Non-perturbative analysis of the Gribov-Zwanziger action

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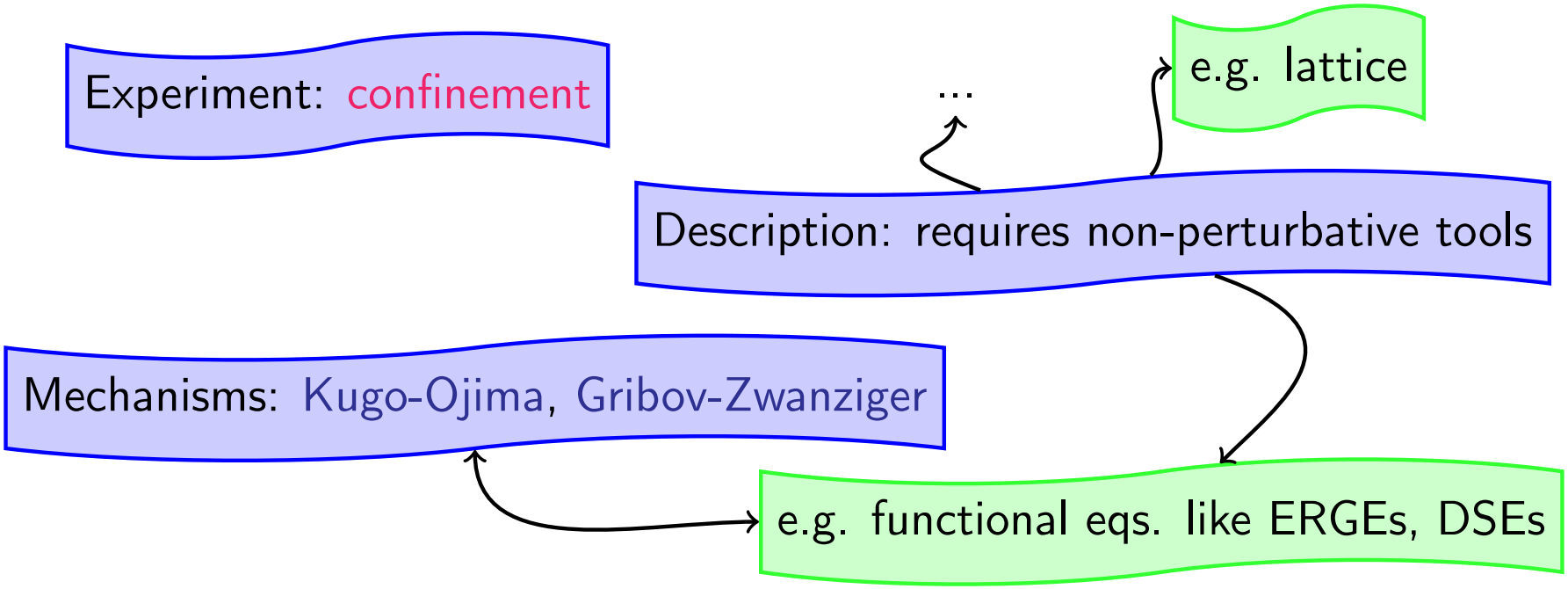
Quark Confinement and the Hadron Spectrum IX, Madrid

Confinement and the Gribov problem



Gribov-Zwanziger

- Gribov problem:
  conventional gauge fixing insufficient for non-perturbative regime
- restriction to Gribov region Ω in Landau gauge:
  - Gribov-Zwanziger action
  - IR is dominated by configurations close to Gribov horizon $\partial\Omega$

Gribov-Zwanziger action

- Landau gauge + restriction of integration in the path integral to the first Gribov region [Zwanziger, NPB323]
- confined gluon at tree-level: vanishes at $p = 0$ like p^2

  $\Rightarrow$ maximal violation of positivity
- horizon condition [Zwanziger, NBP399]

  $\Rightarrow$ IR enhanced ghost propagator, 1-loop: $1/p^4$

Cf. functional equations:

- Faddeev-Popov action (no reference to the Gribov problem)
- boundary condition required

  [Zwanziger, PRD65; Fischer, Maas, Pawlowski, AP324]:

  e.g., horizon condition or the Kugo-Ojima criterion
- scaling solution

  (qualitatively the same, but non-integer exponents for dressings)

  or
- decoupling solution

Gribov-Zwanziger action

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Content of this talk:

- Use functional equations to investigate the Gribov-Zwanziger action.

$\Rightarrow$ Explicit restriction to the Gribov region.

- scaling solution

(qualitatively the same, but non-integer exponents for dressings)

or

- decoupling solution

Dyson-Schwinger equations in the Gribov region

Zwanziger, PRD65,69:

Faddeev-Popov action can be used for DSEs, but boundary condition required $\leftarrow$ horizon condition.

Derivation of DSEs:
Integral of a total derivative vanishes:

$$\int [DA\bar{c}c]\frac{\delta}{\delta A}e^{-S+JA} = \int [DA\bar{c}c]\left(J-\frac{\delta S}{\delta A}\right)e^{-S+JA} = 0.$$

$\Rightarrow$ DSEs for all Green functions (full, connected, 1PI) by further differentiations.

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$$\int_{\Omega} [DA] \left(J - \frac{\delta S}{\delta A} \right) \delta(\partial \cdot A) \boldsymbol{det(M)} e^{-S_{YM}+JA} = 0.$$

$$det(M)\Big|_{\partial\Omega} = 0$$

Local Gribov-Zwanziger action

Add non-local horizon function h
to the Faddeev-Popov action [Zwanziger, NPB323]:

$$\mathcal{L} = \mathcal{L}_{FP} + \gamma^4 h.$$

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Localization with (anti)commuting fields $(\bar{\eta}^{ab}_\mu, \eta^{ab}_\mu)$ V^{ab}_μ:

$$\mathcal{L}_{GZ} = \mathcal{L}_{FP} - \frac{1}{2}\bar{\eta}^{ac}_\mu M^{ab} \eta^{bc}_\mu \frac{1}{2} + V^{ac}_\mu M^{ab} V^{bc}_\mu + i\, g\, \gamma^2 \sqrt{2} f^{abc} A^a_\mu V^{bc}_\mu$$

Faddeev-Popov operator: $-\partial D^{ab}$

Connection to conventional fields:

$\varphi = \frac{1}{\sqrt{2}}(U + i\,V)\,, \quad \bar{\varphi} = \frac{1}{\sqrt{2}}(U - i\,V)$

$c,\ \bar{c},\ U,\ \omega,\ \bar{\omega} \longrightarrow \eta,\ \bar{\eta}$

from scale invariance, e.g., $c \to c e^{\theta}$

Derivation of DSEs (*DoDSE*)

$\Rightarrow$ *DoDSE* [Alkofer, M.Q.H., Schwenzer, CPC 180 (2009)]

Given a structure of interactions, the DSEs are derived symbolically using *Mathematica*.

Example (Landau gauge):

- only input: interactions in Lagrangian (AA, AAA, AAAA, cc, Acc)
- Which DSE do I want?

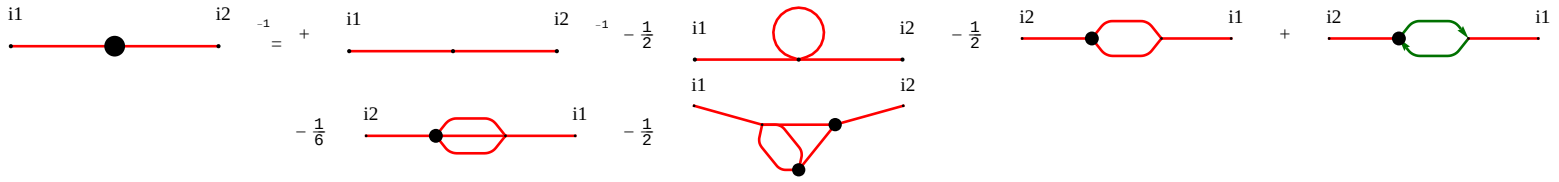
- Step-by-step calculations possible.
- Can handle mixed propagators (then there are really many diagrams; e. g. in Gribov-Zwanziger action).

Upgrade

Provide Feynman rules and get complete algebraic expressions.
$\rightarrow$ E. g. calculate color algebra with *FORM* and integrals with *C*.

Landau Gauge: Propagators

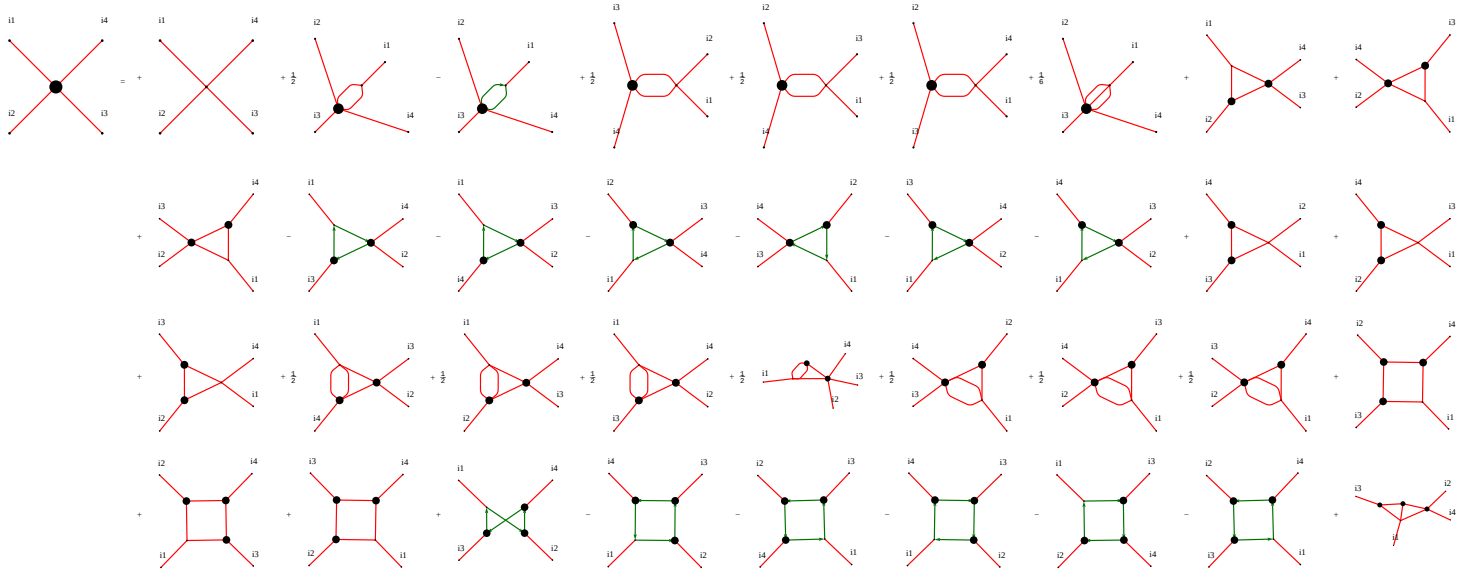
Gluon propagator:



Ghost propagator:

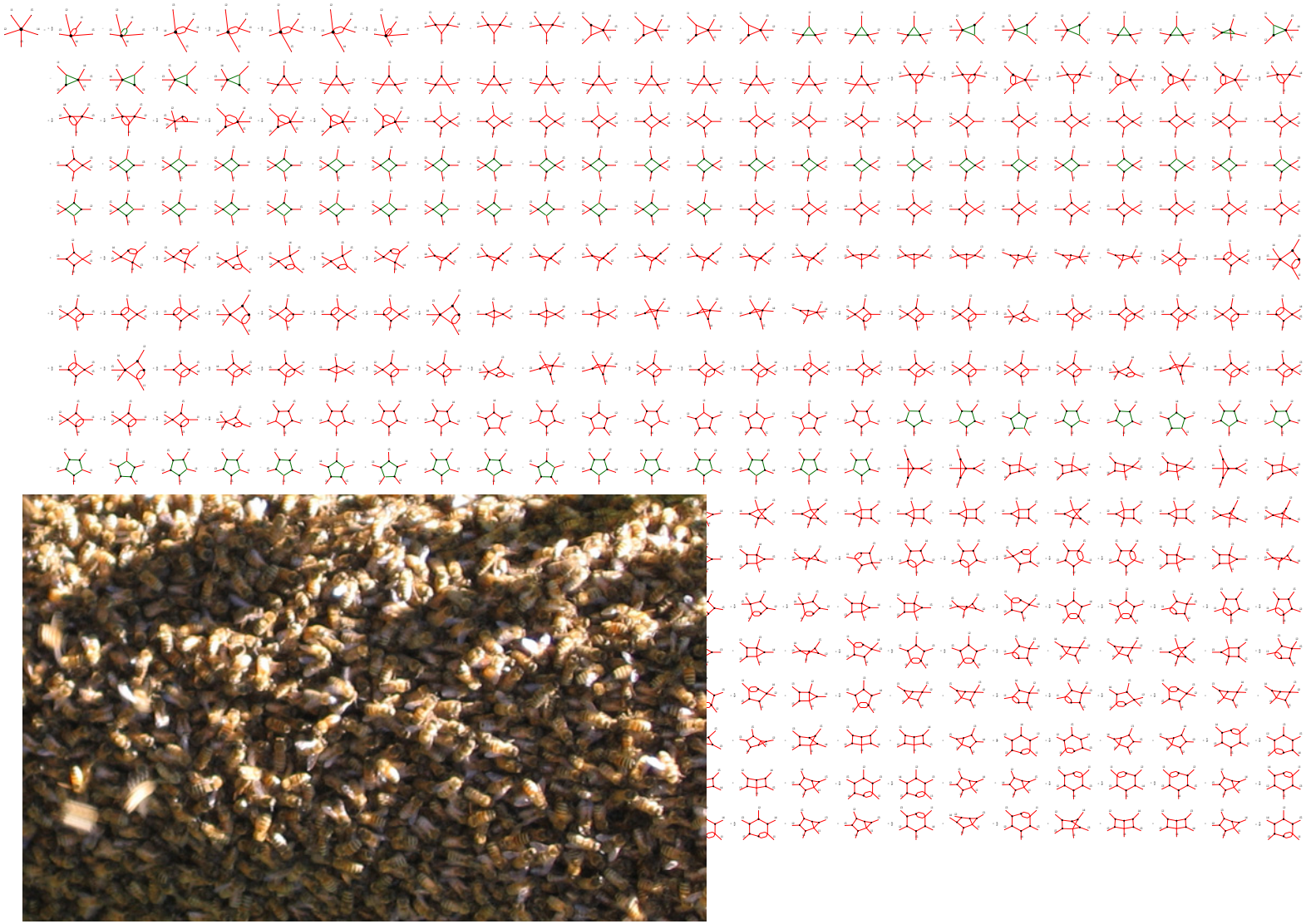
Landau Gauge: Four-Gluon Vertex

66 terms



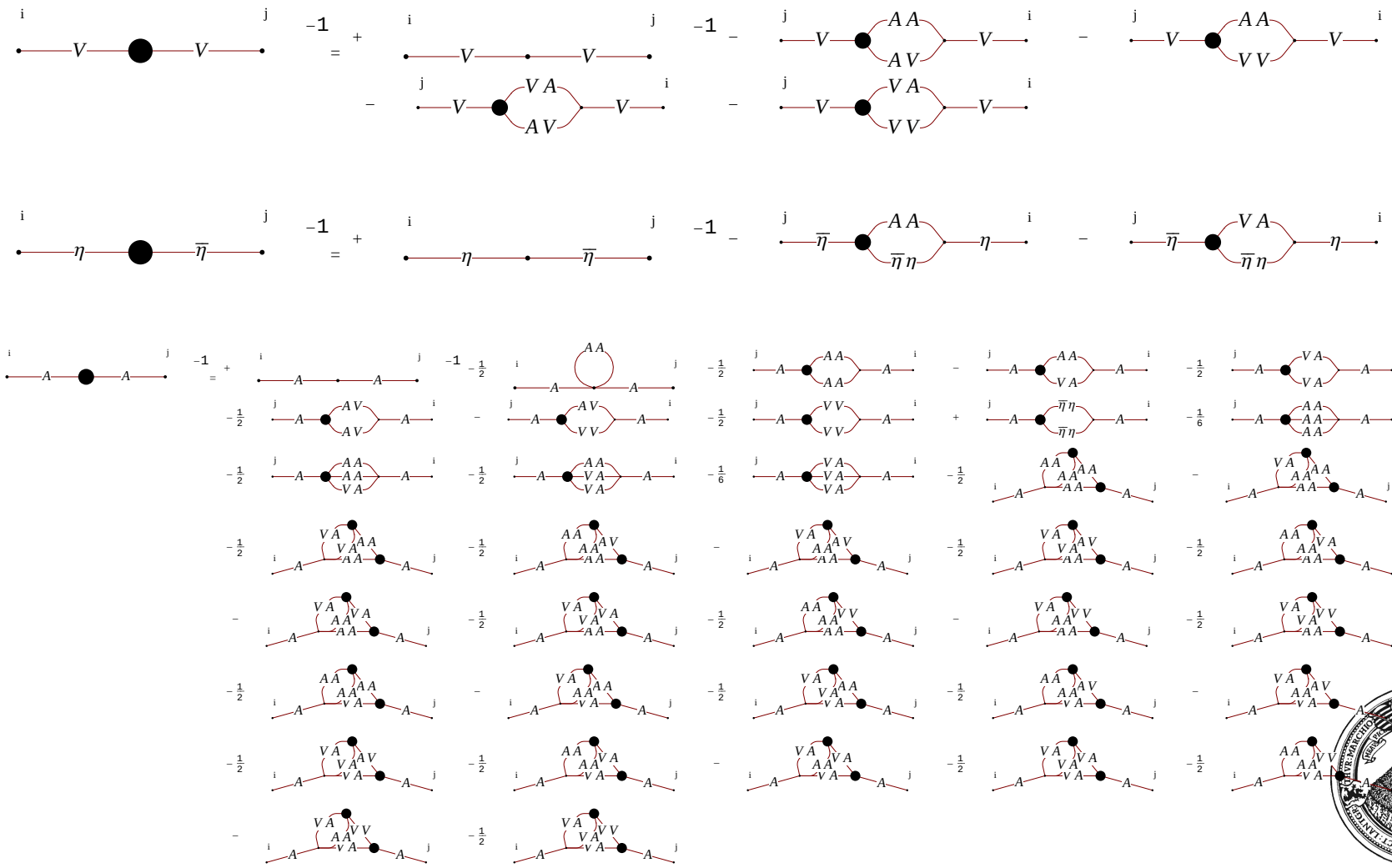
Landau Gauge: Five-Gluon Vertex

434 terms



DSEs of Gribov-Zwanziger action I

Just to give an impression:



DSEs of Gribov-Zwanziger action II

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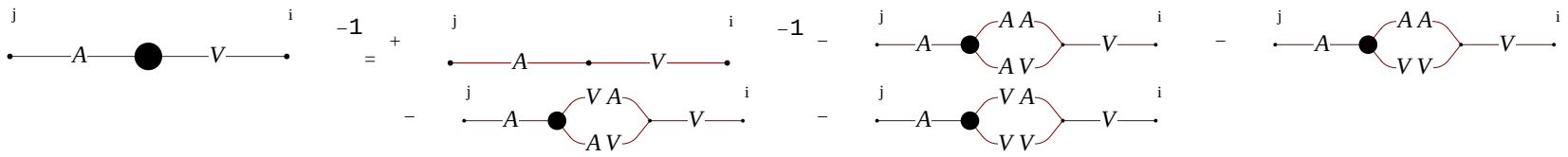
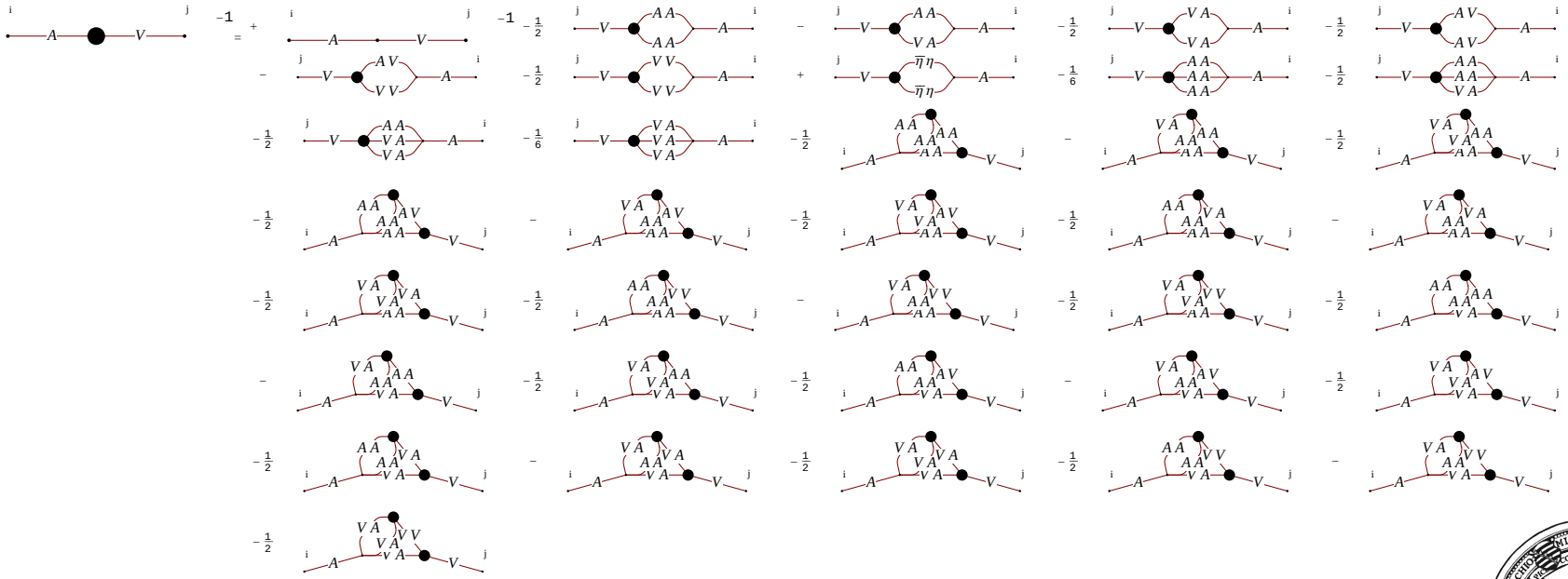




Complete analysis of all diagrams!

Infrared power counting

Generic propagator

$$T_{(ij)} \cdot \frac{D(p^2)}{p^2},$$

assume power law behavior at low p^2

IR exponent

$$D^{IR}(p^2) = A \cdot (p^2)^{\delta}$$

- Vertices also assume power law behavior
  [e.g., Alkofer, Fischer, Llanes-Estrada, PLB 611 (2005) (skeleton expansion)].
- Limit of all momenta approaching zero simultaneously.
- Upon integration all momenta converted into powers of external momenta.

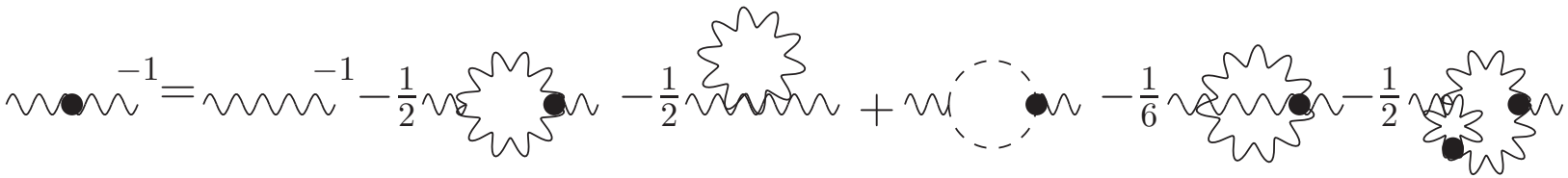
$\Rightarrow$ counting of IR exponents

System of inequalities

- lhs is dominated by at least one diagram on rhs and rhs cannot be more divergent than lhs.

$$\rightarrow \delta_{lhs} \leq \delta_{rhs,any\ diagram}$$

- Not knowing which diagram is leading on the rhs, we can write inequalities from all diagrams.



$$-\delta_{gl} \leq 2\delta_{gl} + \delta_{3g}, \qquad -\delta_{gl} \leq 2\delta_{gh} + \delta_{gg}, \qquad \ldots$$

That's the basic idea. Still, for a large system a lot of work.

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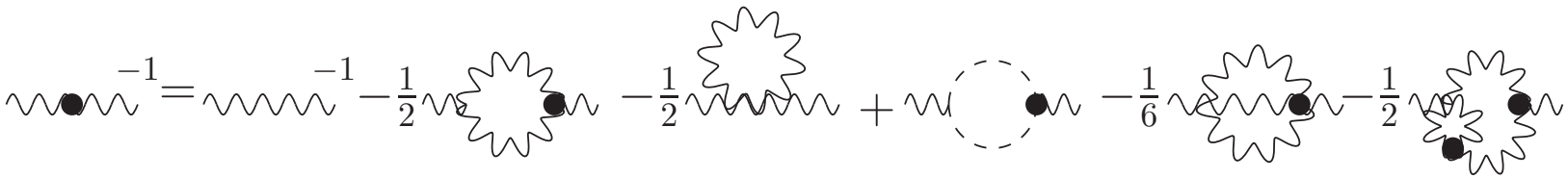

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All inequalities relevant?

Relevant inequalities

Closed form for all relevant inequalities
from 2 independent sets of funct. equations

type		derived from	#
dressed vertices	$C_1 := \delta_{vertex} + \frac{1}{2} \sum_{\substack{\text{legs } j \text{ of} \\ \text{vertex}}} \delta_j \geq 0$	ERGEs	∞
prim. div. vertices	$\boldsymbol{C_2 := \frac{1}{2} \sum_{\substack{\text{legs } j \text{ of} \\ \text{prim. div.} \\ \text{vertex}}} \delta_j \geq 0}$	DSEs+ERGEs	a few

Relevant inequalities

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validity of skeleton expansion basic property of scaling solutions

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Shifting analysis to IR exponents $\rightarrow$ exact from this point on.

Analysis of propagator DSEs

at least for one i: $C_2^i := \frac{1}{2}\sum_j \delta_j = 0$

Example: $\delta_{gluon} + 2\delta_{ghost} = 0$ in Landau gauge

- scaling relation
- one vertex always IR constant (no Taylor argument req.)
- related to bare vertices in DSEs, cf. [Fischer, Pawlowski PRD77]
- necessary condition for scaling solutions
- qualitative behavior of higher Green functions
- valid for a large number of systems

Very useful for complicated actions like the maximally Abelian gauge

(results support hypothesis of Abelian dominance [M.Q.H., Schwenzer, Alkofer, EPJC 68]).

Propagators and two-point functions

Mixing at two-point level: $i\,g\,\gamma^2\sqrt{2}f^{abc}A^a_\mu V^{bc}_\mu$

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Example: VV-two-point function,

$$\Gamma^{VV,abcd}_{\mu\nu} = \delta^{ac}\delta^{bd}p^2 c_V(p^2) g_{\mu\nu}$$

dressing function $c_V(p^2) \xrightarrow{p^2\to 0} d_V \cdot (p^2)^{\kappa_V}$

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VV-propagator:

$$D^{VV,abcd}_{\mu\nu} = \frac{1}{p^2}\frac{1}{c_V(p^2)}\delta^{ac}\delta^{bd}g_{\mu\nu} - f^{abe}f^{cde}\frac{1}{p^2}P_{\mu\nu}\frac{2c^2_{AV}(p^2)}{c^\perp_A(p^2)c^2_V(p^2) + 2N\,c^2_{AV}(p^2)c_V(p^2)}$$

The four possibilities

Which part of the determinant $c_A^{\perp}(p^2)c_V(p^2) + 2N\, c_{AV}^2(p^2)$ dominates in the IR?

$$c_{ij}(p^2) = d_{ij} \cdot (p^2)^{\kappa_{ij}}$$

I: $c_{AV}^2 > c_A c_V \leftrightarrow \kappa_A + \kappa_V > 2\kappa_{AV}$

II: $c_A c_V > c_{AV}^2 \leftrightarrow 2\kappa_{AV} > \kappa_A + \kappa_V$

III: $c_{AV}^2 \sim c_A c_V \leftrightarrow \kappa_A + \kappa_V = 2\kappa_{AV}$, no cancelations

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- Extend method to mixed propagators [M.Q.H., Alkofer, Sorella, PRD 81]: Cases I and IV directly lead to inconsistencies.
- System of equations of III has no solution.
- $\Rightarrow$ Unique solution II with $\kappa_V = \kappa_c = -\kappa_A/2$.

Case II: Solution as from Faddeev-Popov

Scaling relation: $\kappa_V = \kappa_c = -\kappa_A/2 = 0.595353$

- System of equations reduces *in the IR* to the Faddeev-Popov system.
- $\Rightarrow$ Same IR solution, i.e.,
  - IR vanishing gluon propagator,
  - IR enhanced ghost propagator,
  - qualitative behavior of all vertices.
- Mixed propagator is IR suppressed.
- Auxiliary fields are IR enhanced, as in [Zwanziger, PRD81].

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Horizon condition: We allow the ghost propagator to be IR enhanced.

$\Rightarrow$ Consistent solution is obtained.

Summary

→ Derivation of DSEs with *DoDSE*: useful for complicated systems

→ Possible scaling relations directly from Lagrangian

→ Explicit restriction to the Gribov region yields the same non-perturbative IR result as Faddeev-Popov theory.

→ Consistent picture of confinement in the Landau gauge (scaling solution):

- physical state space (Kugo-Ojima),
- role of Gribov region,
- qualitative behavior of all Green functions

The end

Thank you very much for your attention.