The dual superconductor picture of confinement and its relation to other confinement scenarios

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Research Training Group
Quantum and Gravitational Fields

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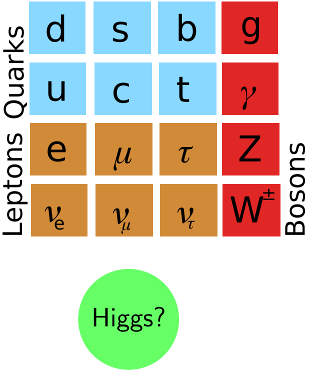
Our view of the world in terms of particles

The standard model:



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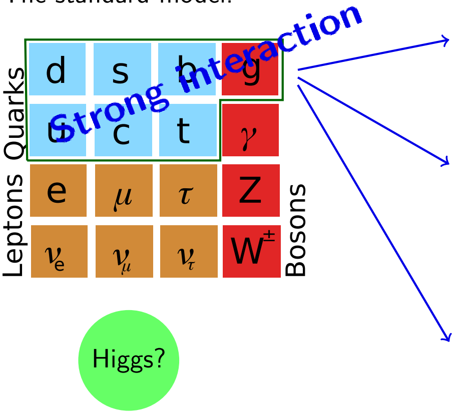
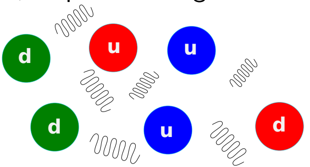
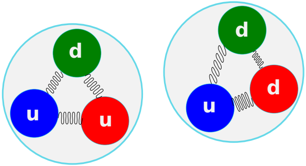


fundamental degrees of freedom:
quarks and gluons, # few
u, d quarks are light $\sim$ MeV



physical degrees of freedom:
hadrons, # many: p, n, $\pi^{0,\pm}$, $K^{0,\pm}$, . . . ,
hadrons are heavy $\gtrsim$ 140 MeV



Hadrons are bound states of quarks,
but they cannot be split into their building blocks.

Confinement

"No one has ever detected a free quark or gluon."

"Absence of free quarks and gluons."

"Quarks and gluons are not part of the physical state space."

We observe hadrons but
use (unobservable) quarks and gluons as elementary fields.

Inelastic scattering:

Atoms $\longrightarrow$ constituents
Hadrons $\longrightarrow$ more hadrons

Without understanding confinement
we do not fully understand hadron physics.

Several confinement scenarios/mechanisms

View the problem from different perspectives $\longrightarrow$ not mutually exclusive but different aspects emphasized.

see e.g. [Alkofer, Greensite, JPG34] for a short review

A few confinement scenarios and approaches

- Magnetic monopoles and dual superconductivity:
  ['t Hooft, Mandelstam, di Giacomo, Suzuki, ...]
- Gribov-Zwanziger scenario: role of Gribov copies?
  [Gribov, Zwanziger, Sorella, Dudal, ...]
- Center vortices: topological objects
  [Faber, Greensite, Langfeld, Olejník, ...]
- Coulomb gauge:
  [Reinhardt, Watson, Zwanziger, ...]
- Landau gauge Green functions:
  [Alkofer, Fischer, von Smekal, Maas, Papavassiliou, Pawlowski, ...]
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Dual superconductor picture of confinement

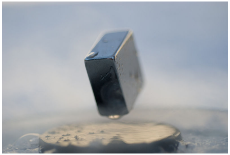
't Hooft 1976, Mandelstam 1976

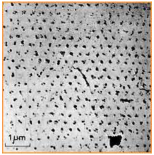
Conventional type-II superconductor:

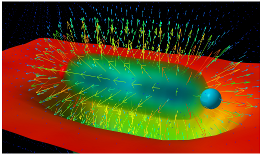
- magnetic flux squeezed into vortices
- condensation of Cooper pairs

Dual superconductor:

- electric flux squeezed into vortices
- condensation of magnetic monopoles







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Theories where confinement was proven:
compact $U(1)$, Georgi-Glashow model,
deformed $\mathcal{N} = 2$ SUSY Yang-Mills

→ In all condensation of magnetic monopoles!

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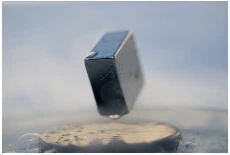
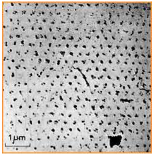
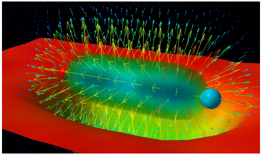
<u>Dual superconductor:</u>

- chromoelectric flux squeezed into vortices
- condensation of chromomagnetic monopoles







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Abelian infrared dominance

Hypothesis of Abelian infrared dominance [Ezawa, Iwazaki, PRD25 (1981)]:
based on dual superconductor picture

Chromomagnetic monopoles live in Abelian part of the theory.

Monopoles dominant for long-range phenomena.

Abelian part dominant in the infrared.

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Abelian part dominant in the infrared.

What is the Abelian part?

$$[A_\mu, A_\nu] = 0\,?$$

Gauge actions

Action is invariant under gauge transformations.
$\rightarrow$ (Infinitely) Many equivalent configurations.
Space of field configurations $\mathcal{A}$ is not the physical space!

Gauge transformations connect physically equivalent configurations.
$\rightarrow$ Physical space is the quotient space over the gauge group G:

$$\mathcal{A}_{phys} = \mathcal{A}/G$$

Realization: Fix a gauge.

Gauge fixing

Restriction to a hypersurface $\Gamma : f[A] = 0$ (gauge fixing condition, e.g., $\partial A = 0$)

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Freedom to choose any gauge!

- Perturbation theory: Feynman gauge convenient, propagator simple ($g_{\mu\nu}/p^2$)
- Dyson-Schwinger equations: Landau gauge, simplification of calculations. Helped us learn the method.
- Deep inelastic scattering: light-cone gauge
- Dual superconductor scenario: Abelian gauges

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(All these gauges are non-perturbatively not complete:
There are still equivalent configurations on Γ. $\rightarrow$ Gribov problem
Physical space $\mathcal{A}_{phys}$ is topological not trivial!

Theorem of Singer 1978:
No unique gauge fixing with a continuous gauge fixing condition.)

The maximally Abelian gauge I

Dual superconductor picture based on Abelian symmetry.

What is it for $SU(3)$?

Approach here:
Split the gauge field into diagonal and off-diagonal parts.

The maximally Abelian gauge II

- The gauge field lives in the algebra $su(3)$: $A_\mu = A_\mu^r T^r$
  (T^r: generators, Gellmann matrices)
- Some of them (λ_3, λ_8) are diagonal.
  $\rightarrow$ Form maximal Abelian subalgebra (Cartan subalgebra).
  $\rightarrow$ "Abelian"/diagonal part: $\boldsymbol{A_\mu^3}$, $\boldsymbol{A_\mu^8}$
- Consider the two parts separately
  $\rightarrow$ diagonal and off-diagonal fields.

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- Consider the two parts separately
  $\rightarrow$ diagonal and off-diagonal fields.
- We use the gauge freedom to choose minimized off-diagonal fields.

$$\Rightarrow \boldsymbol{D_\mu A_\mu} = 0$$

$\rightarrow$ maximally Abelian gauge (MAG)

(Compare Landau gauge: $\partial_\mu A_\mu = 0$)

The maximally Abelian gauge III

Indication of Abelian infrared dominance:

$\longrightarrow$ String tension of quark potential stays almost the same when calculated from diagonal fields only.

[e.g. Suzuki, Yotsuyanagi, PRD42].

Note:

Here the off-diagonal part is kept!

$\longrightarrow$ Investigation of the full theory!

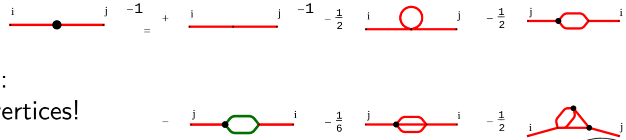
Functional equations

Green functions:

Propagators and vertices $\leftrightarrow$ describe how fields propagate and interact.

Exact relations between Green functions given by, e.g., Dyson-Schwinger and functional renormalization group equations.

DSE for the gluon propagator (Landau gauge):



Valid **non-perturbatively**:
dressed propagators and vertices!

Infinite tower of coupled equations:
every eq. contains higher Green functions.

Applications of functional equations

- condensed matter
- Yang-Mills theories
- QCD and QCD-like theories
- standard model physics
- supersymmetry
- gravity
- ...

$\longrightarrow$ Wide variety of theories.

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- Yang-Mills theories $\longrightarrow$ Investigate Abelian IR dominance with MAG.
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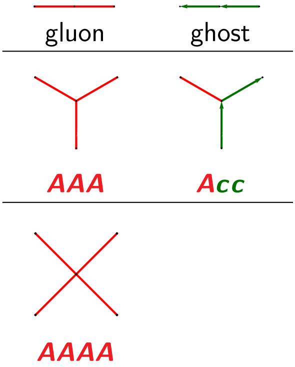
Lagrangian for Yang-Mills theory in the max. Abelian gauge

First example: Landau gauge



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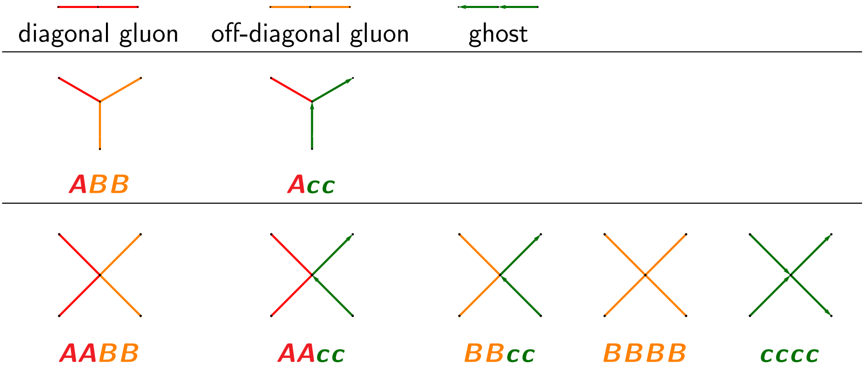

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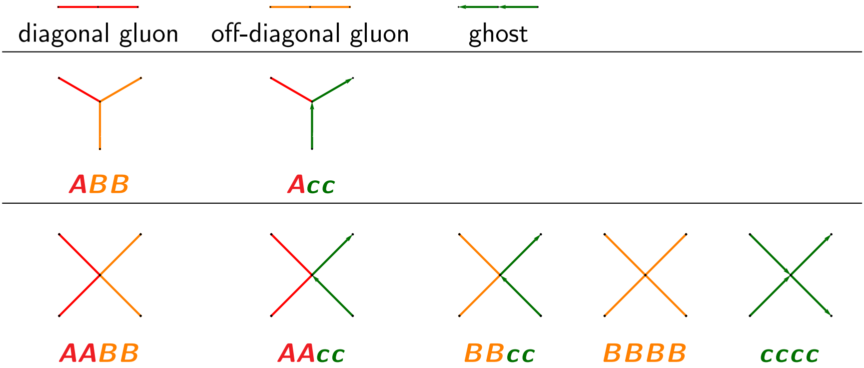

$SU(N > 2)$

BBB Bcc $ABBB$ $ABcc$

Dyson-Schwinger equations of the maximally Abelian gauge

High number of interactions $\rightarrow$ automated derivation very useful.

$\rightarrow$ *Mathematica* application *DoFun* (Derivation of functional eqs.)
[Alkofer, MQH, Schwenzer, CPC180; MQH, Braun, arXiv:1102.5307]

$\downarrow \quad \downarrow \quad \downarrow$

Derivation of full Dyson-Schwinger and functional RG equations.
$\rightarrow$ More complicated theories become accessible,
e.g., Gribov-Zwanziger action [MQH, Alkofer, Sorella, PRD81]

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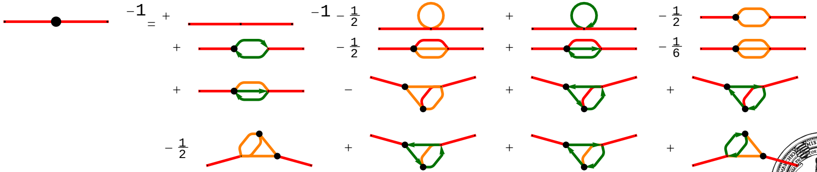
For example: Diagonal gluon



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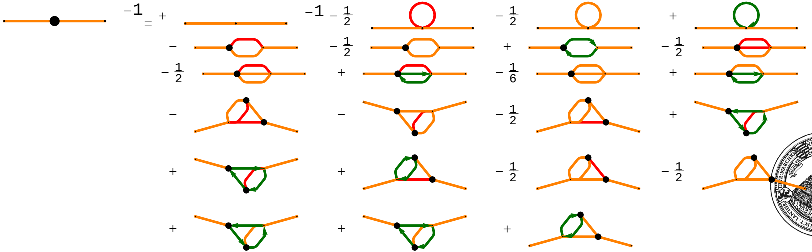
For example: Off-diagonal gluon



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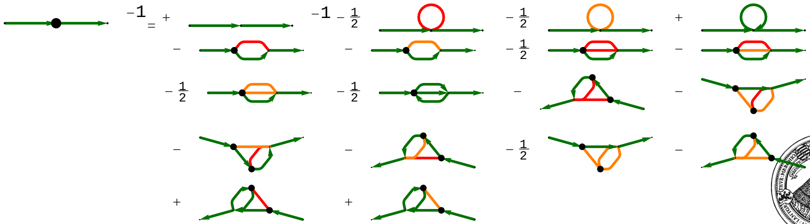
For example: Ghost



Infrared solution for the maximally Abelian gauge I

Infinite towers of functional equations.

But: In the IR the untruncated towers can be solved qualitatively.

[MQH, Schwenzer, Alkofer, EPJC68]

Scaling-type solution:
All dressing functions characterized by power laws in the IR:

$$c_i(p_1^2, p_2^2, \ldots) \stackrel{IR}{=} d_i \cdot (p^2)^{\kappa_i} \qquad \text{(common momentum scale } p\text{)}$$

The exponents κ_i are uniquely related!

Numerical values for the κ_i can be calculated.

Infrared solution for the maximally Abelian gauge II

Qualitative behavior of propagators at low momentum p ($\kappa_{MAG} \geq 0$):

canonical dim.

- Off-diagonal gluon propagator is IR suppressed $\sim (p^2)^{\kappa_{MAG}-1}$.
- Ghost propagator (off-diag.) is IR suppressed $\sim (p^2)^{\kappa_{MAG}-1}$.
- Diagonal gluon propagator is IR enhanced $\sim (p^2)^{-\kappa_{MAG}-1}$.
  $\rightarrow$ Diagrams with most diagonal gluons dominate in DSEs.
- Qualitative solution for the whole tower of Green functions.

$\rightarrow$ IR dominance of diagonal/"Abelian" degrees of freedom.

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($SU(N > 2)$ has more interactions than $SU(2)$,
but the IR solution is the same.)

Value of the infrared exponent κ_{MAG}

Solution for κ_{MAG} is <u>necessary but not sufficient</u>.

Dressing functions of gluons and ghosts:

$$c_A(p^2) \overset{p^2\to 0}{=} d_A \cdot (p^2)^{-\kappa_{MAG}} \qquad c_c(p^2) \overset{p^2\to 0}{=} d_c \cdot (p^2)^{\kappa_{MAG}}$$

$$c_B(p^2) \overset{p^2\to 0}{=} d_B \cdot (p^2)^{\kappa_{MAG}} \qquad 0 \le \kappa_{MAG} \le 1$$

$$\kappa_{MAG} \approx 0.74$$

$\longrightarrow$ Infrared consistent solution exists.

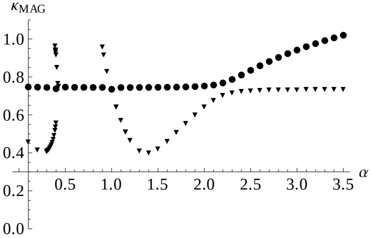
Gauge fixing condition:
$\boldsymbol{D_\mu A_\mu} = 0 \leftrightarrow$ gauge fixing parameter α_{MAG}, MAG: $\alpha_{MAG} = 0$

(cf. linear covariant gauges: $\partial_\mu A_\mu = 0 \leftrightarrow \alpha_{LCG}$,
Landau gauge: $\alpha_{LCG} = 0$)

Gauge fixing parameter dependence

Extension to non-zero α_{MAG} easily possible,
not easy for Landau gauge and linear covariant gauges.

Two solution branches:

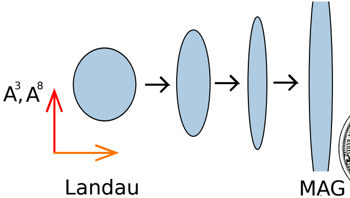


General linear covariant gauges: different from the Landau gauge
$\rightarrow$ The maximally Abelian is the first case where an IR solution is found that seems independent of the gauge fixing parameter.

The maximally Abelian and the Landau gauge

Structures of equations considerably different in the two gauges.

	Landau gauge (ghost dominance)	Maximally Abelian gauge (Abelian dominance)
dominant configurations	Gribov horizon ($\to$ ghosts)	diagonal



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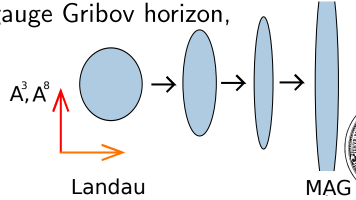
Abelian configurations $\xrightarrow{\text{gauge transf.}}$ Landau gauge Gribov horizon,
i.e., both on same gauge orbit.

[Greensite, Olejník, Zwanziger, PRD69 (2004)]



Confinement scenarios and approaches: a sketch

Dual superconductor & monopoles

...

Abelian infrared dominance

Center vortices

Gribov-Zwanziger

Maximally Abelian gauge

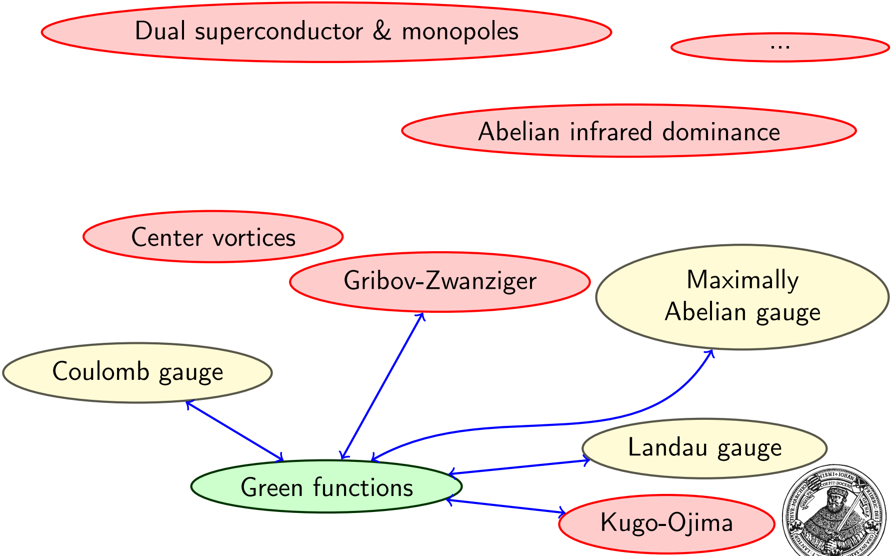
Coulomb gauge

Landau gauge

Green functions

Kugo-Ojima

Confinement scenarios and approaches: a sketch



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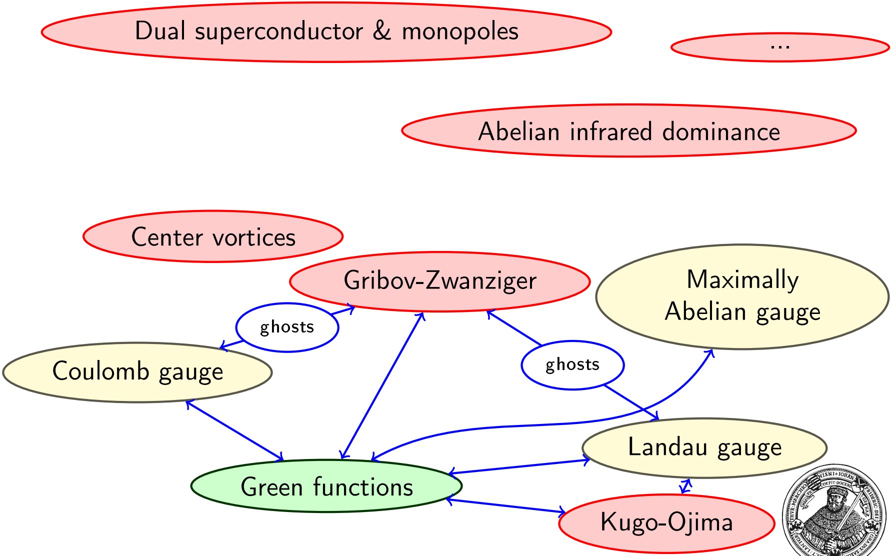

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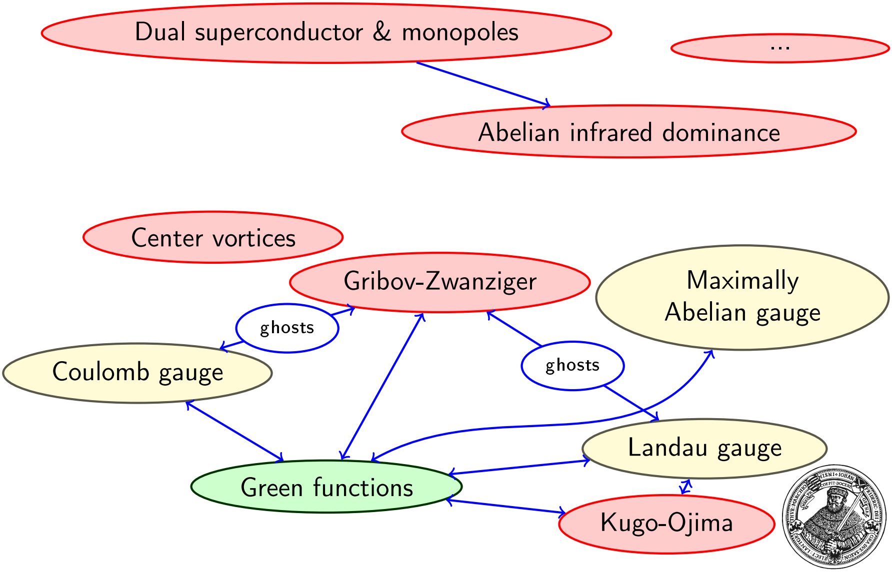
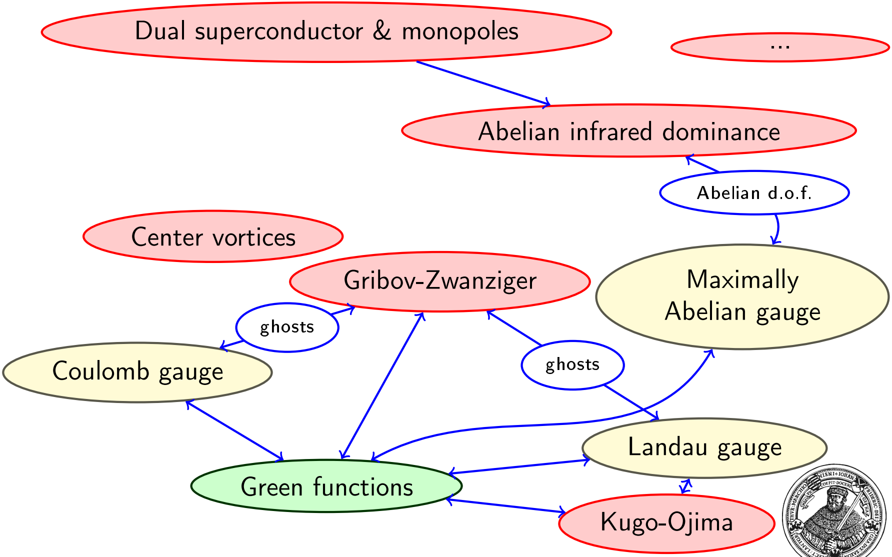


Confinement scenarios and approaches: a sketch



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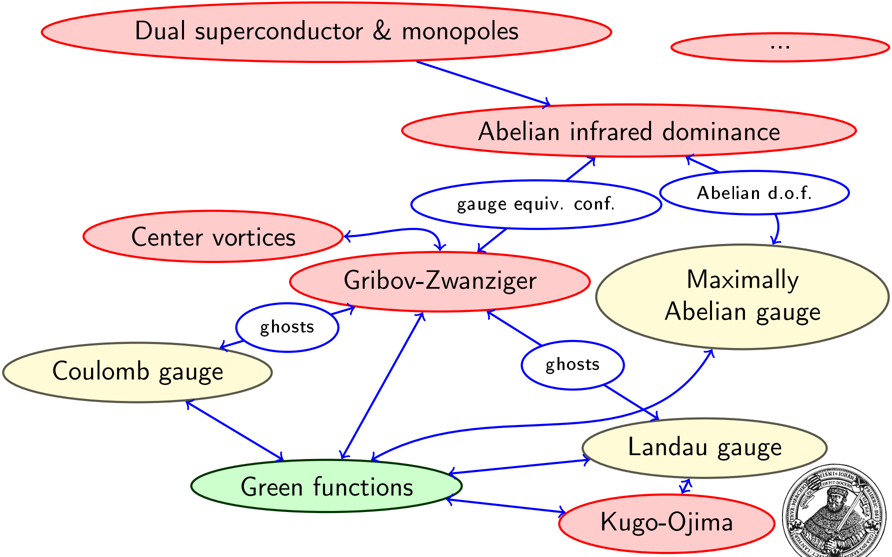

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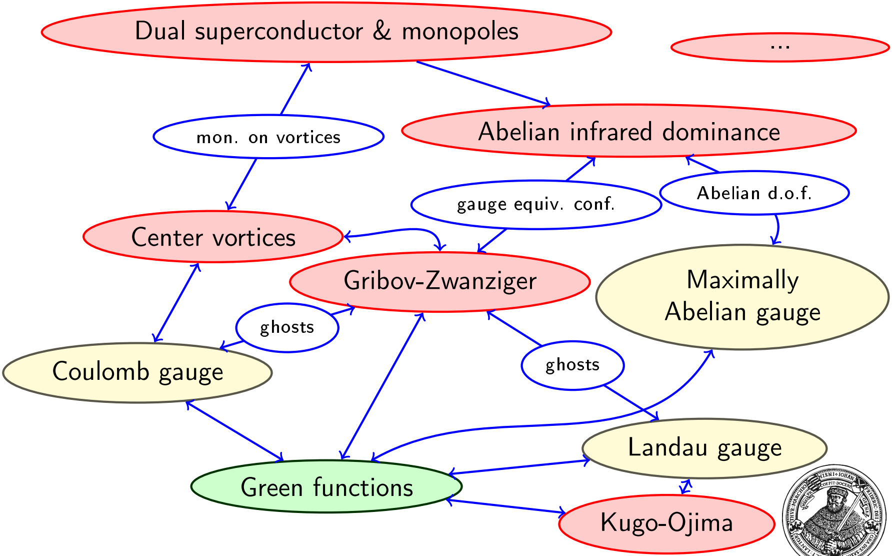

Summary

Understanding of confinement increases:
We put together the jigsaw pieces,
but the big picture needs to be finished.

- Maximally Abelian gauge is the second gauge after the Landau gauge where a qualitative solution for the whole tower of Green functions is known.
- Relation to Landau gauge partly understood:
  ghost $\leftrightarrow$ Abelian dominance
- Maximally Abelian gauge suited for investigation of Abelian IR dominance.
- Results corroborate this scenario, applicable for family of gauges (α_{MAG}).

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