A confining mechanism based on Gribov horizon and BRST soft symmetry breaking in a toy model

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Abstract

A toy model which mimics the effects of the Gribov horizon and of the BRST soft symmetry breaking is constructed in order to investigate a possible mechanism for confinement. We show that the Gribov horizon and the BRST breaking combine in a nice fashion, in such a way that a class of composite operators whose correlation functions exhibit physical cuts only can be obtained as cohomology classes of the BRST operator.

Introduction and Motivation

- The problem of confinement in quantum field theory may be defined by the property that *the fundamental fields appearing in the Lagrangian defining the theory do not describe the physical excitations of the theory.*
- The prime example where this happens is QCD at low energy scales. Since even without quarks the theory is confining, we can focus only on the pure Yang-Mills theory.
- To properly quantize a gauge theory we must take care of the redundancy introduced by the gauge variables. In a path integral quantization this is usually done by the introduction of Faddeev-Popov ghosts fixing the gauge in a consistent way.
- Gauge symmetry turns out to be replaced by a global symmetry involving anticommuting variables known as BRST symmetry. This symmetry has a very important role in defining the physical states of the theory which are identified as the cohomology classes of the BRST generator (physical states $|Phys\rangle$ are the ones annihilated by the BRST generator Q and which are not of the form $Q|something\rangle$, that is, are BRST closed but not exact).
- Unfortunately the Faddeev-Popov procedure does not work properly outside the perturbative domain as pointed out by Gribov [1]. There still remain gauge copies after the introduction of the ghosts. Gribov suggested a way out by restricting the configuration space of integration to what became known as the *Gribov region*. Later Zwanziger [2] was able to implement analytically this restriction in the path integral formulation defining the so called *Gribov-Zwanziger action*.
- The Gribov-Zwanziger action is renormalizable but softly breaks the BRST symmetry. The breaking has its origin in the restriction to the Gribov region implemented in the action by the so called horizon function. This means that there is no straightforward way of defining the physical states of the theory and in particular these states cannot be identified with the fundamental fields anymore. This is exactly what we expect for a confining phase.
- In this work we discuss a toy model which displays the essential elements of this picture. The toy model has many similarities with the Gribov-Zwanziger theory and allows for a more accessible investigation of its physical properties and to get a glimpse of the possible mechanism responsible for the establishment of the confining phase.

Toy Model

In order to define the toy model, we start with the free theory

$$\mathcal{L}_0 = \frac{1}{2}\partial_\mu\psi^a\partial_\mu\psi^a + \partial_\mu\bar{\varphi}^a\partial_\mu\varphi^a - \partial_\mu\bar{\omega}^a\partial_\mu\omega^a, \quad (7)$$

where the real field ψ is in the adjoint representation of $SU(N)$. The BRST quartet φ^a, $\bar{\varphi}^a$, ω^a and $\bar{\omega}^a$, which are also in the adjoint representation, does not change the physical content of the theory since the last two terms are BRST exact.
This lagrangian is invariant under the BRST transformation (c is a constant ghost field)

$$s\,\psi^a = f^{abc}c^b\psi^c; \quad s\,c^a = \frac{1}{2}f^{abc}c^bc^c \quad (8)$$

$$s\,\bar{\omega}^a = \bar{\varphi}^a + f^{abc}c^b\bar{\omega}^c; \quad s\,\varphi^a = \omega^a + f^{abc}c^b\varphi^c, \quad (9)$$

$$s\,\bar{\varphi}^a = f^{abc}c^b\bar{\varphi}^c; \quad s\,\omega^a = f^{abc}c^b\omega^c, \quad (10)$$

We now turn this model into a non-trivial theory by adding a term that explicitly breaks the BRST symmetry

$$\mathcal{L}_0 \to \mathcal{L}_0 + \mathcal{L}_\vartheta, \quad (11)$$

$$\mathcal{L}_\vartheta = \vartheta^2\psi^a(\varphi^a - \bar{\varphi}^a), \quad s\mathcal{L}_\vartheta = \vartheta^2\psi^a\omega^a \neq 0. \quad (12)$$

The added term is the analog of the horizon function of the Gribov-Zwanziger formulation. Here ϑ, a mass dimension parameter, plays the same role as the Gribov parameter γ. Whereas we introduced the breaking of the BRST symmetry here by hand, it appears naturally in the Yang-Mills theory due to the Gribov problem.

Search for the physical spectrum

The theory is now drastically modified and in particular the ψ field propagator assumes the form

$$\langle\psi^a(p)\psi^b(-p)\rangle = \delta^{ab}\frac{p^2}{p^4 + 2\vartheta^4}. \quad (13)$$

This has the same structure as the gluon propagator of the Gribov-Zwanziger formulation. The poles of the propagator are purely imaginary at $\pm i\sqrt{2}\vartheta^2$. Thus the propagator does not describe a physical particle.
We expect that the physical excitations take the form of bound states. This would mimic the behavior of Yang-Mills theory where the physical particles are the glueballs only, the gluons being confined. We are thus searching for composite operator with physical cuts.
The Yang-Mills case was investigated by Zwanziger [2] in terms of the correlation function of two Wilson loops, $\langle F^2(x)F^2(y)\rangle$. He found the interesting result that this quantity has the following structure:

$$\langle F^2(p)F^2(-p)\rangle = G^{un}(p) + G^{ph}(p). \quad (14)$$

The first part possess imaginary cuts and is thus unphysical, whereas the second part has a cut at the real axis with a positive spectral density. Therefore it may describe the physical excitation of a glueball. How to get rid of the unphysical part so that the complete correlation function becomes physical was an unsolved problem. Within the present toy model setting, however, we are able to circumvent this problem and identify a physical operator whose correlation function has no unphysical part.

Gribov-Zwanziger

The Gribov-Zwanziger formulation is a proposal to describe the quantum theory of the Yang-Mills system at low energies. The usual gauge fixed Yang-Mills action

$$S_{YM} = \int dx \left(\frac{1}{4}F^a_{\mu\nu}F^a_{\mu\nu} + i\,b^a\partial_\mu A^a_\mu - \bar{c}^a M^{ab}c^b\right), \quad (1)$$

with (Landau gauge)

$$M^{ab} = -D^{ab}_\mu\partial_\mu = -(\delta^{ab}\Box + g\,f^{abc}A^c_\mu\partial_\mu), \quad (2)$$

is modified in order to take into account the restriction of the integration in configuration space to the Gribov region by the introduction the so-called horizon function $h(x)$

$$h(x) = g^2 f^{ace}A^a_\mu(M^{-1})^{cd}f^{bde}A^b_\mu. \quad (3)$$

The Gribov-Zwanziger action is given by

$$S_{GZ} = S_{YM} + \gamma^4\int dx\,h(x). \quad (4)$$

The mass parameter γ is not free but determined by

$$\langle h(x)\rangle = d(N^2 - 1), \quad (5)$$

where N is the number of colors and d the space-time dimension.
The added horizon function term softly breaks the BRST symmetry of the model and as a result the gluon propagator

$$\langle A_\mu(p)A_\nu(-p)\rangle = \frac{p^2}{p^4 + \gamma^4}\left(\delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}\right). \quad (6)$$

does not have a particle interpretation since the poles are not real and it also does not satisfy the Källén-Lehmann representation with a positive spectral density.

Physical Operators

The idea is to exploit the breaking of BRST symmetry. When a theory is BRST invariant, one can add to any operator another operator that is BRST exact without altering the result. However, if BRST symmetry is broken, this statement is no longer true. The goal is to choose the additional parts such that they cancel the unphysical part of the original operator correlation functions. There is a whole class of operators that can be parameterized by coefficients multiplying the BRST exact terms. In general, an operator $O(x)$ can be modified as

$$O(x) \to O(x) + \sum_i c_i s(\mathcal{Y}_i), \quad (15)$$

where $\mathcal{Y}_i$ are functions of the fields and c_i some coefficients to be chosen to cancel the unphysical part of the correlators. We studied an explicit example:

$$O_2(x) = \frac{1}{2}\partial_\mu\psi^a\partial_\mu\psi^a - s(\partial_\mu\bar{\varphi}^a\partial_\mu\omega^a), \quad (16)$$

and we found that the last term indeed gives the non-trivial required contribution in order to cancel the unphysical part of the correlator $\langle O_2(p)O_2(-p)\rangle$. We found that this correlator can be written in the Källén-Lehmann representation

$$\langle O_2(p)O_2(-p)\rangle = \int_{2\sqrt{2}\vartheta^2}^{\infty} d\tau\,\frac{\rho(\tau)}{p^2 + \tau}, \quad (17)$$

with the spectral function ρ a positive function. For instance, in 4 dimensions it is given by

$$\rho_{d=4}(\tau) = \frac{\tau^2}{(8\pi)^2}\sqrt{1 - \frac{8\vartheta^4}{\tau^2}}, \quad (18)$$

Thus we conclude that a physical spectrum can be unraveled if we explore the BRST symmetry breaking in the construction of physical operators. The hope is that this procedure can be applied to the more involved Yang-Mills system in order to identify glueballs.

References

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