Going beyond the propagators of Landau gauge Yang-Mills theory

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X_{th} Confinement and the Hadron Spectrum

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Non-perturbative Landau gauge Green functions

Non-perturbative propagators of Landau gauge Yang-Mills theory:

- information about confinement
- input for phenomenological calculations (QCD phase diagram, bound states, ...) $\rightarrow$ sessions B, D
- QCD phase diagram:
  phase transitions (e.g. via condensates, Polyakov loop),
  no sign problem for functional methods, but equations get more complicated;
  also vertices change, e.g., ghost-gluon vertex at non-zero temperature [Fister, Pawlowski, 1112.5440]

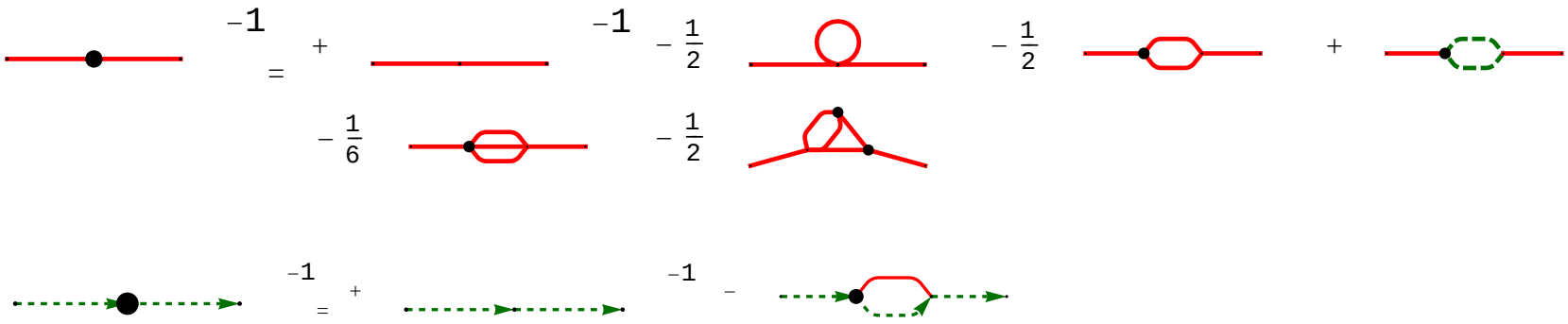
Calculated with

- Monte-Carlo simulations,
- functional renormalization group,
- Dyson-Schwinger equations,
- ...

Dyson-Schwinger equations: Propagators

Dyson-Schwinger equations (DSEs) of gluon and ghost propagators:

$$D_{gl}(p) = \left(g_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}\right)\frac{Z(p^2)}{p^2}, \quad D_{gh} = -\frac{G(p^2)}{p^2}$$



- Equations of motion of correlation functions.
- Infinite tower of coupled integral equations.
- Contain three-point and four-point functions:

  ghost-gluon vertex , three-gluon vertex , four-gluon vertex

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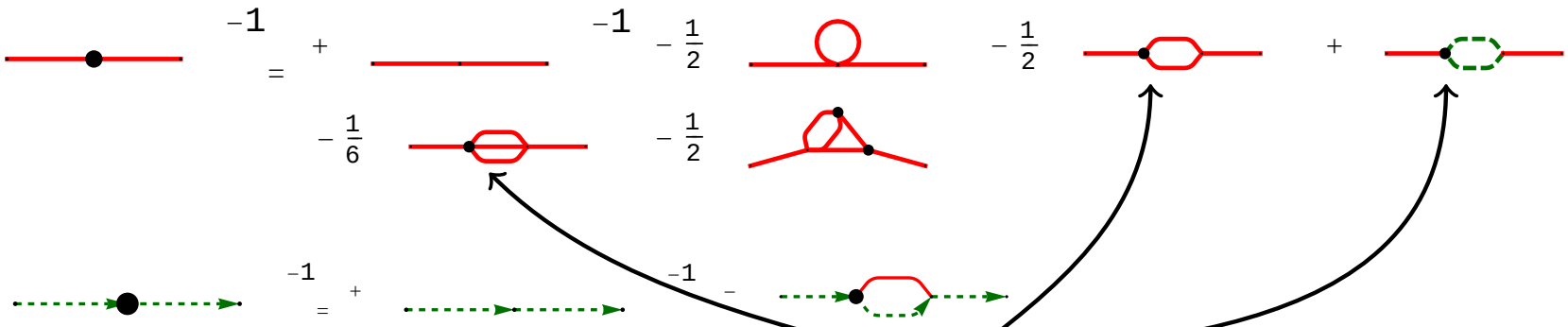

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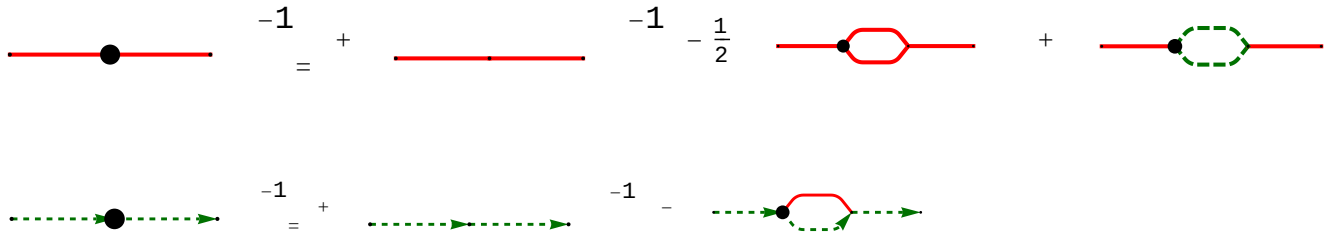
Truncated propagator Dyson-Schwinger equations

Standard truncation:



using bare ghost-gluon vertex and three-gluon vertex model

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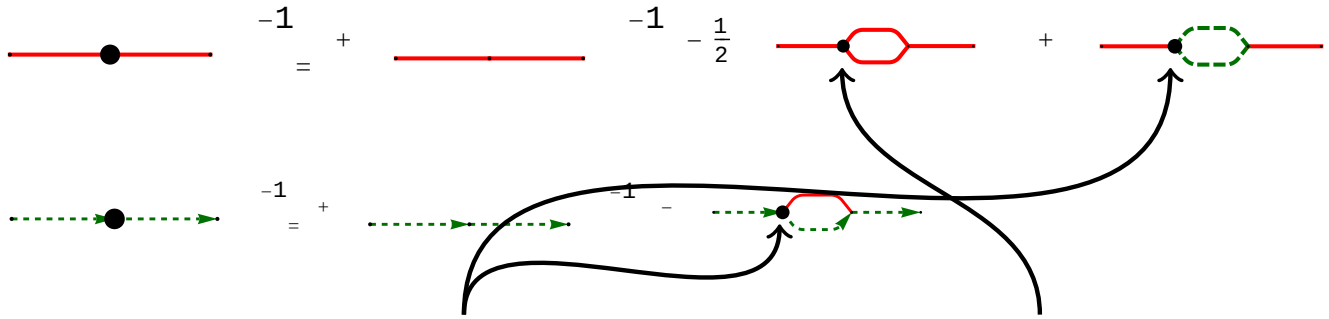

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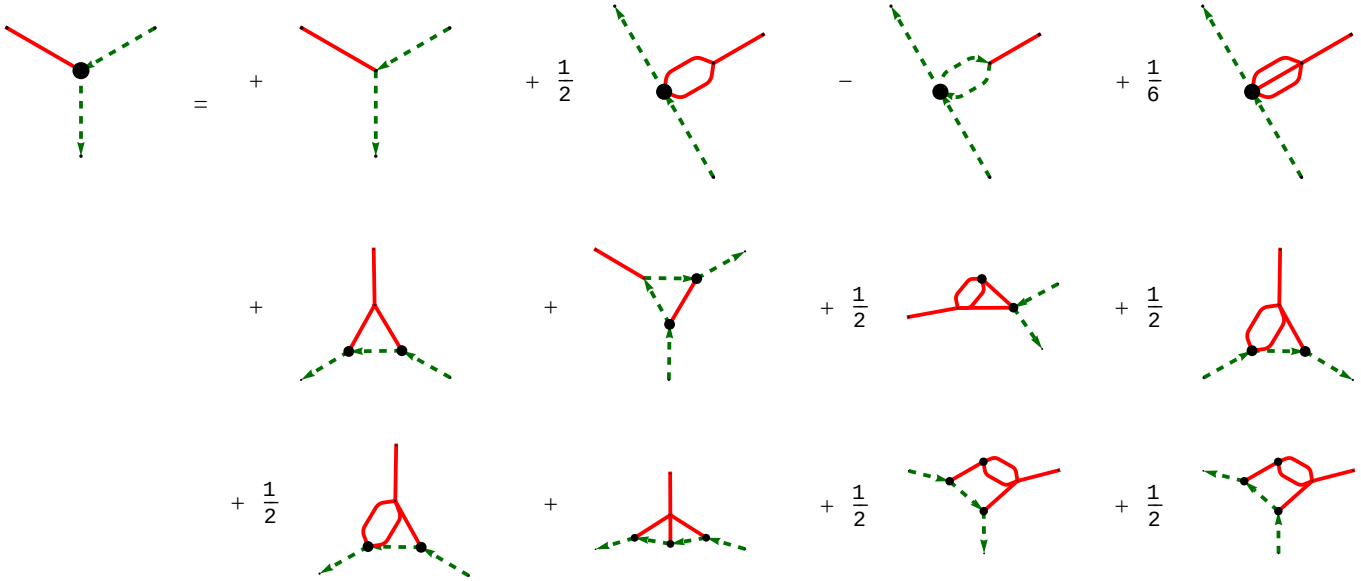
Ghost-gluon vertex



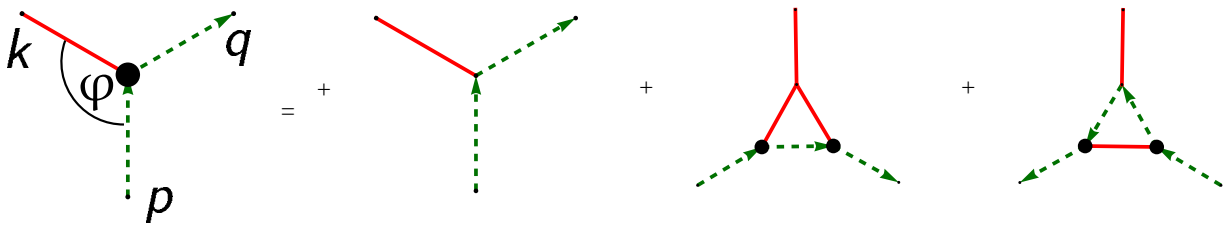
- Various lattice results [Cucchieri, Maas, Mendes, PRD77; Ilgenfritz, BJP37]
- OPE analysis [Boucaud et al., JHEP 1112]
- Modeling via ghost DSE [Dudal, Oliveira, Rodriguez-Quintero, 1207.5118]
- Semi-perturbative DSE analysis [Schleifenbaum et al., PRD72]

Ghost-gluon vertex

$$\Gamma_\mu^{A\bar{c}c,abc}(k;p,q) := i\,g\,f^{abc}\,(p_\mu A(k;p,q) + k_\mu B(k;p,q))$$

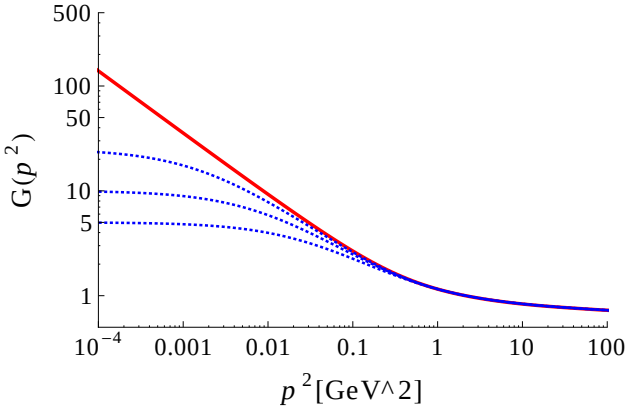
Note:
$B(k;p,q)$ is irrelevant in Landau gauge, but it is not the pure longitudinal part.

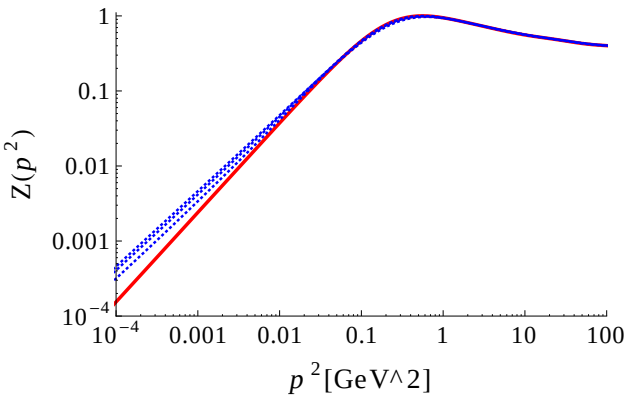
<u>IR and UV consistent truncation:</u>



Solutions of DSEs: Decoupling and scaling

- Two types of solutions with functional methods that differ only in deep IR [Boucaud et al., JHEP 0806, 012; Fischer, Maas, Pawlowski, AP 324]:
  scaling [von Smekal, Alkofer, Hauck PRL97],
  decoupling [Aguilar, Binosi, Papavassiliou PRD78]
- Lattice calculations find only decoupling type solution for $d = 3, 4$ and scaling for $d = 2$





$d = 2$: Analytic and numerical arguments from DSEs for scaling only [Cucchieri, Dudal, Vandersickel, PRD85; MQH, Maas, von Smekal, 1207.0222] as well as from analysis of Gribov region [Zwanziger, 1209.1974].

Ghost-loop-only truncation

Scaling solution: Ghost loop is IR dominant by power counting [von Smekal, Hauck, Alkofer, 79; Alkofer, Fischer, Llanes-Estrada, PLB611]

Decoupling solution: No ghosts $\longrightarrow$ no gluon "mass" [Aguilar, Binosi, Papavassiliou, JHEP1201];

ghost loop alone, i.e., $\Gamma^{A^3} = 0$?

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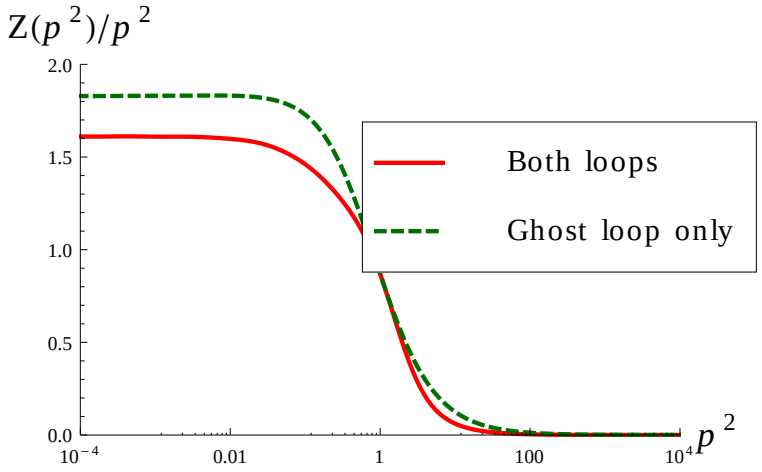

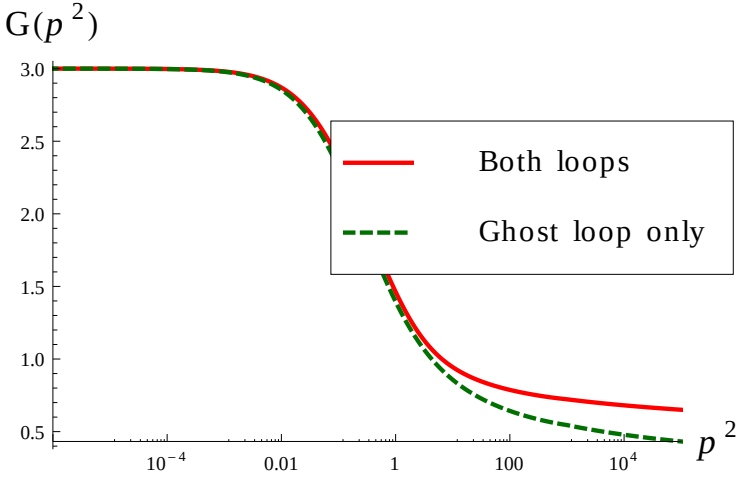

$\Rightarrow$ Ghost loop alone sufficient to obtain decoupling behavior!

Note: Gluon loop only $\longrightarrow$ Mandelstam solution, IR divergent gluon propagator [Mandelstam PRD20]

Three-gluon vertex: Ultraviolet

Ansatz that reproduces the correct UV behavior of the gluon propagator [Fischer, Alkofer, Reinhardt, PRD65]:

$$D^{A^3}(x,y,z) = \frac{1}{Z_1}\frac{[G(y)G(z)]^{1-a/\delta-2a}}{[Z(y)Z(z)]^{1+a}}$$

y and z are the momenta in the gluon loop, i.e., not Bose symmetric in y, z *and* x.

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Bose symmetrized version:

$$D^{A^3,UV}(x,y,z) = \frac{1}{Z_1}G(x+y+z)^{\alpha}Z(x+y+z)^{\beta}$$

Fix α and β:

1. UV behavior of gluon propagator
2. IR behavior of three-gluon vertex?

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2. IR behavior of three-gluon vertex $\rightarrow$ yes, but ...

Three-gluon vertex: Infrared

Hints from lattice data [Cucchieri, Maas, Mendes, PRD77]:

Three-gluon vertex might have a zero crossing.

($d = 2, 3$: zero crossing seen [Cucchieri, Maas, Mendes, PRD77; Maas, PRD75])

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$\Rightarrow$ add IR part:

$$D^{A^3,IR}(x,y,z) = \boldsymbol{h_{IR}} G(x+y+z)^3 (f^{3g}(x) f^{3g}(y) f^{3g}(z))^4$$

IR damping function $f^{3g}(x) := \dfrac{\Lambda_{3g}^2}{\Lambda_{3g}^2 + x}$

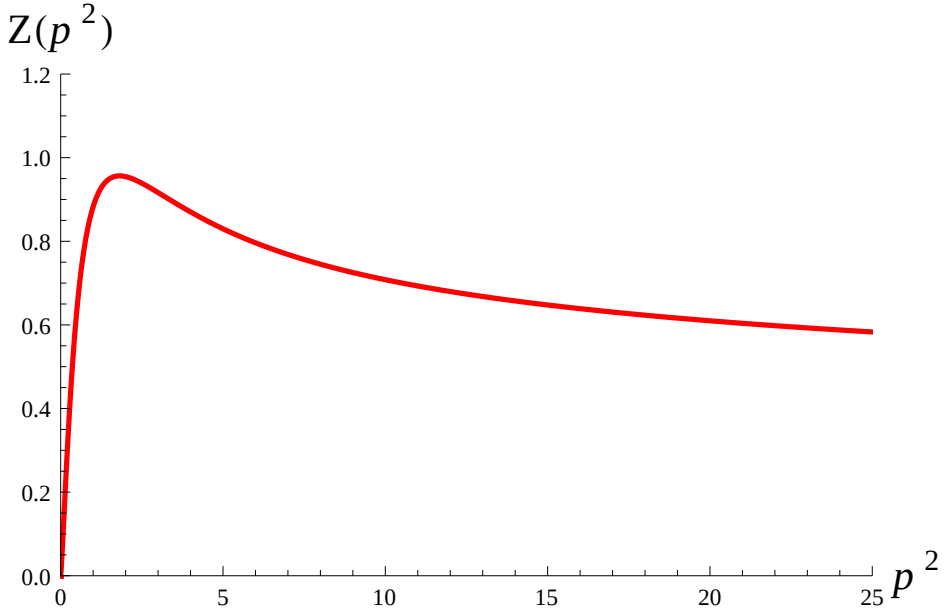
Parameter for zero crossing: $\boldsymbol{h_{IR} < 0}$

New three-gluon vertex:

$$D^{A^3}(x,y,z) = D^{A^3,IR}(x,y,z) + D^{A^3,UV}(x,y,z)$$

Propagators: Improving the three-gluon vertex

ghost-gluon vertex: bare



original three-gluon vertex

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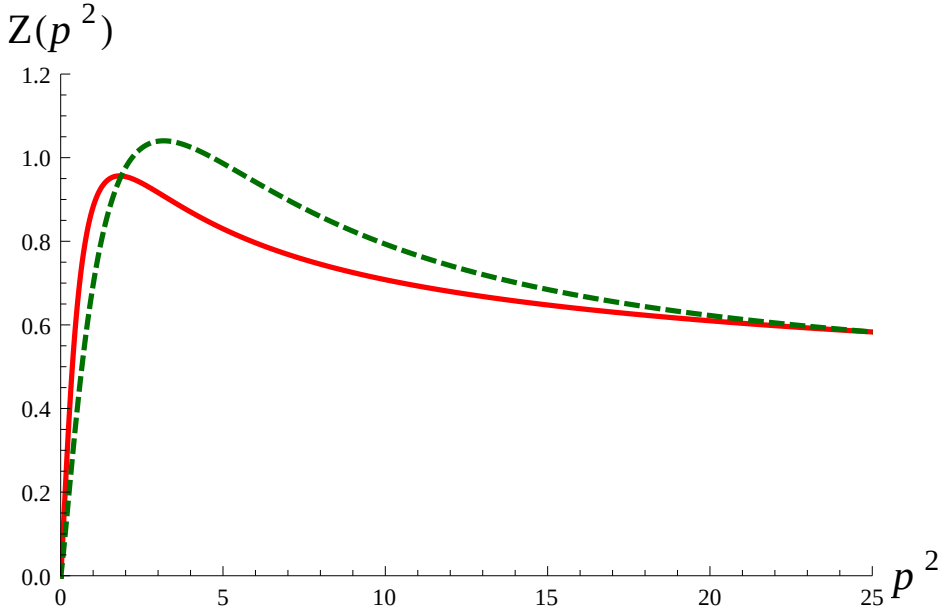

original three-gluon vertex

Bose symmetric three-gluon vertex

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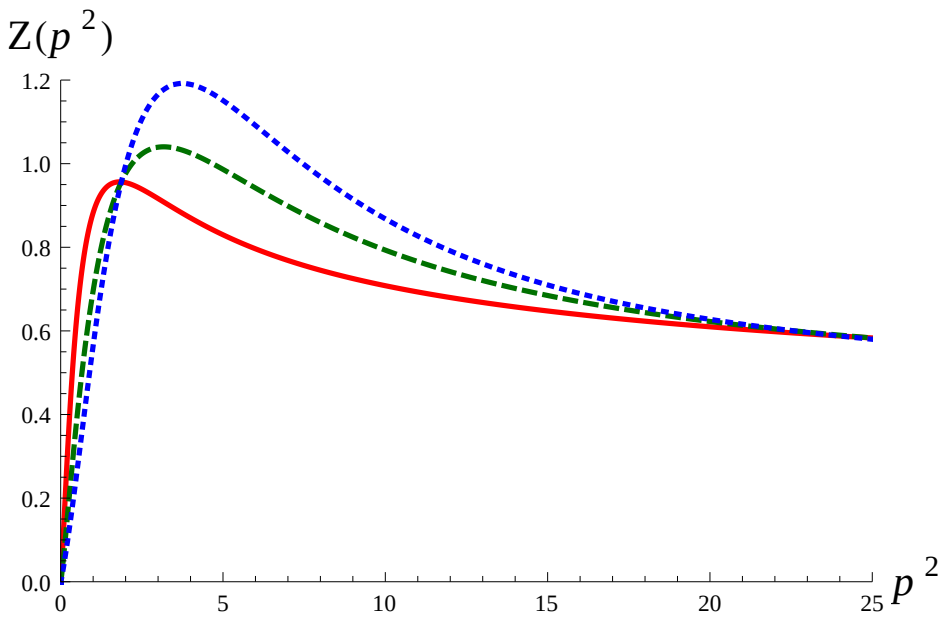

original three-gluon vertex
Bose symmetric three-gluon vertex
Bose symmetric three-gluon vertex with IR part

$\Rightarrow$ Improved three-gluon vertex adds additional strength in the mid-momentum regime.

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basis choice: p^2, k^2, φ

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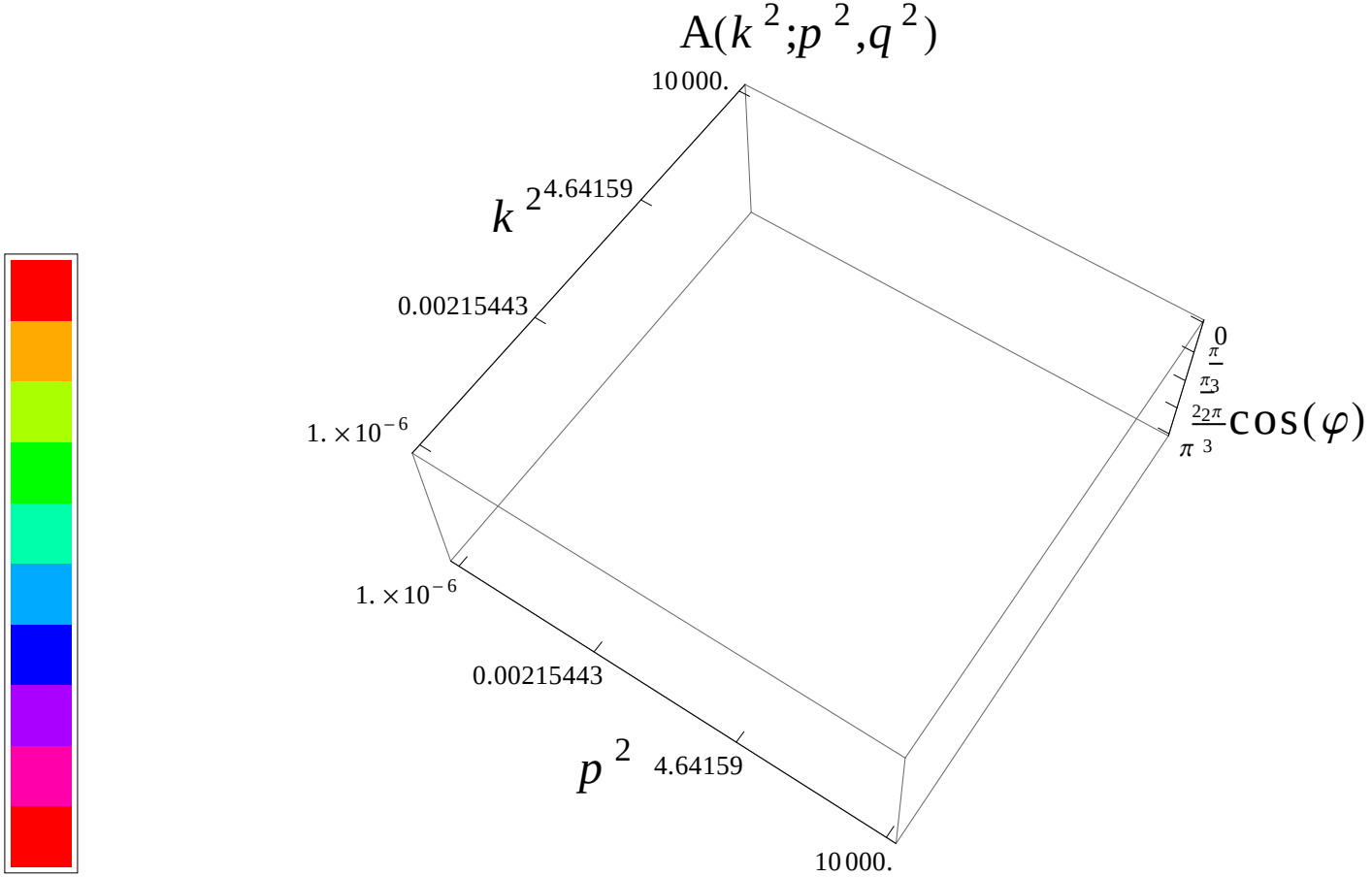

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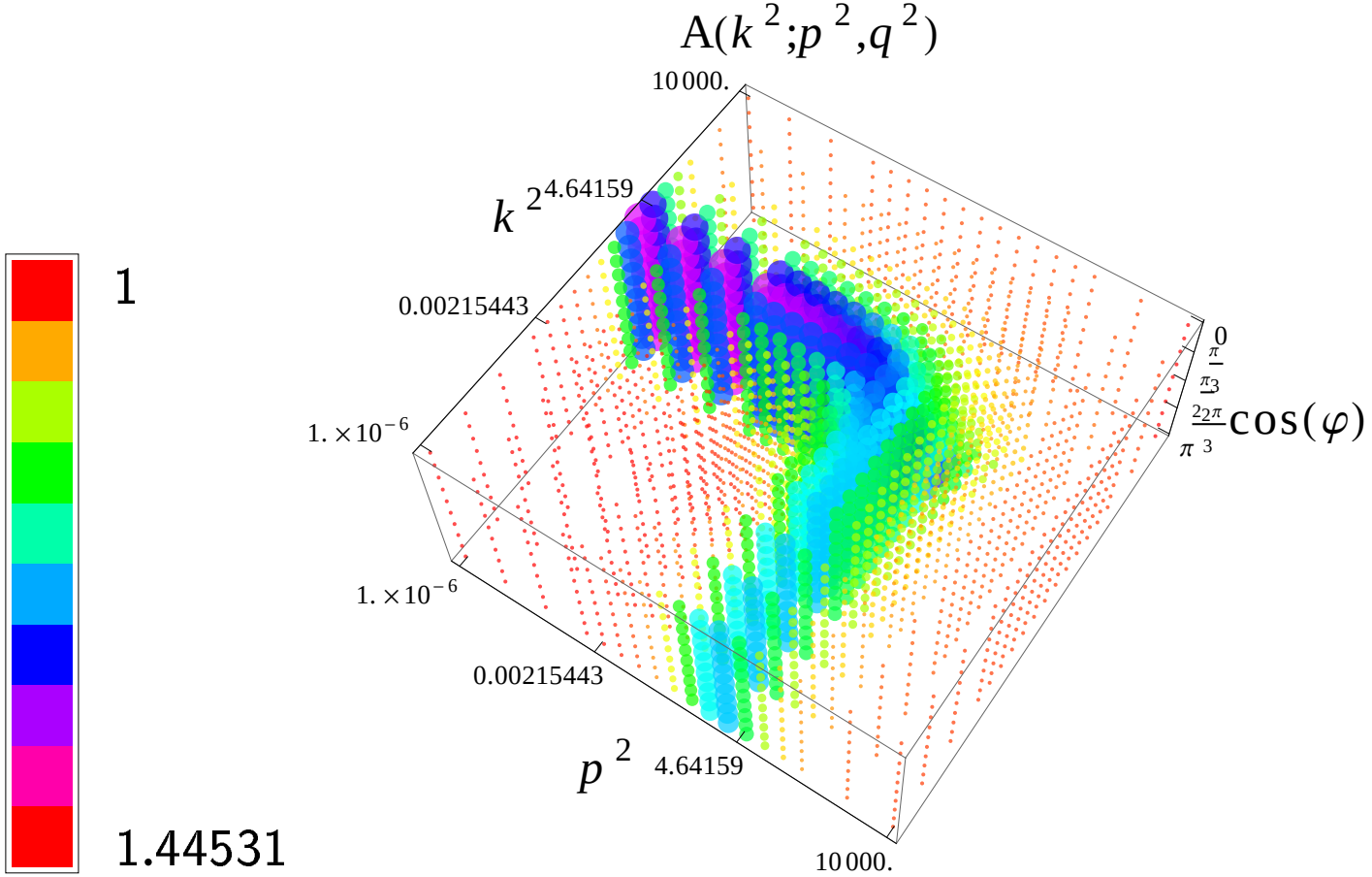
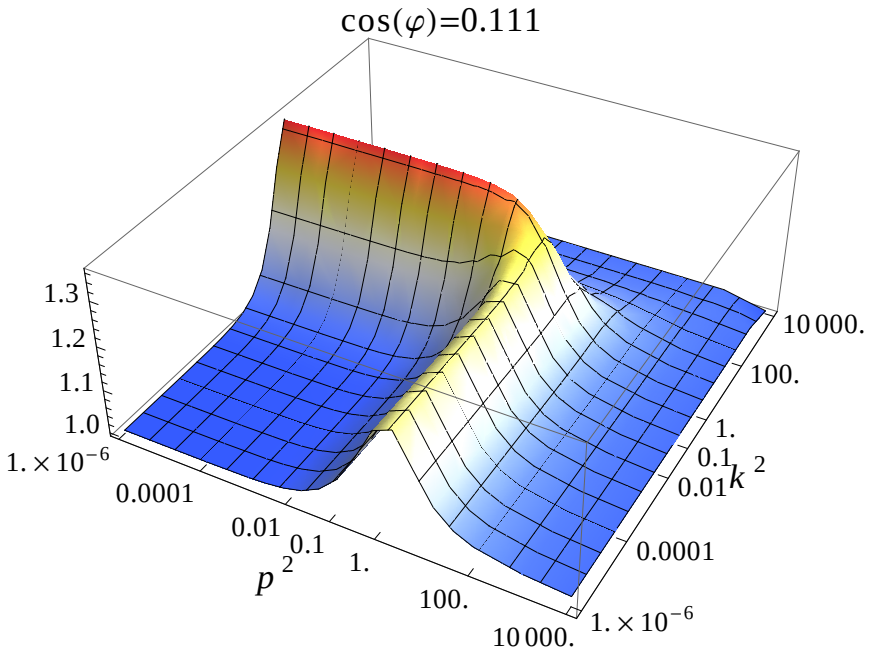
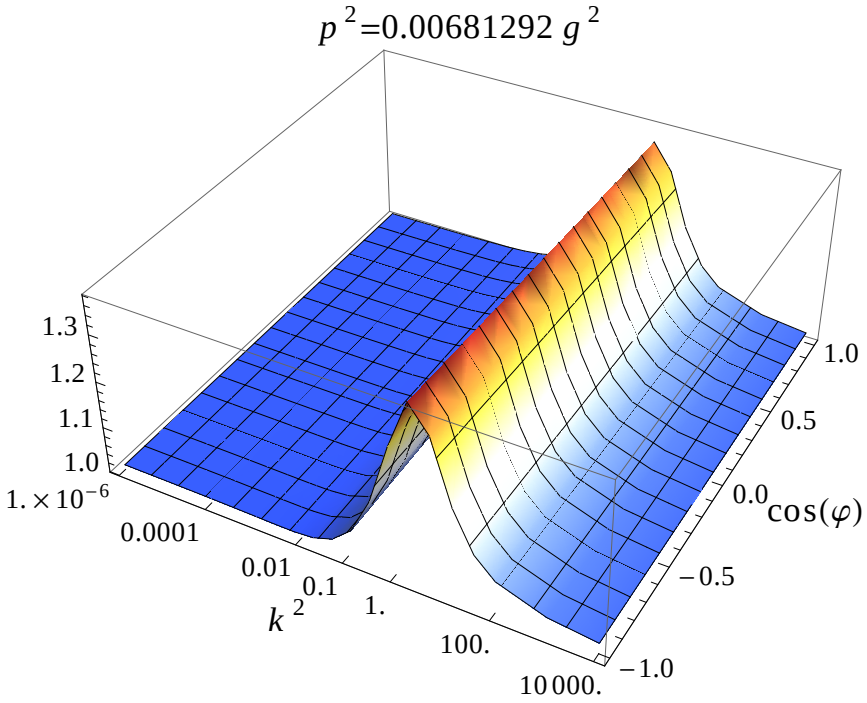


Ghost-gluon vertex: Selected configurations

Fixed angle:



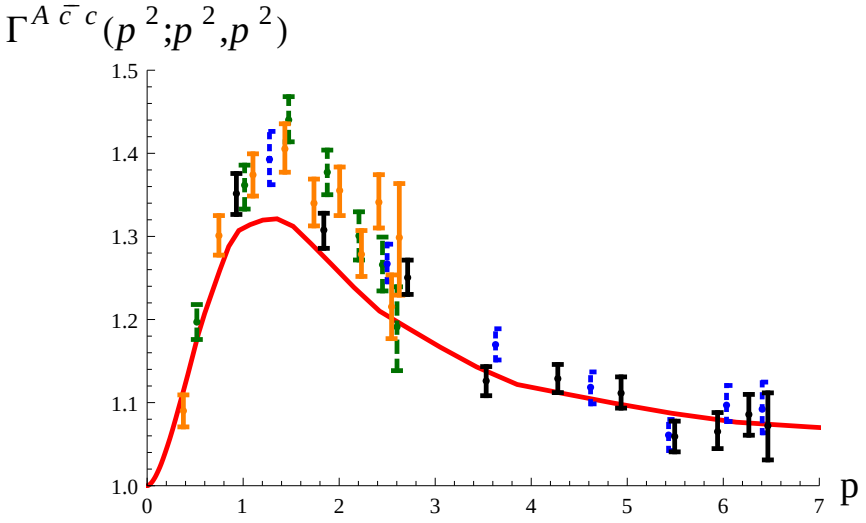
Fixed momentum:



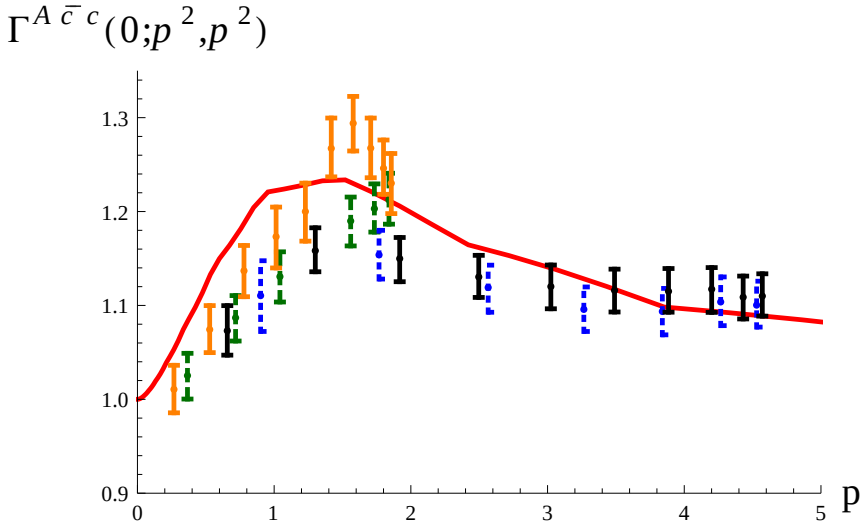
Results for $SU(2)$

Ghost-gluon vertex: Comparison with lattice data

Symmetric configuration $k^2 = q^2 = p^2$:



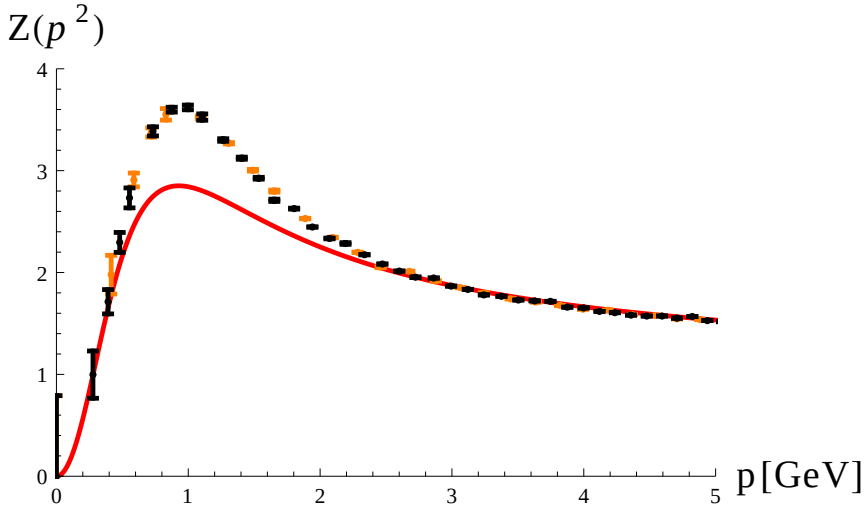
Orthogonal configuration $k^2 = 0$, $q^2 = p^2$:

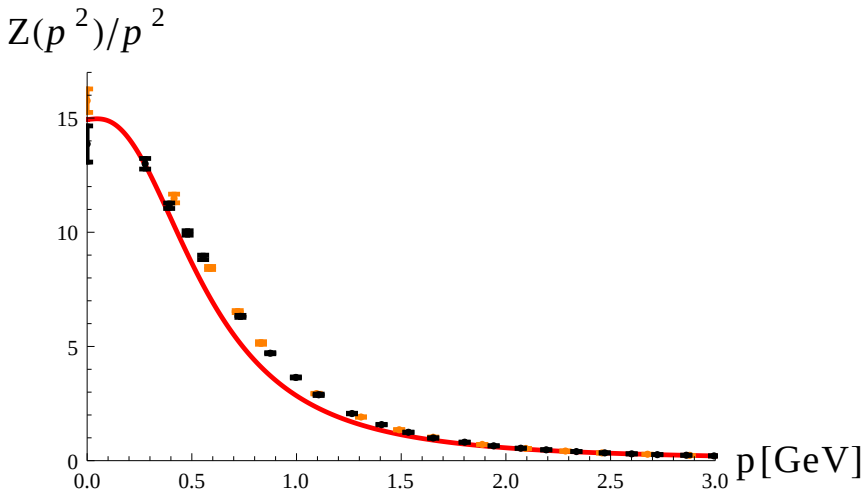


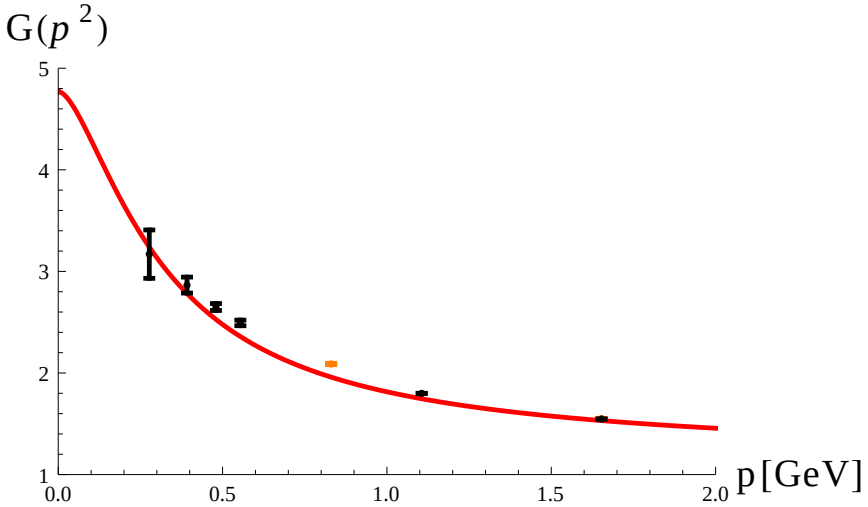
lattice data: [Cucchieri, Maas, Mendes, PRD77]

Results for $SU(2)$

Propagators and dynamic ghost-gluon vertex





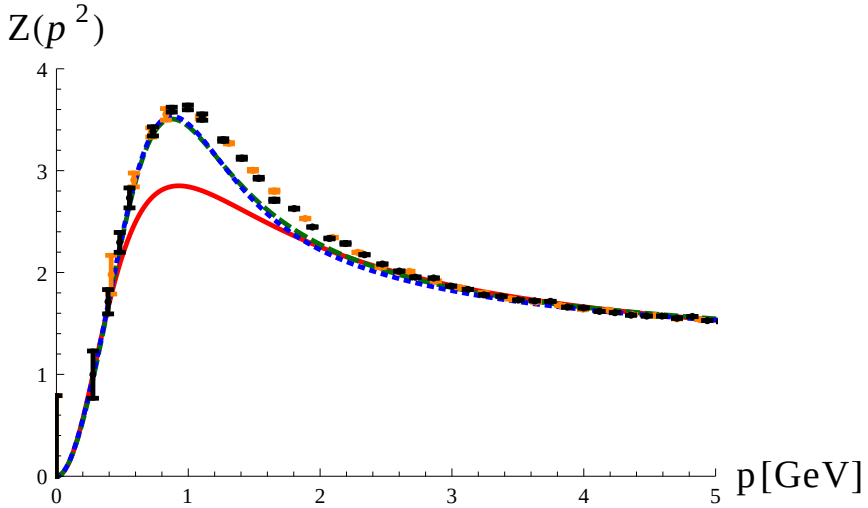


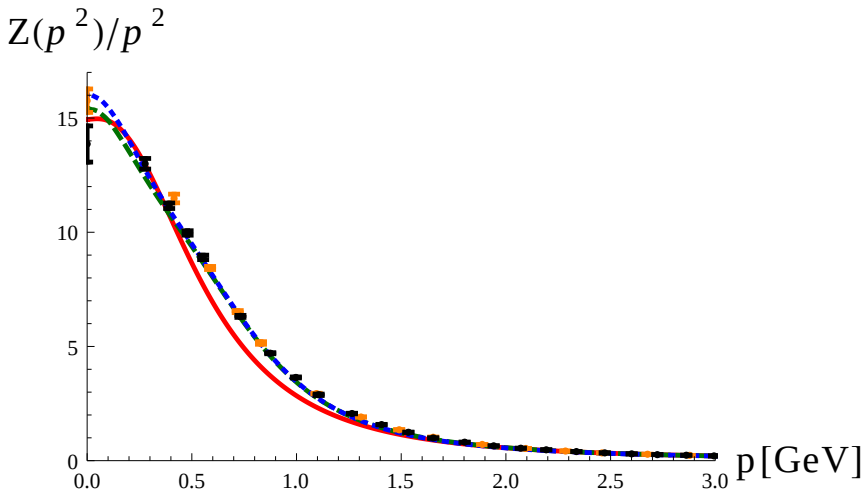
Results for $SU(3)$

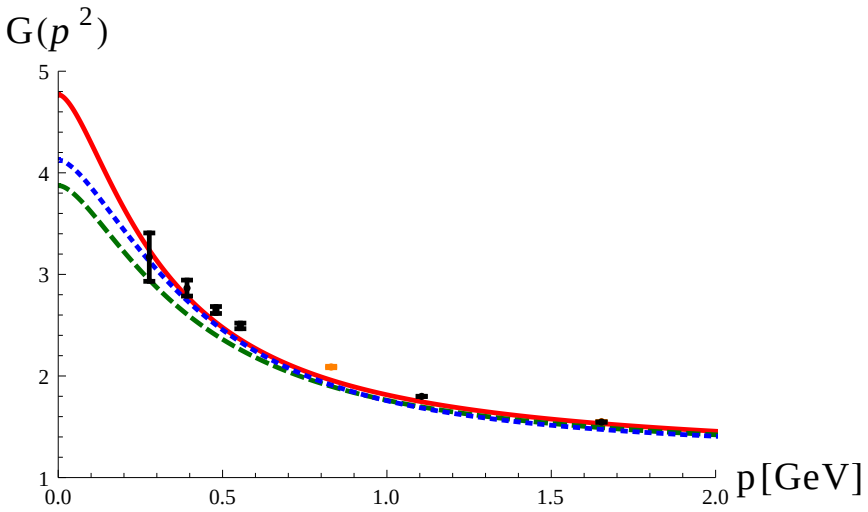
bare ghost-gluon vertex,
Bose non-symmetric three-gluon vertex

lattice: 32^4
lattice: 48^4 [Sternbeck, hep-lat/0609016]

Propagators and dynamic ghost-gluon vertex







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bare ghost-gluon vertex,
improved three-gluon vertex
dynamic ghost-gluon vertex,
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lattice: 32^4
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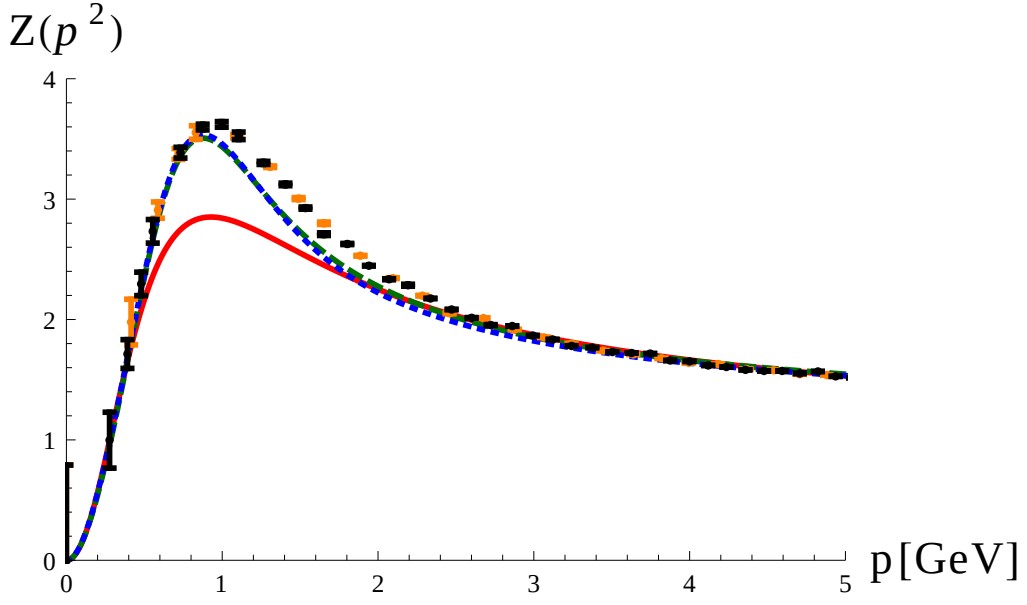
Propagators and dynamic ghost-gluon vertex

- More realistic three-gluon vertex improves mid-momentum behavior.
- Dynamic ghost-gluon vertex influences ghost in IR
  Note: Solution with dynamic ghost-gluon vertex requires improved three-gluon vertex for stability (gluon loop otherwise too strong).
- Missing part around 2 *GeV* probably due to two-loop terms
  $\longrightarrow$ talk of Valentin Mader



Summary & Conclusions

- Ghost dominance for decoupling and scaling solution (ghost-loop-only analysis).
- More realistic three-gluon vertex:
  Bose symmetric and IR features from lattice
  $\longrightarrow$ Yields more support in mid-momentum regime.
  $\longrightarrow$ Reduces gap to lattice results.
- Dynamic inclusion of ghost-gluon vertex:
  - rather small changes for gluon
  - influences ghost in the IR
  - improved three-gluon vertex required
- Ghost-gluon vertex results agree well with available lattice results.

Summary & Conclusions

- Inclusion of three-point functions:
  - test of truncations,
  - required for quantitative results and
  - likely also for some aspects of non-zero temperature and density calculations

- Automatization tools:
  DoFun [Alkofer, MQH, Schwenzer, CPC180; MQH, Braun, CPC183]
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