Landau gauge correlation functions from Dyson-Schwinger equations

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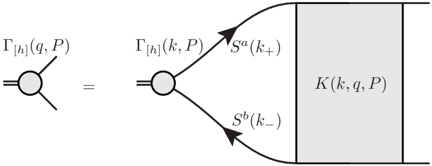
Hadrons from bound state equations

Calculation of bound states from QCD correlation functions:

Meson BSE:



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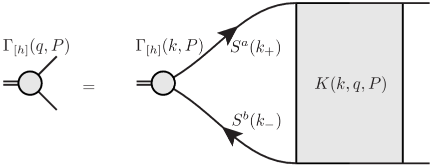

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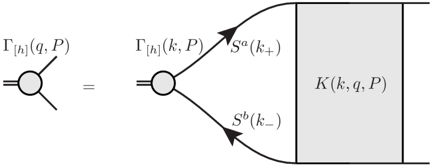

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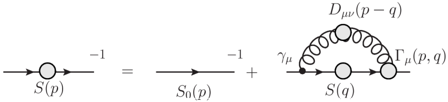

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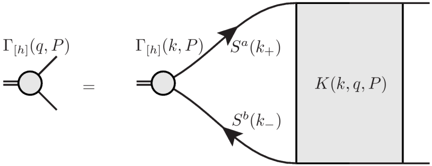

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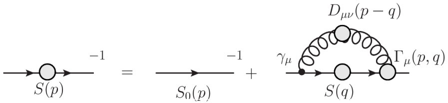

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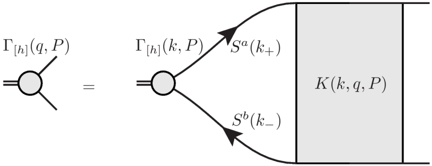

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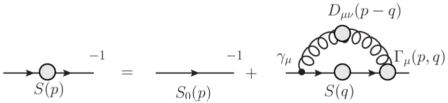


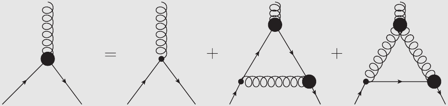
  - Make ansatz for the gluon propagator $D_{\mu\nu}$ and the quark-gluon vertex Γ_μ, e.g., rainbow + effective interaction
- Constraints by chiral symmetry!

Correlation functions from first principles

Widely used truncation: Rainbow + ladder + variant of Maris-Tandy interaction

How to reduce model dependence

- Improve kernel K
- Use explicit gluon propagator + quark-gluon vertex



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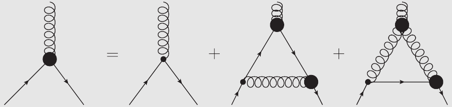

$\longrightarrow$ We need full control over the gluonic sector.

- Gluon propagator
- Three-gluon vertex
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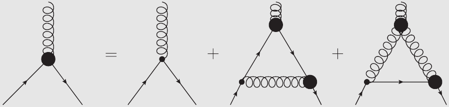

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Side note: QCD phase diagram

- No sign problem ☺
- Truncation problem ☹

$\rightarrow$ Ultimately, we need full control over the gluonic sector.

Landau gauge QCD

$$\mathcal{L} = \bar{q}(-\not{D} + m)q + \frac{1}{2}F^2 + \mathcal{L}_{gf} + \mathcal{L}_{gh}$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + i\,g\,[A_\mu, A_\nu]$$

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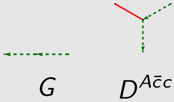
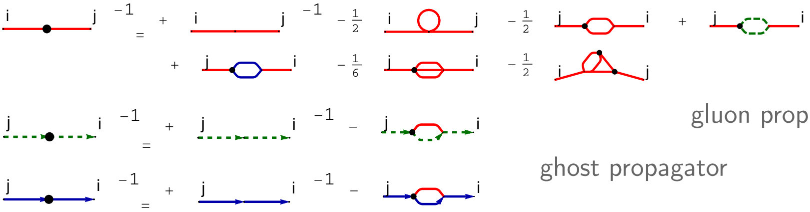
Landau gauge

- simplest one for functional equations
- $\partial_\mu A_\mu = 0$: $\quad \mathcal{L}_{gf} = \frac{1}{2\xi}(\partial_\mu A_\mu)^2, \quad \xi \to 0$
- requires ghost fields: $\quad \mathcal{L}_{gh} = \bar{c}\,(-\Box + g\,A\,\times)\,c$



The tower of DSEs

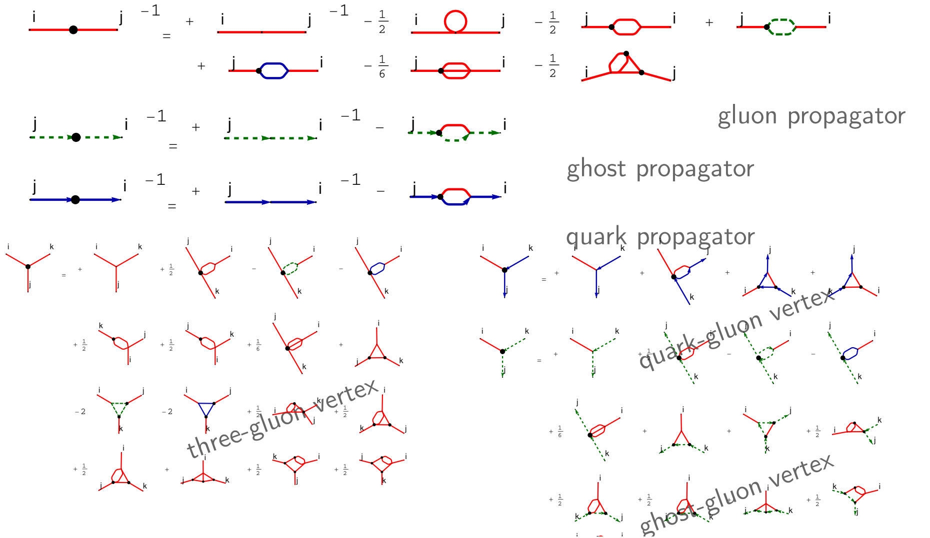


gluon propagator

ghost propagator

quark propagator

The tower of DSEs



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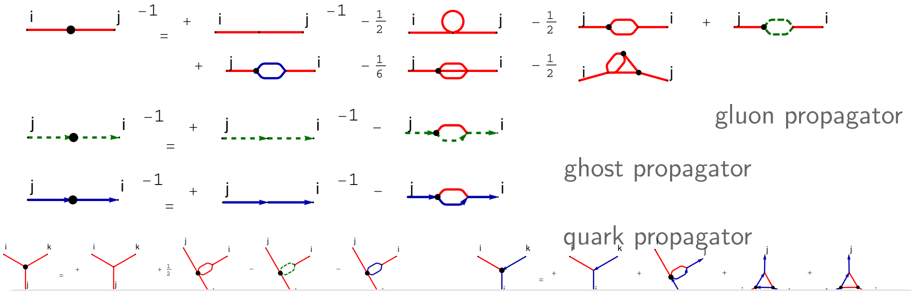
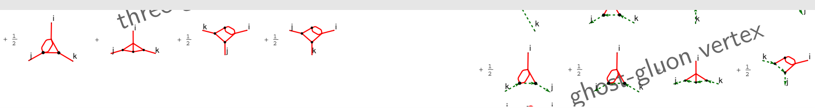


Infinitely many equations. In QCD, every n-point function depends on $(n+1)$- and possibly $(n+2)$-point functions.



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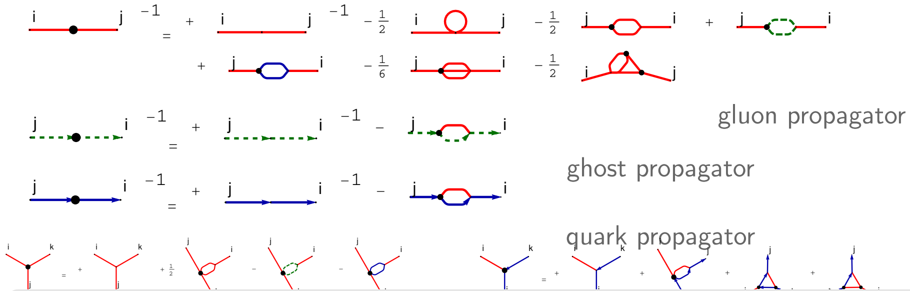

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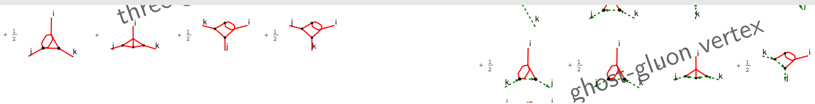
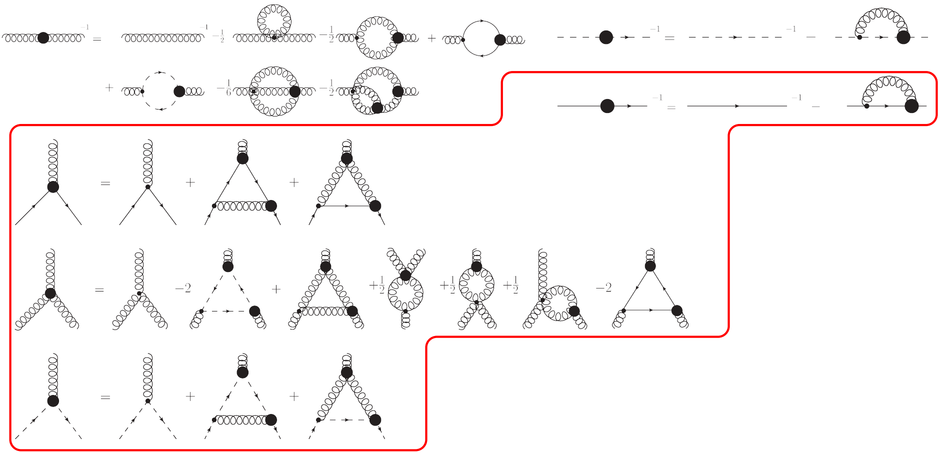
Is it possible to find and solve a truncation with all relevant contributions?

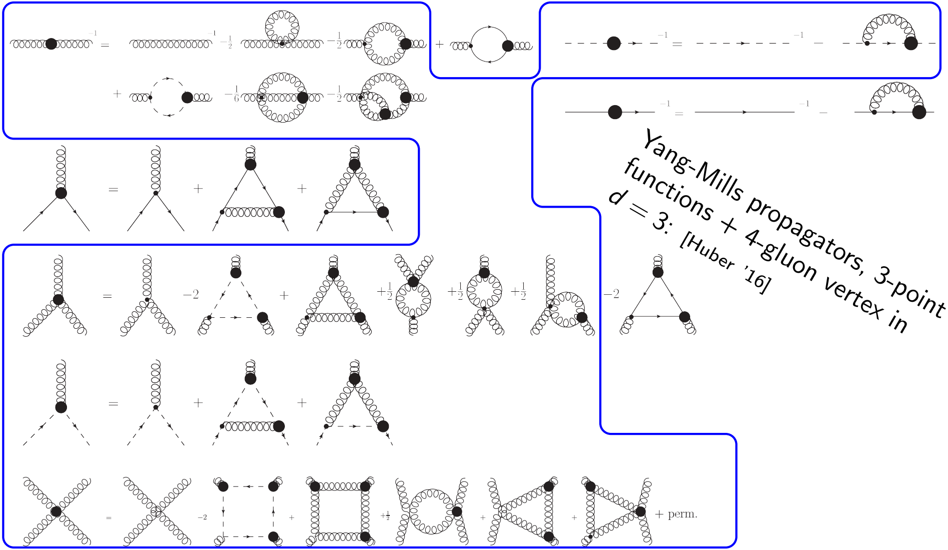


Dyson-Schwinger equations: Truncations



quark propagator + 3-point functions: [Williams, Fischer, Heupel '15]

Dyson-Schwinger equations: Truncations



Yang-Mills theory

- Truncations: From qualitative to quantitative level. $\rightarrow$ Testing required.
- Glueballs!

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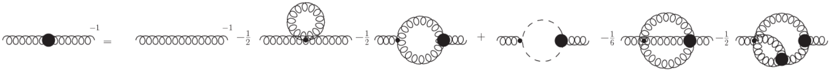
Solving the gluon propagator DSE:



- Requires ghost propagator and three- and four-point functions
- Some hidden obstacles: correct UV behavior, spurious divergences

$d = 3$ Yang-Mills theory as testing ground

Advantages:

- UV finite: no renormalization, no anomalous running
- Spurious divergences easier to handle
- UV behavior 'easier'

$\boxed{\Rightarrow \text{ Many complications from } d = 4 \text{ absent. } \rightarrow \text{ Focus on truncation effects.}}$

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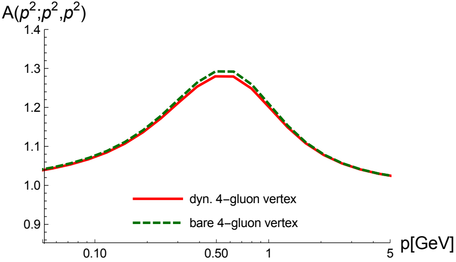
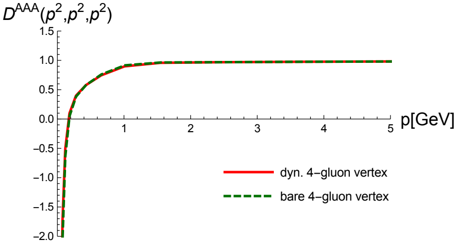
Historically interesting because cheaper on the lattice $\rightarrow$ easier to reach the IR.

However: Numerically not cheaper for functional equations of 2- and 3-point functions.

Three-point functions





Ghost-gluon vertex:

- Deviation from tree-level in midmomentum 'small'

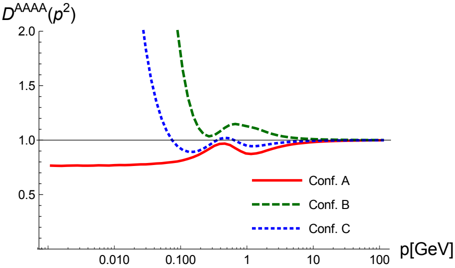
Three-gluon vertex:

- Close to tree-level above 1 GeV
- Non-tree-level dressings negligible

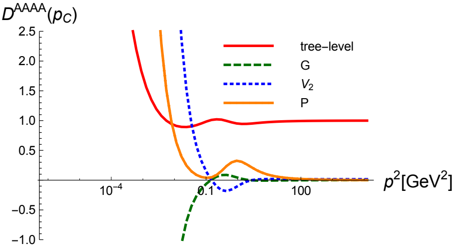
  [d=4: Eichmann, Williams, Alkofer, Vujinovic '14]

Four-gluon vertex

Different momentum configurations:



Different projections:



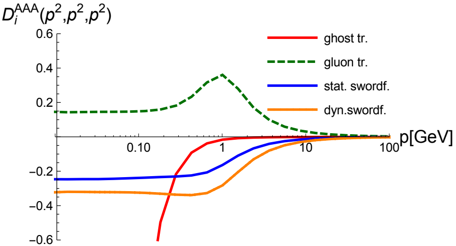
Four-gluon vertex:

- Close to tree-level down to 1 GeV

→ Corrections small individually?

Cancellations in gluonic vertices

Three-gluon vertex:



- Individual contributions large.
- Sum is small!

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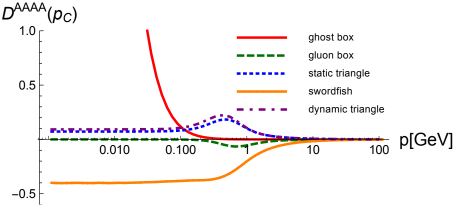

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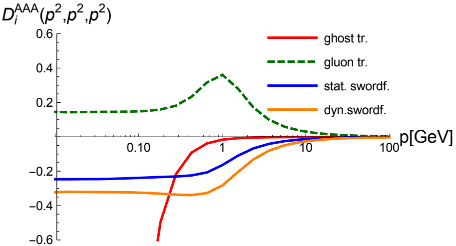

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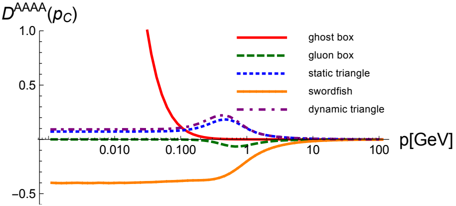


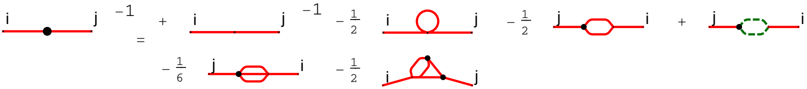
- Individual contributions large.
- Sum is small!

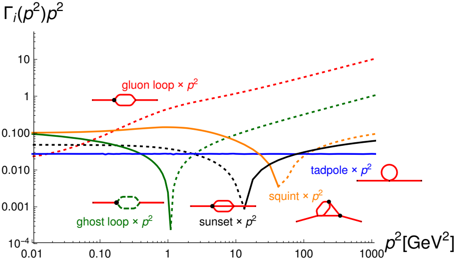
$\Downarrow$

Higher contributions:

- Higher vertices close to 'tree-level'? $\rightarrow$ Small.
- If pattern changes (higher vertices large): cancellations required.

Gluon propagator: Single diagrams





- Squint important in midmomentum regime.
- Sunset contribution small.

Solution from the 3PI effective action

Different set of functional equations:
equations of motion from 3PI effective action (at three-loop level)

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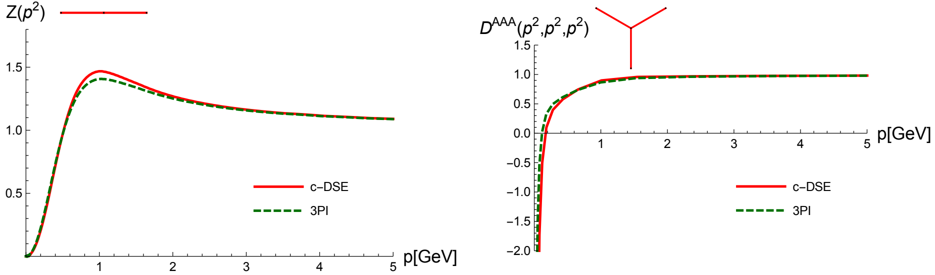
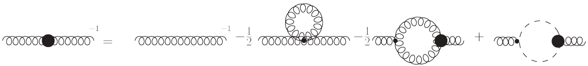


$\rightarrow$ Very similar results. [MQH '16]

UV behavior of the gluon propagator

Resummed one-loop order: anomalous dimension $\gamma = -13/22$
One-loop truncation:



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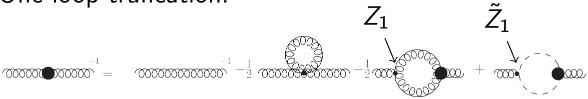


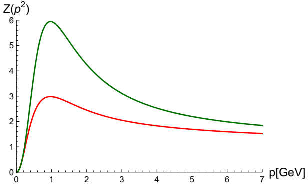
Self-consistent solution puts constraints on UV behavior of vertices [von Smekal, Hauck, Alkofer '97]:

- Ghost-gluon vertex: $\sim$ *const.* $\to \checkmark$
- Three-gluon vertex: $\propto (\log p)^{17/22}$
  Anomalous dimension $\gamma_{3g} = 17/44 \to$ ☹
  Solutions: $Z_1 \to Z_1(p^2) \leftrightarrow$ modified three-gluon vertex model [von Smekal, Hauck, Alkofer '97; Fischer, Alkofer '02]

Truncation artifact!

Resummed behavior

- Resolving the UV behavior within this truncation leads to an additional parameter dependence $\rightarrow$ part of the model
  Extreme example:



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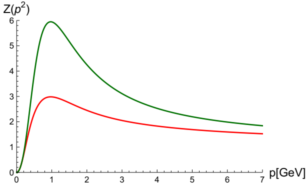

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One-loop anomalous dimension

Origin in resummation of higher order diagrams.

- Some are included in truncation.
- Some are missing, e.g., squint diagram.
- Sunset does not contribute at $O(g^4)$.

Resummed behavior

Minimal requirements to obtain one-loop resummed behavior:

- Squint diagram
- Proper models for three-point functions (with correct anom. dimensions)

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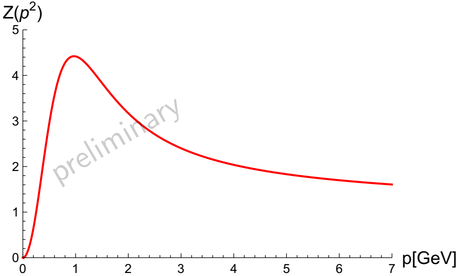
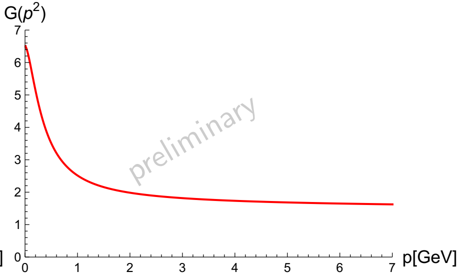




[propagator eqs. full, 3-point models, bare 4-gluon vertex]

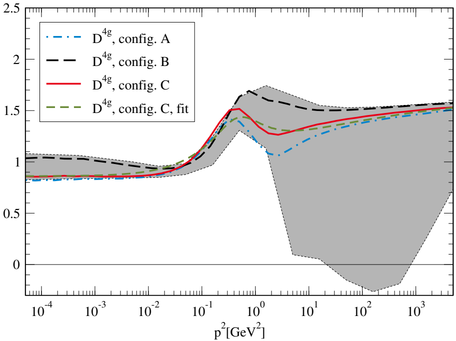
- Resummed behavior is recovered.
- Coupling: Consistent with results for $N_f = 0$ from lattice: [Sternbeck, Maltman, Müller-Preussker, on Smekal '12]

Four-gluon vertex

Full calculation with fixed input:

[Cyrol, MQH, von Smekal ’14]

Computationally expensive!



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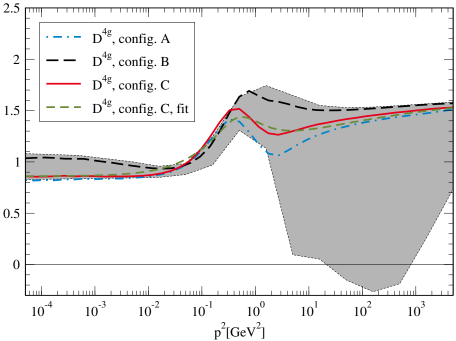

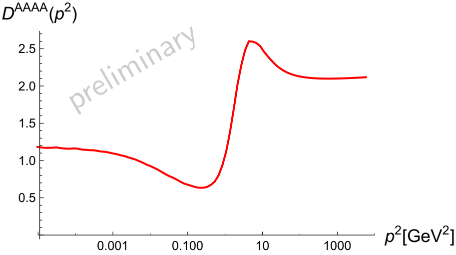

For dynamic inclusion: Resort to a one-momentum approximation (symmetric point).

Effect of four-gluon vertex

Three-gluon vertex: Important for convergence within current truncations in $d = 4$ [Blum, MQH, Mitter, von Smekal '14; Eichmann, Williams, Alkofer, Vujinovic '14]

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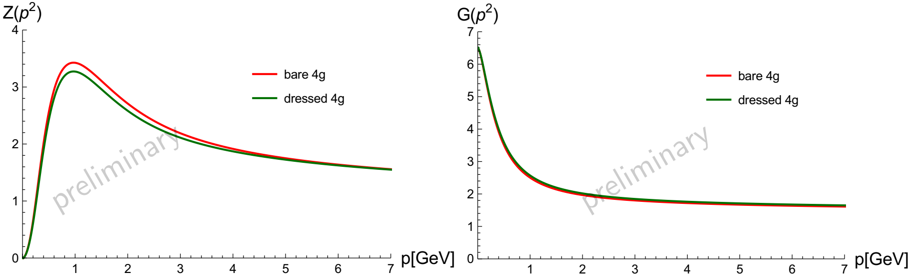
In gluon propagator: Via sunset diagram, small contribution of tree-level dressing



[propagator eqs. full, 3-gluon model, ghost-gluon dyn., 4-gluon fixed]

Summary and conclusions

Test truncation effects in $d = 3$, where spurious divergences and RG resummation are understood [Huber '16]:

- Truncation stable under all tested variations:
  - comparison with 3PI
  - changing the four-gluon vertex
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Extension to $d = 4$:

- Renormalization: Anomalous running
- Two-loop diagrams: Required quantitatively and for self-consistency
- Ghost-gluon vertex, four-gluon vertex included

Outlook:

- Include three-gluon vertex dynamically
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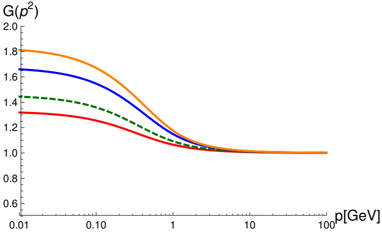
Thank you for your attention!

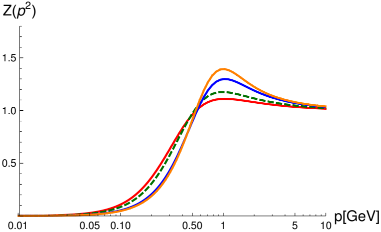
Family of solutions

Cf. FRG results: Bare mass parameter from modified STIs [Cyrol, Fister, Mitter, Pawlowski, Strodthoff '16].

DSEs: Enforce family of solutions by fixing the gluon propagator at $p^2 = 0$.

Simple toy system with bare vertices [MQH, 1606.02068]:





$\Rightarrow$ Possibility of family of solutions.

NB: Effect overestimated here since vertices are fixed.