Landau gauge propagators of two-dimensional Yang-Mills theory

Markus Q. Huber

in collaboration with Axel Maas, Lorenz von Smekal

Institute of Nuclear Physics, Technical University Darmstadt

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Landau gauge Green functions

Green functions are useful quantities for

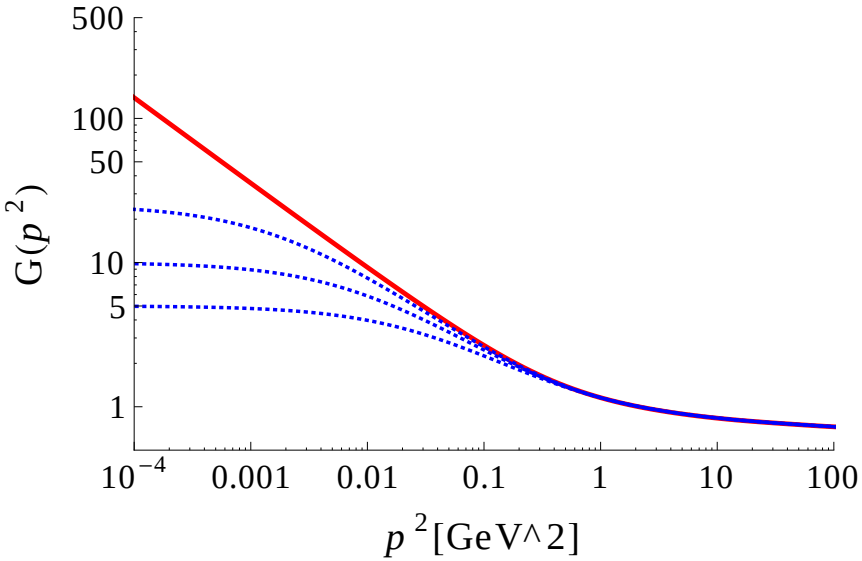
- investigations of phase diagrams
  $\longrightarrow$ see, e.g. (this school), Bonnet, Herbst, Luecker, Mitter, ...
- calculations of bound states
  $\longrightarrow$ see, e.g., Eichmann, Heupel, Sanchis Alepuz, ...
- understanding non-perturbative phenomena like confinement, dynamical creation of mass
  $\longrightarrow$ see, e.g., Alkofer, Hopfer, Mader, Mufti, Schröck, Windisch, ...

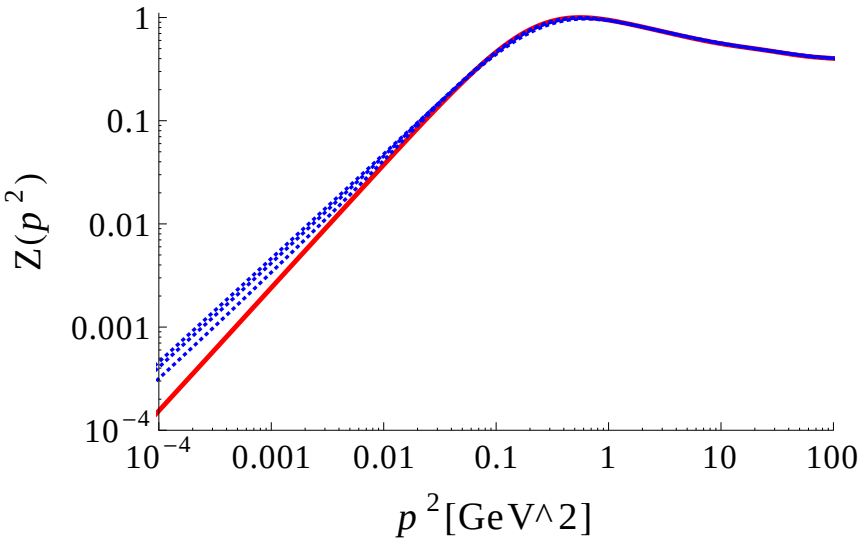
Non-perturbatively calculated, e.g., by Monte-Carlo simulations (finite lattice $\longrightarrow$ momentum cutoffs) or functional methods (continuum approach).

Propagators of four-dimensional Yang-Mills theory

Two qualitatively different solutions from functional methods

[Boucaud et al., JHEP 0806, 012; Fischer, Maas, Pawlowski, AP 324]





Behavior at low momenta:

	ghost dressing	gluon dressing
scaling	$\sim (p^2)^{-\kappa}$	$\sim (p^2)^{2\kappa}$
decoupling	$\sim (p^2)^0$	$\sim (p^2)^1$

Scaling relation:
$2\delta_c + \delta_A = 0$

Why $d < 4$?

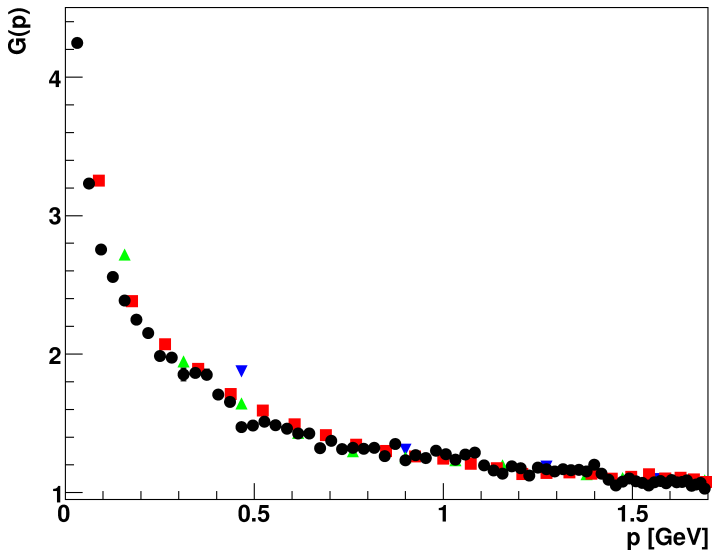
- Lattice calculations: low momenta require large lattices.
- Lattice points (n points in every direction): n^d
- $\Rightarrow$ lower dimensions require (much) less computer power
- $d = 2$ allows really large lattices, e.g., 2560^2 ($L \approx 460\,fm$) [Cucchieri, Mendes, AIP CP 1343, 185].
- Cf. $d = 4$: e.g., 128^4 ($L \approx 27\,fm$) [Cucchieri, Mendes, Pos LAT2007, 297]

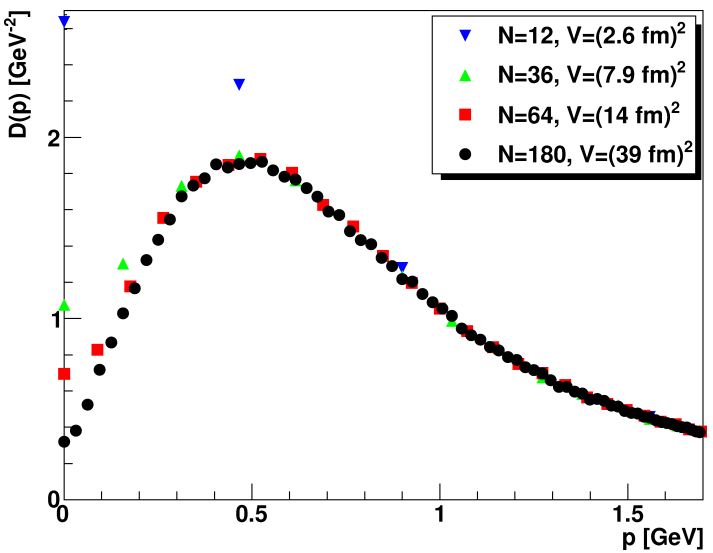
No transverse directions $\rightarrow$ gluons have no degrees of freedom,
but one can investigate Gribov copies, finite size effects, existence of solutions, ...

Lattice results for $d = 2$

Lattice calculations find only the decoupling type of solution for $d = 3, 4$.

$d = 2$ seems different: Only the scaling type solution is found.





[Maas, 1106.3942]

What to expect for $d = 2$

Scaling type solution:

- IR analysis allows two sets of IR exponents $\{\delta_c, \delta_A\}$ [Zwanziger, PRD65]:

$$\{0, 1\} \text{ and } \{-0.2, 1.4\},$$

  always $2\delta_c + \delta_A = -(d-4)/2 = 1$
- Qualitative behavior of all Green functions known [Huber, Alkofer, Fischer, Schwenzer, PLB659].

Decoupling type solutions:

- finite ghost dressing, finite gluon propagator
- until recently existence unclear

Note: Set 1 looks like a decoupling type (peculiar to $d = 2$).

Calculation of DSEs

***DoFun*: Derivation of functional equations** [Huber, Braun, CPC, tbp]**:**

- *Mathematica* application to derive flow equations and DSEs
- Derivation of large and/or complicated systems of DSEs, e.g., maximally Abelian gauge, Gribov-Zwanziger action, alleviated.

***CrasyDSE*: Computation of rather large systems of DSEs** [Huber, Mitter, 1112.5622]**:**

- Combination of *Mathematica* application and *C++* code to handle DSEs
- *CrasyDSE* provides structures for DSEs and modules for handling dressing functions and integrating and solving DSEs.

$\Rightarrow$ Useful tools for systems with many dressing functions and/or complicated kernels.

Ghost equation

Ghost DSE:

input: various gluon propagator ansätze, trivial ghost-gluon vertex

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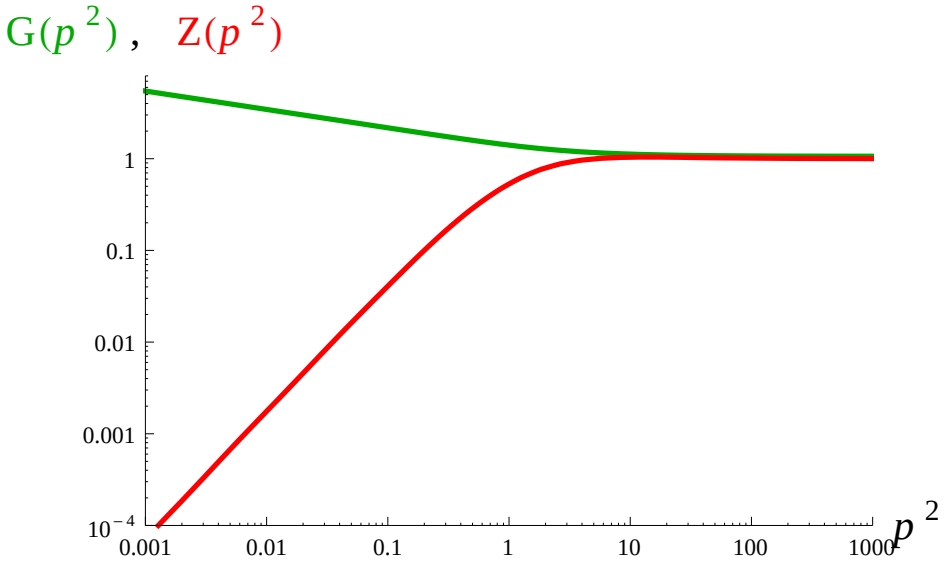
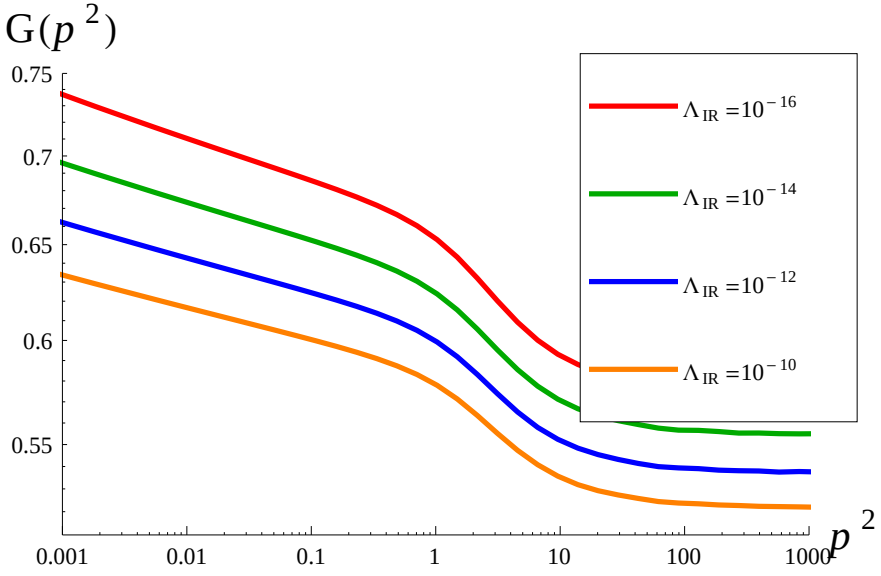


Scaling type solution



Decoupling type solution

→ IR cutoff dependence for decoupling type solutions

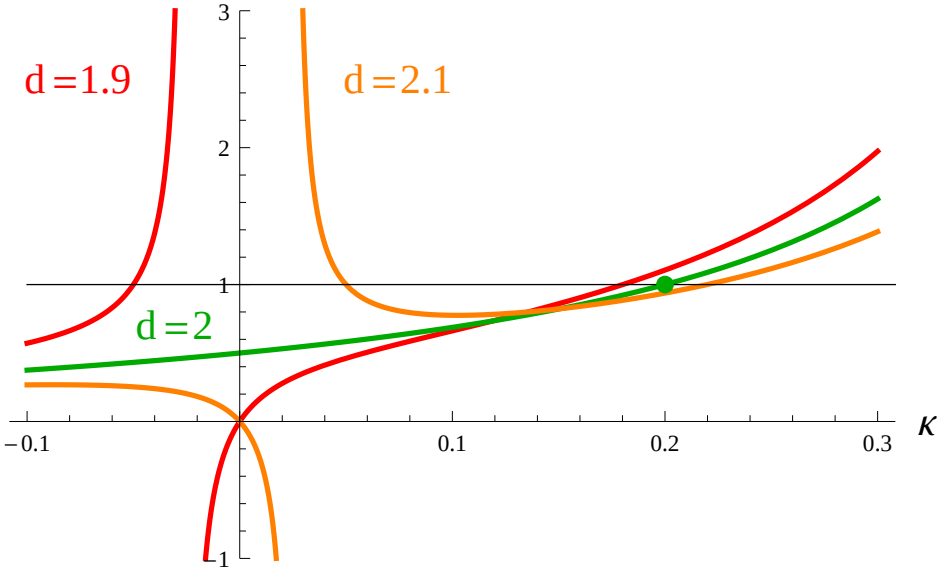
⇒ no decoupling solution (cf. [Cucchieri, Dudal, Vandersickel, 1202.1912])

$\kappa = 0$ revisited

IR exponents can be determined analytically from the IR dominant diagrams:

Value of κ is determined from the equation

$$\frac{\sin(\pi\kappa)\Gamma(d/2-\kappa)\Gamma(\kappa)\Gamma(1+d/2+\kappa)}{2(d-1)\sin(\pi(d/2-2\kappa))\Gamma(d-2\kappa)\Gamma(2\kappa)\Gamma(1+\kappa)} = 1$$



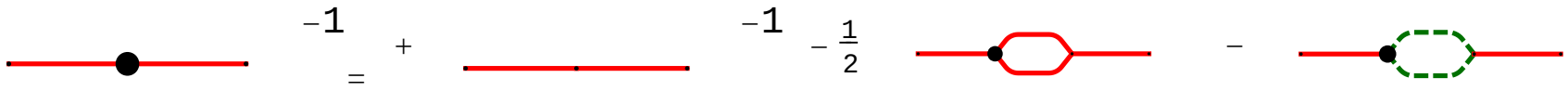
There is no solution $d \to 2$, $\kappa = 0$.

To obtain $\kappa = 0$ an additional prescription is required.

$\to$ Existence of decoupling solution is scheme dependent.

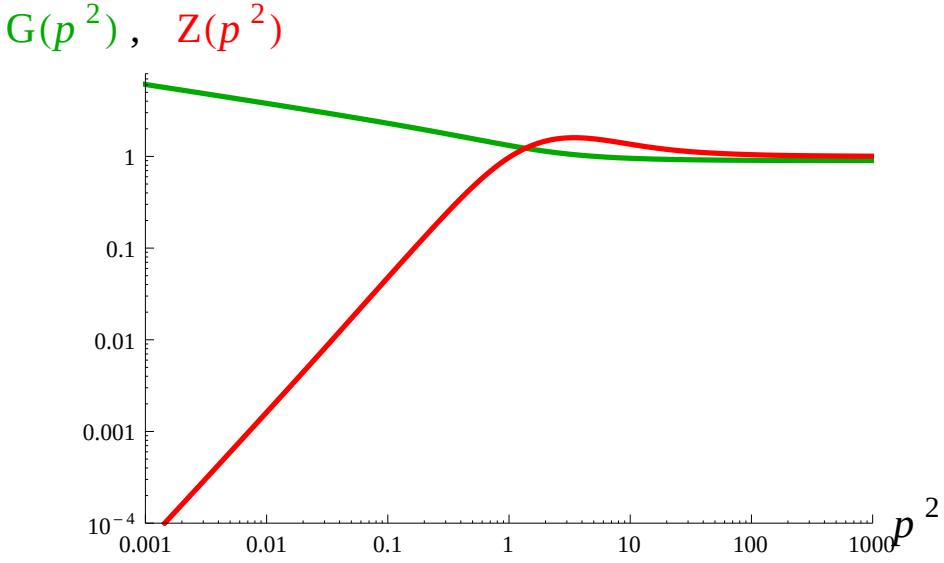
Ghost and gluon DSEs

Gluon DSE:



trivial ghost-gluon vertex, ansatz for three-gluon vertex

Coupled system of equations:



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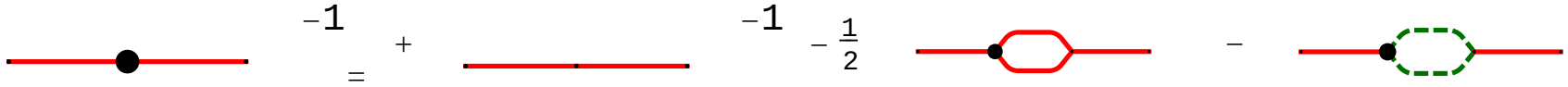

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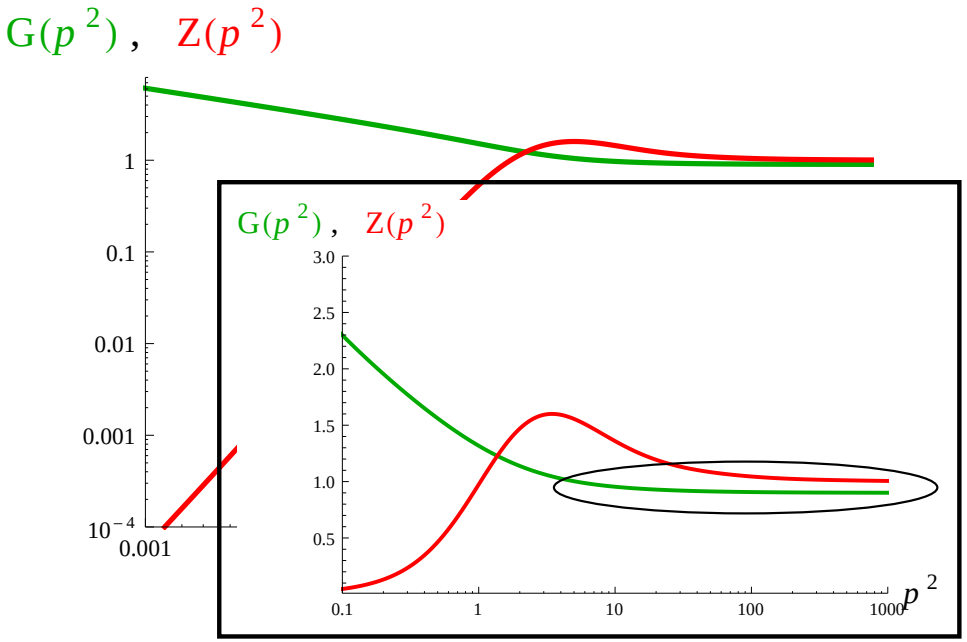

$\Rightarrow$ Ghost does not approach 1 in the UV.

From dim. arguments:

$$G(p^2) \xrightarrow{p^2 \to \infty} \frac{1}{1 + c/p^2}$$

Preliminary results!

Mid-momentum regime and UV behavior

What is the source of this deviation of the ghost propagator in the UV?

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Return to ghost equation:
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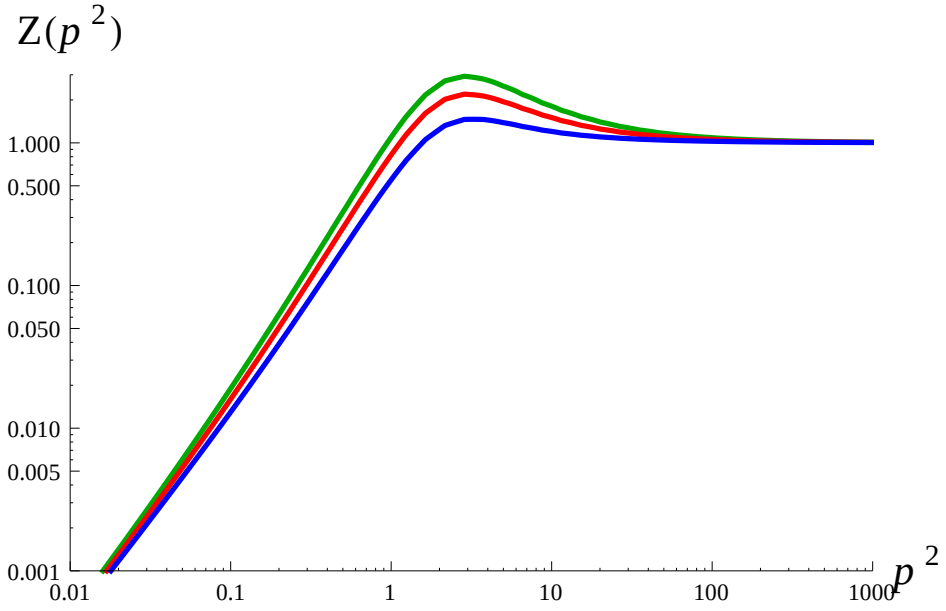

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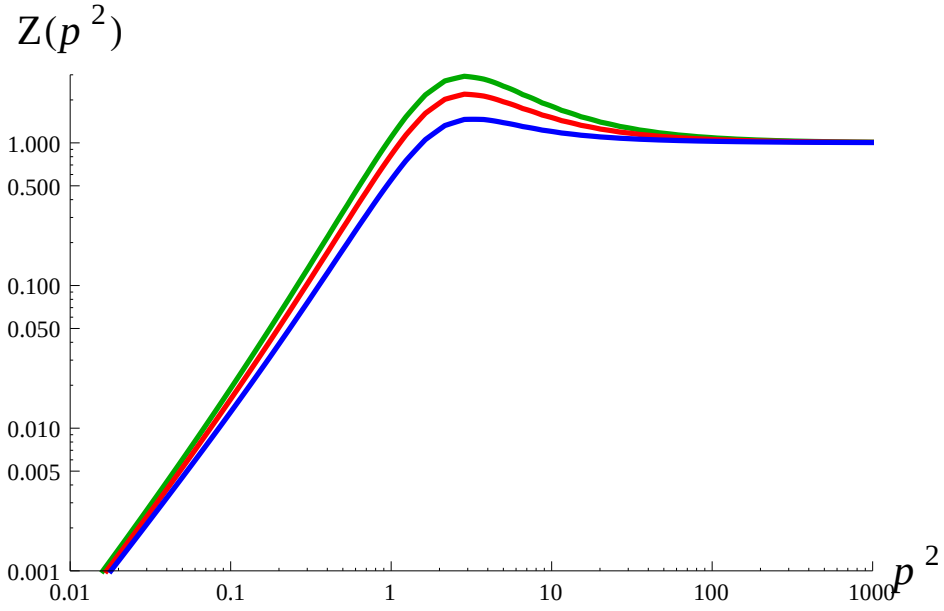
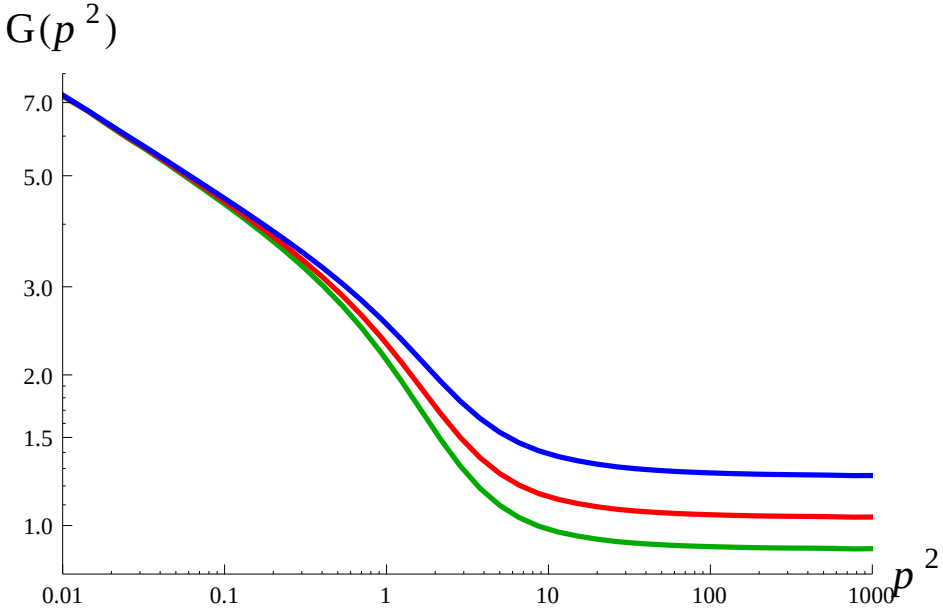
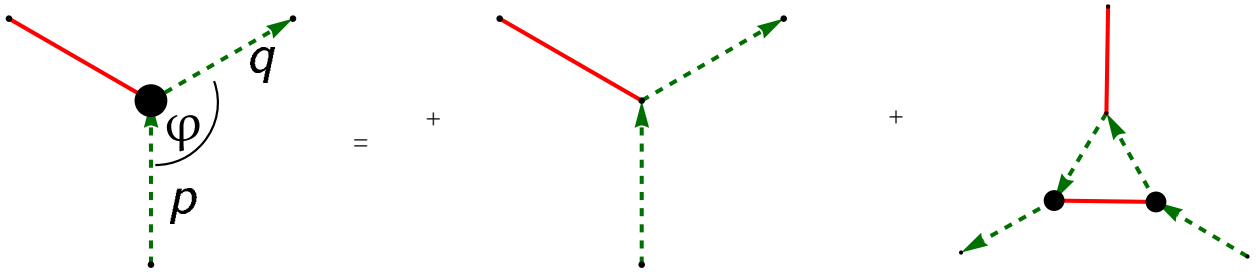




$\Rightarrow$ Mid-momentum regime has (for $d = 2$)
a direct influence on the UV behavior.

Ghost-gluon vertex DSE

(Truncated) ghost-gluon vertex DSE:

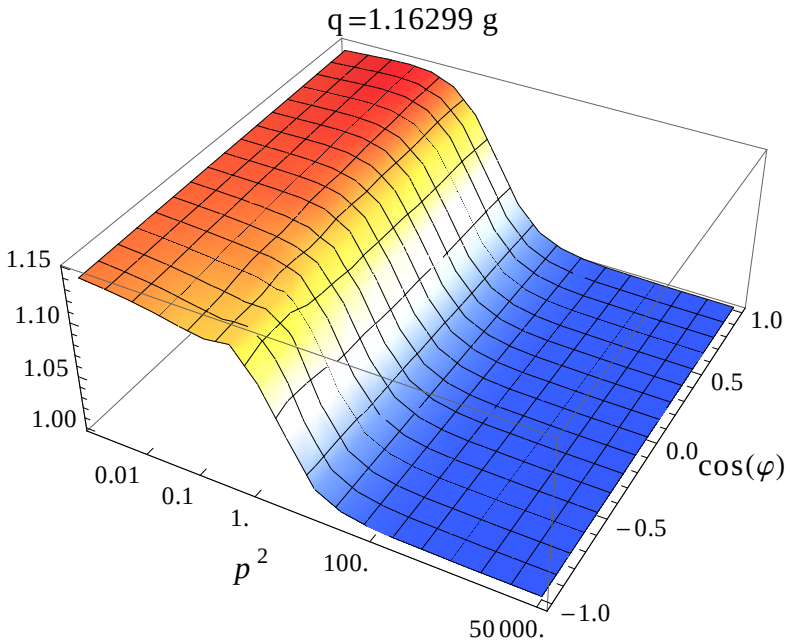


Three-point function depends on 3 variables.
Here: 2 ghost momenta p^2 and q^2, angle φ between them.

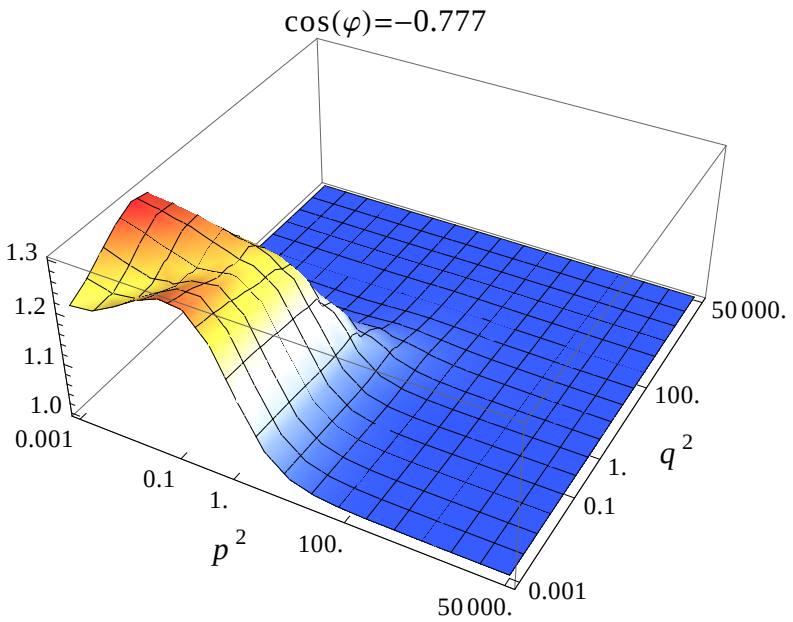
Ghost-gluon vertex

Calculated from fully iterated propagators (bare ghost-gluon vertex used):

Fixed momentum:



Fixed angle:



→ Almost no dependence on angle.

→ IR constant.

Preliminary results!

Summary

- No decoupling type solution exists in $d = 2$.
  (Also not seen on the lattice.)
- Trivial ghost-gluon vertex insufficient
  to obtain correct UV limit of ghost propagator.
- Ghost-gluon vertex influences UV value of the ghost.
- Also three-gluon vertex
  has a more direct influence as in higher dimensions.
- Heading towards
  dynamical inclusion of three-point functions in Yang-Mills theory.

The end

Thank you very much for your attention.