Studying the non-perturbative regime of QCD with Dyson-Schwinger equations

Markus Q. Huber

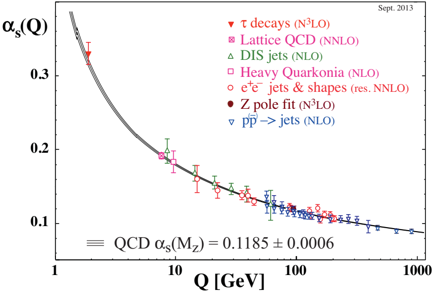
Institute of Physics, University of Graz

53. Internationale Universitätswochen für Theoretische Physik, Schladming

March 1, 2015

How to investigate QCD

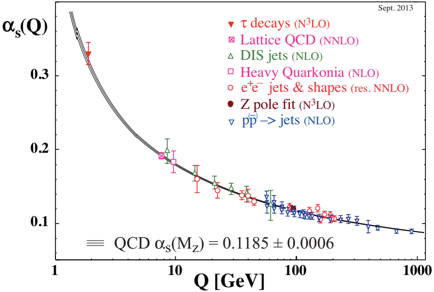
Asymptotic freedom (Nobel prize 2004):



Perturbative description at high energies.
Plenty of applications.

How to investigate QCD

Asymptotic freedom (Nobel prize 2004):



- Perturbative series is not convergent.
- Non-perturbative phenomena?
  - E.g., no mass creation to every order in perturbation theory.

$\Rightarrow$ Non-perturbative methods required.

Perturbative description at high energies.
Plenty of applications.

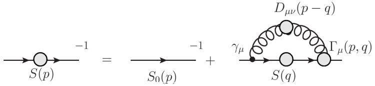
The family of functional equations

Coupled integro-differential/integral equations.

The family of functional equations

Coupled integro-differential/integral equations.

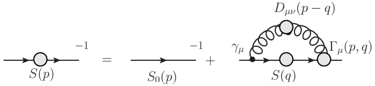
- Dyson-Schwinger equations: eqs. of motion for correlation functions
  [e.g. talks by Alkofer R., Hilger, posters by Blum, Vujinovic]



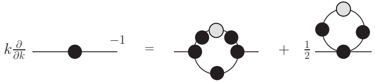
The family of functional equations

Coupled integro-differential/integral equations.

- Dyson-Schwinger equations: eqs. of motion for correlation functions
  [e.g. talks by Alkofer R., Hilger, posters by Blum, Vujinovic]



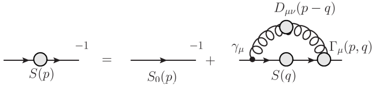
- Functional renormalization group: flow equations, RG scale k, regulator
  [e.g. talks by Herbst, Mitter, Rennecke, Roscher, poster by Khan]



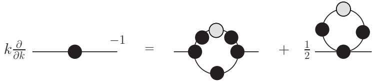
The family of functional equations

Coupled integro-differential/integral equations.

- Dyson-Schwinger equations: eqs. of motion for correlation functions
  [e.g. talks by Alkofer R., Hilger, posters by Blum, Vujinovic]



- Functional renormalization group: flow equations, RG scale k, regulator
  [e.g. talks by Herbst, Mitter, Rennecke, Roscher, poster by Khan]

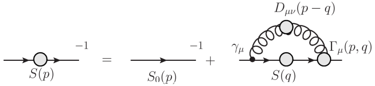


- N-PI effective action
  [e.g. talk by Alkofer R.]

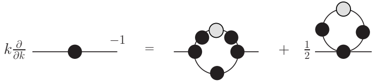
The family of functional equations

Coupled integro-differential/integral equations.

- Dyson-Schwinger equations: eqs. of motion for correlation functions
  [e.g. talks by Alkofer R., Hilger, posters by Blum, Vujinovic]



- Functional renormalization group: flow equations, RG scale k, regulator
  [e.g. talks by Herbst, Mitter, Rennecke, Roscher, poster by Khan]



- N-PI effective action
  [e.g. talk by Alkofer R.]

Non-perturbative in the sense:

- Exact equations.
- No small coupling required.

From Green functions to 'observables'

Functional equations are expressed in terms of
Green functions/correlation functions/n-point functions $\Gamma_{i_1 \ldots i_n}$.

The effective action is the generating functional of 1PI Green functions.

$$\Gamma[\Phi] = \sum_{n=0}^{\infty} \frac{1}{n!} \Phi_1 \ldots \Phi_n \Gamma_{i_1 \ldots i_n}$$

$\longleftrightarrow$

The set of **all** Green functions describes the theory completely.

$$\Gamma_{ij} = \frac{\delta^2 \Gamma[\Phi]}{\delta\Phi_i \delta\Phi_j},$$

$$\Gamma_{ijk} = \frac{\delta^3 \Gamma[\Phi]}{\delta\Phi_i \delta\Phi_j \delta\Phi_k}, \quad \ldots$$

From Green functions to 'observables'

Functional equations are expressed in terms of
Green functions/correlation functions/n-point functions $\Gamma_{i_1 \ldots i_n}$.

The effective action is the generating functional of 1PI Green functions.

$$\Gamma[\Phi] = \sum_{n=0}^{\infty} \frac{1}{n!} \Phi_1 \ldots \Phi_n \Gamma_{i_1 \ldots i_n}$$

$$\longleftrightarrow$$

The set of **all** Green functions describes the theory completely.

$$\Gamma_{ij} = \frac{\delta^2 \Gamma[\Phi]}{\delta \Phi_i \delta \Phi_j},$$

$$\Gamma_{ijk} = \frac{\delta^3 \Gamma[\Phi]}{\delta \Phi_i \delta \Phi_j \delta \Phi_k}, \quad \ldots$$

Green functions $\rightarrow$ 'observables'?

Examples:

- Bound state equations $\rightarrow$ masses and properties of hadrons
- Analytic properties of Green functions $\rightarrow$ confinement
- (Pseudo-)Order parameters

Dyson-Schwinger/Schwinger-Dyson-equations

1. Start from path integral: Integral of derivative vanishes.

$$0 = \int D[\phi] \frac{\delta}{\delta\phi} e^{-S+\int dy \phi(y) J(y)}$$

Details and example of scalar theory:
`http://tinyurl.com/dsenotes`

Dyson-Schwinger/Schwinger-Dyson-equations

1. Start from path integral: Integral of derivative vanishes.

$$0 = \int D[\phi] \frac{\delta}{\delta\phi} e^{-S+\int dy\phi(y)J(y)}$$

2. Go to effective action $\Gamma[\phi_{cl}]$ (Legendre transform of $W[J] = \ln Z[J]$).

Details and example of scalar theory:
`http://tinyurl.com/dsenotes`

Dyson-Schwinger/Schwinger-Dyson-equations

1. Start from path integral: Integral of derivative vanishes.

$$0 = \int D[\phi] \frac{\delta}{\delta\phi} e^{-S+\int dy \phi(y) J(y)}$$

2. Go to effective action $\Gamma[\phi_{\text{cl}}]$ (Legendre transform of $W[J] = \ln Z[J]$).
3. Master equation:

$$\left. \frac{\delta S}{\delta\phi(x)} \right|_{\phi(x')=\phi_{\text{cl}}(x')+\int dz\, D(x',z)^{J}\, \delta/\delta\phi_{\text{cl}}(z)} = \frac{\delta\Gamma[\phi_{\text{cl}}]}{\delta\phi_{\text{cl}}(x)}$$

Details and example of scalar theory:
http://tinyurl.com/dsenotes

Dyson-Schwinger/Schwinger-Dyson-equations

1. Start from path integral: Integral of derivative vanishes.

$$0 = \int D[\phi] \frac{\delta}{\delta\phi} e^{-S+\int dy\phi(y)J(y)}$$

2. Go to effective action $\Gamma[\phi_{cl}]$ (Legendre transform of $W[J] = \ln Z[J]$).
3. Master equation:

$$\left.\frac{\delta S}{\delta\phi(x)}\right|_{\phi(x')=\phi_{\mathbf{cl}}(x')+\int dz\, D(x',z)^{J}\, \delta/\delta\phi_{\mathbf{cl}}(z)} = \frac{\delta\Gamma[\phi_{cl}]}{\delta\phi_{cl}(x)}$$

4. DSEs for Green functions by differentiating wrt fields.

Details and example of scalar theory:
`http://tinyurl.com/dsenotes`

Landau gauge Yang-Mills theory

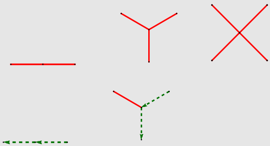
Gluonic sector of quantum chromodynamics: Yang-Mills theory

$$\mathcal{L} = \frac{1}{2}F^2 + \mathcal{L}_{gf} + \mathcal{L}_{gh}$$

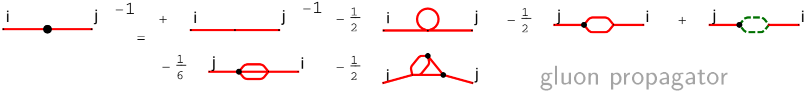
$$F_{\mu\nu} = \partial_\mu \boldsymbol{A}_\nu - \partial_\nu \boldsymbol{A}_\mu + i\,g\,[\boldsymbol{A}_\mu, \boldsymbol{A}_\nu]$$

Landau gauge

- simplest one for functional equations
- $\partial_\mu \boldsymbol{A}_\mu = 0$: $\quad \mathcal{L}_{gf} = \frac{1}{2\xi}(\partial_\mu \boldsymbol{A}_\mu)^2, \quad \xi \to 0$
- requires ghost fields: $\quad \mathcal{L}_{gh} = \bar{\boldsymbol{c}}\,(-\Box + g\,\boldsymbol{A}\,\times)\,\boldsymbol{c}$



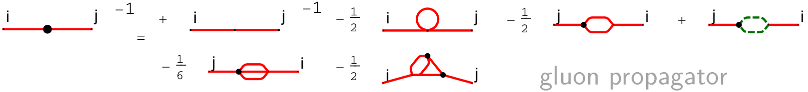
The tower of DSEs



gluon propagator

ghost propagator

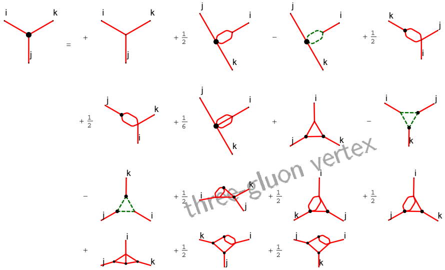
The tower of DSEs

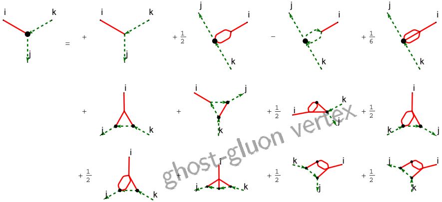


gluon propagator

ghost propagator

Self-consistence!





Infinitely many equations. In QCD, every n-point function depends on $(n+1)$- and possibly $(n+2)$-point functions.

Taming the equations

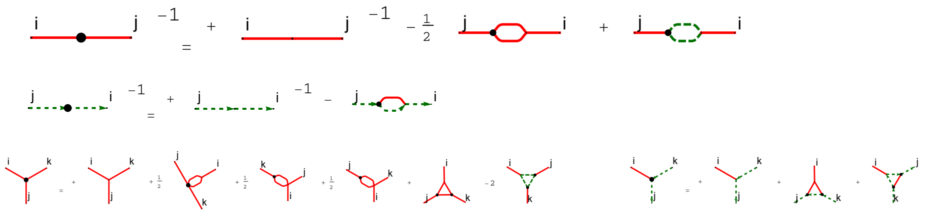
Keep most important parts!

- Drop quantities
- Model quantities

Taming the equations

Keep most important parts! The art. . .

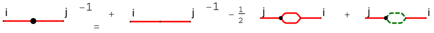
- Drop quantities
- Model quantities



Most important parts

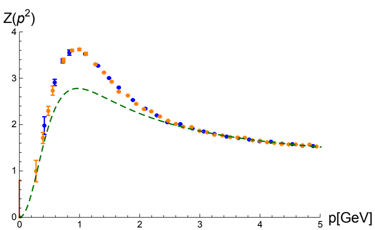
- UV leading (perturbation theory)
- IR leading (analytic, lattice)

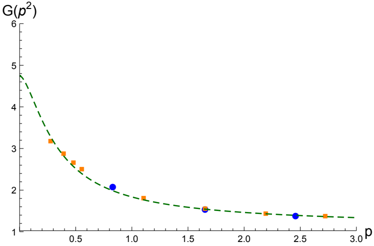
Propagators I



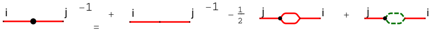
Long-time standard truncation

- No four-gluon vertex
- Ghost-gluon vertex: bare
- Three-gluon vertex: model



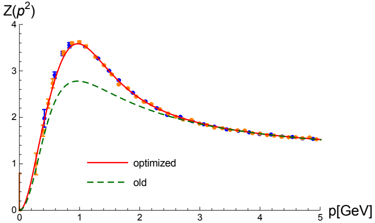


Propagators I

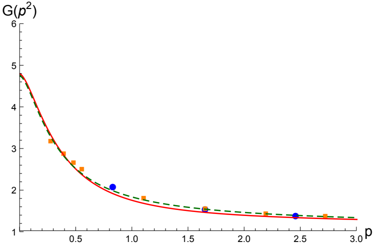


Long-time standard truncation

- No four-gluon vertex
- Ghost-gluon vertex: bare $\rightarrow$ dressed (dynamic)
- Three-gluon vertex: model $\rightarrow$ optimized model



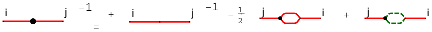
[MQH, von Smekal '12; lattice; Sternbeck '06]



$\rightarrow$ Role of three-gluon vertex?

$\rightarrow$ Use as input in other calculations.

Propagators I

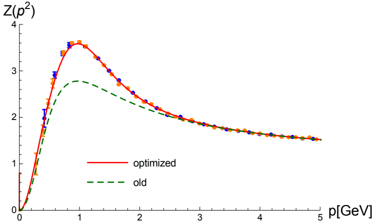


Long-time standard truncation

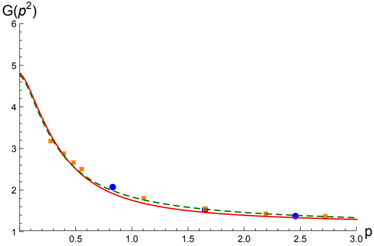
- No four-gluon vertex
- Ghost-gluon vertex: bare $\rightarrow$ dressed (dynamic)
- Three-gluon vertex: model $\rightarrow$ optimized model

Missing strength in mid-momentum regime:

- neglected diagrams?
- vertices?



[MQH, von Smekal '12; lattice; Sternbeck '06]

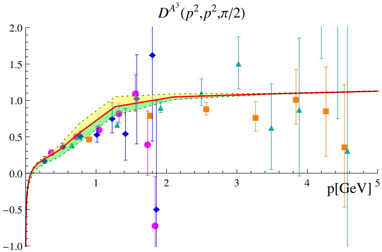


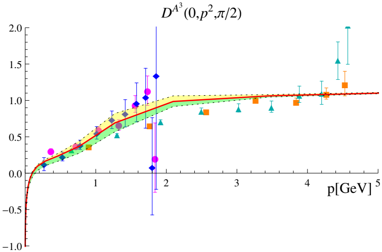
$\rightarrow$ Role of three-gluon vertex?

$\rightarrow$ Use as input in other calculations.

The three-gluon vertex

[See also poster by Blum.]



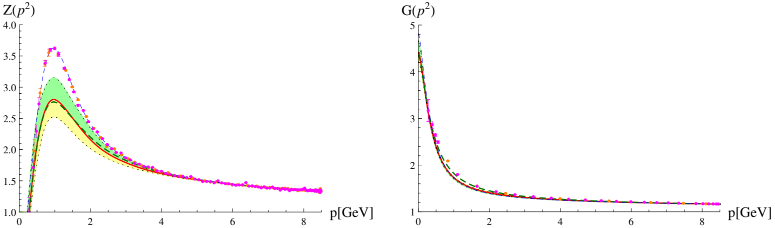


[Blum, MQH, Mitter, von Smekal '14; lattice: Cucchieri, Maas, Mendes '08]

→ Truncation reliable. Neglected terms, including two-loop, suppressed.

See also results by [Eichmann, Williams, Alkofer, Vujinovic '14], esp. other dressings, and [Peláez, Tissier, Wschebor '13].

Propagators II: Limits of one-loop truncation



[Blum, MQH, Mitter, von Smekal '13]

- Use calculated three-gluon vertex.
- Ghost almost unaffected.
- Gap in midmomentum regime must be due to missing two-loop diagrams!

Explicit two-loop studies [Bloch '03; Mader, Alkofer '12; Meyers, Swanson '14]:
squint $\gg$ sunset diagram

Four-gluon vertex

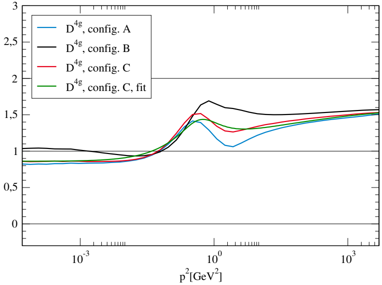
- 6 external variables
- 4 integration variables

Cf. propagator: 1 ext., 2 int.
2 propagators $\rightarrow$ laptop
four-gluon vertex $\rightarrow$ > 100 cores on cluster

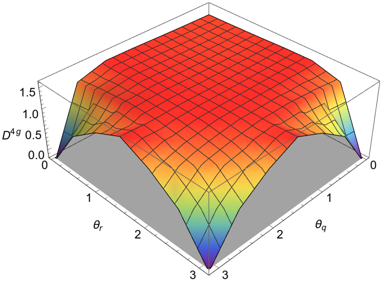
Four-gluon vertex

- 6 external variables
- 4 integration variables

Cf. propagator: 1 ext., 2 int.
2 propagators $\rightarrow$ laptop
four-gluon vertex $\rightarrow$ $>$ 100 cores on cluster



[Cyrol, MQH, von Smekal '14]



2-parameter fit:

$$D^{4g,\,\text{dec}}_{\text{model}}(p,\,q,\,r,\,s) = \left(a \tanh\left(b/\bar{p}^2\right) + 1\right) D^{4g}_{\text{RG}}(p,\,q,\,r,\,s)$$

Beyond Landau gauge: Coulomb gauge

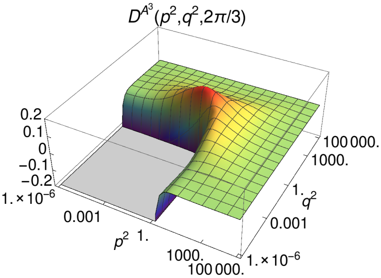
Why the Landau gauge is convenient

- Minimum number of terms in DSEs.
- Transversality $\rightarrow$ longitudinal part decouples.
- Historically ghost-gluon vertex provided the entry point (special here).

Beyond Landau gauge: Coulomb gauge

Why the Landau gauge is convenient

- Minimum number of terms in DSEs.
- Transversality $\rightarrow$ longitudinal part decouples.
- Historically ghost-gluon vertex provided the entry point (special here).



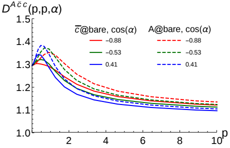
[MQH, Campagnari, Reinhardt '14]

Three-gluon vertex:

- Zero crossing
- IR divergent like p^{-3}

Ghost-gluon vertex:

- Different truncations quite similar

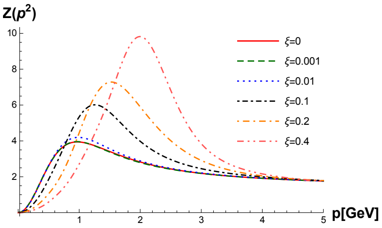


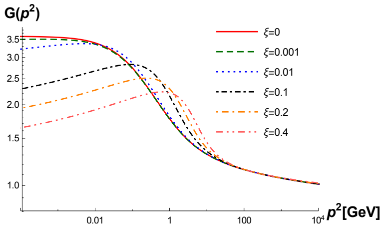
Beyond Landau gauge: Linear covariant gauges

- Gaussian distribution $e^{-\frac{1}{2\xi}(\partial A)^2}$ around Landau gauge in path integral
- Test of gauge (in)dependence of observables possible.
- Well-known Landau gauge is endpoint: $\xi = 0$
- Special choices convenient perturbatively, e.g., Feynman gauge $\xi = 1$; non-pertubatively no advantage

Beyond Landau gauge: Linear covariant gauges

- Gaussian distribution $e^{-\frac{1}{2\xi}(\partial A)^2}$ around Landau gauge in path integral
- Test of gauge (in)dependence of observables possible.
- Well-known Landau gauge is endpoint: $\xi = 0$
- Special choices convenient perturbatively, e.g., Feynman gauge $\xi = 1$; non-pertubatively no advantage





[MQH '15]

- Ghost vanishes logarithmically in IR, see also [Aguilar, Binosi, Papavassiliou '15].
- Gluon propagator IR finite.

Summary

Examples for applications of functional methods:

- Description of hadron properties from first principles
- QCD phase diagram: explore non-zero density

Summary

Examples for applications of functional methods:

- Description of hadron properties from first principles
- QCD phase diagram: explore non-zero density
- Testing gauge dependence in various gauges: Coulomb, linear covariant

Summary

Examples for applications of functional methods:

- Description of hadron properties from first principles
- QCD phase diagram: explore non-zero density
- Testing gauge dependence in various gauges: Coulomb, linear covariant

Technical challenges!

Summary

Examples for applications of functional methods:

- Description of hadron properties from first principles
- QCD phase diagram: explore non-zero density
- Testing gauge dependence in various gauges: Coulomb, linear covariant

Technical challenges!

Truncation:

- 2-, 3- and 4-point functions calculated
- Truncation effects understood (after more than 30 years!)
- Two-loop terms important in 2- but not in higher n-point functions
- System of DSEs closes with this truncation

$\rightarrow$ self-contained, quantitative description.

Summary

Examples for applications of functional methods:

- Description of hadron properties from first principles
- QCD phase diagram: explore non-zero density
- Testing gauge dependence in various gauges: Coulomb, linear covariant

Technical challenges!

Truncation:

- 2-, 3- and 4-point functions calculated
- Truncation effects understood (after more than 30 years!)
- Two-loop terms important in 2- but not in higher n-point functions
- System of DSEs closes with this truncation
  $\rightarrow$ self-contained, quantitative description.

Thank you for your attention.