Landau gauge propagators of two-dimensional Yang-Mills theory

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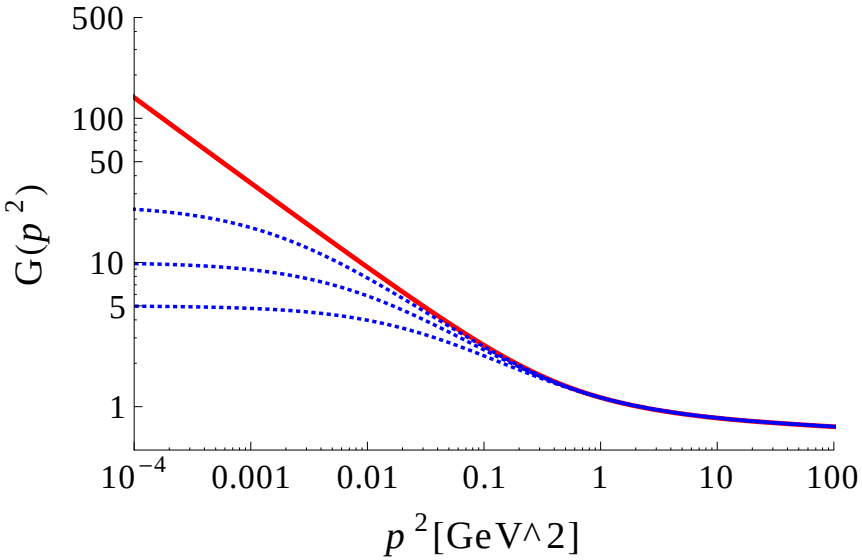
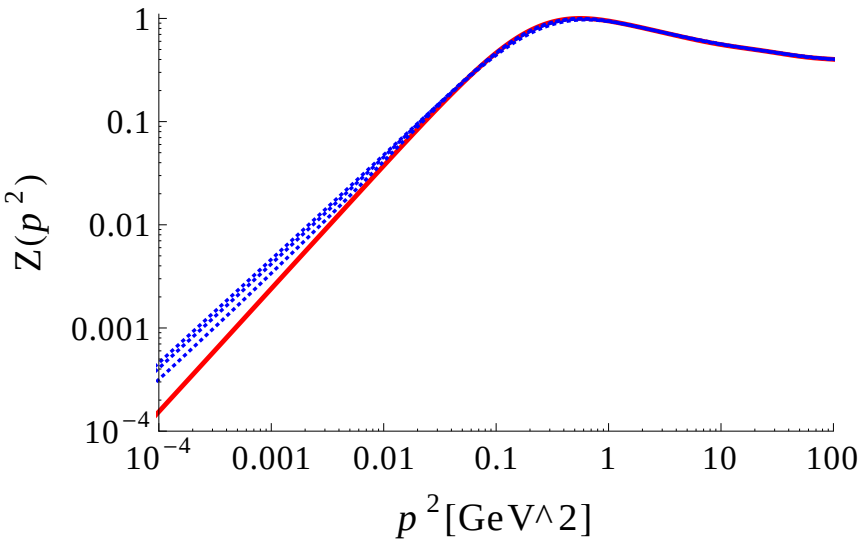
Propagators of four-dimensional Yang-Mills theory

Two qualitatively different solutions from functional methods

[Boucaud et al., JHEP 0806, 012; Fischer, Maas, Pawlowski, AP 324]





Behavior at low momenta:

	ghost dressing	gluon dressing
scaling	$\sim (p^2)^{-\kappa}$	$\sim (p^2)^{2\kappa}$
decoupling	$\sim (p^2)^0$	$\sim (p^2)^1$

Scaling relation:
$2\delta_c + \delta_A = 0$

Non-perturbative gauge choice [Maas, PLB 689]?

Why $d < 4$?

- n lattice points per direction: n^d total lattice points
  $\Rightarrow$ lower dimensions require (much) less computer power
- Example lattices:
  $d = 4$: 128^4 ($L \approx 27\,fm$) [Cucchieri, Mendes, Pos LAT2007, 297]
  $d = 2$: 2560^2 ($L \approx 460\,fm$) [Cucchieri, Mendes, AIP CP 1343, 185]

$d = 2$

- No transverse directions $\rightarrow$ gluons have no degrees of freedom
- Gribov problem
- Ambiguity of solutions?
- Good lattice results, even for three-point functions

Lattice results for $d = 2$

Lattice calculations find only the decoupling type of solution for $d = 3, 4$.

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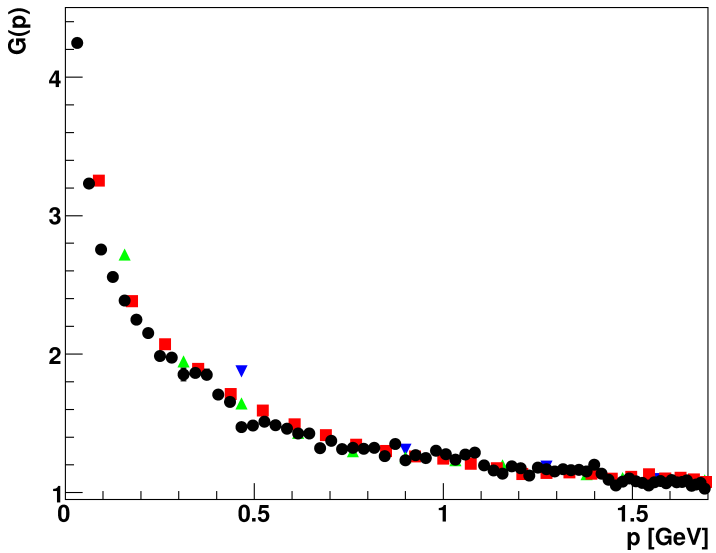
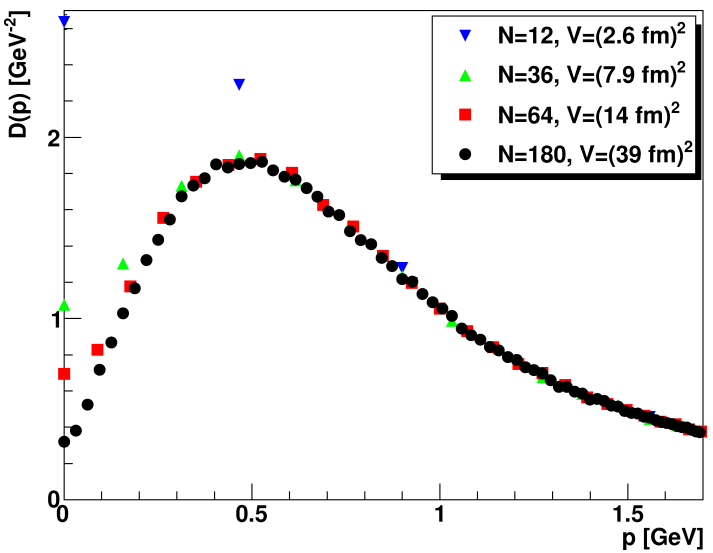
$d = 2$ seems different: Only the scaling type solution is found.





[Maas, 1106.3942]

What to expect for $d = 2$

Scaling type solution:

- IR analysis allows two sets of IR exponents $\{\delta_c, \delta_A\}$ [Zwanziger, PRD65]:

$$\{0, 1\} \text{ and } \{-0.2, 1.4\},$$

  always $2\delta_c + \delta_A = -(d-4)/2 = 1$
- Qualitative behavior of all Green functions known [Huber, Alkofer, Fischer, Schwenzer, PLB659].

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Decoupling type solutions:

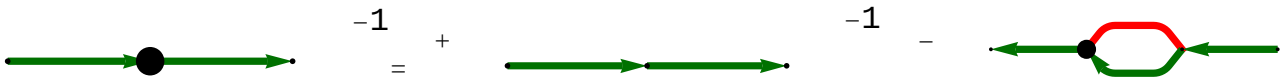
- finite ghost dressing, finite gluon propagator

Note: Set 1 looks like a decoupling type (peculiar to $d = 2$).

Ghost equation

Ghost DSE:

(Presented results have been obtained with *CrasyDSE* [Huber, Mitter, 1112.5622].)



input: various gluon propagator ansätze, trivial ghost-gluon vertex

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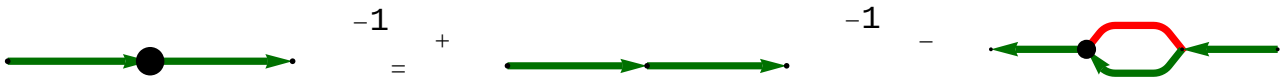

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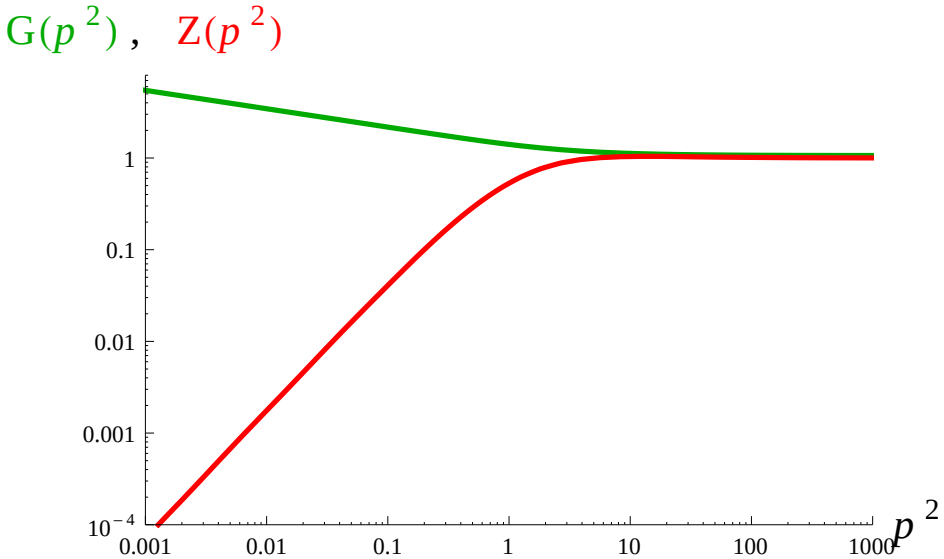
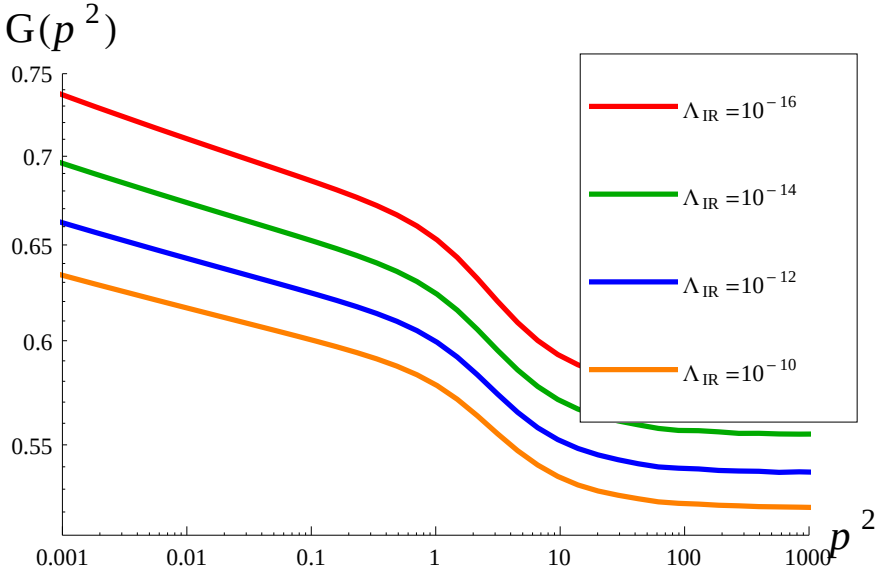


Scaling type solution



Decoupling type solution

$\longrightarrow$ IR cutoff dependence for decoupling type solutions

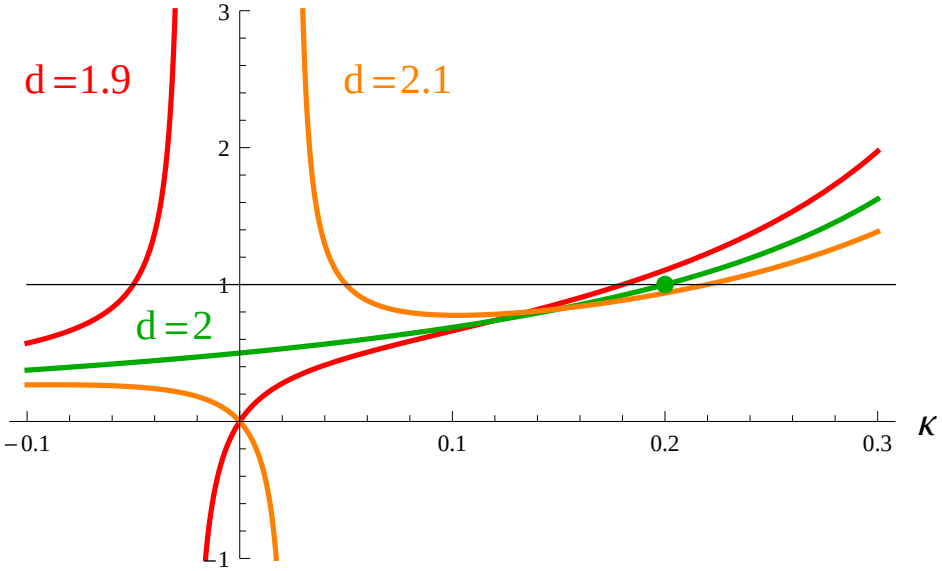
$\Rightarrow$ no decoupling solution (cf. [Cucchieri, Dudal, Vandersickel, 1202.1912])

$\kappa = 0$ revisited

IR exponents can be determined analytically from the IR dominant diagrams:

Value of κ is determined from

$$\frac{\sin(\pi\kappa)\Gamma(d/2-\kappa)\Gamma(\kappa)\Gamma(1+d/2+\kappa)}{2(d-1)\sin(\pi(d/2-2\kappa))\Gamma(d-2\kappa)\Gamma(2\kappa)\Gamma(1+\kappa)} = 1$$



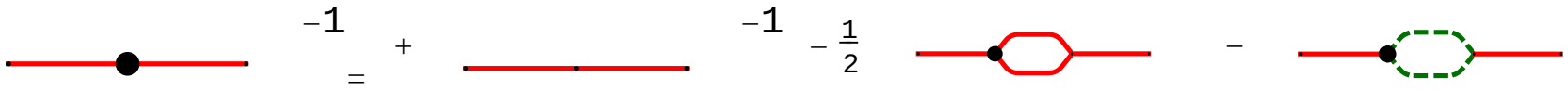
There is no solution $d \to 2$, $\kappa = 0$.

To obtain $\kappa = 0$ an additional prescription is required.

$\to$ Existence of decoupling solution is scheme dependent.

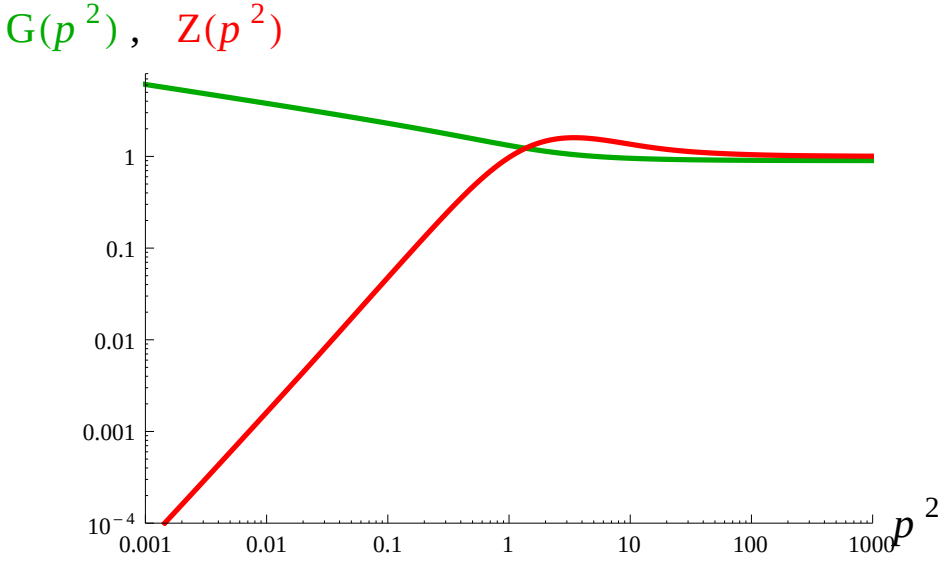
Ghost and gluon DSEs

Gluon DSE:



trivial ghost-gluon vertex, ansatz for three-gluon vertex

Coupled system of equations:



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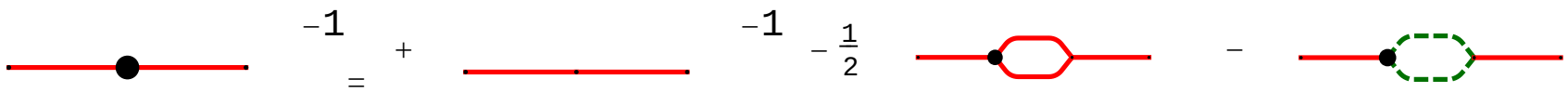

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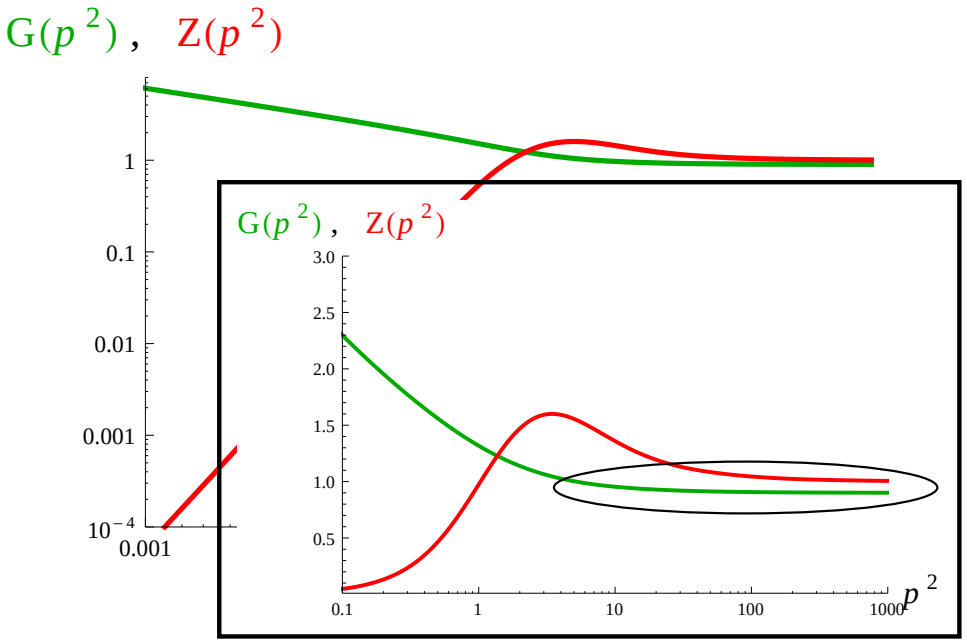

$\Rightarrow$ Ghost does not approach 1 in the UV.

From dim. arguments:

$$G(p^2) \xrightarrow{p^2 \to \infty} \frac{1}{1 + c/p^2}$$

Preliminary results!

Mid-momentum regime and UV behavior

What is the source of this deviation of the ghost propagator in the UV?

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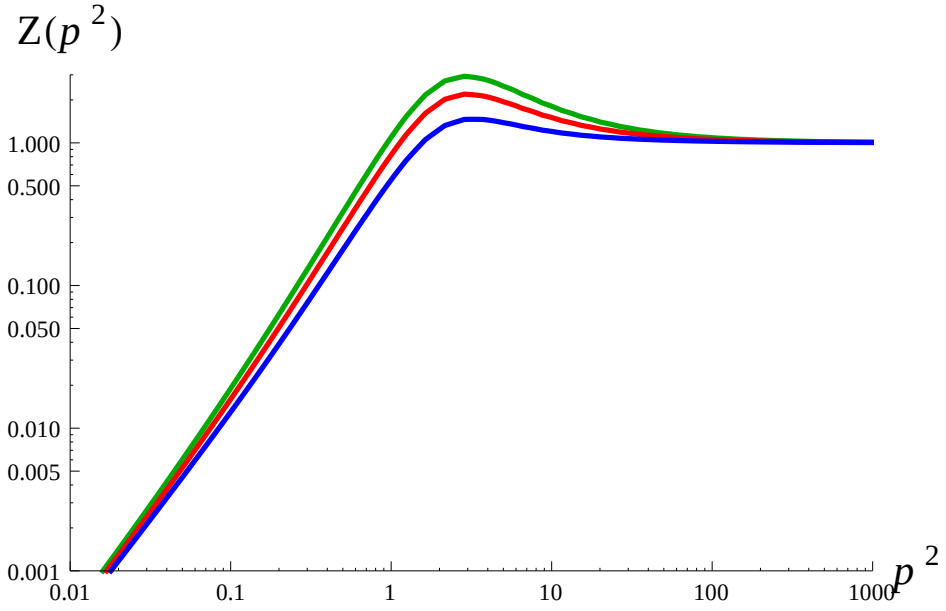
Use gluon ansätze which differ in mid-momentum regime.



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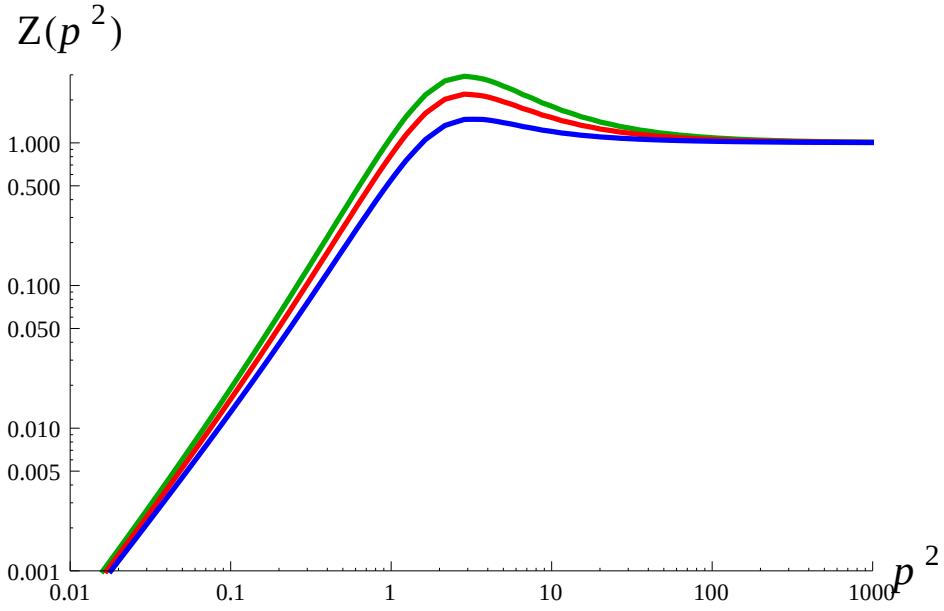
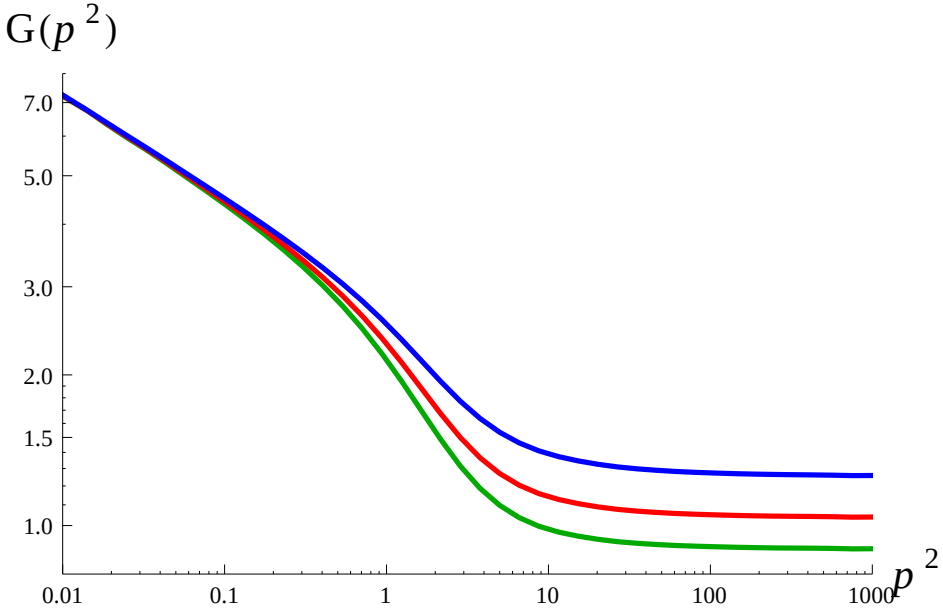
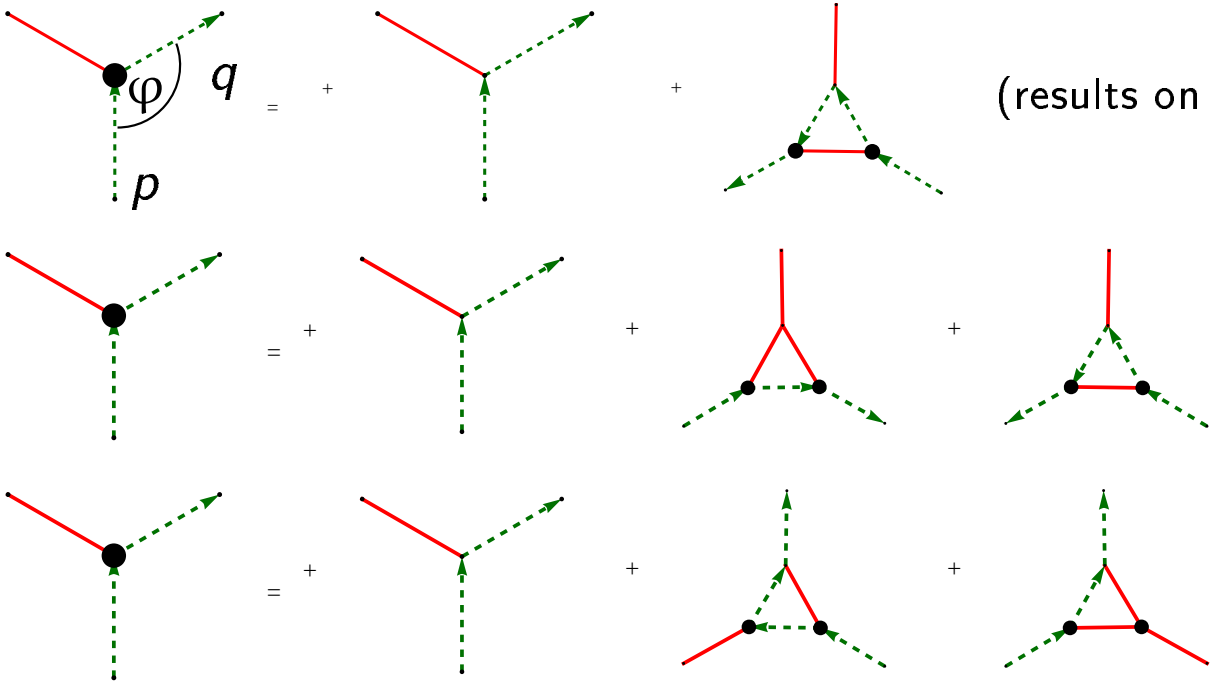




$\Rightarrow$ Mid-momentum regime has (for $d = 2$ and contrary to $d = 4$) a direct influence on the UV behavior.

Ghost-gluon vertex DSE

Various truncations for ghost-gluon vertex DSEs:



(results on next slide from this truncation)

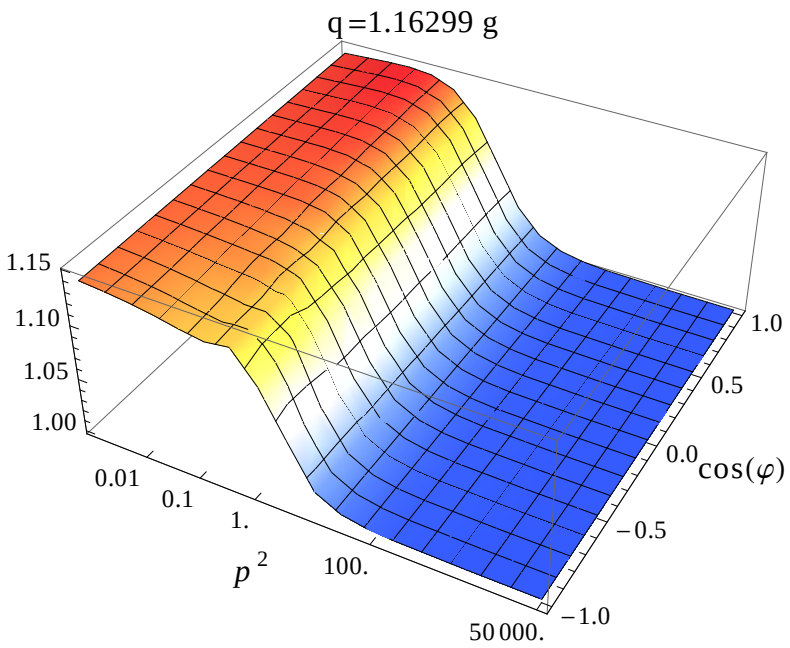
Three-point function depends on 3 variables.
Here: 2 ghost momenta p^2 and q^2, angle φ between them.

Ghost-gluon vertex

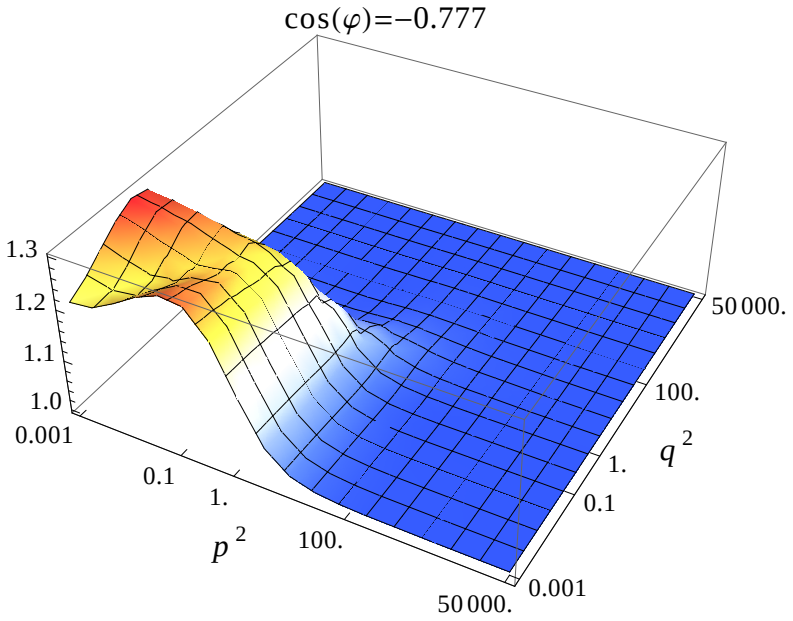
Calculated from fully iterated propagators (bare ghost-gluon vertex used):

Preliminary results!

Fixed momentum:



Fixed angle:



→ 1 in the UV

→ IR constant

→ Almost no dependence on angle

→ Direct influence on propagators

Conclusions

Correct UV behavior of ghost propagator in two dimensions
depends on mid-momentum regime.

For the full solution more information needed than in $d = 4$:

- $\longrightarrow$ Bare ghost-gluon vertex insufficient
- $\longrightarrow$ Three-gluon vertex important as well
- $\longrightarrow$ Even two-loop diagrams important

- $\longrightarrow$ No decoupling type solution exists in $d = 2$
  in agreement with lattice results:
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Thank you very much for your attention.