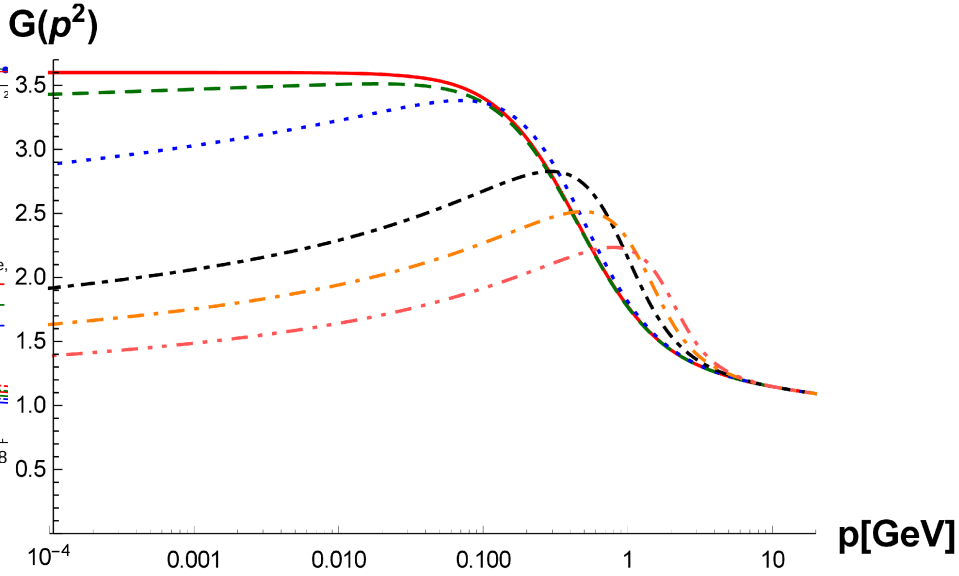






Coulomb and

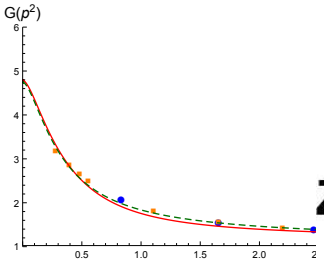


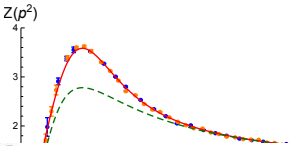


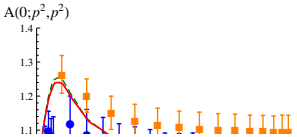
[MQH '15]

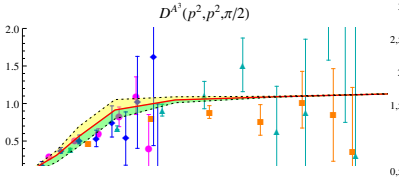
Go ahead and calculate . . .

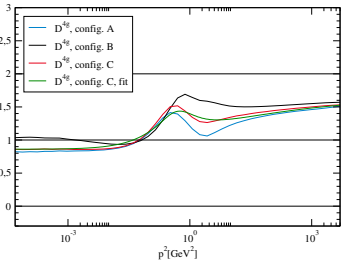
Landau gauge, vacuum:



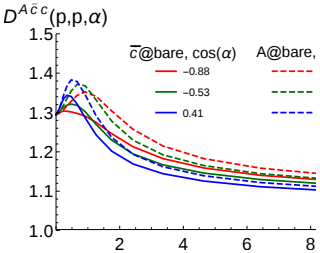


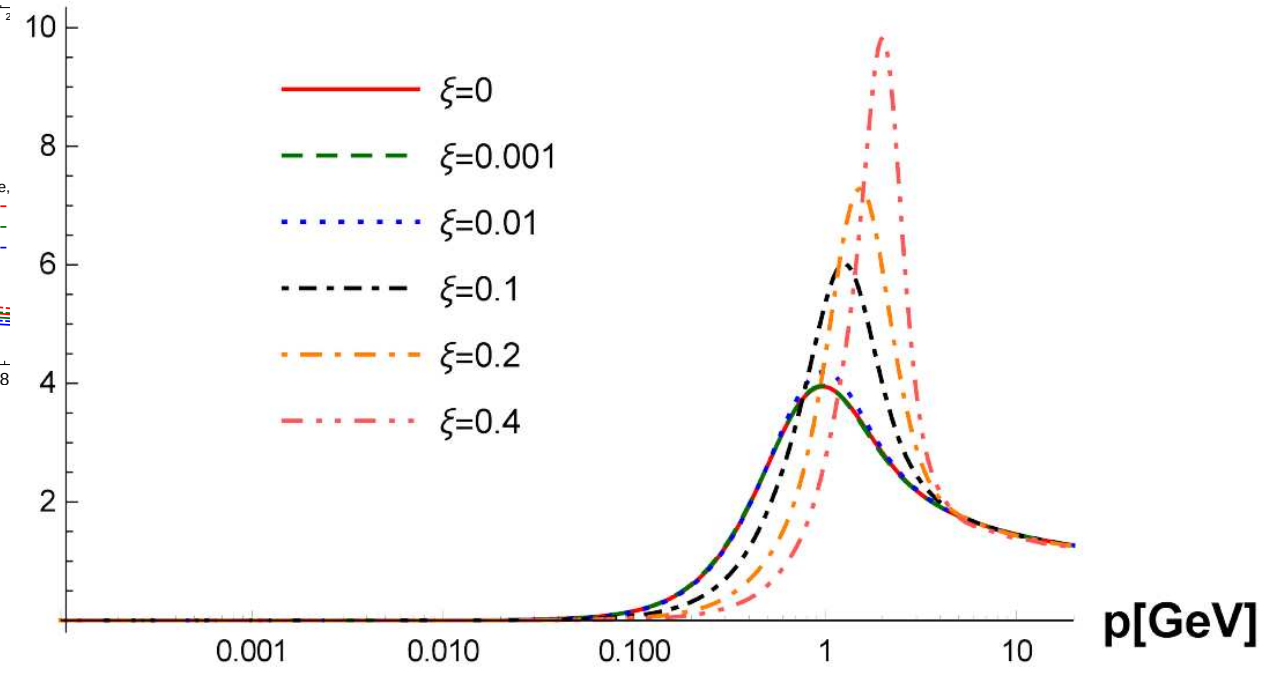






Coulomb and

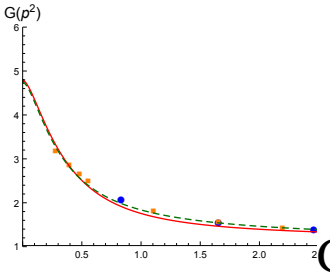


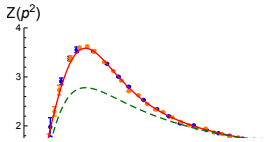


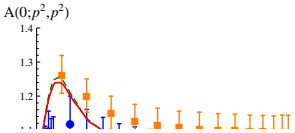
[MQH '15]

Go ahead and calculate . . .

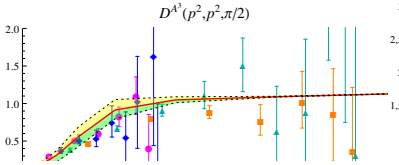
Landau gauge, vacuum:

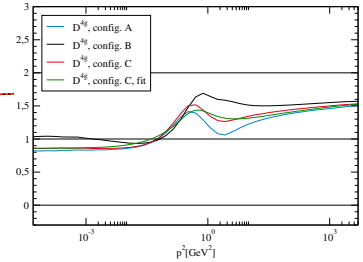




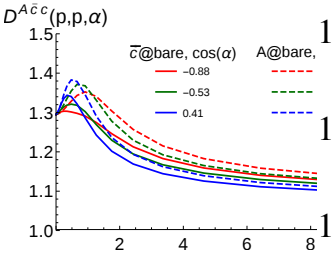


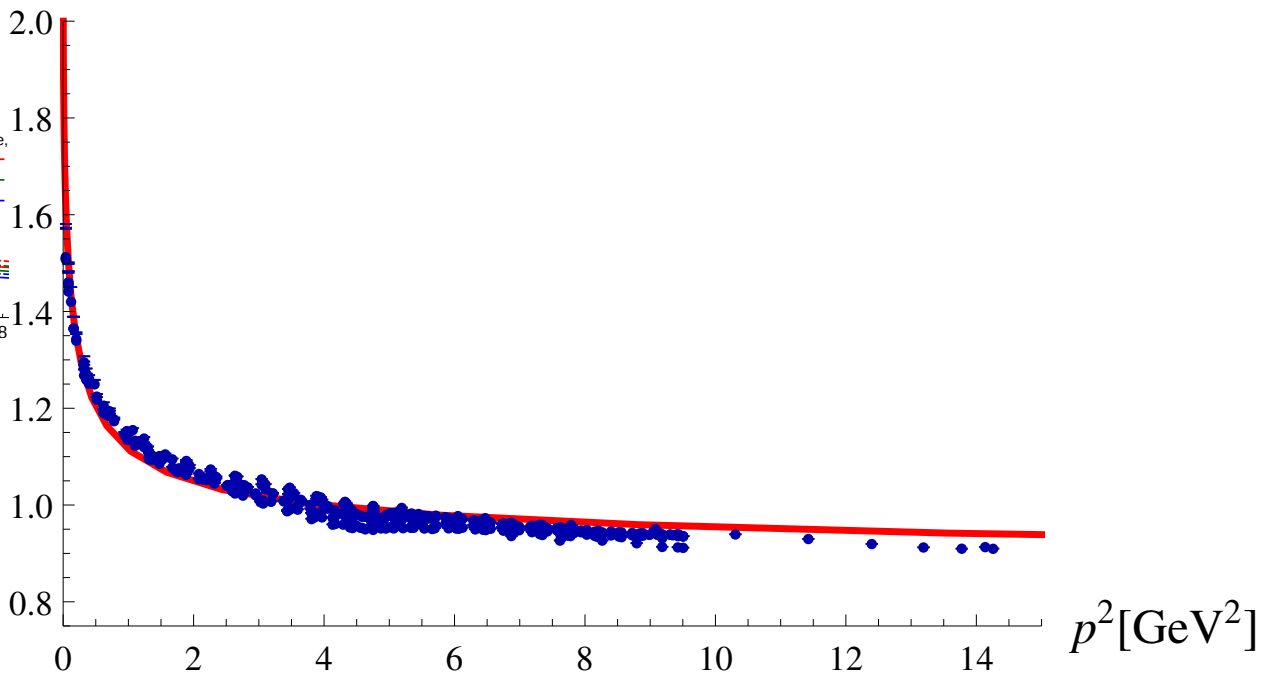
T=0.98T_c





Coulomb and

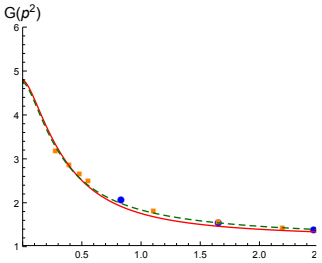


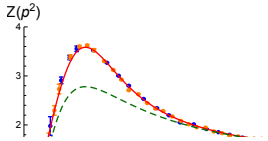


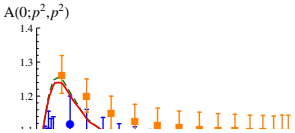
[MQH, von Smekal '13]

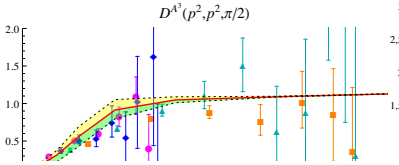
Go ahead and calculate . . .

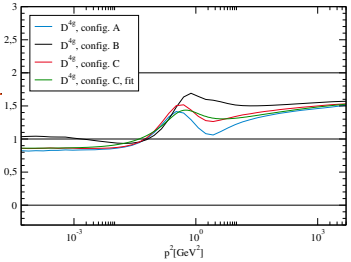
Landau gauge, vacuum:





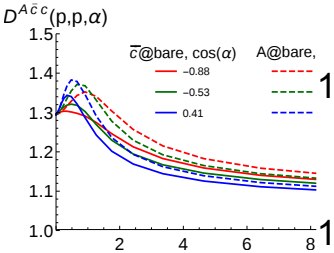




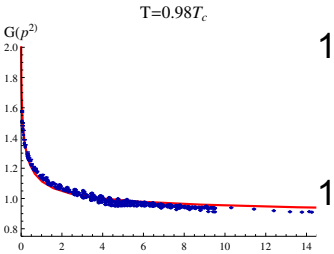


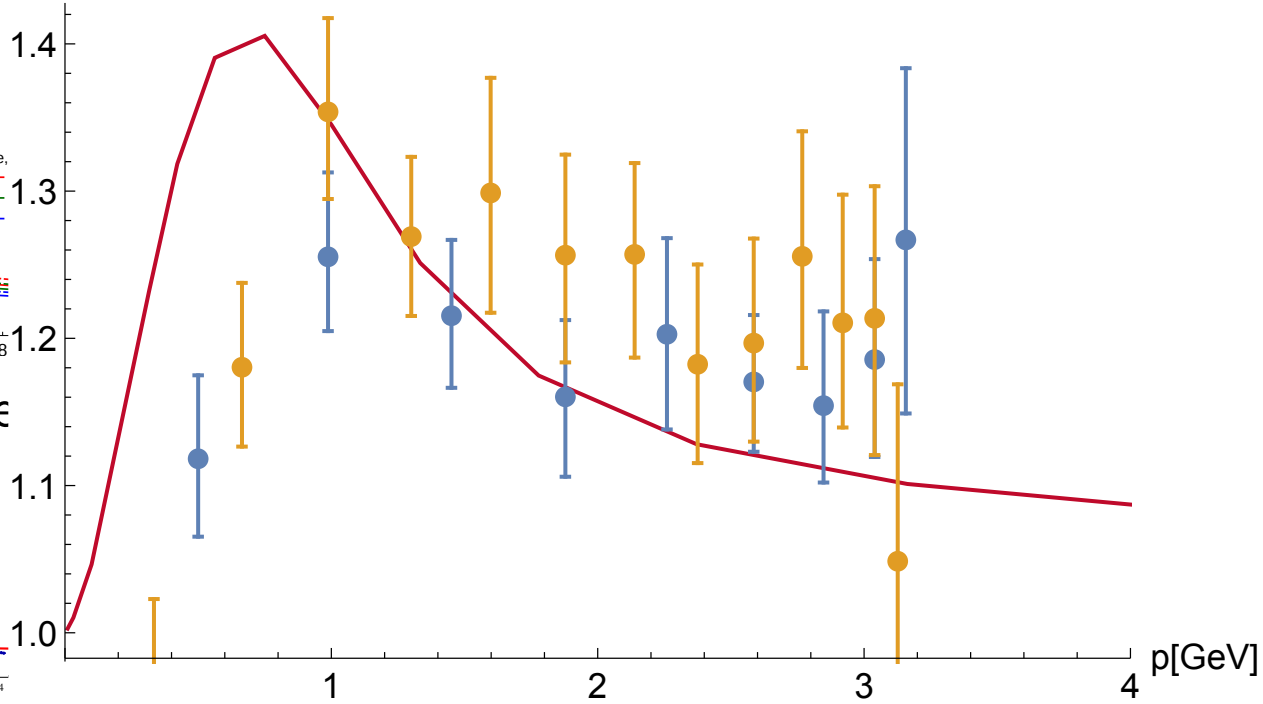
$D^{A\,\bar{c}\,c}(p^2,p^2,2\pi/3)$ T=0.94Tc

Coulomb and



Landau gauge

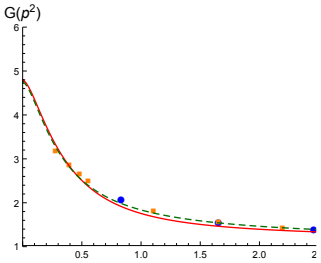


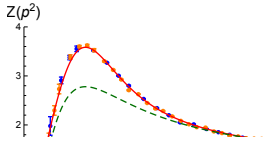


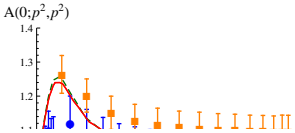
[MQH, von Smekal '13]

Go ahead and calculate . . .

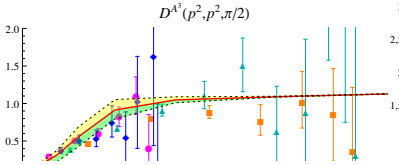
Landau gauge, vacuum:

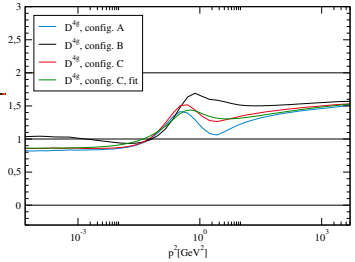




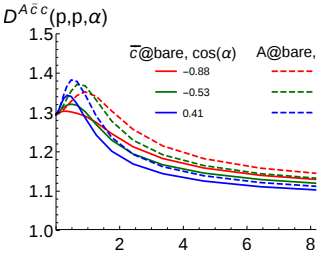


$D^{AAA}(p^2,p^2,2\pi/3)$ T=2.Tc

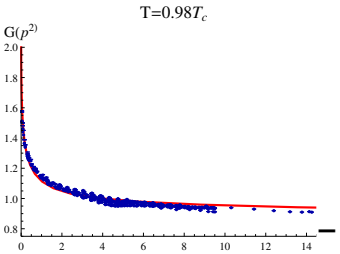


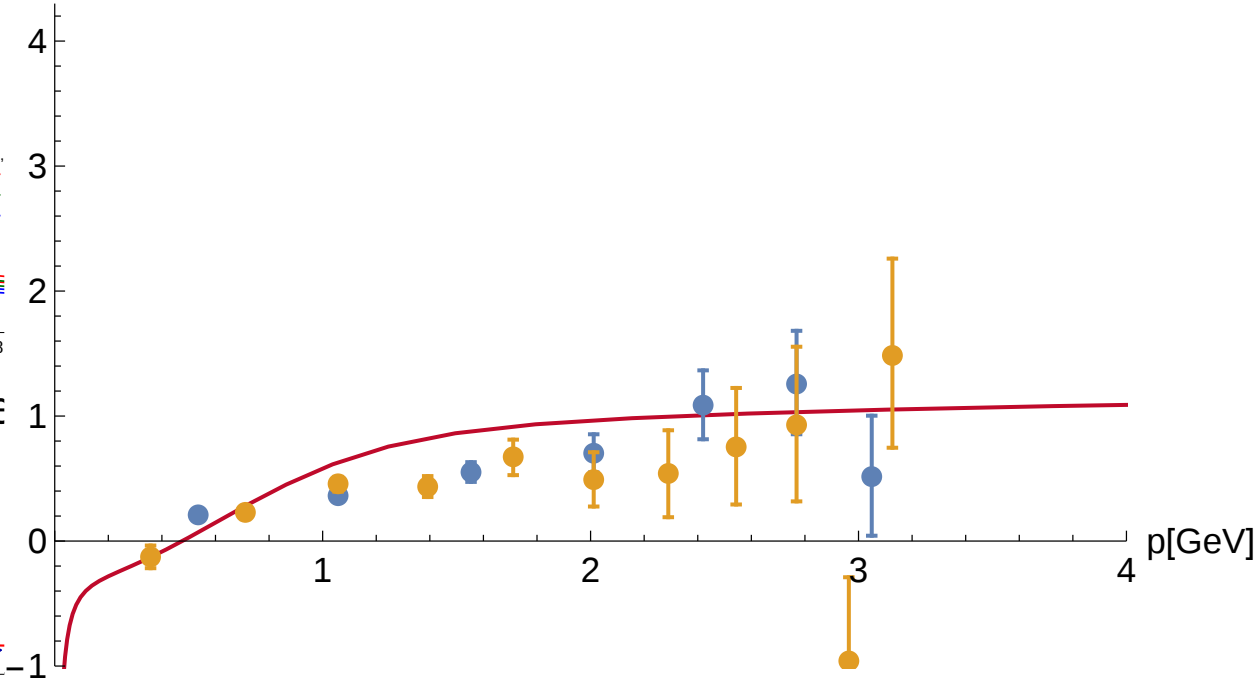


Coulomb and



Landau gauge

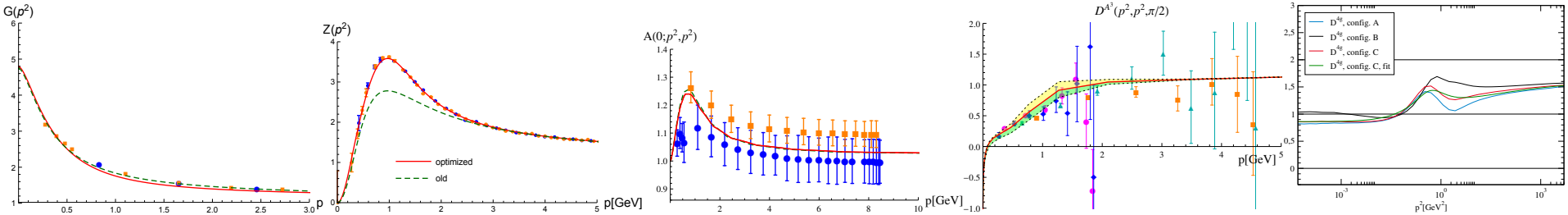




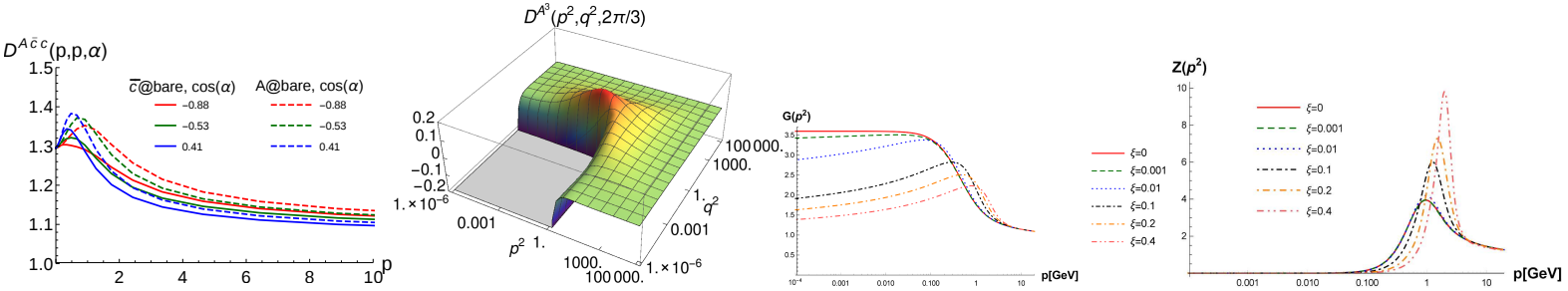
[MQH unpub.]

Go ahead and calculate . . .

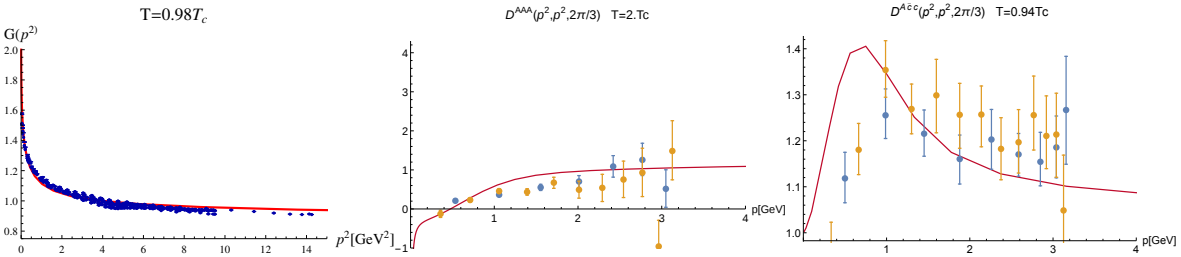
Landau gauge, vacuum:



Coulomb and linear covariant gauges, vacuum:



Landau gauge, non-zero temperatures:



But...

...how do we know that the results are trustworthy?

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$\rightarrow$ Compare with other methods.

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...how do we know that the results are trustworthy?
→ Compare with other methods.

Lattice results

Available for

- Vacuum

But...

...how do we know that the results are trustworthy?
→ Compare with other methods.

Lattice results

Available for

- Vacuum
- Propagators

But...

...how do we know that the results are trustworthy?
$\rightarrow$ Compare with other methods.

Lattice results

Available for

- Vacuum
- Propagators
- $T > 0$

But...

...how do we know that the results are trustworthy?
$\rightarrow$ Compare with other methods.

Lattice results

Available for

- Vacuum
- Propagators
- $T > 0$
- Three-point functions (restricted kinematics)

But...

...how do we know that the results are trustworthy?
$\rightarrow$ Compare with other methods.

Lattice results

Available for

- Vacuum
- Propagators
- $T > 0$
- Three-point functions (restricted kinematics)
- $\mu > 0$?

But...

...how do we know that the results are trustworthy?
$\rightarrow$ Compare with other methods.

Lattice results

Available for

- Vacuum
- Propagators
- $T > 0$
- Three-point functions (restricted kinematics)
- $\mu > 0$?
- Four-point functions?

But...

...how do we know that the results are trustworthy?
$\rightarrow$ Compare with other methods.

Lattice results

Available for

- Vacuum
- Propagators
- $T > 0$
- Three-point functions (restricted kinematics)
- $\mu > 0$?
- Four-point functions?

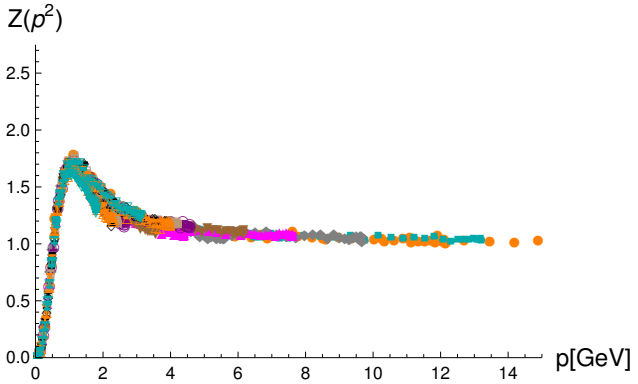
$\rightarrow$ Comparison with lattice is helpful, but finally self-consistent checks are required.

Two words of caution:

- One cannot assume naturally that the hierarchy is the same for all T and μ.
- Even the effect of a single correlation function is difficult to estimate.

Yang-Mills theory in 3 dimensions

Historically interesting because cheaper on the lattice → easier to reach the IR, e.g., [Cucchieri '99; Cucchieri, Mendes, Taurines '03; Cucchieri, Maas, Mendes, '08; Maas '08, '14; Maas, Pawlowski, Spielmann, Sternbeck, von Smekal '09; Cucchieri, Dudal, Mendes, Vandersickel '11; Bornyakov, Mitrjushkin, Rogalyov '11, '13; Cucchieri, Dudal, Mendes, Vandersickel '16]



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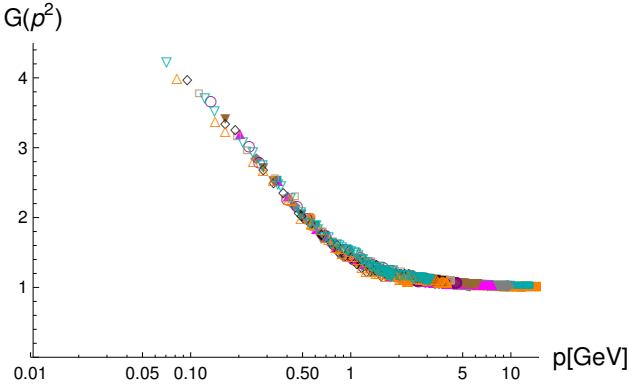

[Maas '14]

Continuum results:

- Coupled propagator DSEs: [Maas, Wambach, Grüter, Alkofer '04]
- (R)GZ: [Dudal, Gracey, Sorella, Vandersickel, Verschelde '08]
- YM + mass term: [Tissier, Wschebor '10, '11]
- DSEs of PT-BFM: [Aguilar, Binosi, Papavassiliou '10]

Yang-Mills theory in 3 dimensions: Motivation

NB: Numerically not cheaper for functional equations of 2- and 3-point functions.

Yang-Mills theory in 3 dimensions: Motivation

NB: Numerically not cheaper for functional equations of 2- and 3-point functions.

Advantages:

- UV finite: no renormalization, no anomalous running
- Spurious divergences easier to handle

$\Rightarrow$ Many complications from $d = 4$ absent!

Quantitative study of truncation effects possible:

Vary equations and truncations.

Regularization and UV behavior

Ways of regularization

- Lattice
- Pauli-Villars
- Analytic reg., proper time, ...; useful for analytic calculations
- Dimensional regularization → numerical difficult, esp. for power law divergences [Phillips, Afnan, Henry-Edwards '99]
- UV cutoff:

$$\int_0^\infty dq \to \int_0^\Lambda dq$$

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Several methods for subtraction of spurious divergences used.
Prerequisite for a quantitatively accurate description: a good understanding of how to subtract them.

Subtraction of divergences of gluon propagator ($d = 4$)

1. Logarithmic divergences handled by subtraction at p_0.
2. Quadratic divergences also subtracted, coefficient C_{sub}.

$$Z(p^2)^{-1} := Z_\Lambda(p^2)^{-1} - C_{\mathrm{sub}} \left(\frac{1}{p^2} - \frac{1}{p_0^2} \right)$$

calculated right-hand side (log. divergences handled)

How to determine C_{sub}?

Calculation of C_{sub} $(d = 4)$

Can be calculated analytically!

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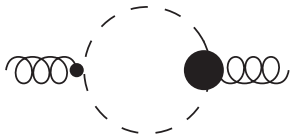
Use projector $P^{\zeta}_{\mu\nu}(p) = g_{\mu\nu} - \zeta p_\mu p_\nu / p^2$ for gluon propagator DSE.

Consider ghost loop:



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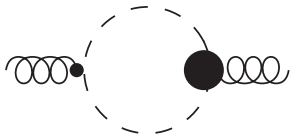

Approximation: $G\left((p+q)^2\right) \to G\left(q^2\right) \Rightarrow$ perform angle integrals

$$I_{gh}(x) = \frac{N_c\, g^2}{192\pi^2} \int_x^{\Lambda^2} dy \Big(\underbrace{x\,(\zeta - 2)}_{\text{log. div. } \checkmark} - \underbrace{(\zeta - 4)y}_{\text{quad. div.}} \Big) \frac{G(y)^2}{xy} + \ldots$$

$x = p^2, y = q^2, z = (p+q)^2$

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$$I_{gh}^{spur}(x) \propto \frac{1}{x} \int_{x_1}^{\Lambda^2} dy\, G_{UV}^2(y)$$

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What about the finite part?

- Perturbatively no mass term should be generated.
- Independent of UV parametrization.
- Dim. reg. calculation yields the same finite part.

Form of spurious divergences

Up to now approximated analytic calculation.

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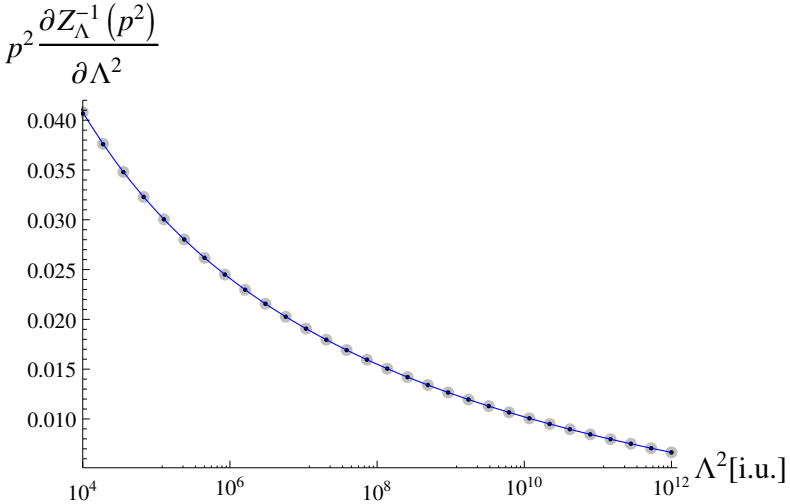
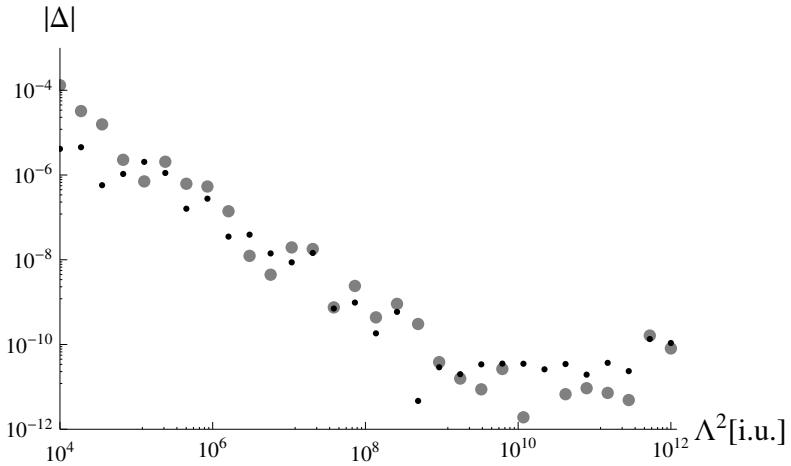
Compare derivatives of analytic result with *full* numeric calculation:



[MQH, von Smekal '14]



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- Independent of external momentum → Purely perturbative origin.
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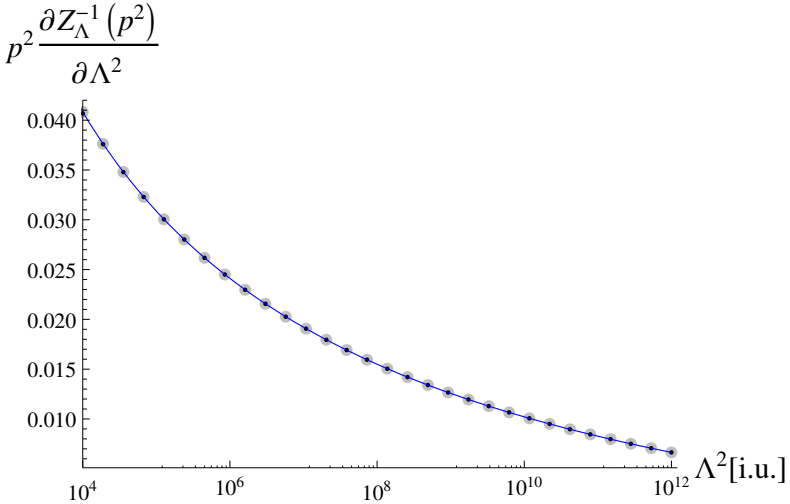

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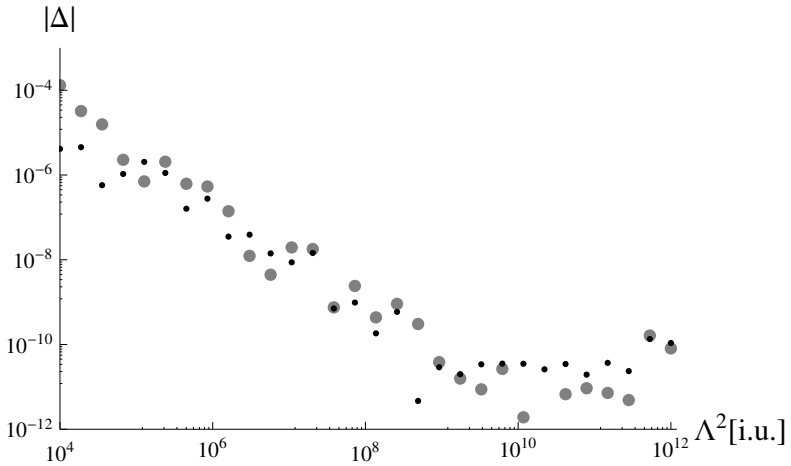

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FRG [Cyrol, Fister, Mitter, Pawlowski, Strodthoff '16]: Relation between finetuning of UV gluon mass and decoupling/scaling solution?

Spurious divergences in $d = 3$

Simplification in $d = 3$:

$$C_{sub} = a\,\Lambda + b\,\ln\Lambda$$

$\rightarrow$ fit (works for numeric vertices and two-loop diagrams)

NB: In $d = 4$ it is *not* $C_{sub} = a\,\Lambda^2$.

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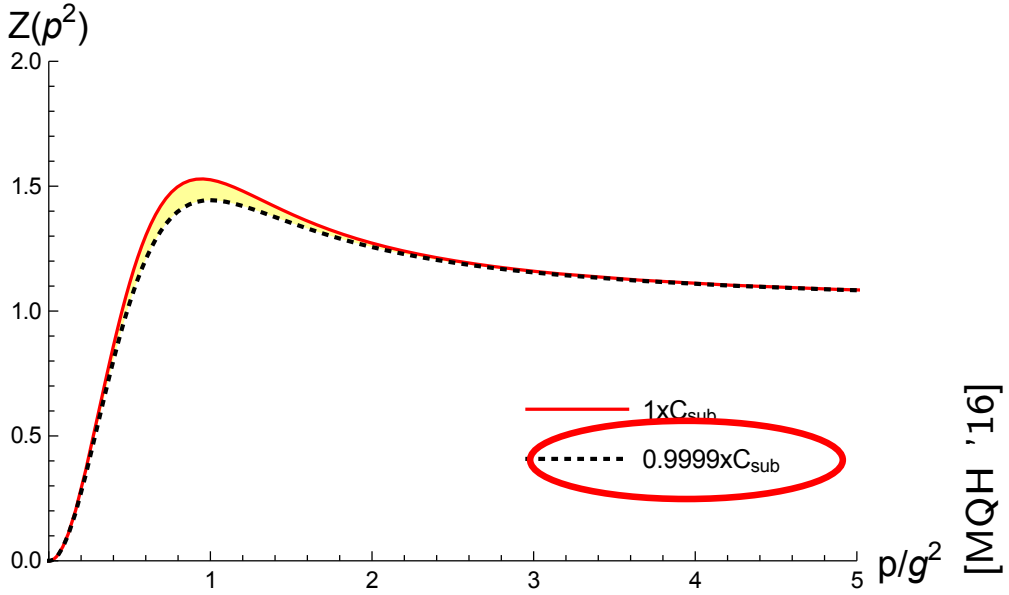
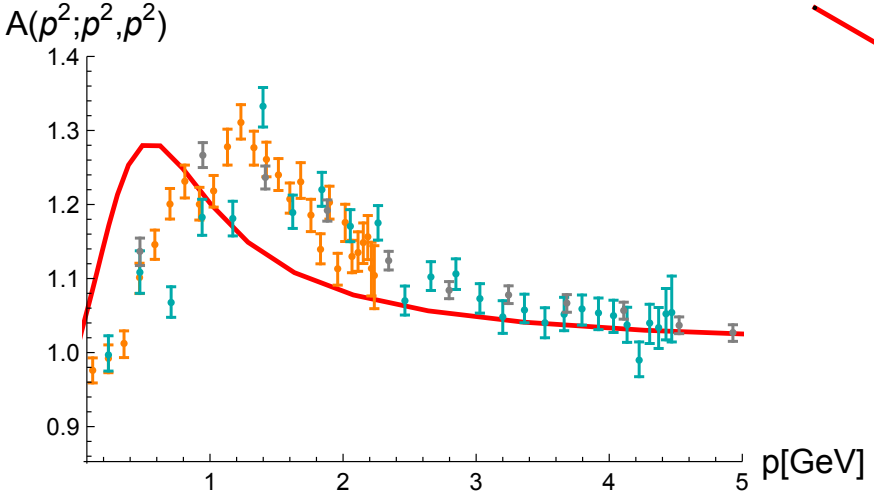
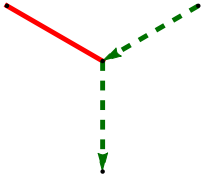


Small deviations $\rightarrow$ large effect.

Results: Three-point functions

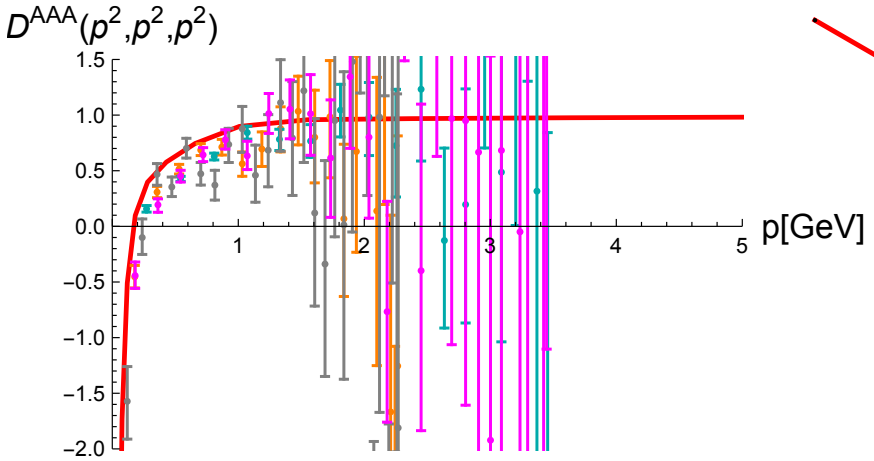
Dressings:

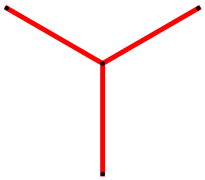




[MQH '16; lattice: Maas, unpublished]

- Maximum position shifted.
- Bump height ok.



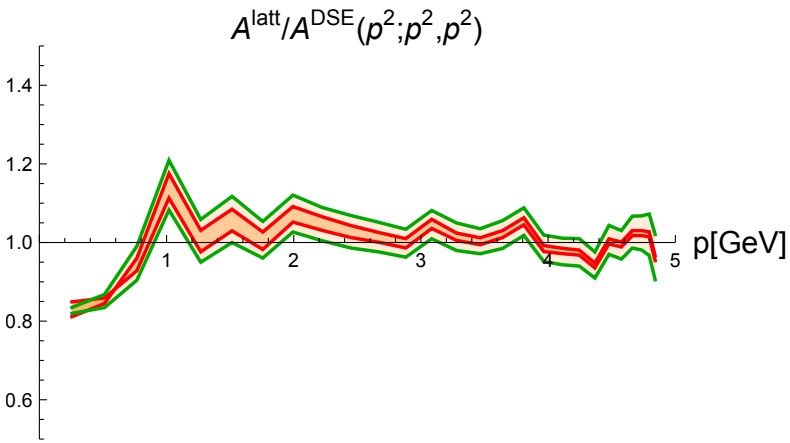


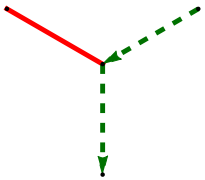
[MQH '16; lattice: Cucchieri, Maas, Mendes

- Good agreement with lattice data.
- Linear IR divergence.

Results: Three-point functions

Ratio lattice/DSE results:





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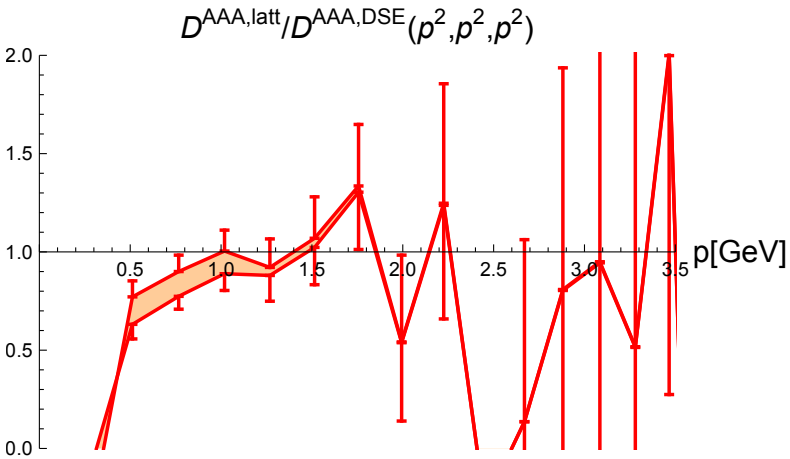

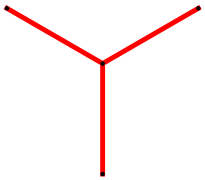

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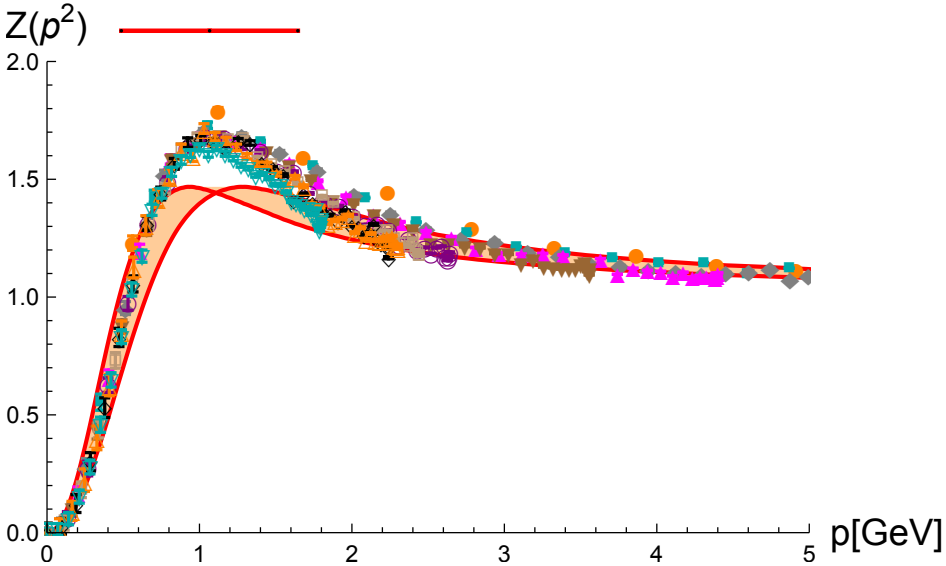
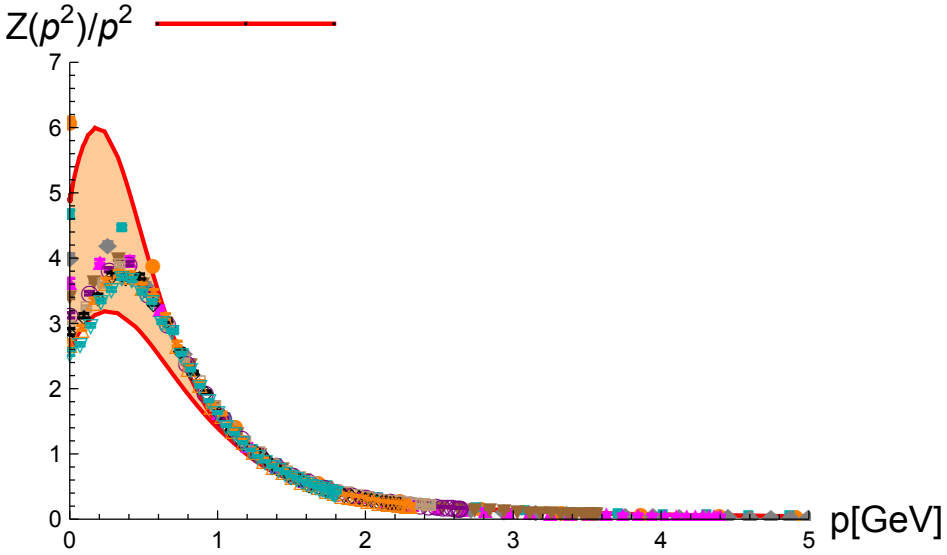
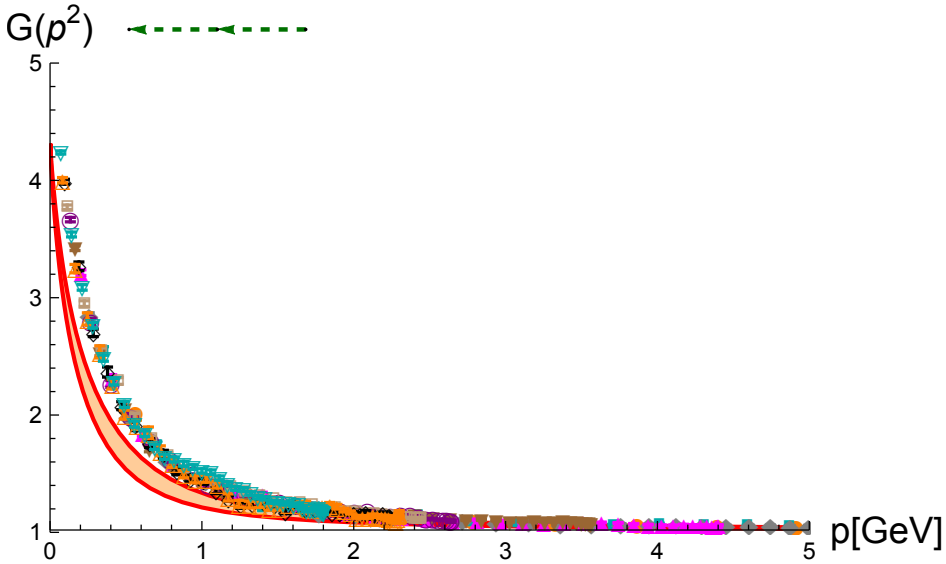
Results: Propagators







Bands from uncertainty in setting the physical scale.

[MQH '16; lattice: Maas '14]

Non-perturbative gauge fixing

Gribov copies: Gauge equivalent configurations that fulfill the Landau gauge condition $\partial A = 0$.

Up to here the minimal Landau gauge was shown for lattice data.

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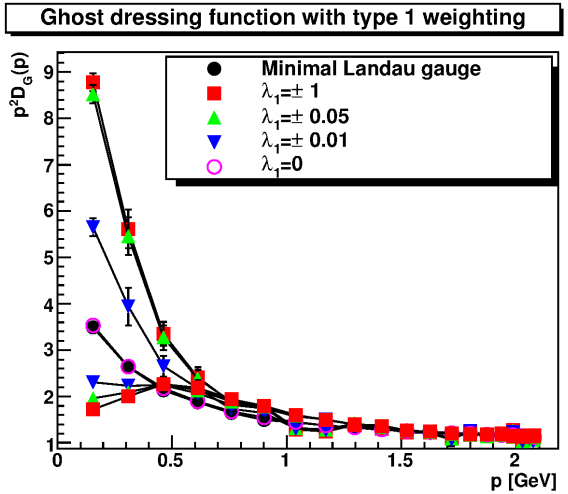
Another possibility: Absolute Landau gauge (global minimum of gauge fixing functional)

$\rightarrow$ Different solutions on the lattice,

e.g. [Maas '09, '11; Cucchieri '97; Bogolubsky et al. '05; Sternbeck, Müller-Preussker '12].

NB: Different solutions also from functional equations [Boucaud et al. '08; Fischer, Maas, Pawlowski '08].

Different solutions



[Maas '13]

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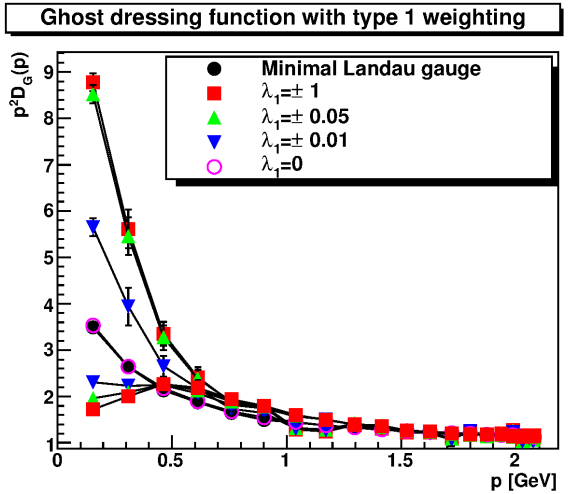

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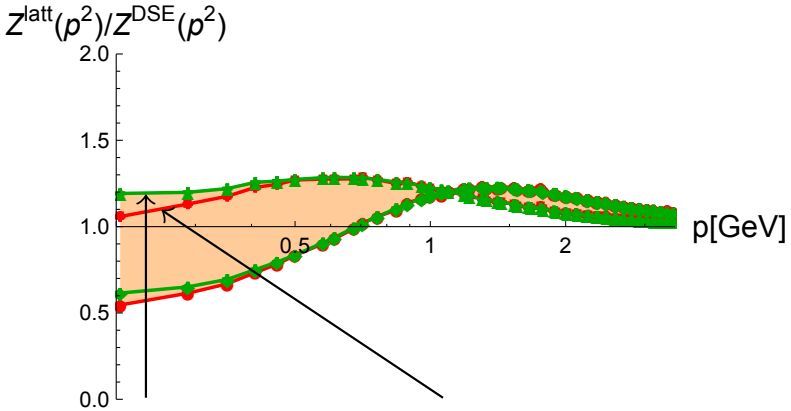

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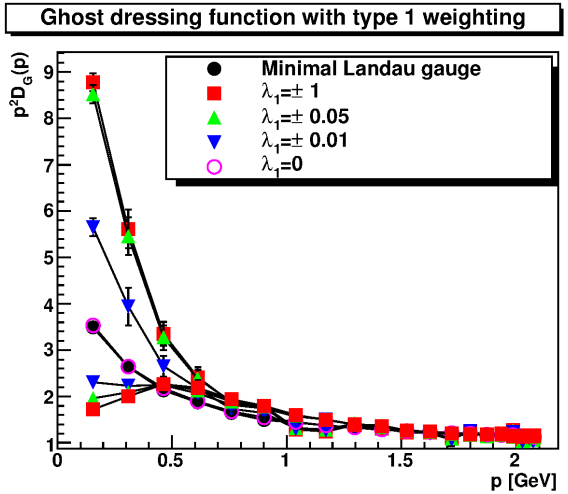

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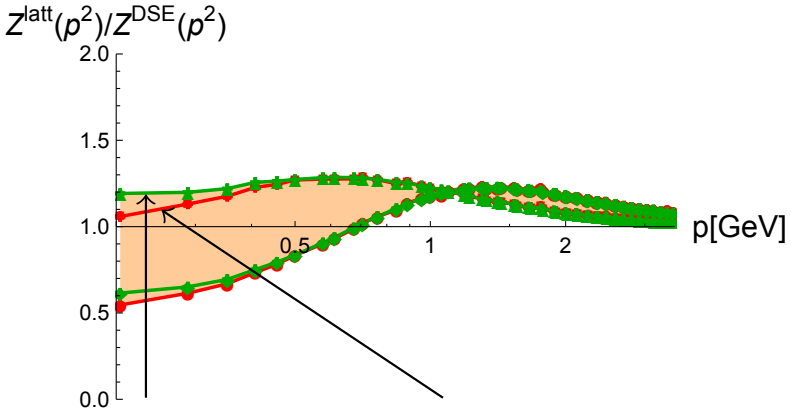

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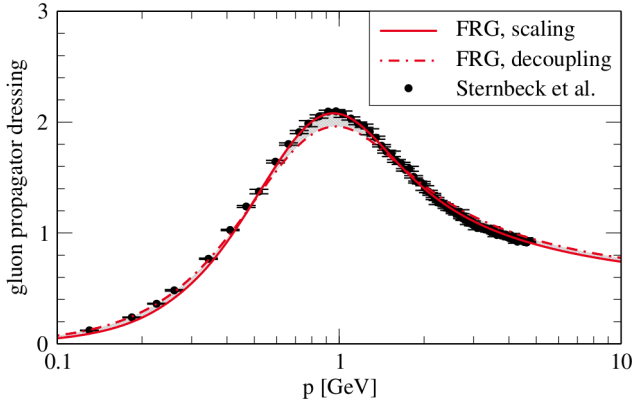
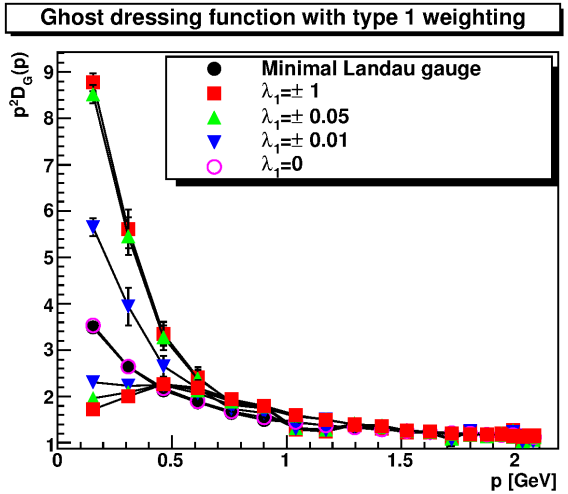


[Cyrol, Fister, Mitter, Pawlowski, Strodthoff '16]

→ Scaling solution has the highest peak in the gluon dressing!

Different solutions



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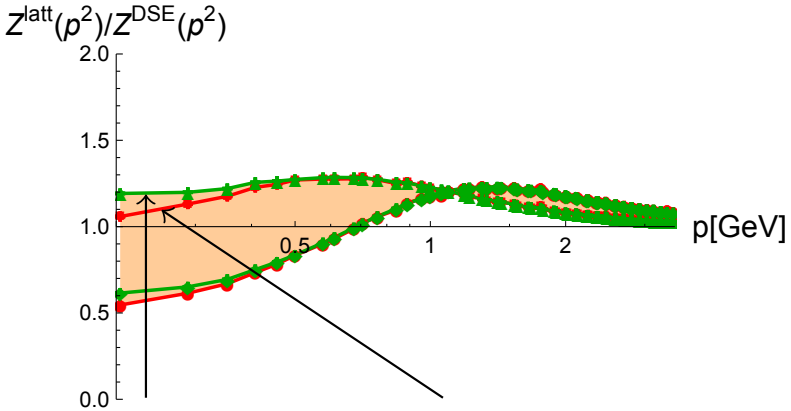

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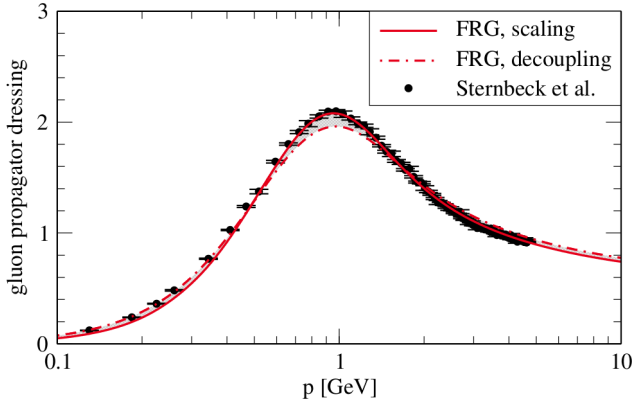
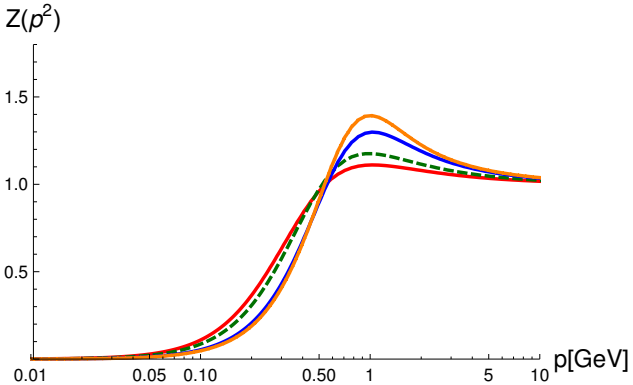


[Cyrol, Fister, Mitter, Pawlowski, Strodthoff '16]

Varying the renormalization prescription:
Propagators with bare vertices [MQH '15]:
Similar effect (but vertex effects neglected!)



Varying the four-gluon vertex

How stable is the truncation?

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Compare:

- full four-gluon vertex
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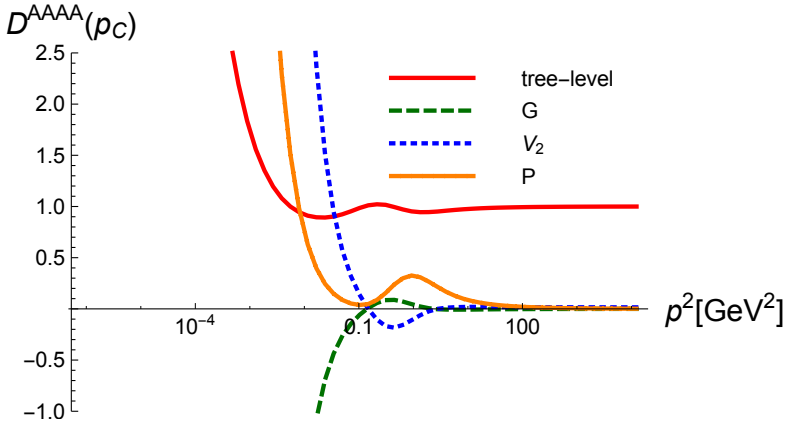

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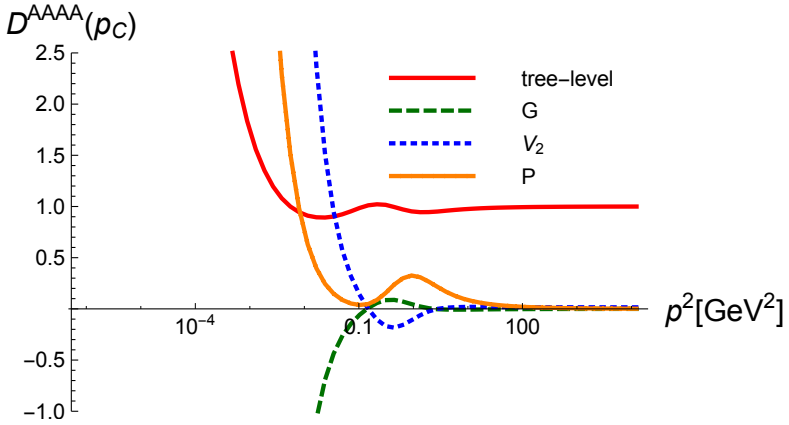

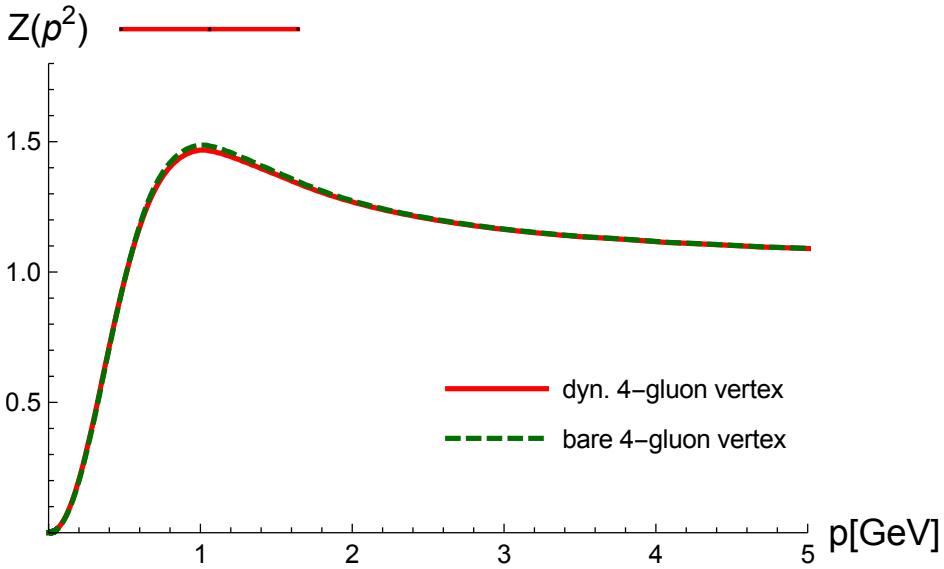

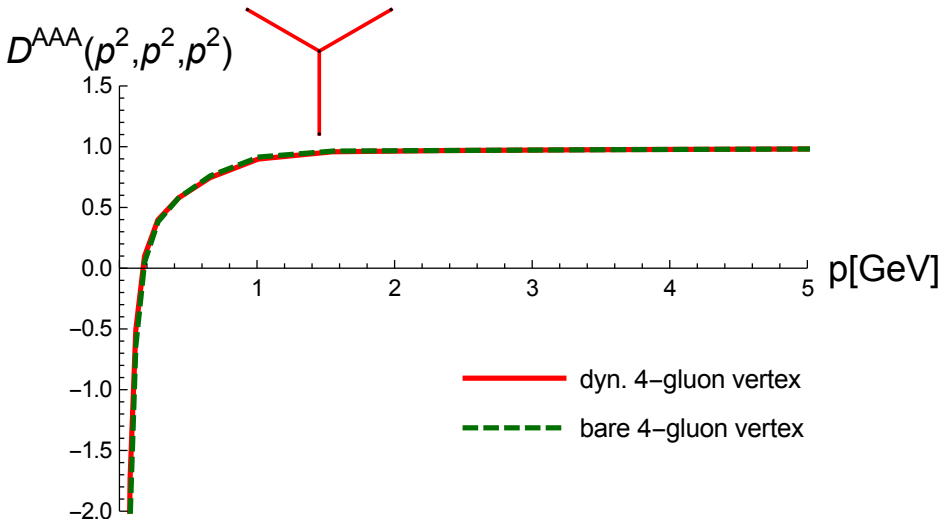

→ Very similar results.

[MQH '16]

Solution from the 3PI effective action

Different set of functional equations:
equations of motion from 3PI effective action (at three-loop level)

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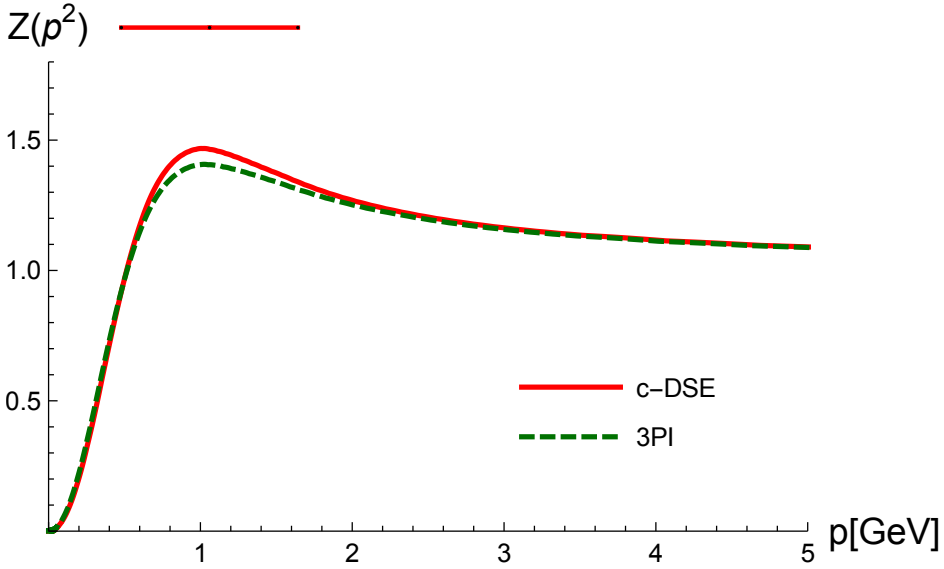

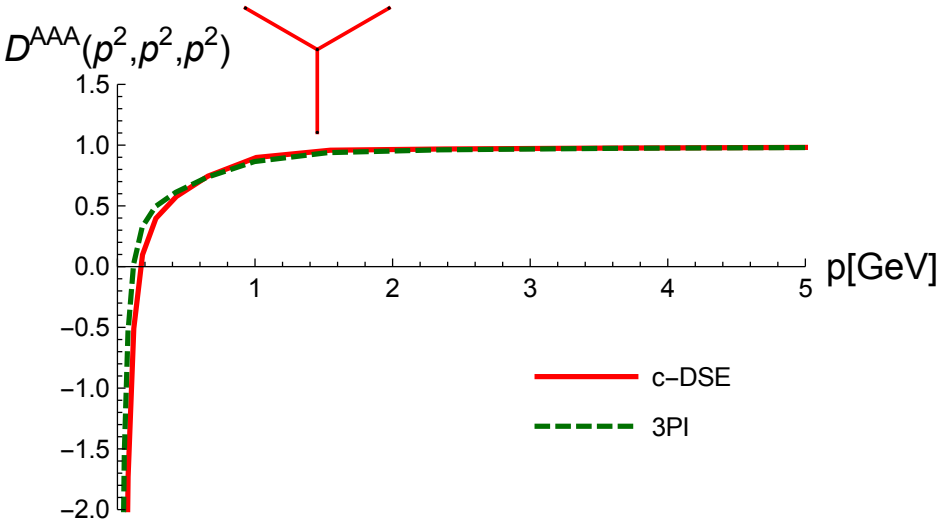

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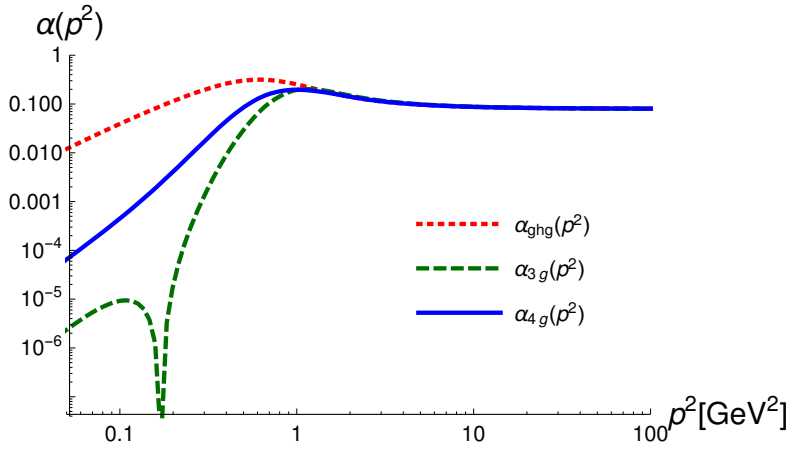
$$\alpha_{\mathrm{ghg}}(p^2) = \frac{g^2}{4\pi} D^{A\bar{c}c}(p^2,p^2,p^2)^2 G(p^2)^2 Z(p^2)$$

$$\alpha_{3\mathrm{g}}(p^2) = \frac{g^2}{4\pi} D^{AAA}(p^2,p^2,p^2)^2 Z(p^2)^3$$

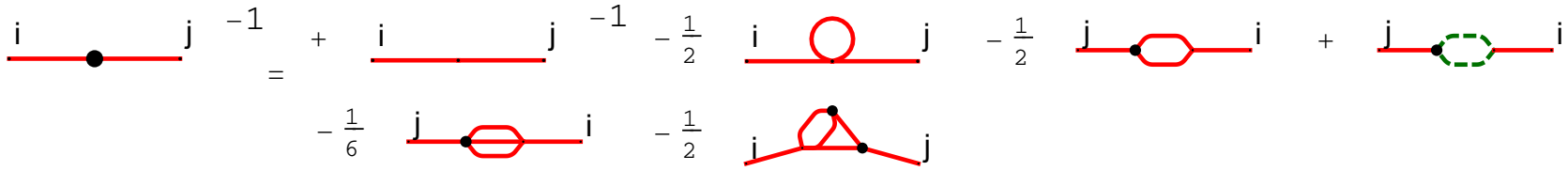
$$\alpha_{4\mathrm{g}}(p^2) = \frac{g^2}{4\pi} D^{AAAA}(p^2,p^2,p^2) Z(p^2)^2$$

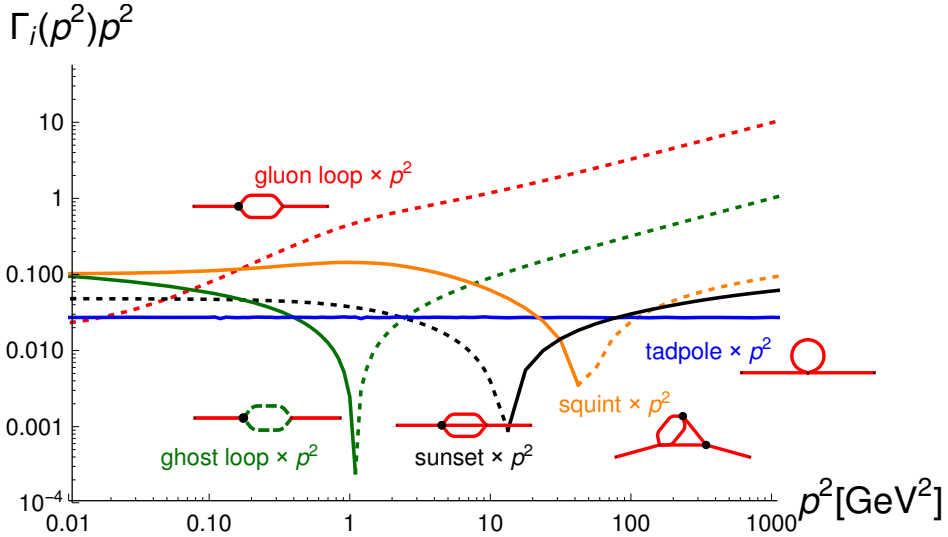


$\Rightarrow$ Good agreement down to a few GeV!

[MQH '16]

Gluon propagator: Single diagrams



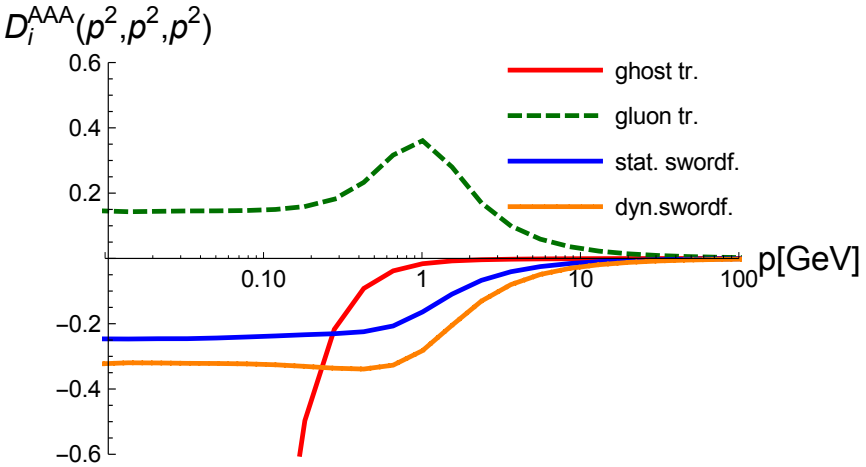


- Squint important in midmomentum regime.
- Sunset contribution small.

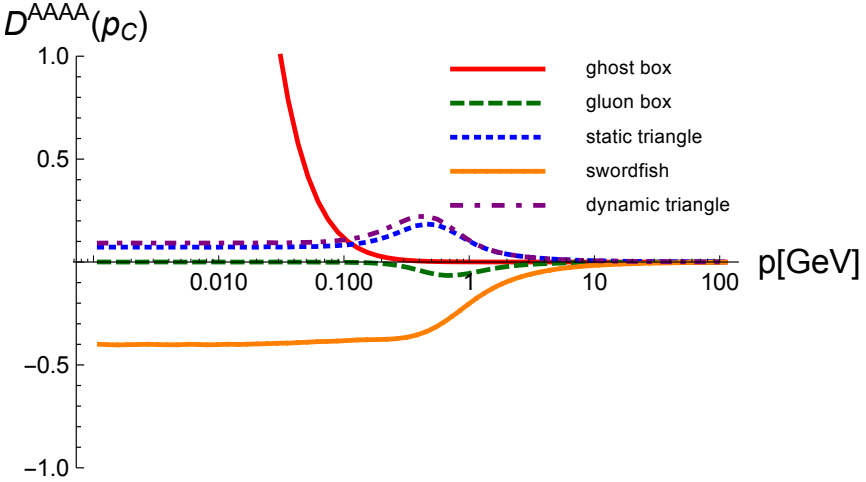
[MQH '16]

Cancellations in gluonic vertices

Three-gluon vertex:



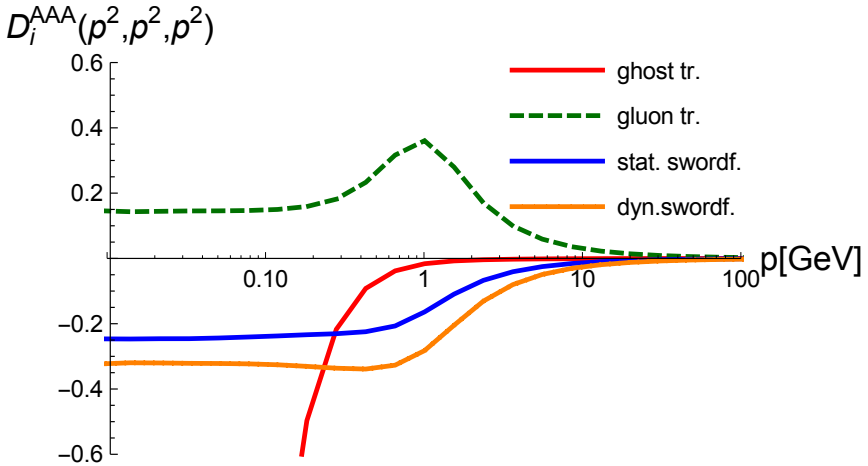
Four-gluon vertex:



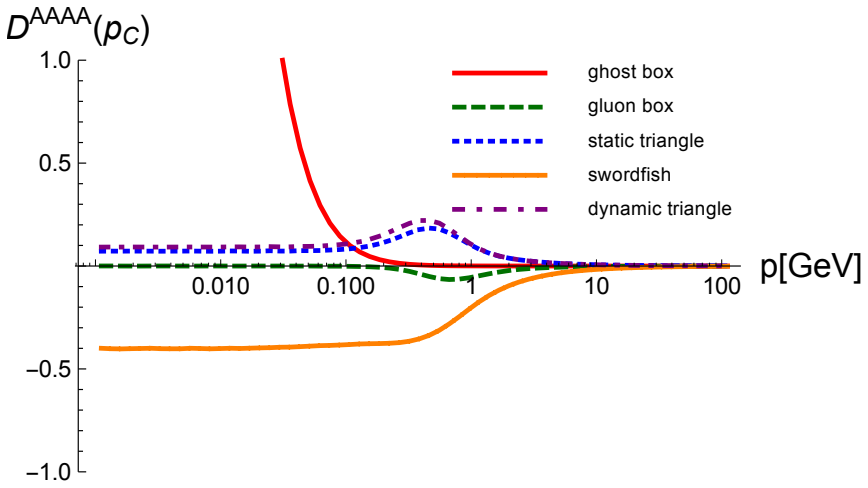
- Individual contributions large.
- Sum is small.

Cancellations in gluonic vertices

Three-gluon vertex:



Four-gluon vertex:



- Individual contributions large.
- Sum is small.

$\Downarrow$

Higher contributions:

- Higher vertices close to 'tree-level'? $\rightarrow$ Small.
- If pattern changes (higher vertices large): cancellations required.

[MQH '16]

Comparison $d = 3$ and $d = 4$

- Two-loop diagrams important in gluon propagator.

  [Blum, MQH, Mitter, von Smekal '14; Meyers, Swanson '14]

- Two-loop diagrams not important in three-gluon vertex.

  [Blum, MQH, Mitter, von Smekal '14; Eichmann, Williams, Alkofer, Vujinovic '14]

- Vertices deviate only mildly from tree-level above 1 GeV.

  [Blum, MQH, Mitter, von Smekal '14; Eichmann, Williams, Alkofer, Vujinovic '14; Binosi, Ibanez, Papavassiliou '14; Cyrol, MQH, von Smekal '14]

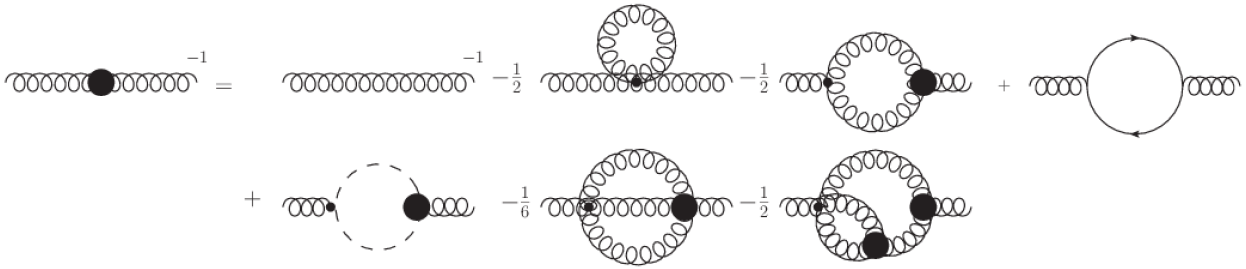
- RG improvement irrelevant in $d = 3$. Role in $d = 4$?

  [Eichmann, Williams, Alkofer, Vujinovic '14]

Unquenching: Propagators

Quark-gluon vertex

Model



Spurious divergences

Handle via analytically calculated subtraction term.

Quark-gluon vertex

Model [Fischer, Alkofer '03]:

$$\Gamma^a_{\mu,ij} = i\,g\,T^a_{ij}\gamma_\mu V(k^2,p^2,q^2)W(k^2,p^2,q^2)$$

$$W(k^2,p^2,q^2) = G(f(x,y,z))^2,\, V(k^2,p^2,q^2) = \frac{A(p^2)+A(q^2)}{2}$$

Momentum arguments of W different for quark and gluon DSEs.

Quark-gluon vertex

Model [Fischer, Alkofer '03]:

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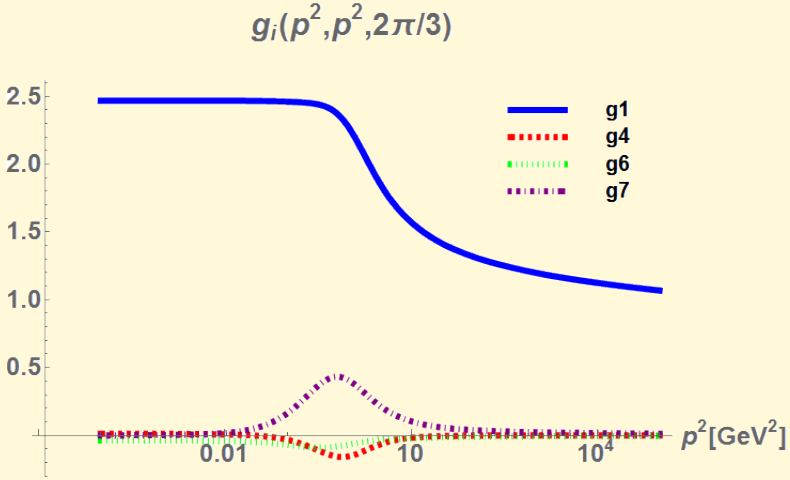
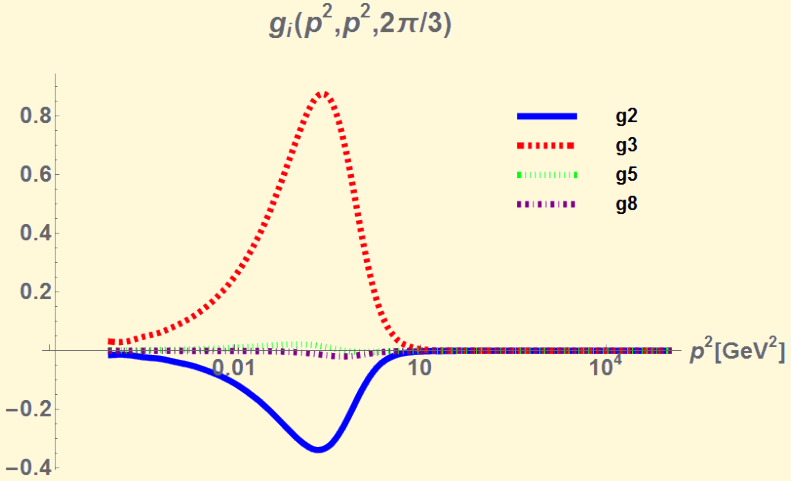
Momentum arguments of W different for quark and gluon DSEs.

Cf. quark-gluon vertex solution

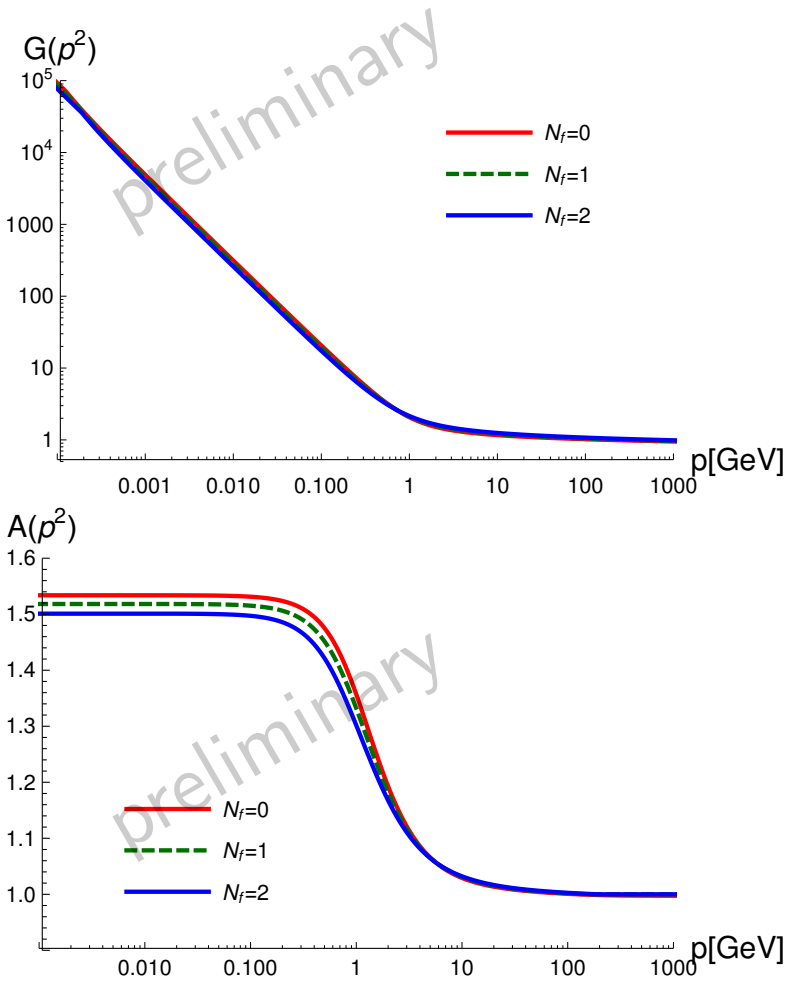
[Hopfer '14; Windisch '14; Mitter, Pawlowski, Strodthoff '14; Williams, Fischer, Heupel '15]

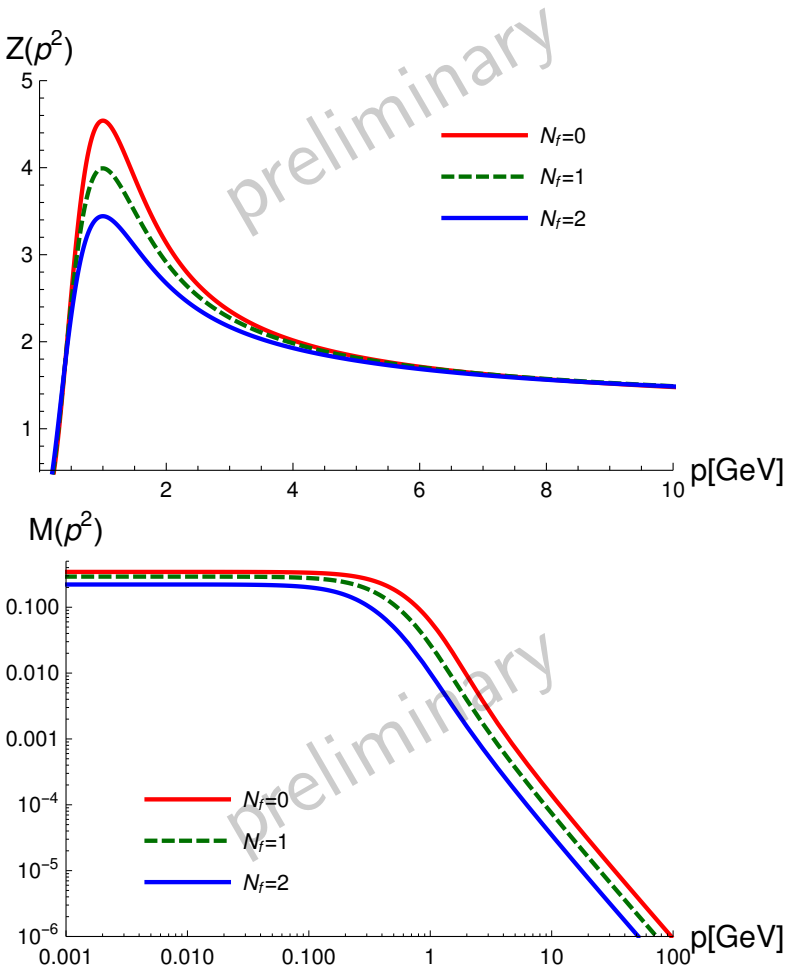




[Blum, Alkofer, MQH, Windisch '15]

Results for propagators



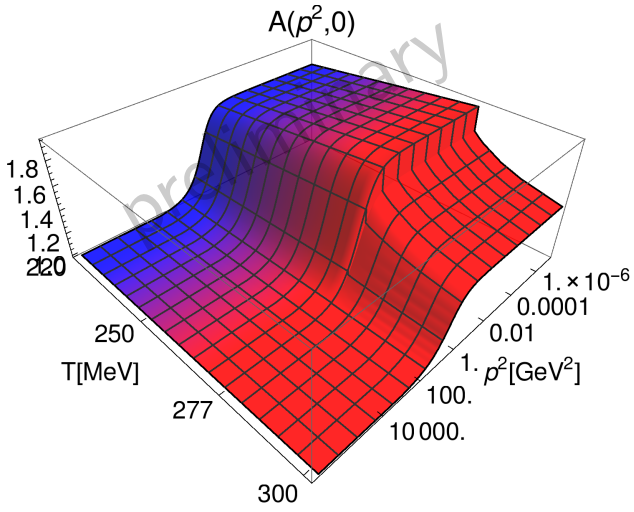


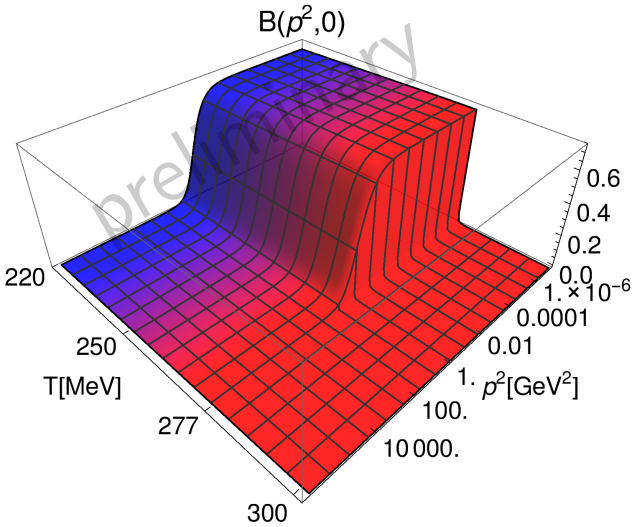
[Contant, MQH, unpublished]

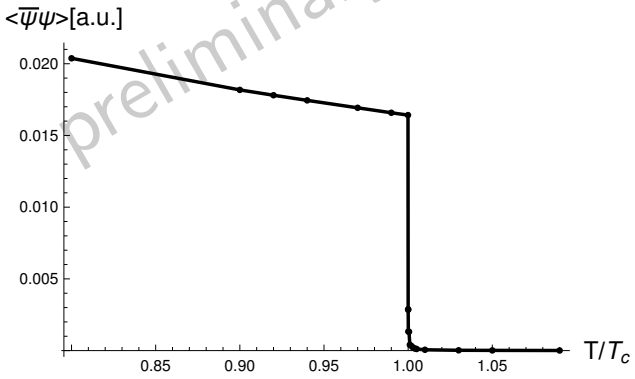
Quenched quark propagator at non-zero temperatures

Input

- Model for quark-gluon vertex [Fischer, Müller '09]
- Fits from lattice gluon propagators [Fischer, Maas, Müller '10; Maas, Pawlowski, von Smekal, Spielmann '10]
  $\rightarrow$ Sets the scale and also determines the position of the phase transition







[Contant, MQH, unpublished]

Summary and conclusions

- Calculations of propagators, vertices and partially mixed systems show a coherent picture.
- Gluon propagator: basic quantity, still challenging: spurious divergences, RG resummation.
- Lessons from $d = 3$:
  - cancellations between diagrams
  - small deviations from tree-level for higher correlations
  - RG resummation
- Yang-Mills vertices 'simple', quark-gluon vertex 'complicated'

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Thank you for your attention.