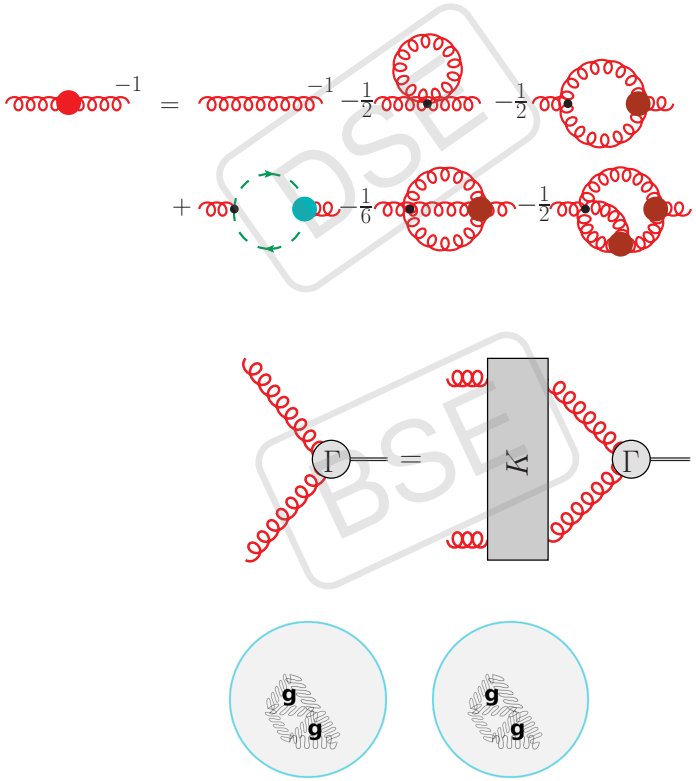
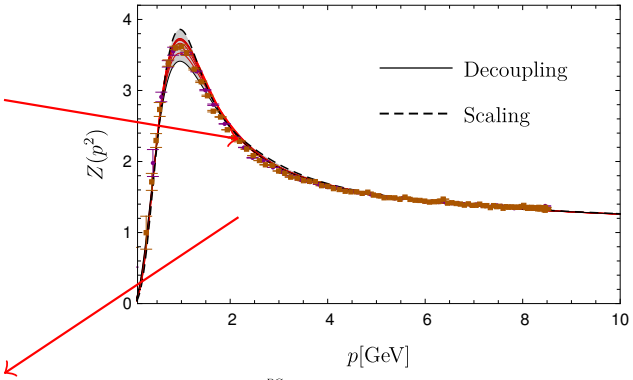
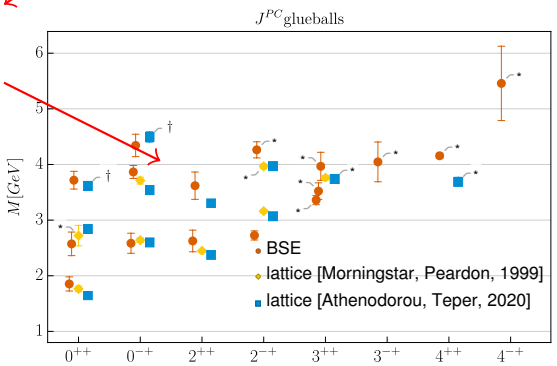
Quenched glueball spectrum from functional equations







JUSTUS-LIEBIG-UNIVERSITÄT GIESSEN

DFG Deutsche Forschungsgemeinschaft

Markus Q. Huber

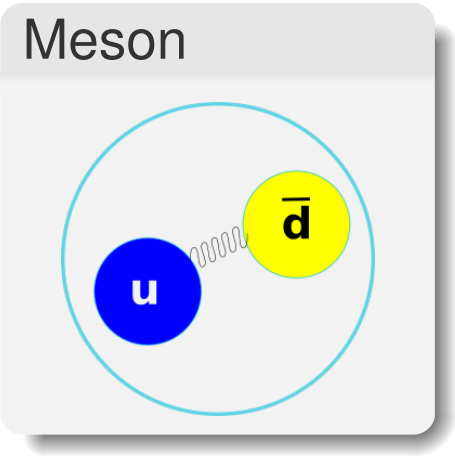
Institute of Theoretical Physics

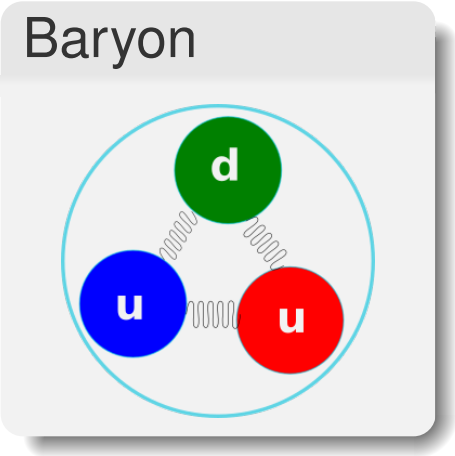
Giessen University

In collaboration with
Christian S. Fischer, Hèlios Sanchis-Alepuz:
Eur.Phys.J.C 80, arXiv:2004.00415
Eur.Phys.J.C 80, arXiv:2110.09180
vConf21, arXiv:2111.10197
HADRON2021, arXiv:2201.05163

Bound states of the strong interaction

Quark model 1964: abundance of known states





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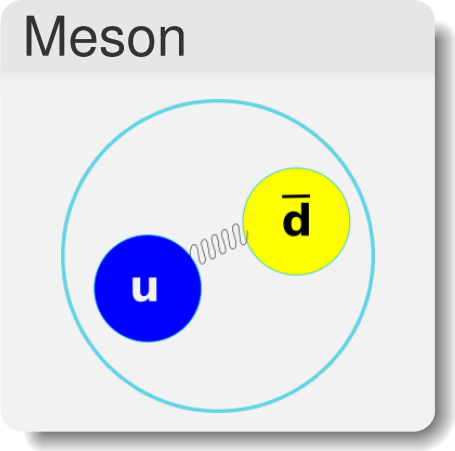
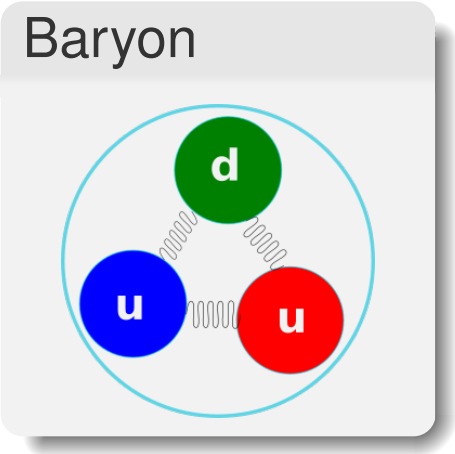
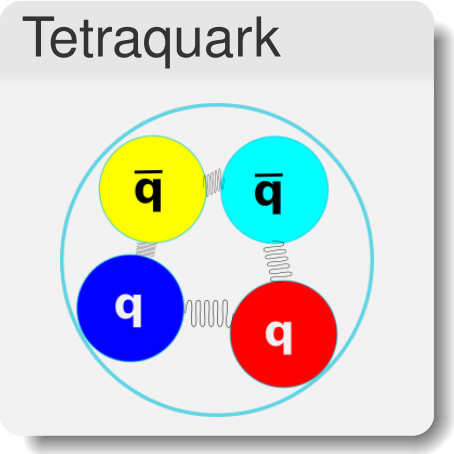
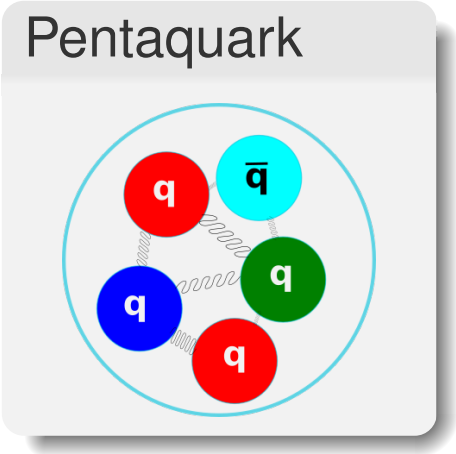
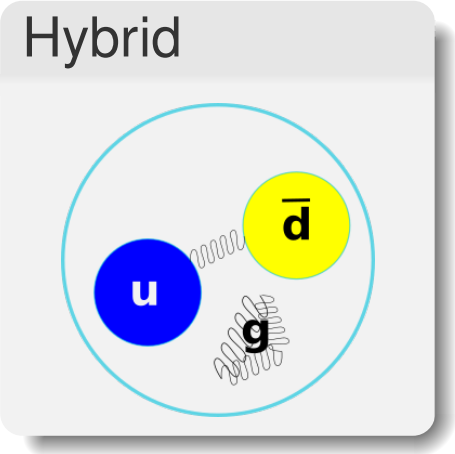




Exotics:





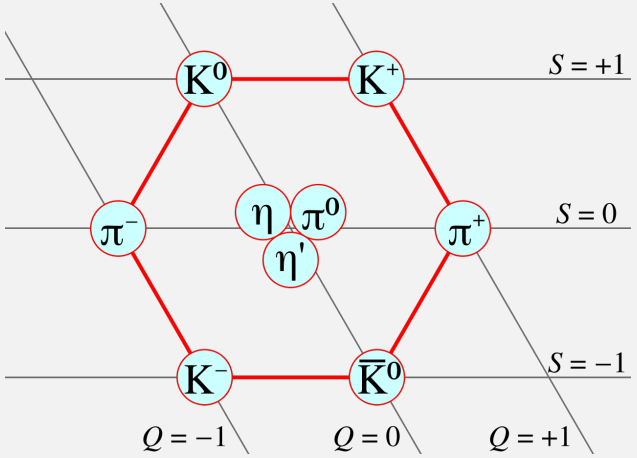


Multiplets

Quark model

Classification in terms of mesons or baryons $\rightarrow$ multiplets

Outside this classification $\rightarrow$ exotics



$J^{PC} = 0^{-+}$

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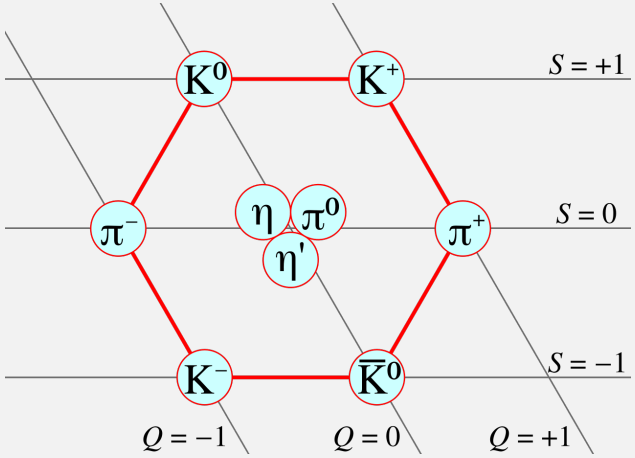


$J^{PC} = 0^{-+}$

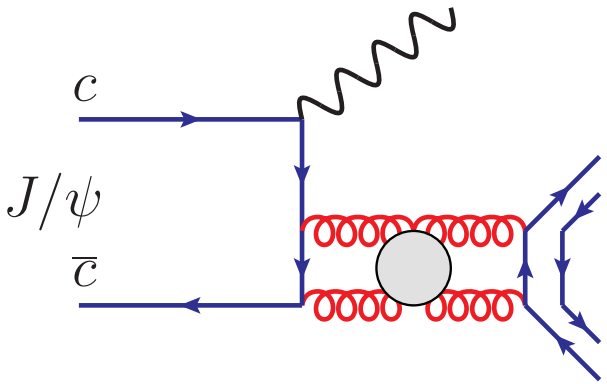
Classification not always easy, e.g., scalar sector $J^{PC} = 0^{++}$:

glueball candidates

$f_0(500)$
$f_0(980)$
$f_0(1370)$
$f_0(1500)$
$f_0(1710)$

+ more states not considered established

Glueballs from J/ψ decay



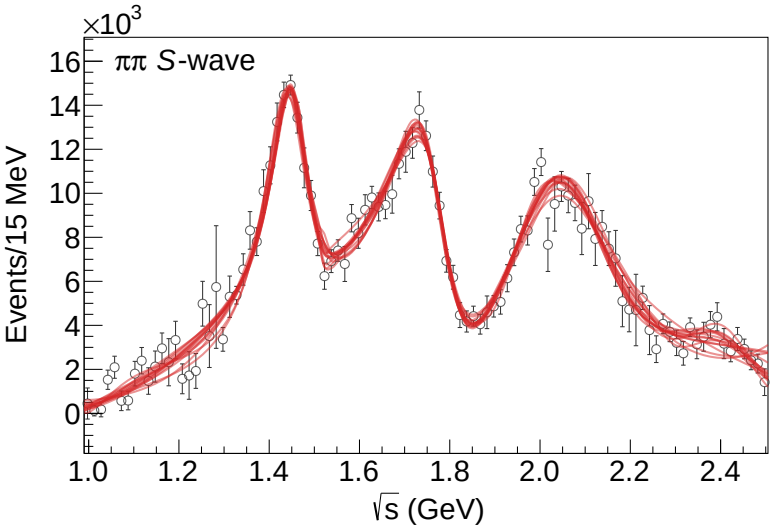
Coupled-channel analyses of exp. data (BESIII):

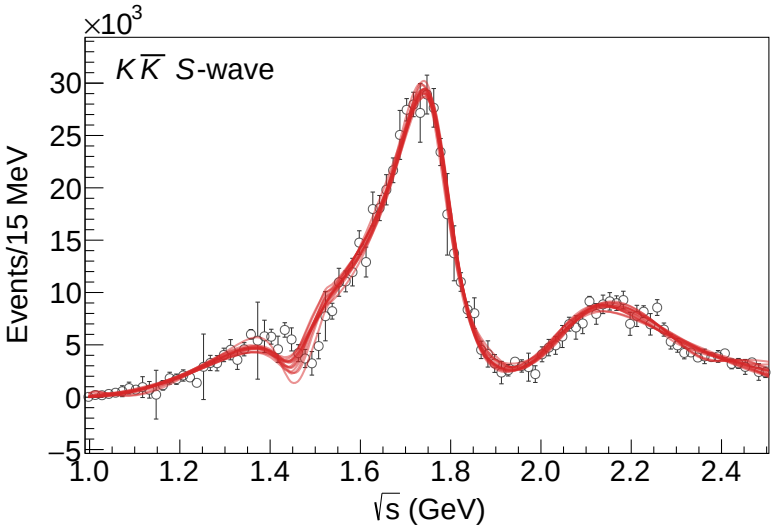
- +add. data, largest overlap with $f_0(1770)$

[Sarantsev, Denisenko, Thoma, Klempt, Phys. Lett. B 816 (2021)]

- largest overlap with $f_0(1710)$

[Rodas et al., Eur.Phys.J.C 82 (2022)]



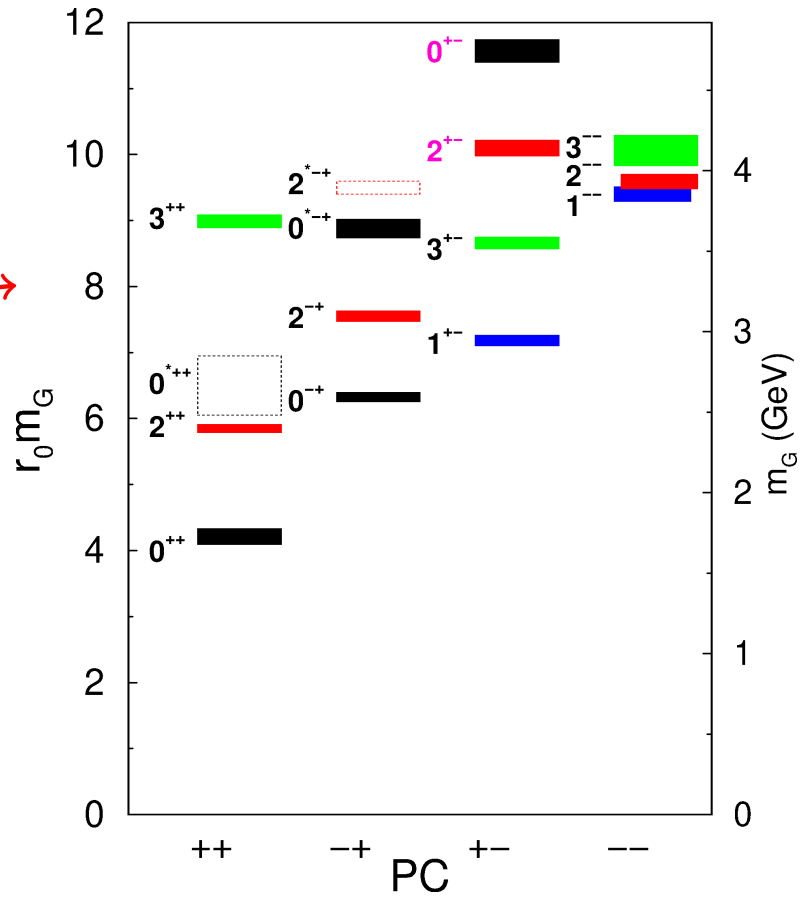


Glueball calculations

Lattice methods

Pure gauge theory:
No dynamic quarks.
→ "Pure" glueballs

- [Morningstar, Peardon, Phys. Rev. D60 (1999)]: standard reference
- [Athenodorou, Teper, JHEP11 (2020)]: improved statistics, more states



[Morningstar, Peardon, Phys. Rev. D60 (1999)]

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"Real QCD":

- [Gregory et al., JHEP10 (2012)]

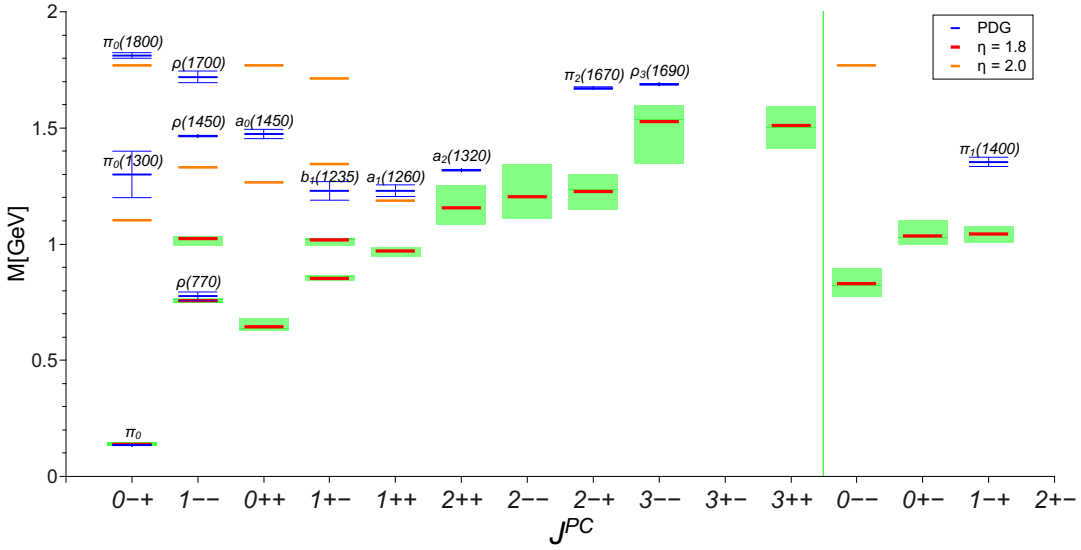
Challenging:

- poor signal-to-noise ratio
- continuum extrapolation
- operator basis incomplete
- large pion masses (360 MeV)

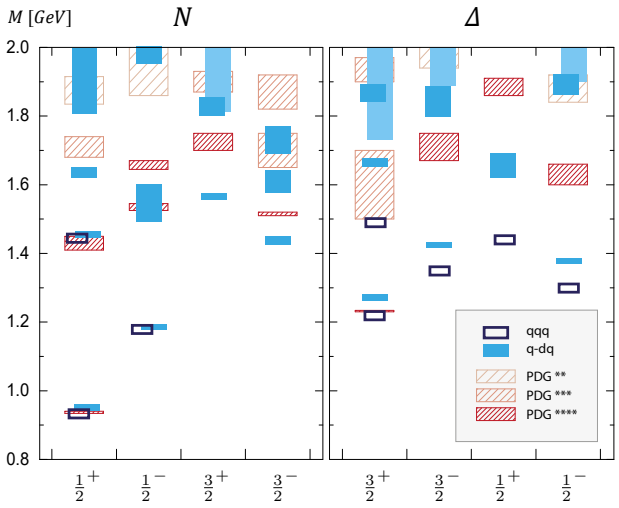
No quantitative results yet.

Functional glueball calculations

Functional methods successful in describing many aspects of the hadron spectrum qualitatively and quantitatively!



[Fischer, Kubrak, Williams, Eur.Phys.J.A50 (2014)]



[Eichmann, Fischer, Sanchis-Alepuz, Phys.Rev.D94 (2016)]

Glueballs?

Functional glueball calculations

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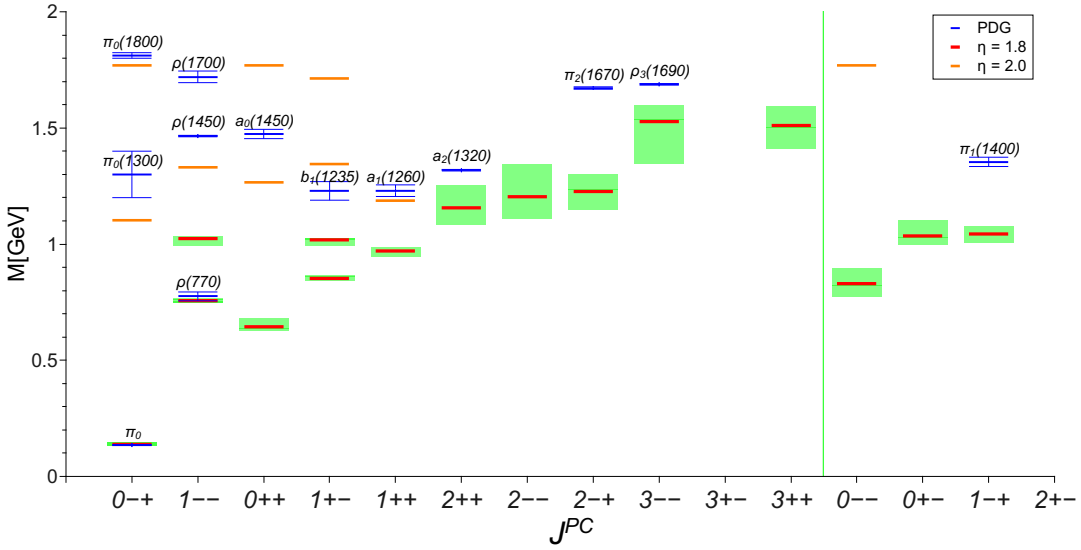

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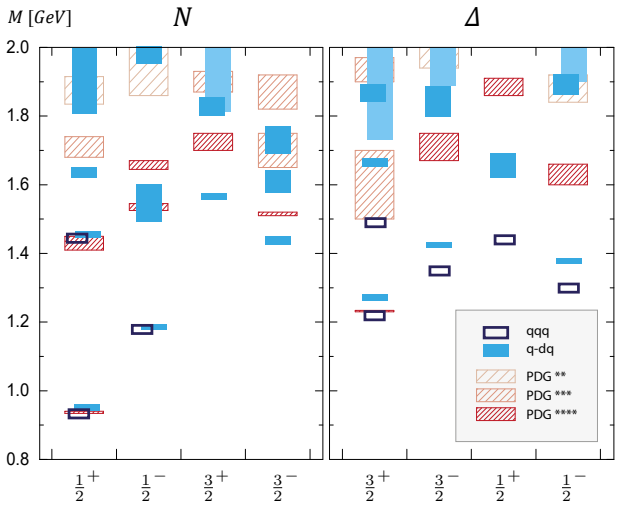
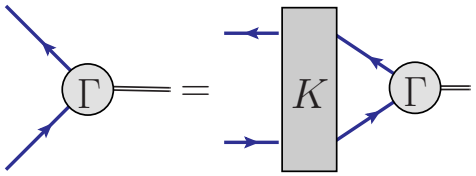


[Eichmann, Fischer, Sanchis-Alepuz, Phys.Rev.D94 (2016)]

Glueballs?

Extreme sensitivity on input!

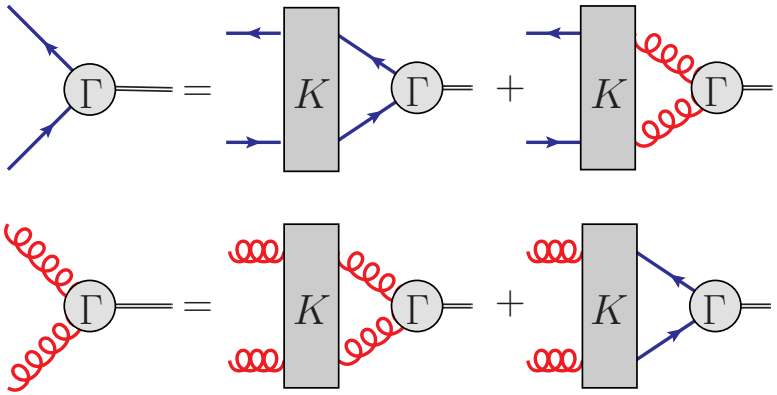
Bound state equations for QCD



- Require scattering kernel K and propagator.

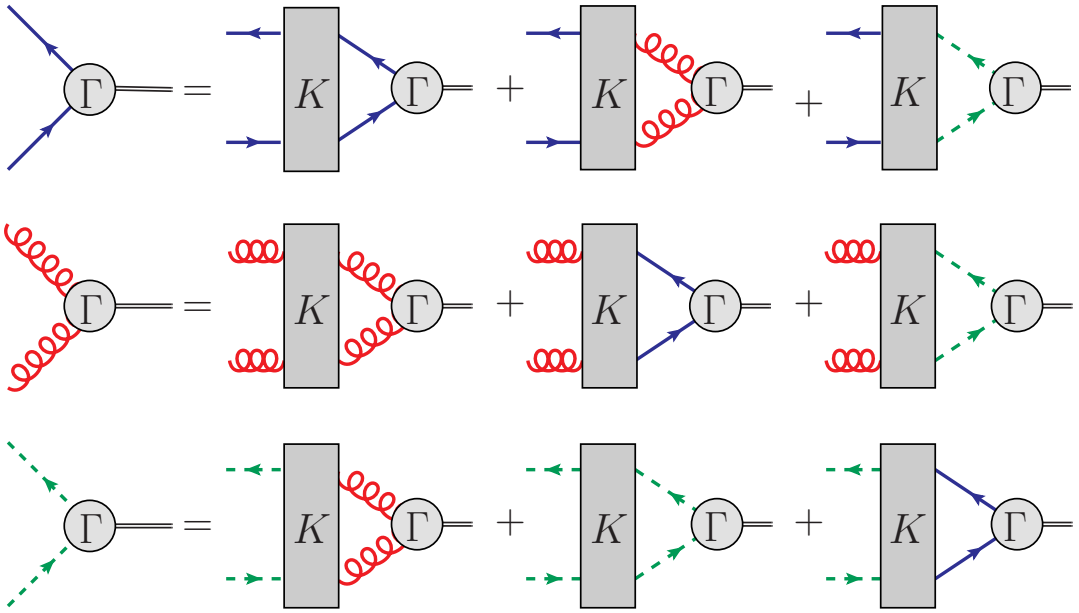
  [→ Hagel, HK19.4]

Bound state equations for QCD



- Require scattering kernels K and propagators.
- Quantum numbers determine which amplitudes Γ couple.

Bound state equations for QCD



- Require scattering kernels K and propagators.
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- Ghosts from gauge fixing

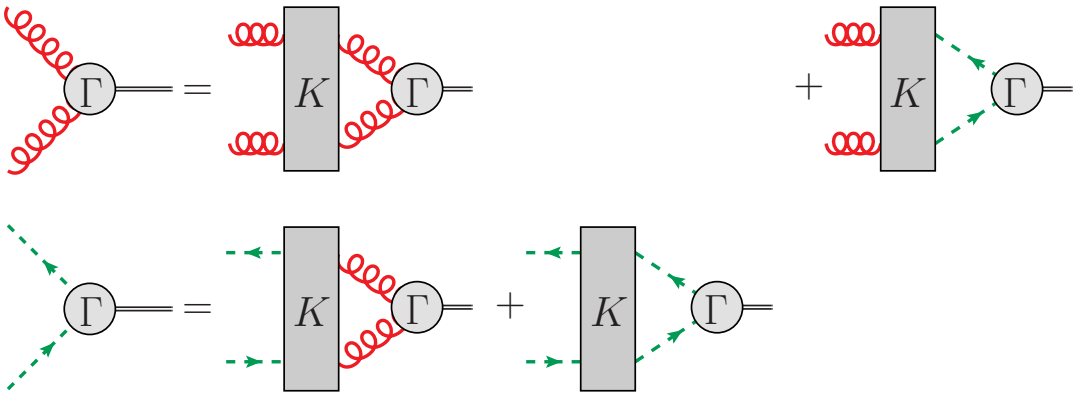
One framework

- Natural description of mixing.
- Similar equations for hadrons with more than two constituents

[→ Hoffer, HK19.6].

Bound state equations for QCD

Focus on pure glueballs.



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One framework

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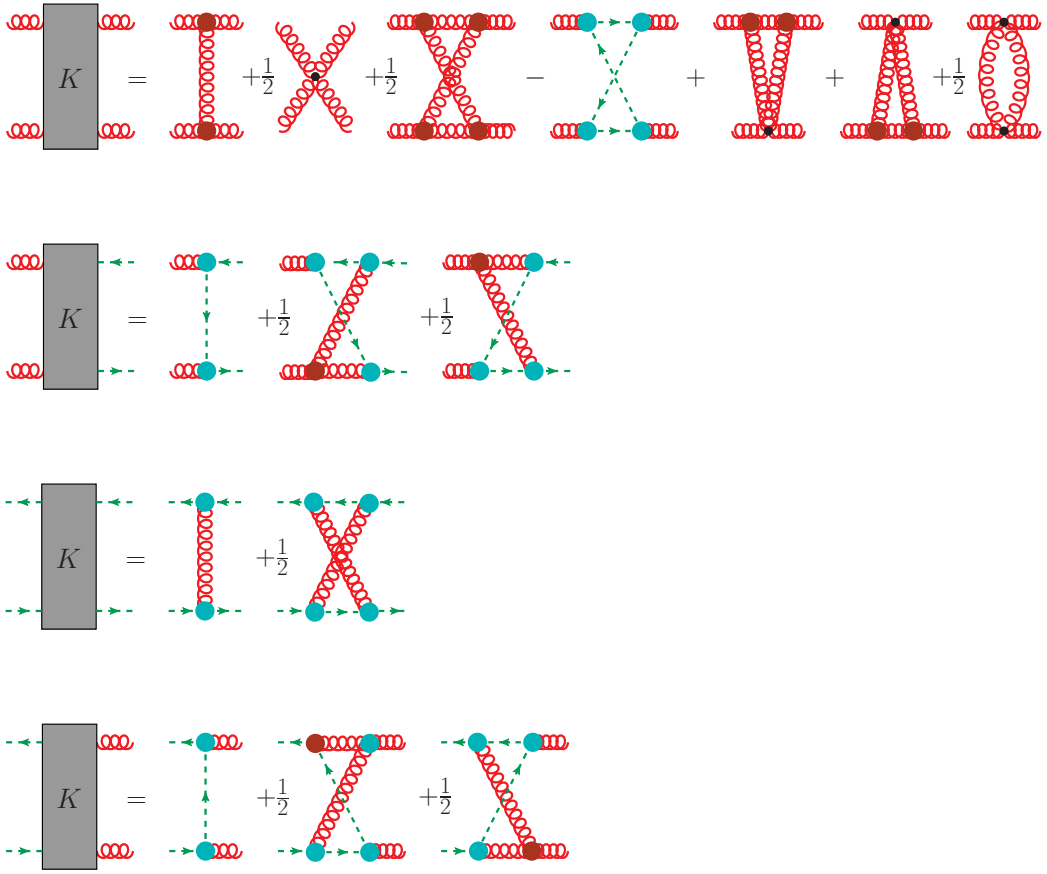
[→ Hoffer, HK19.6].

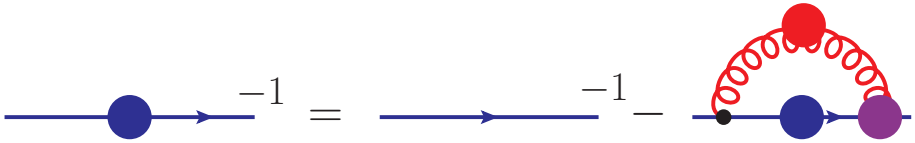
Construction of kernels

Systematic derivation from 3PI effective action: Self-consistent treatment of 3-point functions requires 3-loop expansion.



Correlation functions and their equations of motion

Example: Equation of motion (Dyson-Schwinger equations) for the quark propagator



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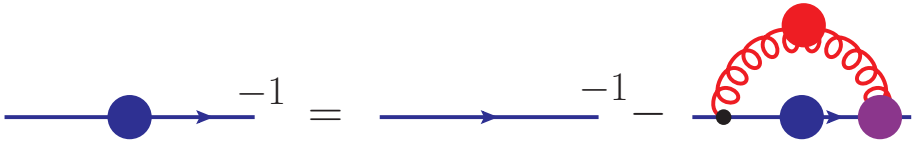
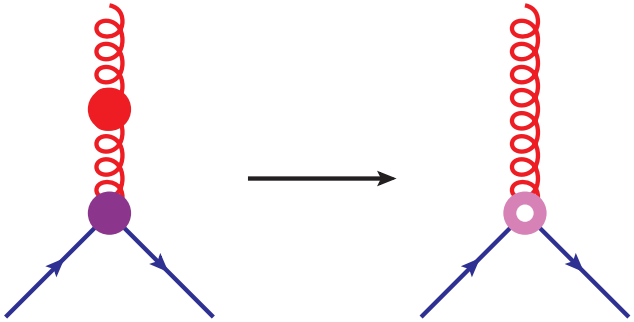


Dealing with the unknowns:

<u>Model</u>



$D_{\mu\nu}(k)\Gamma_\nu(k;p,q) \to \mathcal{G}(D,\Lambda,k^2)\gamma_\mu^{\mathrm{T}}$

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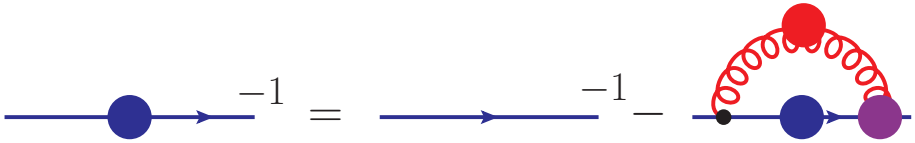

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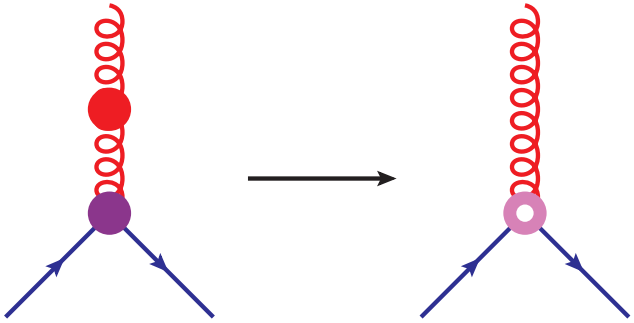
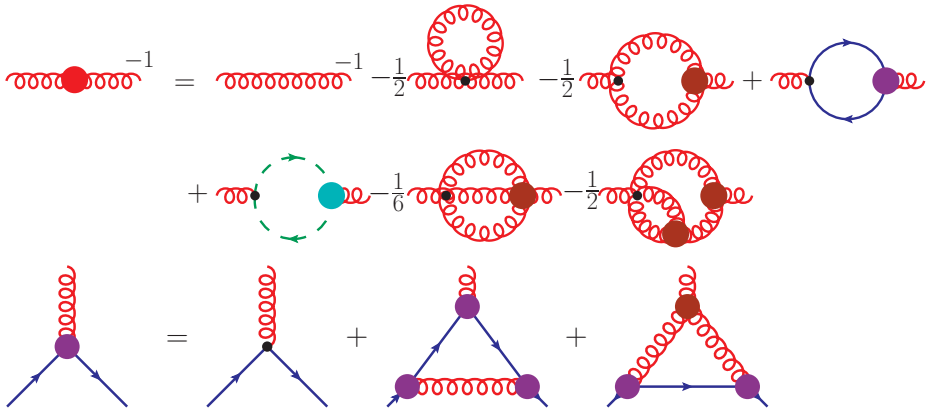


$D_{\mu\nu}(k)\Gamma_\nu(k;p,q) \to \mathcal{G}(D,\Lambda,k^2)\gamma_\mu^{\mathrm{T}}$

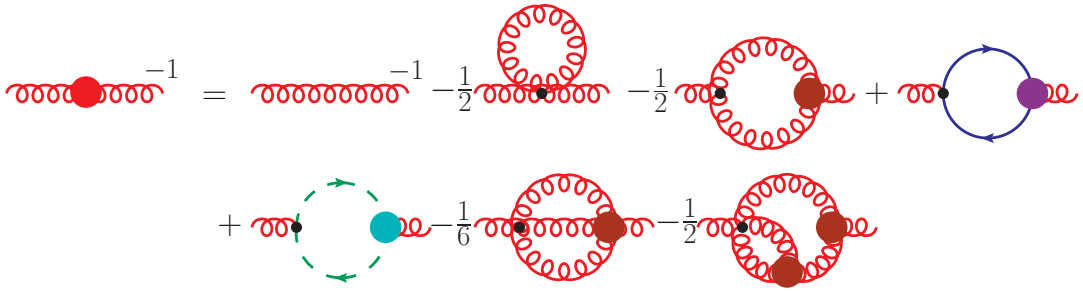
Calculate

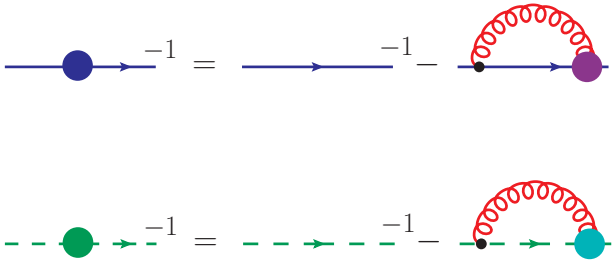


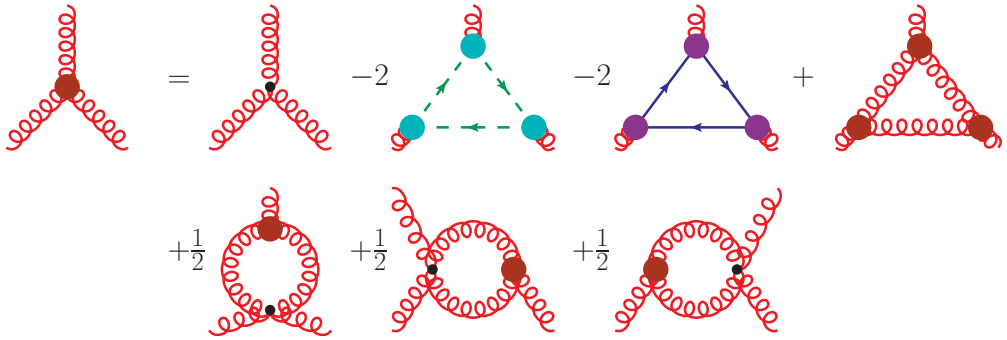
Correlation functions of quarks and gluons

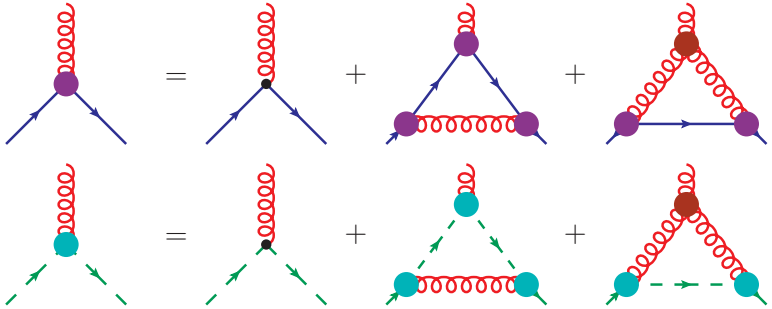
Equations of motion:

→ [Review: MQH, Phys.Rept. 879 (2020)]









Truncation: 3-loop 3PI effective action

- Self-contained: Only parameters are the strong coupling and the quark masses!

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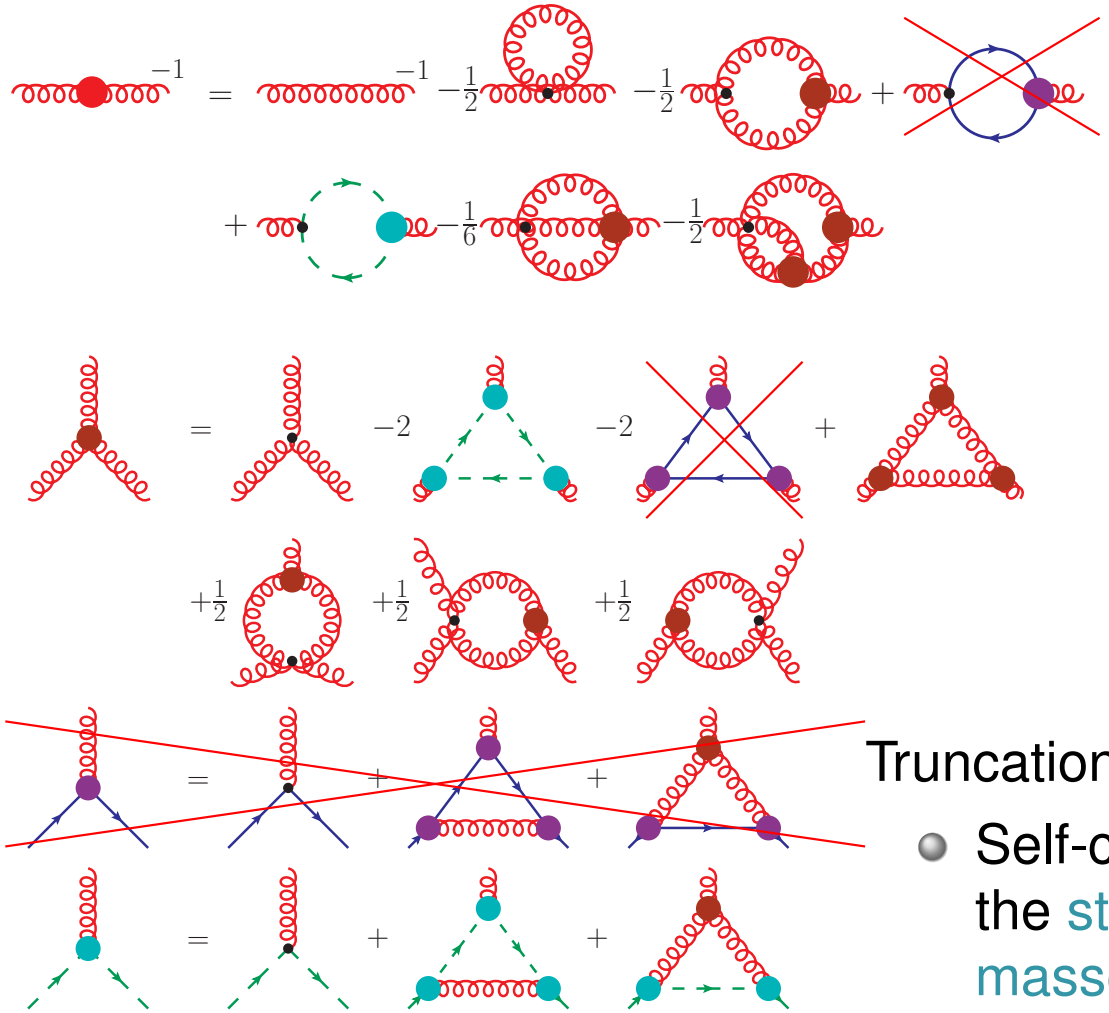

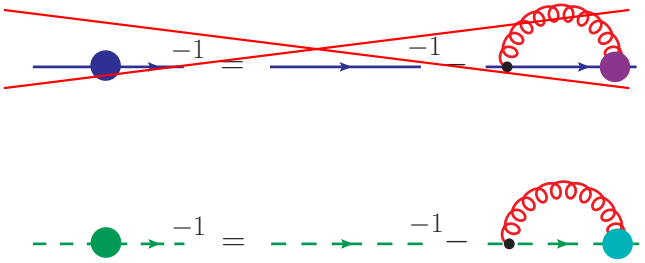

Truncation: 3-loop 3PI effective action

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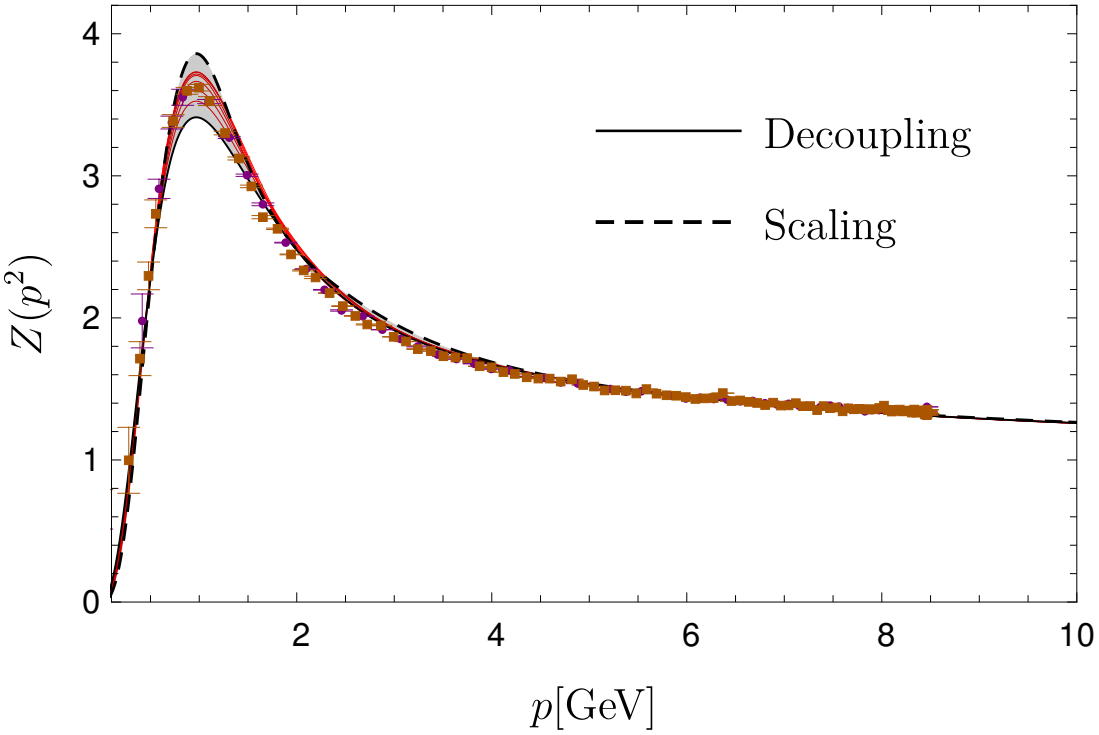
Self-consistent solution

Self-contained: Only external input is the coupling! [MQH, Phys.Rev.D 101 (2020)]

Gluon dressing function:



Conceptual and technical challenges: nonperturbative renormalization, two-loop diagrams, convergence, size of kernels, ...

Stability of the solution

- Agreement with lattice results.

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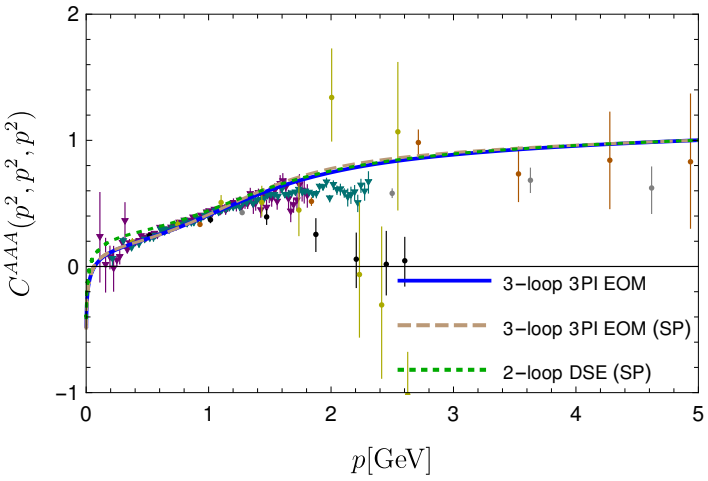
- Concurrence between functional methods:

3PI vs. 2-loop DSE:



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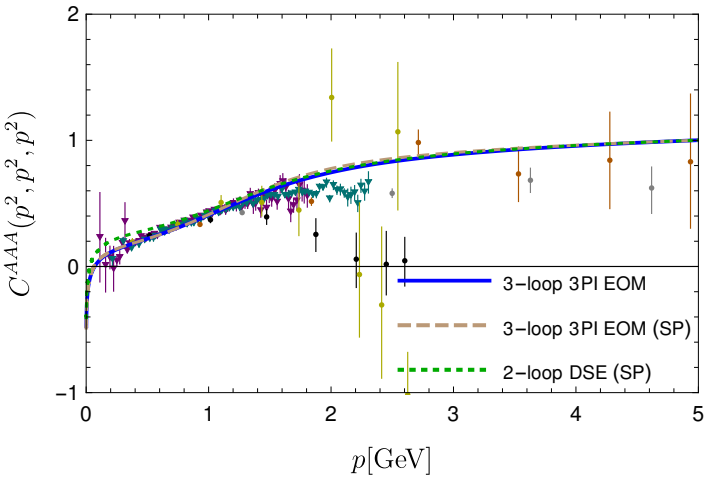
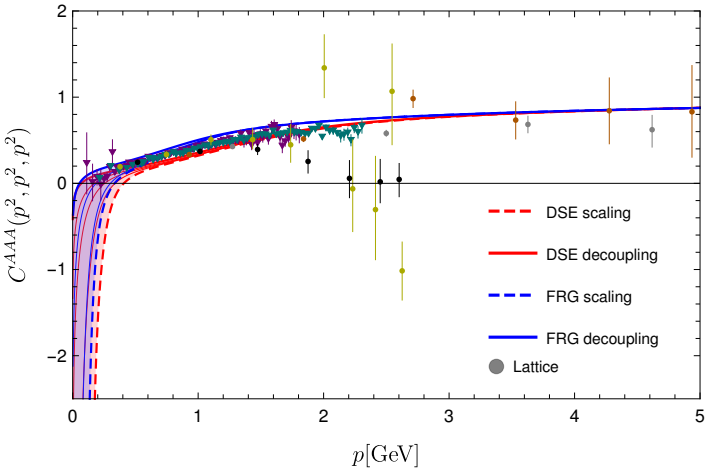


DSE vs. FRG:



[Cucchieri, Maas, Mendes, Phys.Rev.D77 (2008); Sternbeck et al., Proc.Sci. LATTICE2016 (2017); Cyrol et al., Phys.Rev.D 94 (2016); MQH, Phys.Ref.D101 (2020)]

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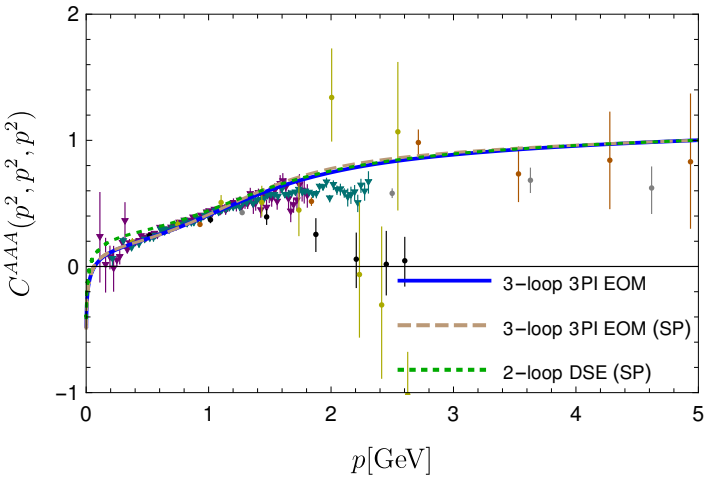

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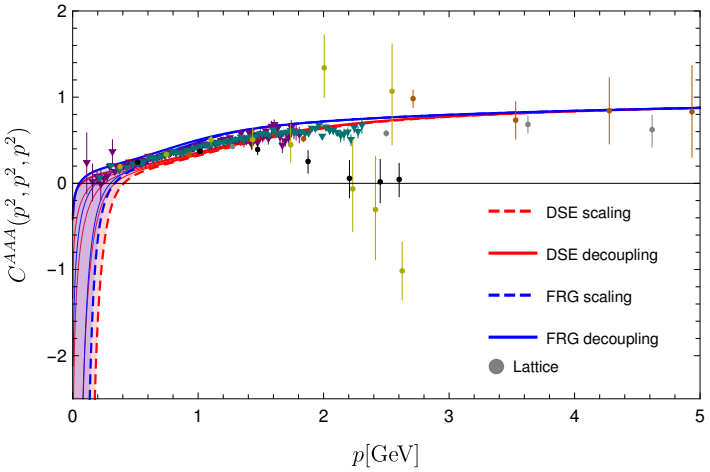

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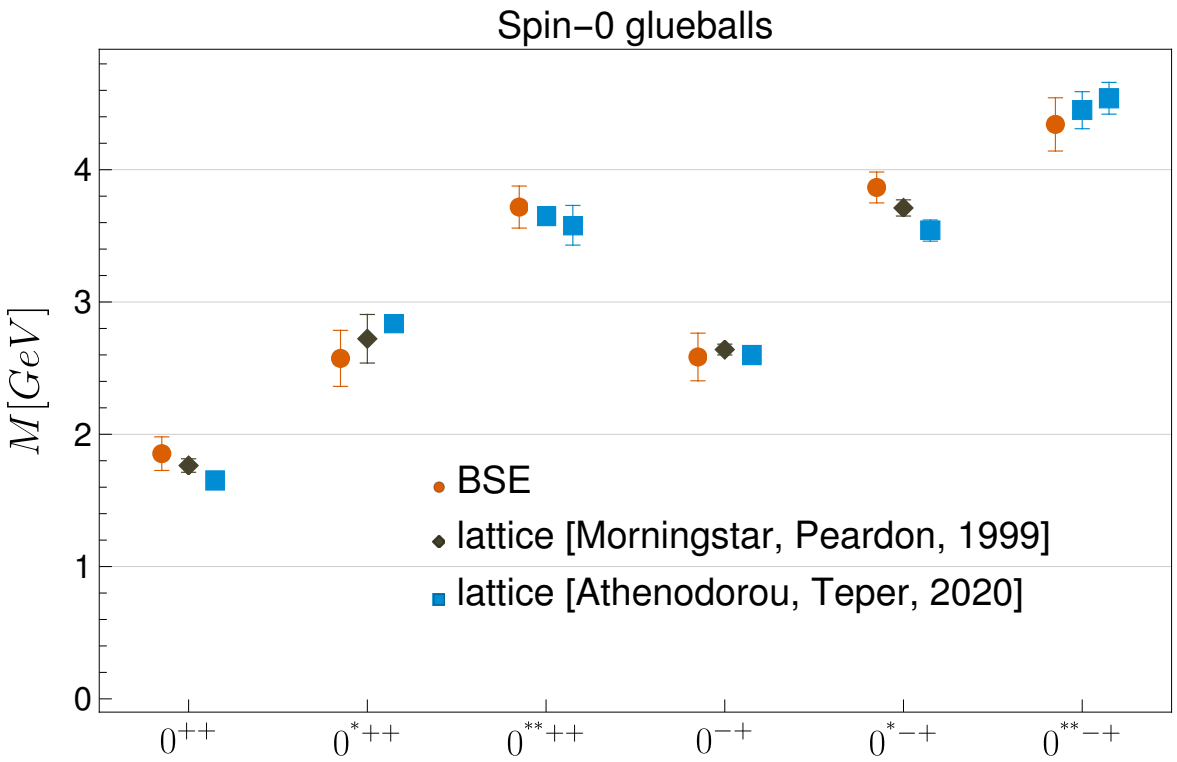
- Stable against extensions: Four-point functions

[MQH, Phys.Rev.D93 (2016); MQH, Eur.Phys.J.C 77 (2017); Corell, SciPost Phys. 5 (2018); MQH, Phys.Rept. 879 (2020)]

Glueball results J=0



Lattice $0^{**\pm+}$: Conjectured based on irred. rep. of octahedral group

All results for $r_0 = 1/418(5)$ MeV.

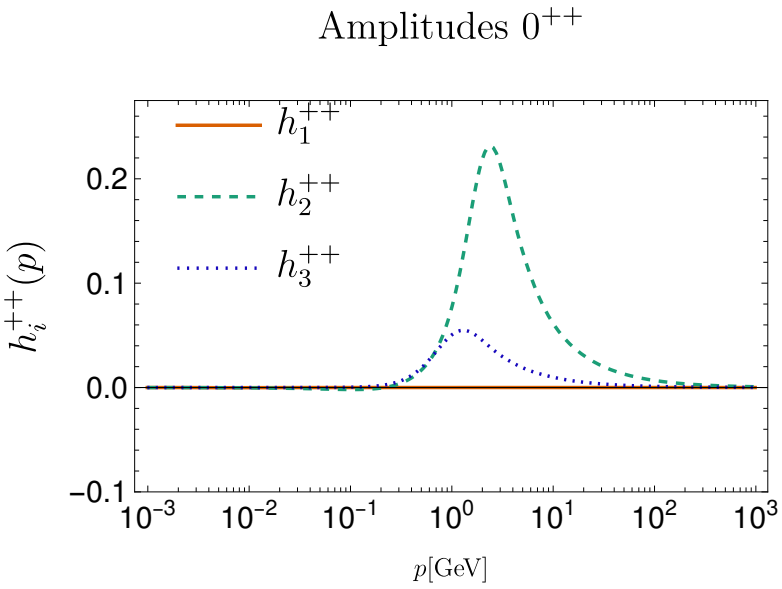
$J = 0$ best investigated case:

- Leading kernel contributions [MQH, Fischer, Sanchis-Alepuz, Eur.Phys.J.C 80 (2020)]
- Subleading effects: none (0^{-+}), tiny ($< 2\%$, 0^{++}): [MQH, Fischer, Sanchis-Alepuz, EPJ Web Conf. 258 (2022); MQH, Fischer, Sanchis-Alepuz, HADRON2021, arXiv:2201.05163]

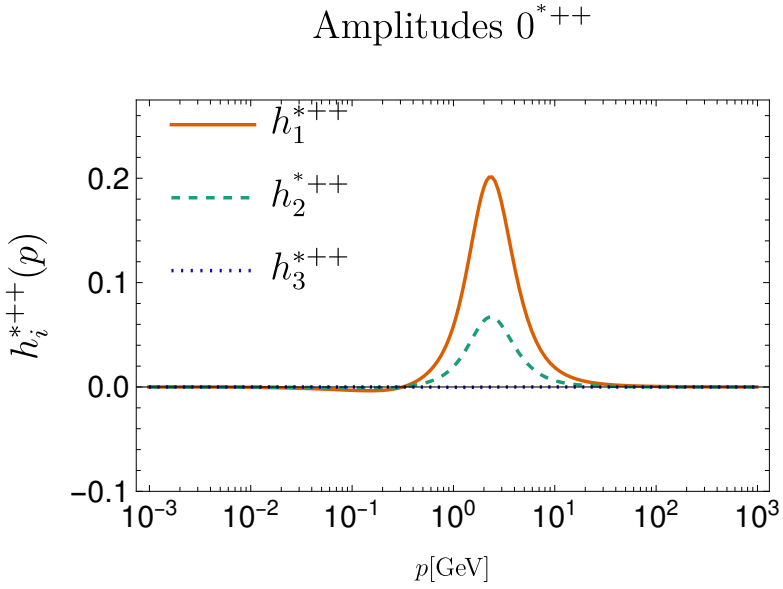
Amplitudes

Information about significance of single parts.

Ground state scalar glueball:



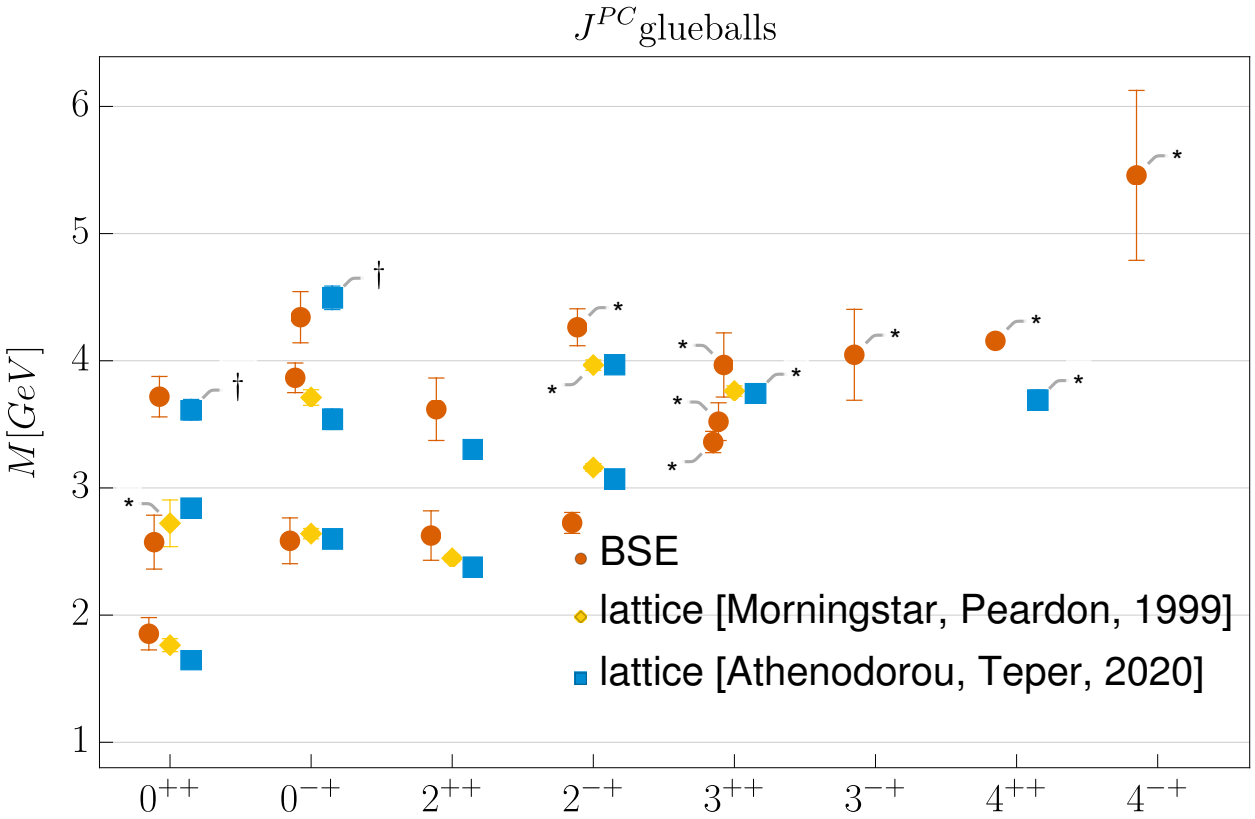
Excited scalar glueball:



→ Amplitudes have different behavior for ground state and excited state. Useful guide for future developments.

→ Meson/glueball amplitudes: Information about mixing.

Glueball results

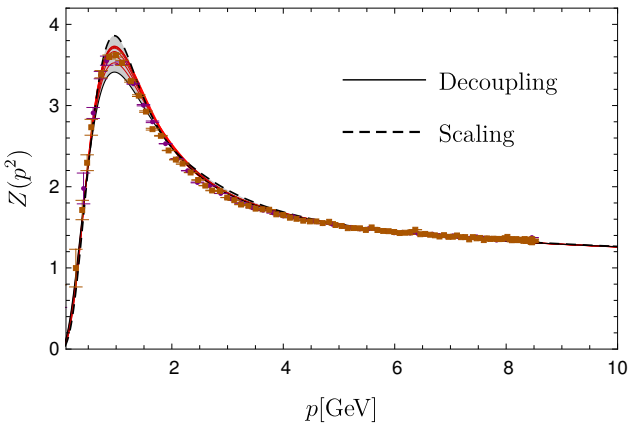


Lattice:
*: identification with some uncertainty
†: conjecture based on irred. rep of octahedral group

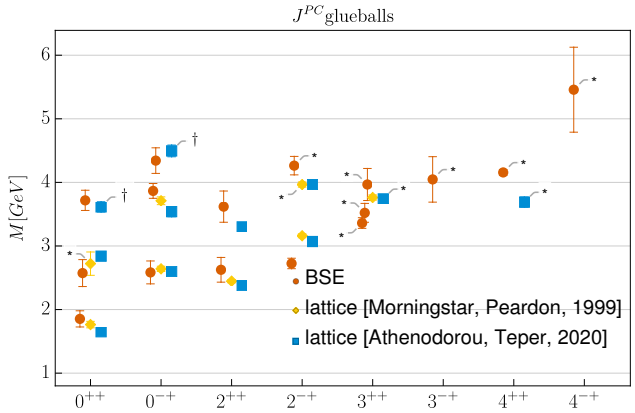
[MQH, Fischer, Sanchis-Alepuz, Eur.Phys.J.C81 (2021)]

- Agreement with lattice results
- New states: 0^{**++}, 0^{**-+}, 3^{-+}, 4^{-+}

Summary and outlook

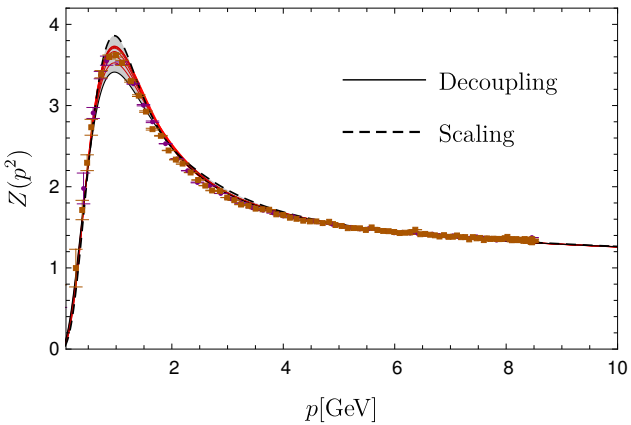


Reliable input: stable against extensions, agreement between different methods

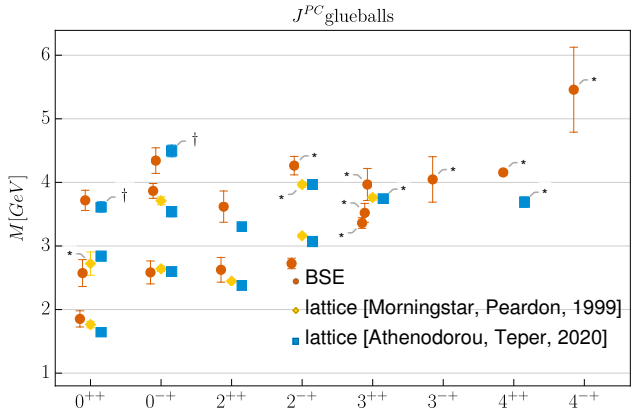


Pure glueball spectrum from first principles.

Summary and outlook



Reliable input: stable against extensions, agreement between different methods



Pure glueball spectrum from first principles.

Outlook: Inclusion of quarks

- Gluon sector fully back-coupled $\rightarrow$ Glueballs/mesons mixing
- $\rightarrow$ Roles of $f_0(1370)$, $f_0(1500)$, $f_0(1710)$

Thank you for your attention.

$J = 1$ glueballs

Landau-Yang theorem

Two-photon states cannot couple to $J^{\mathrm{P}} = \mathbf{1}^{\pm}$ or $(2n+1)^{-}$

[Landau, Dokl.Akad.Nauk SSSR 60 (1948); Yang, Phys. Rev. 77 (1950)].

($\rightarrow$ Exclusion of $J = 1$ for Higgs because of $h \rightarrow \gamma\gamma$.)

Applicable to glueballs?

$\rightarrow$ Not in this framework, since gluons are not on-shell.

$\rightarrow$ Presence of $J = 1$ states is a dynamical question.

$J = 1$ not found here.

Extrapolation

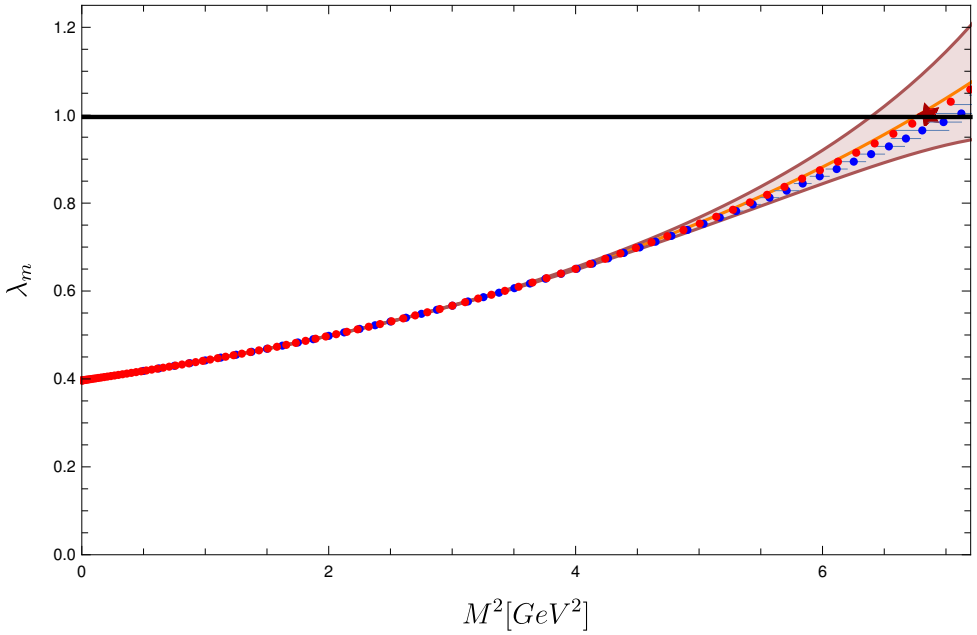
Input only for $P^2 > 0 \rightarrow$ extrapolation of eigenvalue curve. Solution when $\lambda(P^2) = 1$.

Schlessinger's continued fraction method [Schlessinger, Phys.Rev.167 (1968)]

Superior performance compared to other extrapolations in this context.

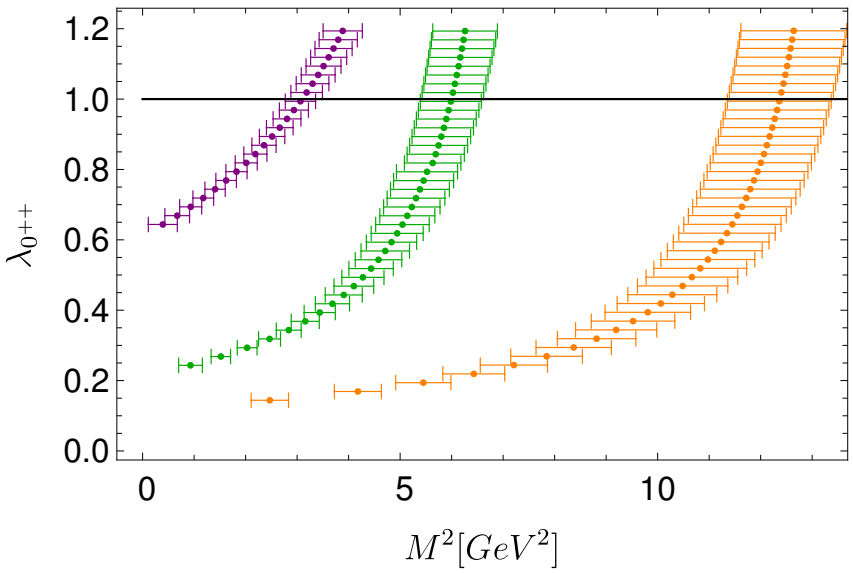
Test for solvable system: Heavy meson

[MQH, Fischer, Sanchis-Alepuz, Eur.Phys.J.C 80 (2020)]

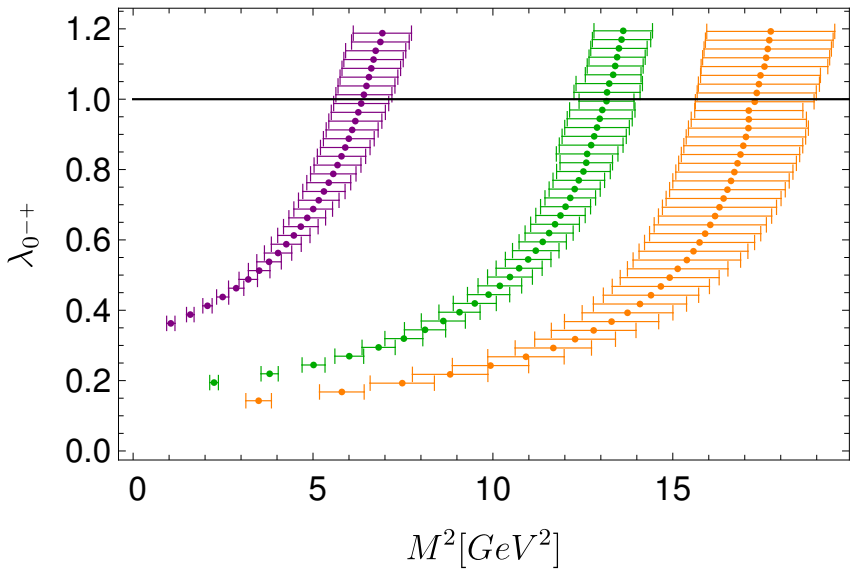


Extrapolation for glueball eigenvalue curves

Scalar glueball (0^{++}):



Pseudoscalar glueball (0^{-+}):



Several curves: ground state and excited states.