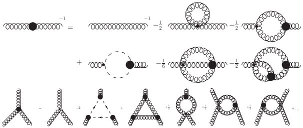
With functional methods from propagators and vertices to glueballs



10th International Conference on Exact Renormalization Group 2020

Kyoto, Japan

Markus Q. Huber

Institute of Theoretical Physics, Giessen University

November 4, 2020

M. Q. Huber, Phys. Rev. D 101 [arXiv:2003.13703]

M. Q. Huber, C. S. Fischer, H. Sanchis-Alepuz, arXiv:2004.00415

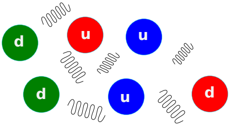
FWF Der Wissenschaftsfonds.

JUSTUS-LIEBIG-UNIVERSITÄT GIESSEN

DFG Deutsche Forschungsgemeinschaft

Bound states of QCD

QCD Lagrangian: Quarks and gluons



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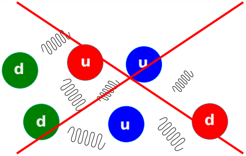
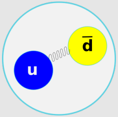
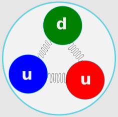


Hadrons

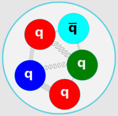
Mesons



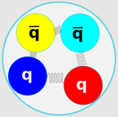
Baryons



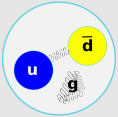
Pentaquarks



Tetraquarks



Hybrids

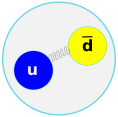


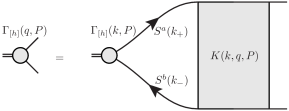
Glueballs

Calculate their properties?

Hadrons from bound state equations

Example: Meson



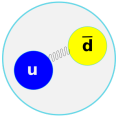


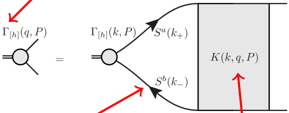
Integral equation: $\Gamma(q,P) = \int dk\,\Gamma(k,P)\,S(k_+)\,S(k_-)\,K(k,q,P)$

Hadrons from bound state equations

Bethe-Salpeter amplitude

Example: Meson





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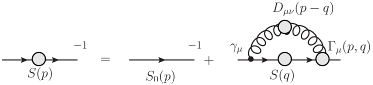
Ingredients:

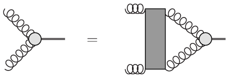
- Quark propagator S



Nonperturbative diagram: full momentum dependent dressings $\rightarrow$ numerical solution

- Interaction kernel K
- Constrained by symmetries

Glueballs as bound states



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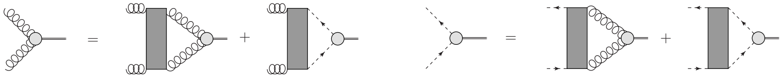

Gluons couple to ghosts $\rightarrow$ Include 'ghostball'-part.

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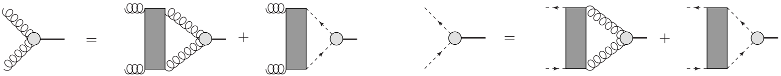

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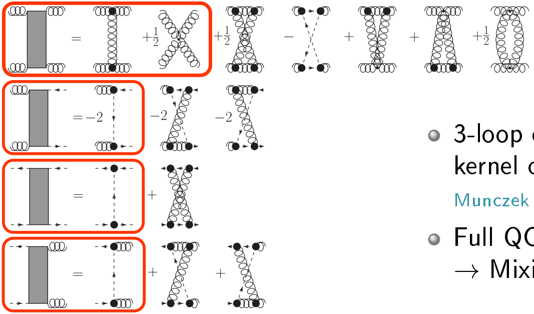
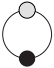
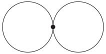
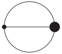
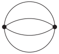
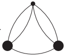
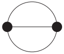
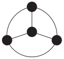
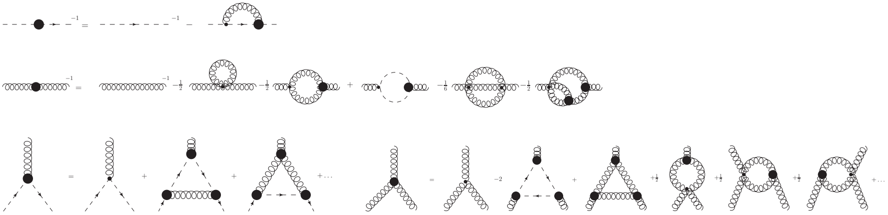


- 3-loop expanded 3PI effective action for kernel construction [Fukuda '87; McKay, Munczek '89; Sanchis-Alepuz, Williams '15]
- Full QCD: Same for quarks $\rightarrow$ Mixing with mesons.

Functional methods

Functional RG equations (FRGEs)	m-loop expanded nPI effective action
effective average action $\Gamma^k[\phi]$	nPI effective action $\Gamma[\phi, D, R^{(3)}, \ldots, R^{(n)}]$
infinitely many equations	finite number of equations
$k\frac{\partial}{\partial k}\Gamma^k[\phi] =$ 	$\Gamma_2^0[\Phi, D, \Gamma^{(3)}] = \frac{1}{8}$  $+\frac{1}{6}$  $+\frac{1}{48}$  $+\frac{1}{8}$  $\Gamma_2^{\text{int}}[D, \Gamma^{(3)}] = -\frac{1}{12}$  $+\frac{1}{24}$ 
1-loop	1- & 2-loop (equations of motion)
	Similarities to Dyson-Schwinger equations (1PI) (resummations)

Equations of motion from 3-loop 3PI effective action



Four-gluon vertex: bare

Self-contained system of equations with the scale as the only input.

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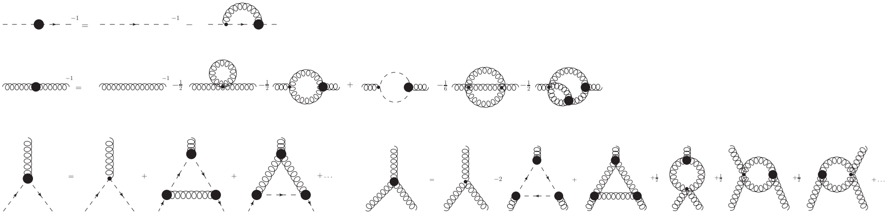

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Self-contained system of equations with the scale as the only input.

Some nontrivialities:

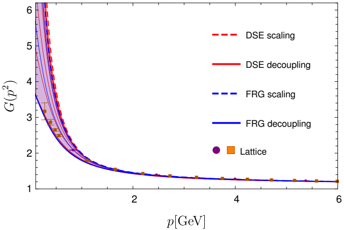
- Renormalization incl. quadratic divergences
- Slavnov-Taylor identities
- Momentum dependence of vertices

Nontrivialities of quark sector

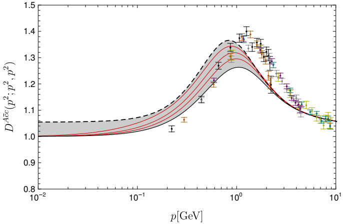
$\rightarrow$ Talk by R. Alkofer

Comparison with other results

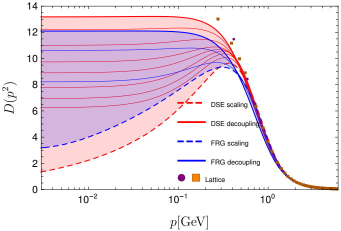
Ghost dressing function:



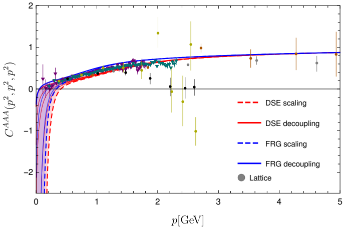
Ghost-gluon vertex dressing function:



Gluon propagator:



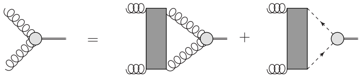
Three-gluon vertex dressing function:

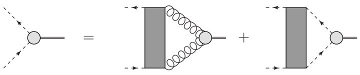


- Lattice: [Sternbeck '06; Cucchieri, Maas, Mendes '08; Sternbeck et al. '17; Maas '19]
- functional: FRG [Cyrol et al. '16], 3PI [MQH '20]

Solving the bound state equation

'2-gluon'-component: Bound state equation analogous to mesons.





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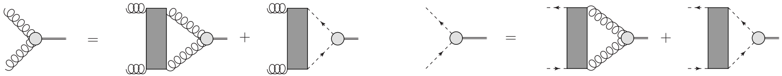

- Solution procedure: Solve eigenvalue problem

$$\mathcal{K} \cdot \Gamma(P,p) = \lambda(P)\,\Gamma(P,p) \qquad \lambda(P) = 1 \to P^2 = -M^2$$

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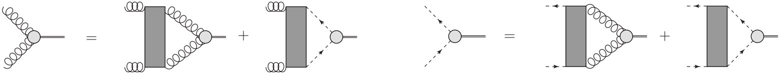

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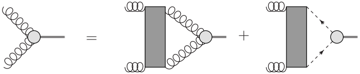

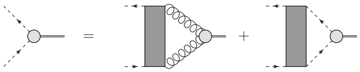

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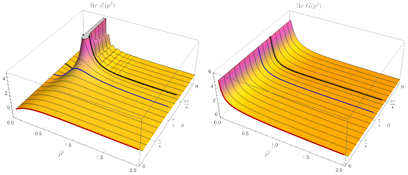

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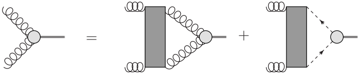

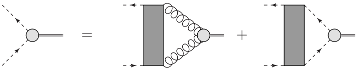

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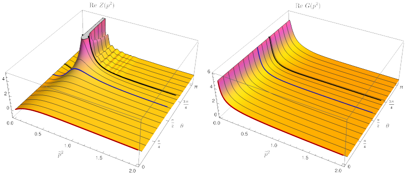

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- Alternative: Analytically continue $\lambda(P)$.

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1. Calculation of eigenvalues for $P^2 \in [10^{-4}, 0.25]\,\mathrm{GeV}^2$.

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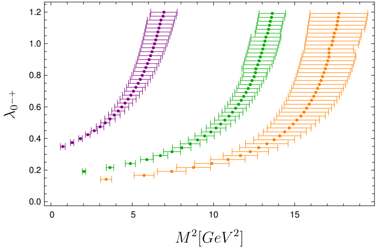

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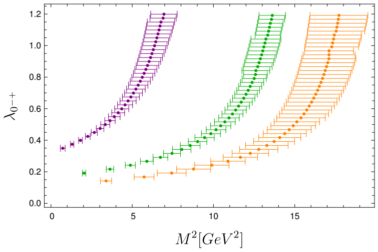
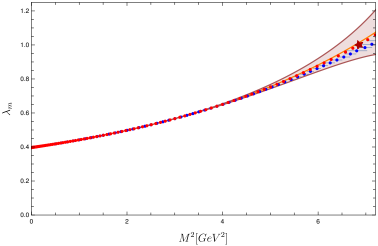


Meson:



Results

Masses of 0^{PC} glueballs in MeV:

State	[Morningstar, Peardon, 1999]	[Chen et al., 2005]	[Athenodorou, Teper, 2020]	[Huber, Sanchis-Alepuz, Fischer, 2020]
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0^{*-+}	3710(60)	–	3540(80)	3870(120)
0^{**-+}	–	–	$4450(140)^{\dagger}$ $4540(120)^{\dagger}$	4340(200)

All results for $r_0 = 1/418(5)$ MeV.
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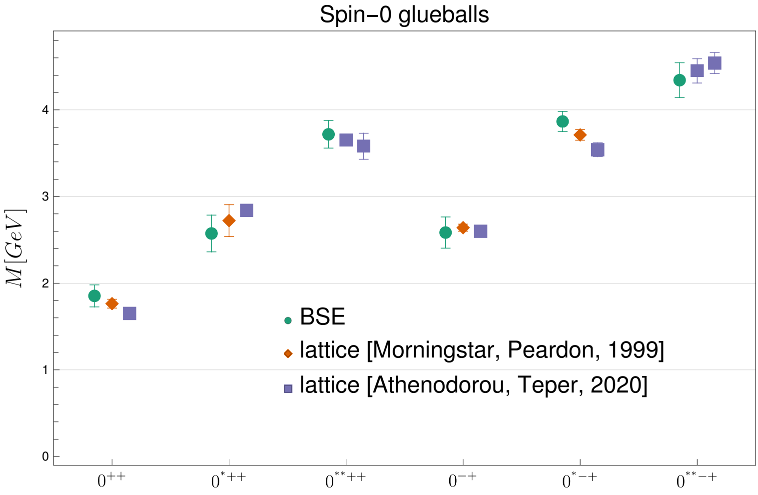

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- Possible extensions: Dynamic quarks, higher spin glueballs

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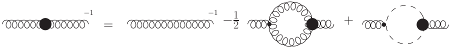
Thank you for your attention!

Complex plane

Simpler truncation: 

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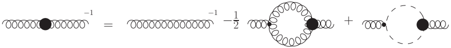
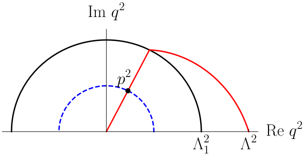


Integration in complex plane:

$$\int_\Lambda \frac{d^d q}{(2\pi)^d} \frac{1}{q^2 (p+q)^2} \rightarrow \int_0^{\Lambda^2} dq^2\, q^{d-4} \int_0^\pi d\theta \frac{(\sin\theta)^{d-2}}{p^2 + q^2 + 2\sqrt{p^2 q^2}\cos\theta}$$

Integration in θ creates cuts at $q^2 = p^2 e^{\pm 2 i \theta}$.



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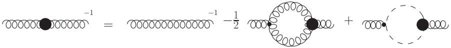

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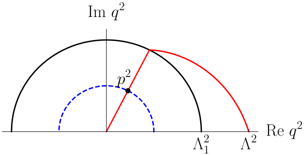

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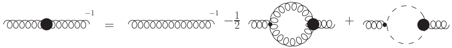
Applications of contour deformation: QED3 [Maris '95], quark propagator [Alkofer et al. '04], Yang-Mills propagators [Strauss, Fischer, Kellermann '12; Fischer, MQH '20], glueball correlators [Windisch, MQH, Alkofer '12], meson decays [Weil et al. '17; Williams '18], quark-photon vertex [Miramontes, Sanchis-Alepuz '19], finite-T spectral functions of $O(N)$ model: [Pawlowski,Strodthoff, Wink '18], scattering amplitudes scalar theory [Eichmann et al. '19]

Landau gauge Yang-Mill propagators in the complex plane

Simpler truncation:



Propagators in the complex plane: $p^2 = \widetilde{p}^2 e^{i\,\theta}$ [Fischer, MQH '20]

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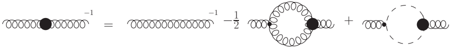
Simpler truncation: 

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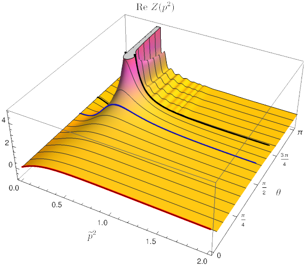
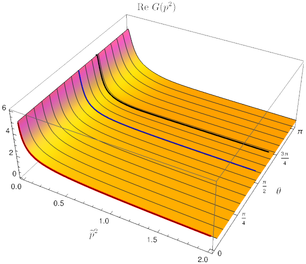


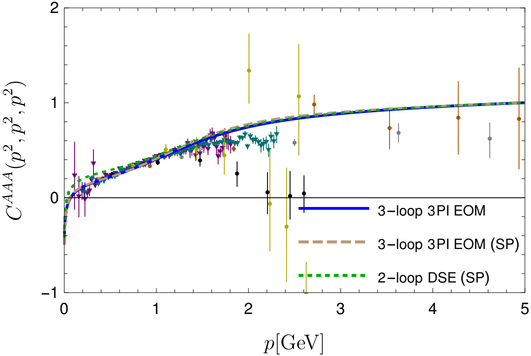


- Method works,
- but current truncation leads to a pole-like structure in the gluon propagator.
- Analyticity up to 'pole' confirmed by various tests.
- Warning: No proof of existence of complex conjugate poles.

DSE vs. 3PI

Three-gluon vertex:

- DSE: 1- & 2-loop
- 3-loop 3PI: 1-loop



[Cucchieri et al. '08; Sternbeck et al. '17; MQH '20]

$\rightarrow$ Use 3PI because it is simpler.

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Hadron masses from correlation functions of color singlet operators.

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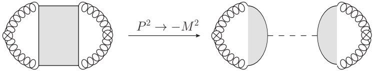
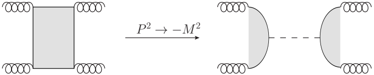
A^4-part of $D(x-y)$, total momentum on-shell:



For bound state equations, consider general four-point function:
$\rightarrow$ Bethe-Salpeter wave functions



Note: '2-gluon'-component sufficient to calculate mass for $J^{PC} = 0^{\pm +}$.