Infrared Behavior of the Three-Gluon Vertex in d-Dimensional Yang-Mills Theory

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Motivation

Diploma thesis: IR-exponents of vertices in three dimensions

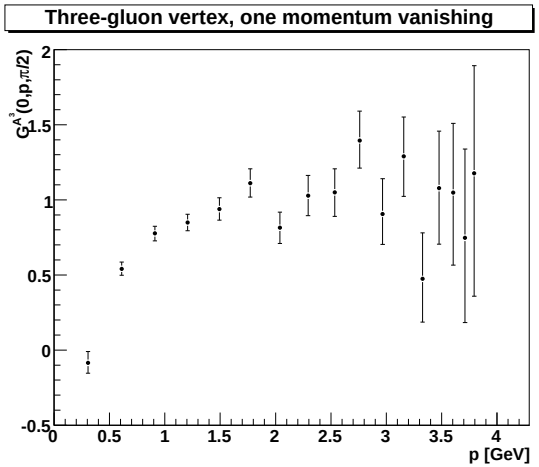
Why **d** space-time dimensions?

	4 dimensions	3 dimensions
Ordering scheme	skeleton expansion Alkofer, Fischer, Llanes-Estrada	ordering scheme?
Lattice	hardly feasible in IR	Cucchieri, Maas, Mendes

Later?:

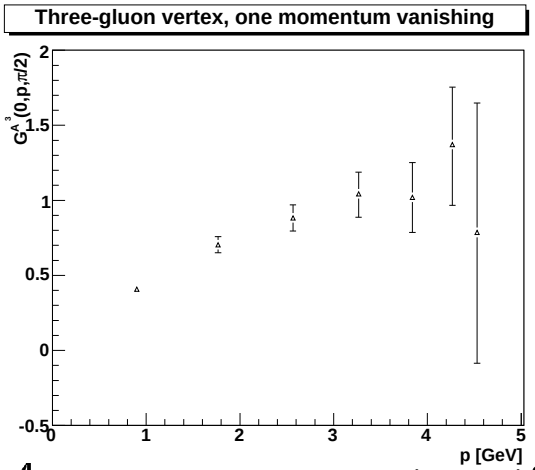
- Three-dimensional Yang-Mills (YM) theory can be compared to the infinite-temperature limit of four-dimensional YM theory.
- A connection between 4d YM in Coulomb gauge and 3d YM in Landau gauge exists.

Available Lattice Data



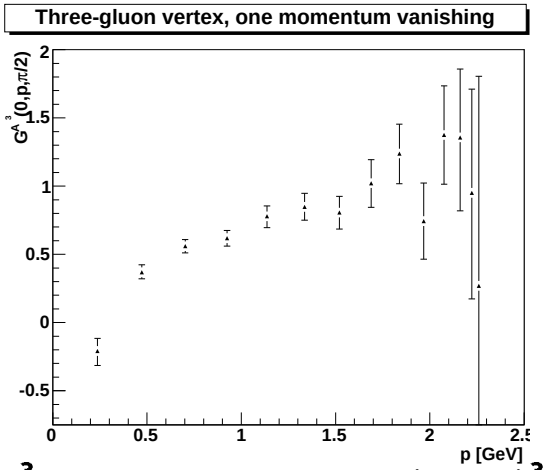
40^2 lattice, $\beta = 30$, $V = (4.0fm)^2$

Maas, in preparation



16^4 lattice, $\beta = 2.5$, $V = (1.4fm)^4$

Cucchieri et al., hep-lat/0610006



30^3 lattice, $\beta = 4.2$, $V = (5.2fm)^3$

Cucchieri et al., PRD74, 014503, 2006

The infrared region is easier to access in two and three dimensions.

What is an infrared exponent?

Infrared exponent expresses the behavior of a Green function at small external momenta ($\ll \Lambda_{QCD}$).

Green functions can have additional momentum dependence,
e.g. ghost propagator: $D(p^2) = -\frac{G(p^2)}{p^2}$

Prominent picture in QCD: Gribov-Zwanziger scenario

- gluon propagator dressing function vanishes in IR: $Z(p^2) \propto (p^2)^{2\kappa}$
- ghost propagator dressing function is divergent in IR: $G(p^2) \propto (p^2)^{-\kappa}$
- $\kappa_{d=4} = 0.595$

Outline

Transversality of gluon propagator

$\Downarrow$

Bare ghost-gluon vertex

$\Downarrow$

Infrared exponents of the propagators from the ghost Dyson-Schwinger equation (DSE)

$\Downarrow$

Skeleton expansion of other DSEs

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Infrared exponents of all dressed Green functions of Yang-Mills theory

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The Bare Ghost-Gluon Vertex

Starting point is transversal gluon propagator in Landau gauge:

$$k_\mu D_{\mu\nu}(k) = k_\mu \frac{Z(k)}{k^2}\left[\delta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}\right] = 0$$

$$(q-p)_\mu D_{\mu\nu}(q-p) = 0 \Rightarrow q_\mu D_{\mu\nu}(q-p) = p_\mu D_{\mu\nu}(q-p)$$

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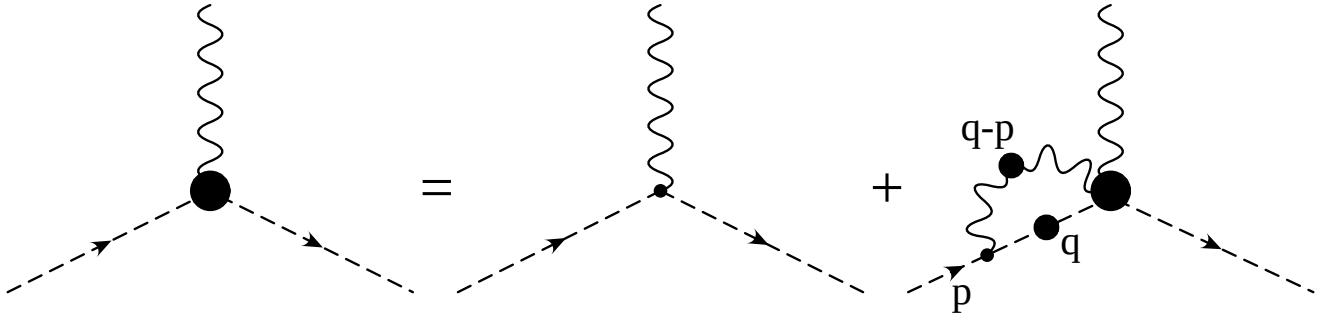
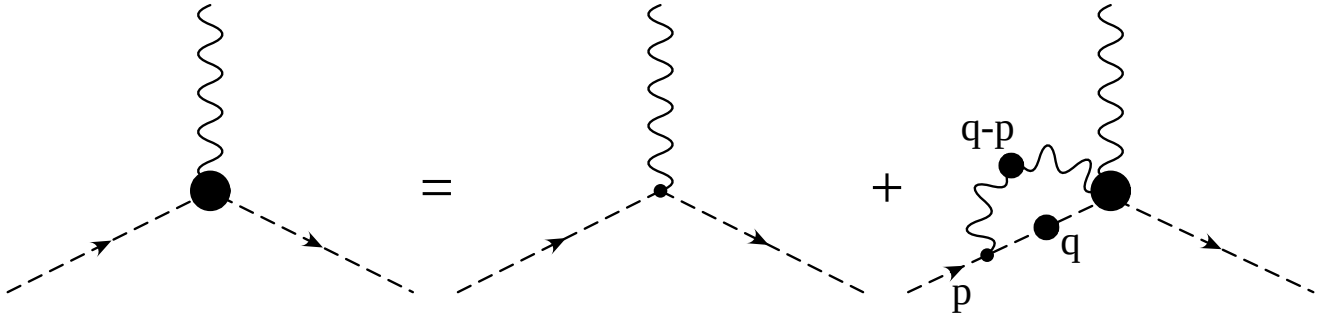


loop momentum q and external momentum p

The Bare Ghost-Gluon Vertex

$$q_\mu D_{\mu\nu}(q-p)$$



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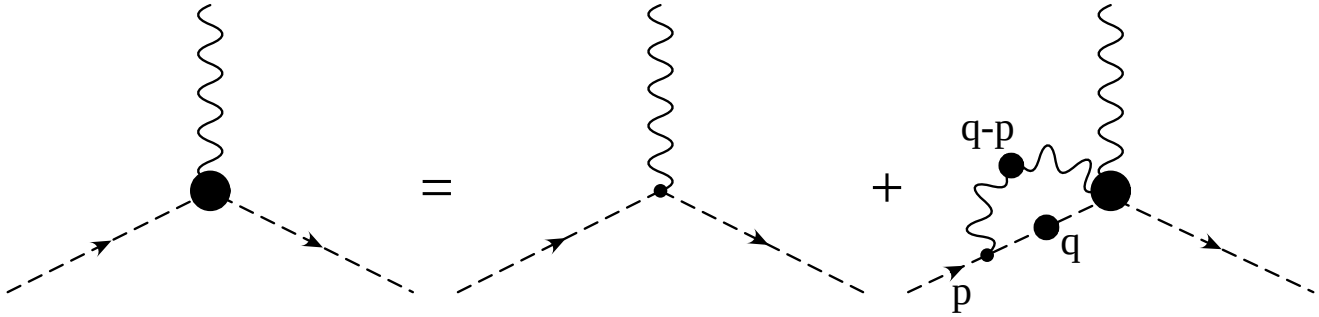

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The Bare Ghost-Gluon Vertex

$$q_\mu D_{\mu\nu}(q-p) \to 0 \text{ for } p \to 0$$

- Consequence 1: Ghost-gluon vertex stays bare in the IR.
- Consequence 2: Its renormalization constant $\tilde{Z}_1$ can be set to 1.

Definition of renormalization constant:

$$G_\mu(q,p)|_{p_1^2=p_2^2=p_3^2=0} = \tilde{Z}_1 G_\mu^{(bare)}(q,p).$$

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The Propagators

Power law ansatz for dressing functions:

$$Z(p^2) = A \cdot (p^2)^\alpha \qquad\qquad G(p^2) = B \cdot (p^2)^\beta$$

Comparison of IR exponent of lhs and rhs:

$$1 - \beta = \frac{d}{2} + \alpha - 1 + \beta - 1 + \frac{1}{2} + \frac{1}{2}$$

$$\alpha = -2\beta + 2 - \frac{d}{2}$$

Set $\beta = -\kappa$: $\rho_{2,1} = 0$, $\rho_{2,0} = -\kappa$, $\rho_{0,2} = 2\kappa + 2 - \frac{d}{2}$

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n-Ghost-n-Antighost-m-Gluon Vertex

- l number of loops
- m_i number of internal gluons
- n_i number of internal ghosts
- $v_{0,3}$ number of dressed three-gluon vertices
- $v_{0,3}^b$ number of bare three-gluon vertices
- $v_{2,1}$ number of ghost-gluon vertices, dressed or bare
- $v_{0,4}^b$ number of bare four-gluon vertices
- v momentum dimension of the bare n-point function itself, e.g. three-gluon vertex: $\frac{1}{2}$

$$\rho_v = (l - m_i + v_{0,3})\frac{d}{2} + (2m_i - n_i - 3v_{0,3})\kappa + \\ + \frac{1}{2}(2m_i - 2n_i - 3v_{0,3} + v_{0,3}^b + v_{2,1} - 2v)$$

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Higher Orders and Other Diagrams

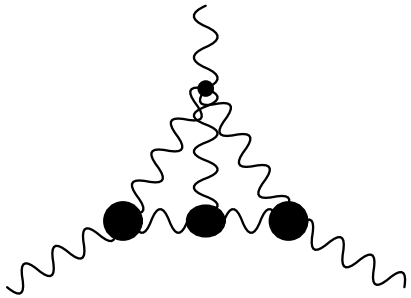
The general formula still contains bare vertices:

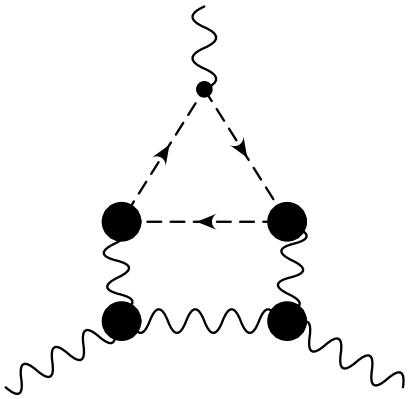
$$\rho_{n,m} = (-n+1-v^b_{0,4}-v^b_{0,3})\frac{d}{2} + (4v^b_{0,4}+3v^b_{0,3}-m+n)\kappa + (2v^b_{0,4}+2v^b_{0,3}+2n-2)$$

Contributions of these,

$$v^b_{0,3}(3\kappa - \frac{d}{2} + 2)$$

$$v^b_{0,4}(4\kappa - \frac{d}{2} + 2),$$





are always positive $\Rightarrow$ diagrams not dominant

$$\rho_{n,m} = (n-m)\kappa + (1-n)(\frac{d}{2}-2)$$

d=4: Alkofer et al., Phys. Lett. B611, 2005

Fischer 2006, unpublished

Differences Between the Dimensions

IR exponent:

$$\rho_{n,m} = (-m+n)\kappa + (1-n)(\frac{d}{2} - 2)$$

The full scaling:

Dimension	d	4	3	2
κ		0.6[1]	0.4/0.5[2]	0.2/0[2]
Ghost	$-\kappa - 1$	-1.6	$-1.4/1.5$	$-1.2/1$
Gluon	$2\kappa + 1 - \frac{d}{2}$	0.2	0.3/0.5	0.4/0
3-gluon	$-3\kappa + \frac{d}{2} - \frac{3}{2}$	-1.3	$-1.2/1.5$	$-1.1/0.5$
4-gluon	$-4\kappa + \frac{d}{2} - 2$	-2.4	$-2.1/2.5$	$-1.8/1$

$\Longrightarrow$ Qualitative behavior is the same in two, three and four dimensions.

[1] Lerche 2001; Zwanziger, Phys. Rev. D6, 2002

[2] Maas et al., Eur. Phys. J. C37, 2004; Zwanziger, Phys. Rev. D6, 2002

Conclusions

- The sekeleton expansion is valid in all dimensions.
- The qualitative behavior in the infrared is the same in two, three and four dimensions, namely dominated by ghost contributions.