Non-perturbative analysis of Landau gauge Yang-Mills theory taking into account the Gribov horizon

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in collaboration with:
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SIC!QFT

Contents of the talk

- Infrared of Yang-Mills theory: What can we learn from it?
- Landau gauge: Does (partly) solving the Gribov problem change the infrared behavior?
- Non-perturbative tool: Dyson-Schwinger equations; is there an easy way to derive them?

Confinement of quarks and gluons

- Confinement is a long-range $\leftrightarrow$ IR phenomenon: We do not see individual $\sim$ infinitely separated quarks or gluons.
  What's the mechanism behind it?
- One expects that the property of being confined is encoded in the particles' propagators.

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  What's the mechanism behind it?
- One expects that the property of being confined is encoded in the particles' propagators.
- Different confinement criteria for the propagators:
  - Positivity violations: negative norm contributions $\rightarrow$ not a particle of the physical state space
  - Kugo-Ojima: quartet mechanism, e. g. Gupta-Bleuler formalism in QED: time-like and longitudinal photon cancel each other.
    Landau gauge Yang-Mills, $p^2 \rightarrow 0$: $D_{gluon} \rightarrow 0$, $p^2 D_{ghost} \rightarrow \infty$
  - Gribov-Zwanziger (Landau gauge, Coulomb gauge): IR suppression of the gluon propagator due to Gribov horizon $\rightarrow$ no long-distance propagation.
    Already manifest at perturbative level with Gribov-Zwanziger Lagrangian!

Dyson-Schwinger equations (DSEs) for investigating QCD

Equations of motion of Green functions

Infinitely large tower of equations

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Pros:

- Exact equations
  $\longrightarrow$ non-perturbative regime accessible
- Continuum, different scales accessible
  $\longrightarrow$ complement lattice method

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- Gauge-dependent

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Pros:

- Exact equations
  $\longrightarrow$ non-perturbative regime accessible
- Continuum, different scales accessible
  $\longrightarrow$ complement lattice method

Cons?:

- Truncations (not for all tasks)
- Gauge-dependent
  $\longrightarrow$ Exploit advantages of different gauges

Deriving Dyson-Schwinger equations

Integral of a total derivative vanishes:

$$\int [D\phi] \frac{\delta}{\delta\phi} e^{-S+J\,\Phi} = \int [D\phi] \left(J - \frac{\delta S}{\delta\phi} \right) e^{-S+J\,\Phi} = 0.$$

$\Rightarrow$ DSEs for all Green functions (full, connected, 1PI) by further differentiations.

Doing it by hand?

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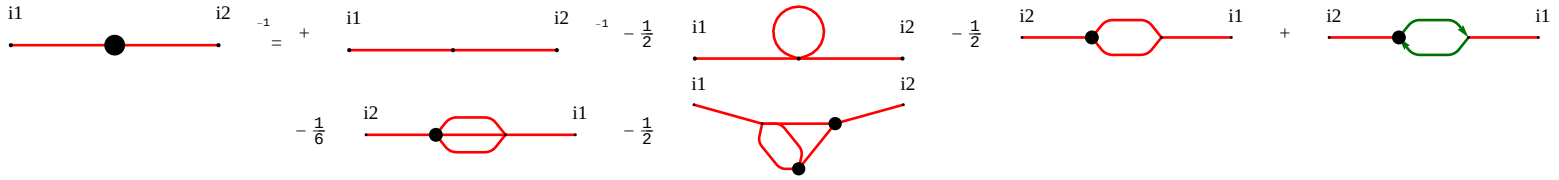
Doing it by hand?

Example: Landau gauge, only 2 propagators (***AA***, ***cc***), 3 interactions (***Acc***, ***AAA***, ***AAAA***)

Landau Gauge: Propagators

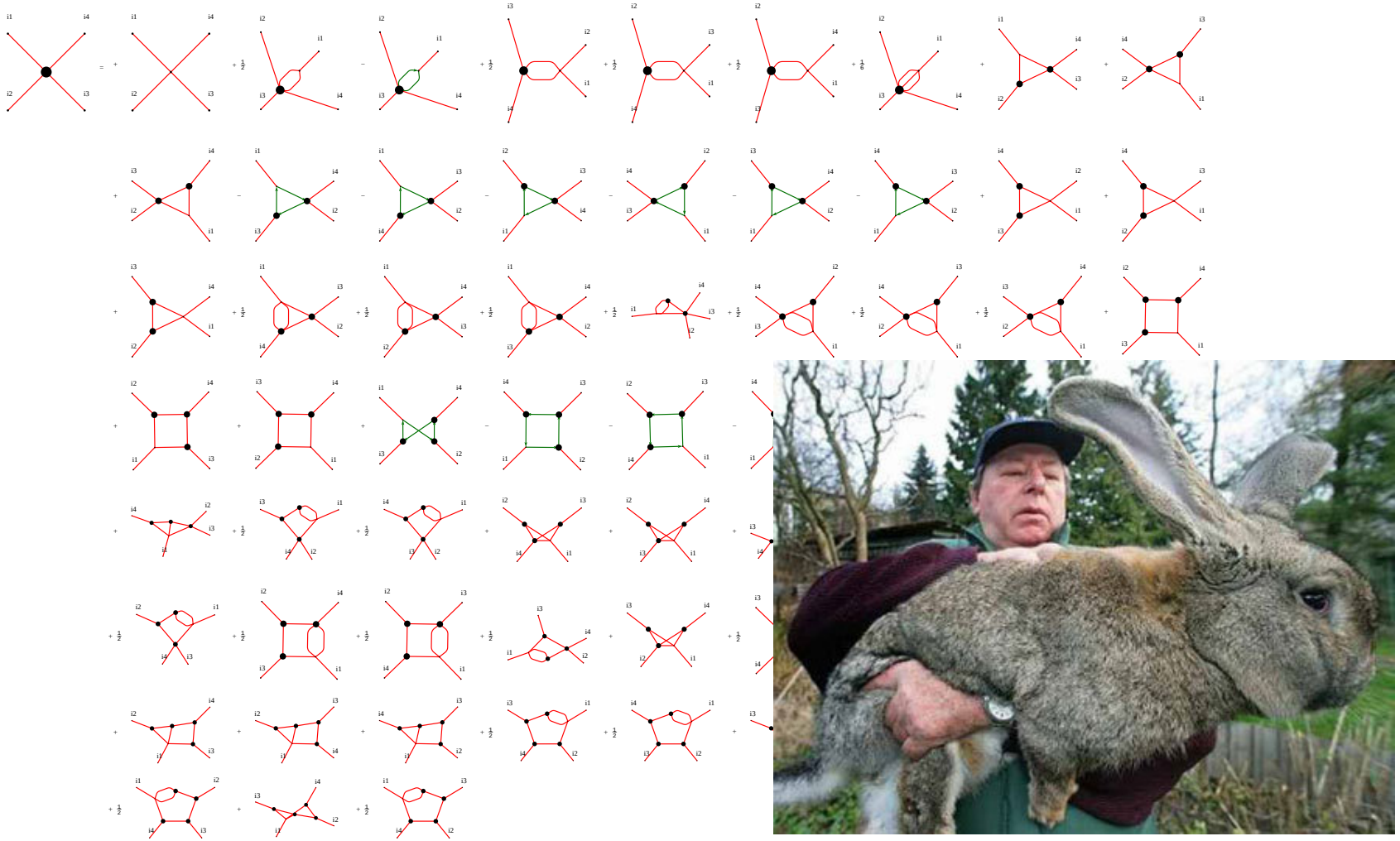
Gluon propagator:



Ghost propagator:

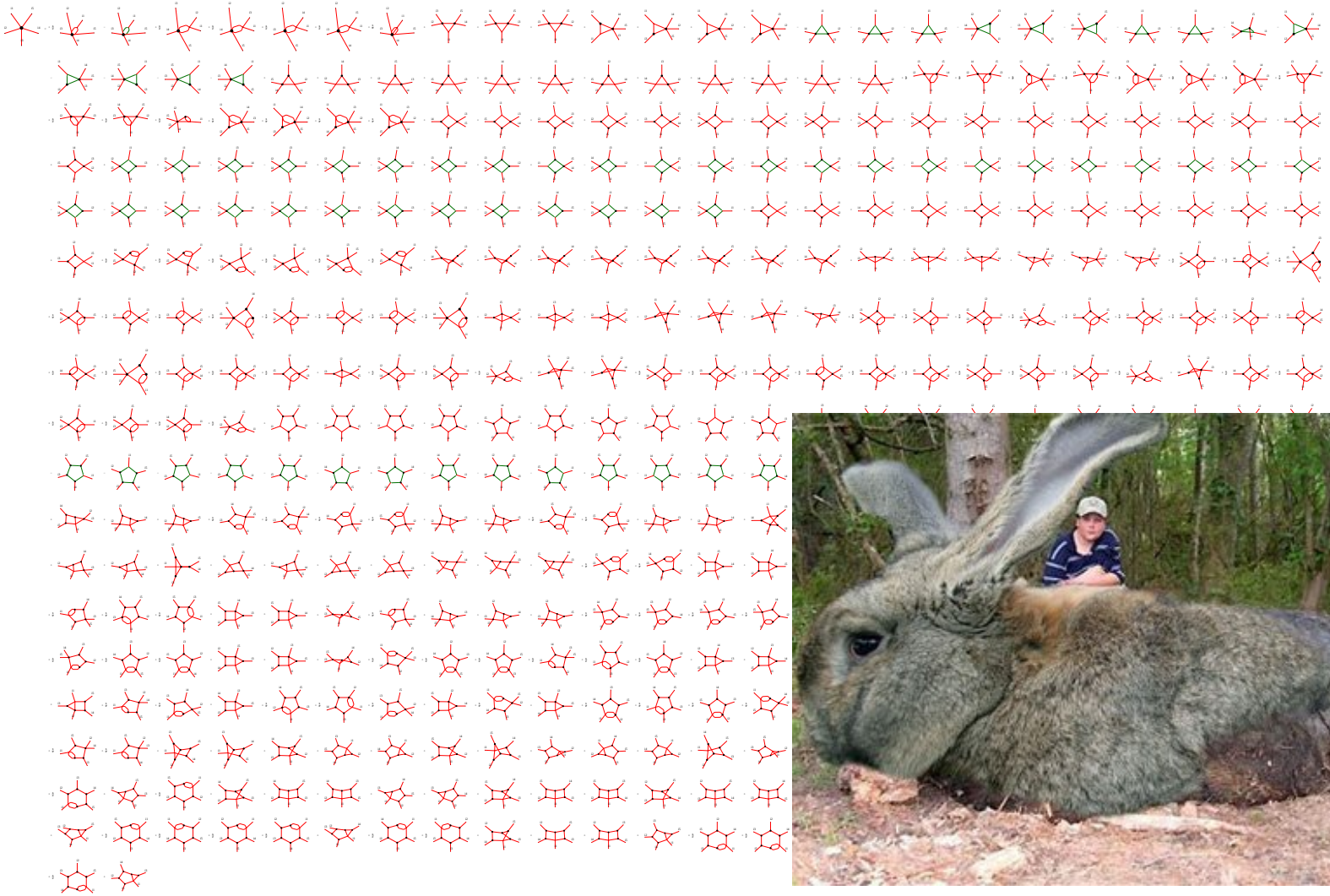
Landau Gauge: Four-Gluon Vertex

66 terms



Landau Gauge: Five-Gluon Vertex

434 terms



Derivation of DSEs (*DoDSE*)

⇒ *DoDSE* [Alkofer, M.Q.H., Schwenzer, CPC 180 (2009)]

Given a structure of interactions, the DSEs are derived symbolically using *Mathematica*.

Example (Landau gauge):

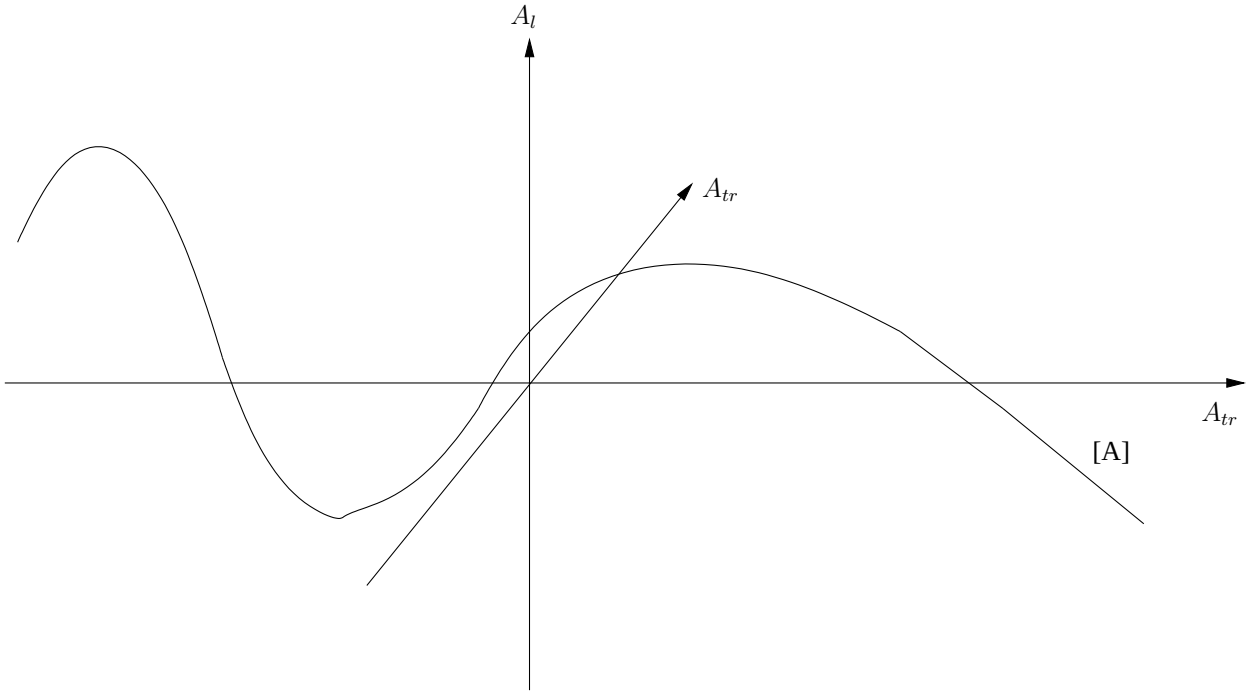
- only input: interactions in Lagrangian (AA, AAA, AAAA, cc, Acc)
- Which DSE do I want?

- Step-by-step calculations possible.
- Can handle mixed propagators (then there are really many diagrams; e. g. in Gribov-Zwanziger action).

Upgrade: *Symb2Alg*

Provide Feynman rules and get complete algebraic expressions.
→ E. g. calculate color algebra with *FORM* and integrals with *C*.

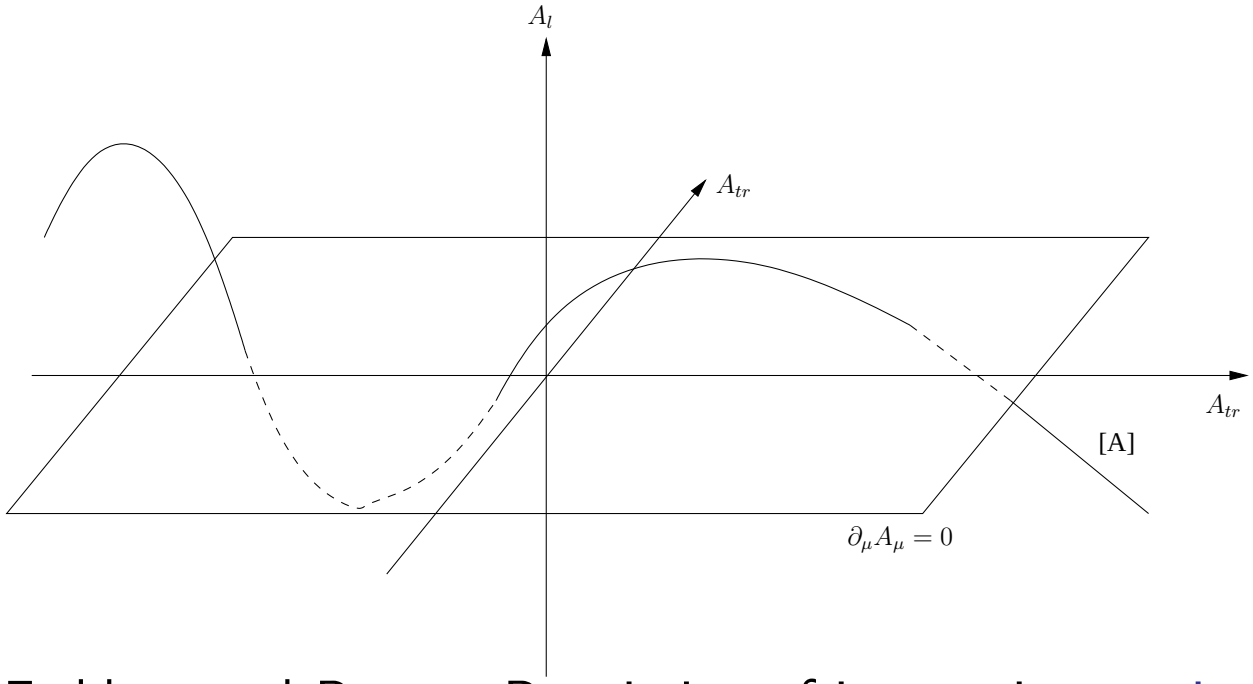
Gauge orbits and Gribov copies



Gauge equivalent configurations (gauge orbit $[A]$) $\Rightarrow$ integration in path integral is overcomplete:

$$Z[J] = \int [D\phi] e^{-S+\phi J}$$

Gauge orbits and Gribov copies

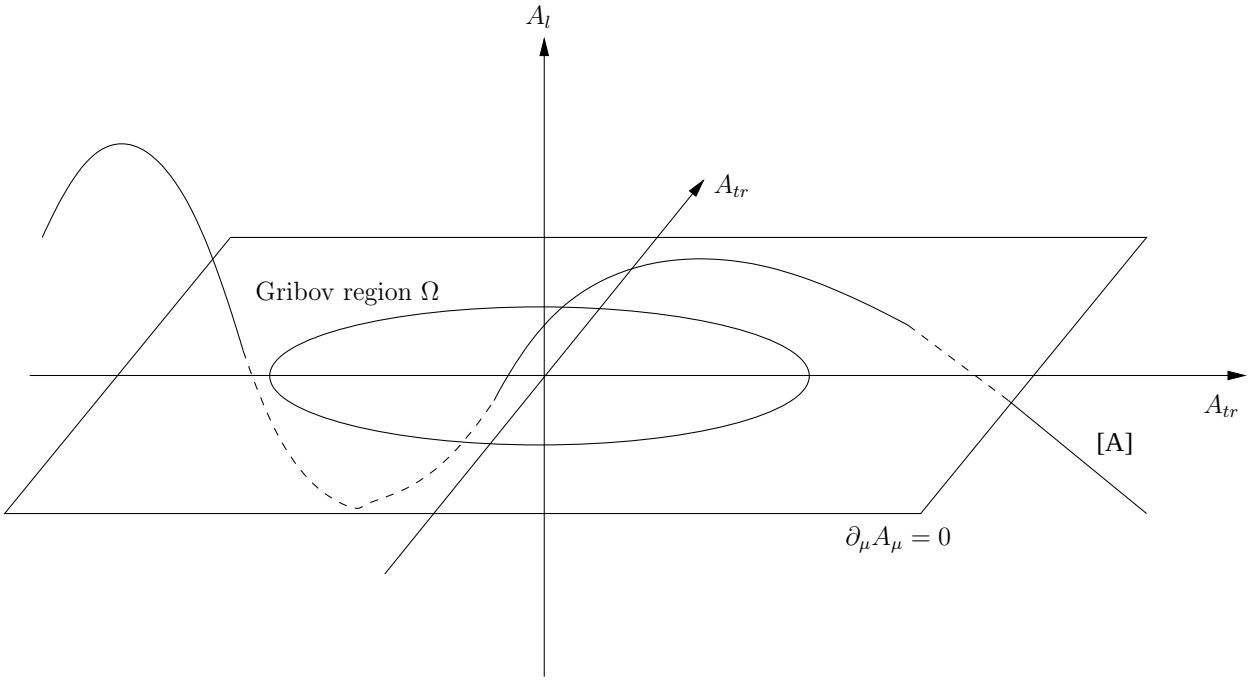


Faddeev and Popov: Restriction of integration to single representative of each gauge orbit possible? Gauge symmetry replaced by BRST symmetry!

Faddeev-Popov operator

$$Z[J] = \int [D\phi]\delta(\partial_\mu A_\mu)\det M e^{-S+\phi J}$$

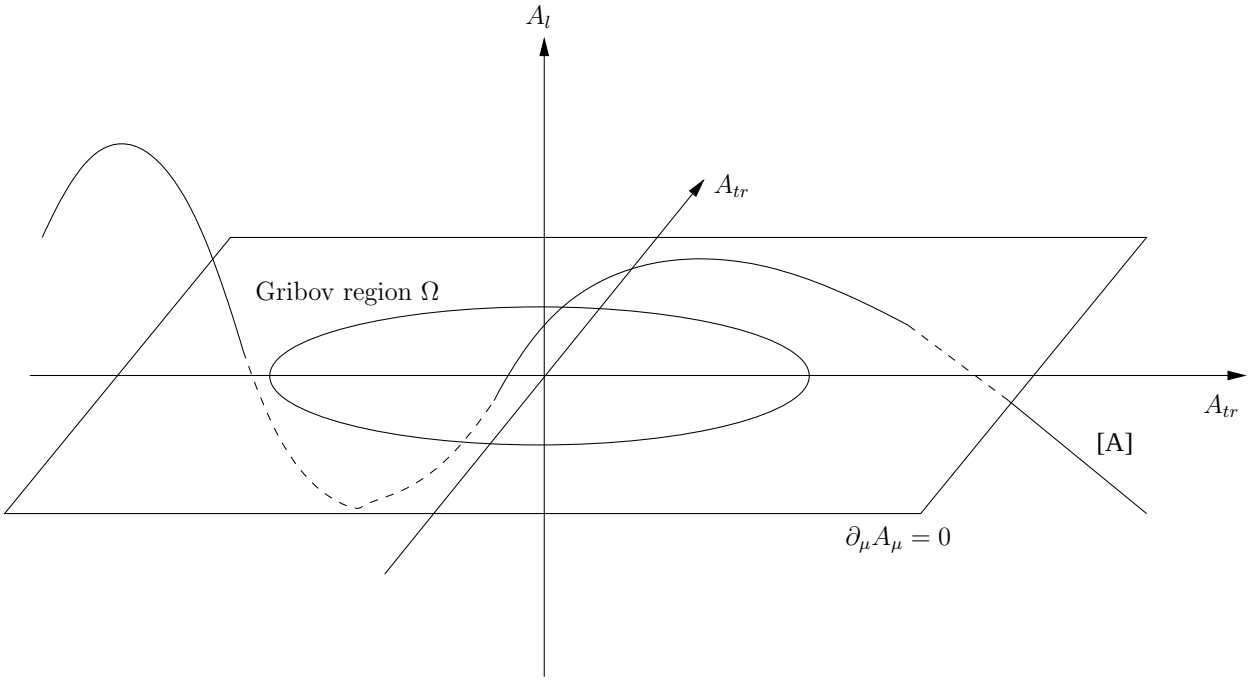
Gauge orbits and Gribov copies



Restriction to Gribov region Ω: almost unique gauge fixing.

$$\Omega := \{A;\ \partial_\mu A_\mu = 0,\ M > 0\}$$

Gauge orbits and Gribov copies



Restriction to Gribov region is done via adding a non-local term to the Lagrangian. $\longrightarrow$ New parameter γ, determined by horizon condition.

How do DSEs usually deal with this?

Integral of a total derivative vanishes:

$$\int [D\phi] \frac{\delta}{\delta\phi} e^{-S+J\,\Phi} = \int [D\phi] \left(J - \frac{\delta S}{\delta\phi} \right) e^{-S+J\,\Phi} = 0.$$

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$$\int_{\Omega} [D\phi] \left(J - \frac{\delta S}{\delta\phi} \right) \delta(\partial \cdot A) \boldsymbol{det(M)} e^{-S_{YM}+J\,\Phi} = 0.$$

$$det(M)\Big|_{\Omega} = 0$$

Local renormalizable action

Non-local term can be localized with auxiliary fields $(\bar{\varphi}^{ab}_\mu,\ \varphi^{ab}_\mu,\ \bar{\omega}^{ab}_\mu,\ \omega^{ab}_\mu) \rightarrow$ local Gribov-Zwanziger action:

$$\mathcal{L}_{GZ} = \mathcal{L}_{FP} + \bar{\varphi}^{ac}_\mu M^{ab} \varphi^{bc}_\mu - \bar{\omega}^{ac}_\mu M^{ab} \omega^{bc}_\mu + \gamma^2 g\, f^{abc} A^a_\mu (\varphi^{bc}_\mu - \bar{\varphi}^{bc}_\mu)$$

- Mixing at the level of two-point functions, e. g. $\langle A^a_\mu \varphi^{bc}_\nu \rangle$.
  $\Rightarrow$ (3x3)-matrix relation between propagators and two-point functions:
  $$D^{\Phi\Phi} = (\Gamma^{\Phi\Phi})^{-1}, \qquad \Phi \in \{A, \varphi, \bar{\varphi}\}$$

More fields . . .

Simplify to (2x2)-matrix relation by splitting into real and imaginary part [Zwanziger, 0904.2380]:

$$\varphi = \frac{1}{\sqrt{2}}\,(U + i\,V)\,, \quad \bar{\varphi} = \frac{1}{\sqrt{2}}\,(U - i\,V)\,.$$

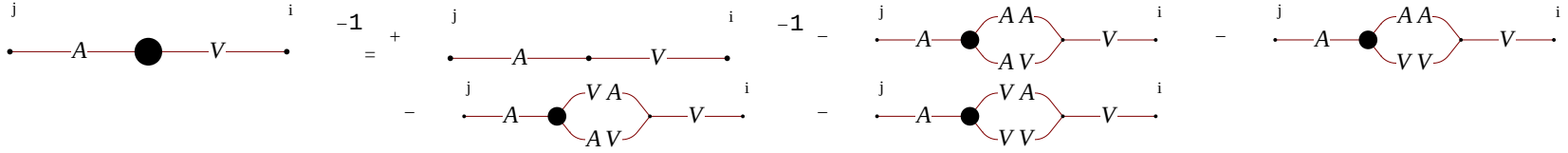
$$\begin{aligned}
\mathcal{L}_{GZ} &= \mathcal{L}_{FP} + \mathcal{L}_U + \mathcal{L}_V + \mathcal{L}_{UV} - \bar{\omega}^{ac}_{\mu} M^{ab} \omega^{bc}_{\mu}\,, \\
\mathcal{L}_U &= \frac{1}{2} U^{ac}_{\mu}\, M^{ab}\, U^{bc}_{\mu}\,, \\
\mathcal{L}_V &= \frac{1}{2} V^{ac}_{\mu}\, M^{ab}\, V^{bc}_{\mu} + \boldsymbol{i\, g\, \gamma^2 \sqrt{2} f^{abc} A^a_{\mu} V^{bc}_{\mu}}\,, \\
\mathcal{L}_{UV} &= \frac{1}{2} i\, g f^{abc} U^{ad}_{\mu} V^{bd}_{\mu} \partial_{\nu} A^c_{\nu} \overset{LG}{=} 0,
\end{aligned}$$

Simplify even further:

$c,\ \bar{c},\ U,\ \omega,\ \bar{\omega} \longrightarrow \eta,\ \bar{\eta}$

DSEs of Gribov-Zwanziger action

Just to give an impression:



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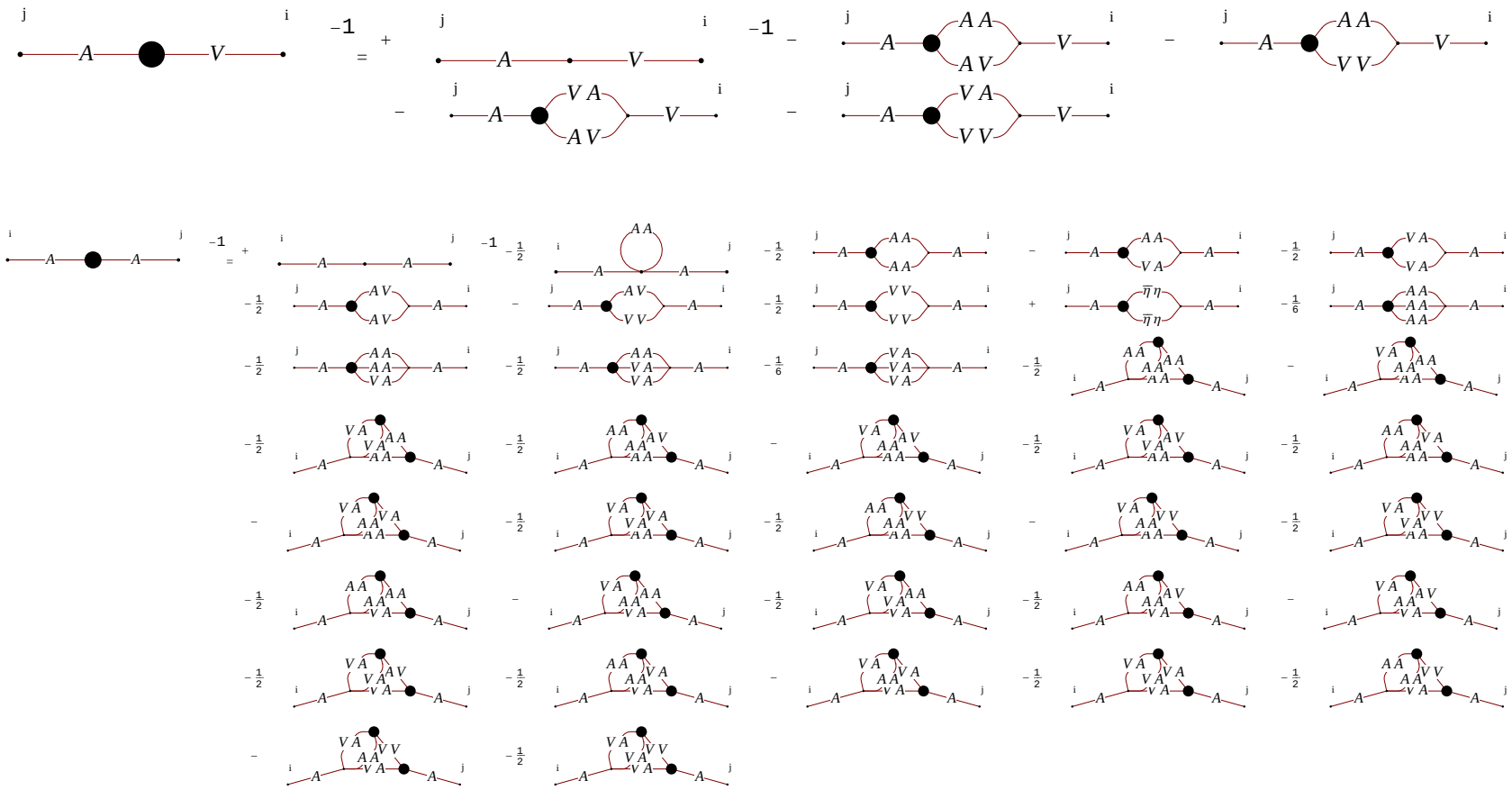

Complete analysis of all diagrams!

Propagators and two-point functions

Mixing at two-point level:

$$D^{\phi\phi} = (\Gamma^{\phi\phi})^{-1}, \qquad \phi \in \{A, V\}$$

$\Rightarrow$ Non-trivial relationship between propagators and two-point functions.

Example: VV-two-point function,

$$\Gamma^{VV,abcd}_{\mu\nu} = \delta^{ac}\delta^{bd} p^2 \boldsymbol{c_V(p^2)} g_{\mu\nu}$$

dressing function $c_V(p^2) \xrightarrow{p^2 \to 0} d_V \cdot (p^2)^{\kappa_V}$

infrared exponent

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VV-propagator:

$$D^{VV,abcd}_{\mu\nu} = \frac{1}{p^2}\frac{1}{\boldsymbol{c_V(p^2)}}\delta^{ac}\delta^{bd} g_{\mu\nu} - f^{abe}f^{cde}\frac{1}{p^2}P_{\mu\nu}\frac{2\boldsymbol{c^2_{AV}(p^2)}}{\boldsymbol{c^\perp_A(p^2)c^2_V(p^2)} + 2N\,\boldsymbol{c^2_{AV}(p^2)}c_V(p^2)}$$

The four possibilities

Which part of the determinant $c_A^{\perp}(p^2)c_V(p^2) + 2N\, c_{AV}^2(p^2)$ dominates in the IR?

$$c_{ij}(p^2) = d_{ij} \cdot (p^2)^{\kappa_{ij}}$$

I: $c_{AV}^2 > c_A c_V \leftrightarrow \kappa_A + \kappa_V > 2\kappa_{AV}$

II: $c_A c_V > c_{AV}^2 \leftrightarrow 2\kappa_{AV} > \kappa_A + \kappa_V$

III: $c_{AV}^2 \sim c_A c_V \leftrightarrow \kappa_A + \kappa_V = 2\kappa_{AV}$, no cancelations

IV: $c_{AV}^2 \sim c_A c_V \leftrightarrow \kappa_A + \kappa_V = 2\kappa_{AV}$, cancelations

Cancelations: Leading contributions cancel and some less dominant term takes over.

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Two solutions lead to inconsistencies [M.Q.H., R. Alkofer, S. P. Sorella, PRD 81].

General method available to determine the qualitative IR behavior, e. g. used for the maximally Abelian gauge $\rightarrow$ results support hypothesis of Abelian dominance [M.Q.H., Schwenzer, Alkofer, arXiv:0904.1873]

Conclusions: Qualitative behavior of the solutions

[M.Q.H., Alkofer, Sorella, PRD 81]

- Scaling relation between FP ghost and gluon unaltered:

$$\kappa_A + 2\kappa_c = 0.$$

  - Gluon propagator is IR suppressed.
  - Propagators of ghost and auxiliary fields are IR enhanced.
  - Mixed propagators are IR suppressed.
- IR exponents of all vertices are obtained.
- Input for numerical solution of the equations.
- Qualitatively the IR behavior of Faddeev-Popov theory is reproduced (Case II corresponds in the IR *exactly* to the Faddeev-Popov theory.)

$\downarrow$ $\downarrow$ $\downarrow$

in agreement with scenarios of Gribov-Zwanziger and Kugo-Ojima.

The end

Thank you very much for your attention.