

Landau gauge correlation functions from Dyson-Schwinger equations



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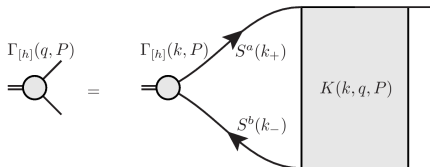


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Hadrons from bound state equations

Calculation of bound states from QCD correlation functions:

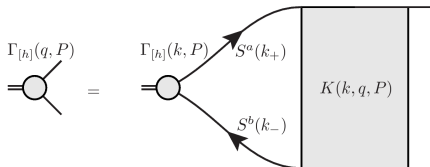
Meson BSE:



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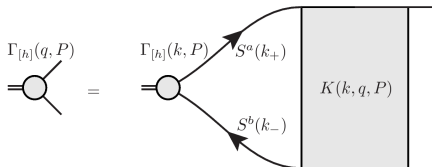


- Make an ansatz for the kernel K , e.g., ladder-type.

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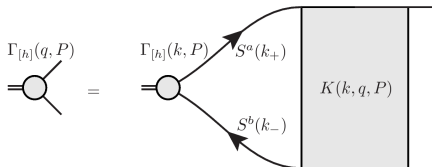
- Make an ansatz for the kernel K , e.g., ladder-type.
- Solve the quark gap equation.

The diagram illustrates the quark gap equation. A quark propagator $S(p)$ is equal to the free propagator $S_0(p)$ plus a loop diagram. The loop diagram consists of a quark propagator $S(q)$ and a gluon propagator $D_{\mu\nu}(p-q)$ connected by vertices γ_μ and $\Gamma_\mu(p, q)$.

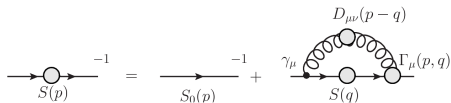
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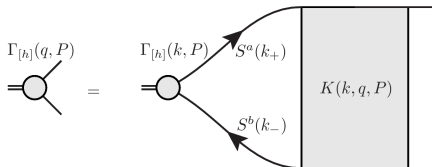


- Make ansatz for the **gluon propagator** $D_{\mu\nu}$ and the **quark-gluon vertex** Γ_μ , e.g., rainbow + effective interaction

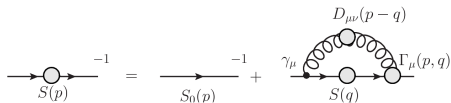
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- Make ansatz for the **gluon propagator** $D_{\mu\nu}$ and the **quark-gluon vertex** Γ_μ , e.g., rainbow + effective interaction
- Constraints by chiral symmetry!

Correlation functions from first principles

Widely used truncation: Rainbow + ladder + variant of Maris-Tandy interaction

How to reduce model dependence

- Improve kernel K
- Use explicit **gluon propagator** + **quark-gluon vertex**



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→ We need full control over the gluonic sector.

- Gluon propagator
- Three-gluon vertex
- ...?

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→ We need full control over the gluonic sector.

- Gluon propagator
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Side note: QCD phase diagram

- No sign problem ☺
- Truncation problem ☹

→ Ultimately, we need full control over the gluonic sector.

Landau gauge QCD

$$\mathcal{L} = \bar{q}(-\not{D} + m)q + \frac{1}{2}F^2 + \mathcal{L}_{gf} + \mathcal{L}_{gh}$$

$$F_{\mu\nu} = \partial_\mu \mathbf{A}_\nu - \partial_\nu \mathbf{A}_\mu + ig [\mathbf{A}_\mu, \mathbf{A}_\nu]$$



Landau gauge QCD

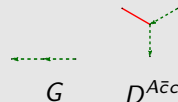
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Landau gauge

- simplest one for functional equations
- $\partial_\mu \mathbf{A}_\mu = 0$: $\mathcal{L}_{gf} = \frac{1}{2\xi}(\partial_\mu \mathbf{A}_\mu)^2$, $\xi \rightarrow 0$
- requires ghost fields: $\mathcal{L}_{gh} = \bar{\mathbf{c}}(-\square + g \mathbf{A} \times) \mathbf{c}$



The tower of DSEs

$$\begin{aligned}
 & \text{quark propagator} \quad i \text{---} \bullet \text{---} j \quad^{-1} = + \text{quark propagator} \quad i \text{---} j \quad^{-1} - \frac{1}{2} \text{quark loop} \quad i \text{---} j \quad^{-1} - \frac{1}{2} \text{quark-gluon loop} \quad i \text{---} i + \text{quark-ghost loop} \quad i \text{---} i \\
 & \quad \quad \quad + \text{quark-gluon loop} \quad i \text{---} i \quad^{-1} - \frac{1}{6} \text{quark-gluon loop} \quad i \text{---} i \quad^{-1} - \frac{1}{2} \text{quark-gluon loop} \quad i \text{---} j \\
 & \text{gluon propagator} \quad j \text{---} \bullet \text{---} i \quad^{-1} = + \text{gluon propagator} \quad j \text{---} i \quad^{-1} - \text{gluon loop} \quad j \text{---} i \\
 & \text{ghost propagator} \quad j \text{---} \bullet \text{---} i \quad^{-1} = + \text{ghost propagator} \quad j \text{---} i \quad^{-1} - \text{ghost loop} \quad j \text{---} i \\
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 \end{aligned}$$

The tower of DSEs

$$\begin{aligned}
 \text{quark propagator} &= \text{tree} + \text{gluon loop} - \frac{1}{2} \text{gluon self-energy} + \text{ghost loop} - \frac{1}{6} \text{ghost self-energy} - \frac{1}{2} \text{quark loop} \\
 &+ \text{quark-gluon vertex} - \frac{1}{2} \text{quark-gluon vertex self-energy}
 \end{aligned}$$

$$\text{gluon propagator} = \text{tree} - \text{ghost loop}$$

$$\text{ghost propagator} = \text{tree} - \text{ghost loop}$$

$$\text{quark-gluon vertex} = \text{tree} + \text{gluon loop} + \text{ghost loop} + \text{quark loop} + \text{quark-gluon vertex self-energy} + \text{ghost-gluon vertex} + \text{quark-ghost vertex}$$

Infinitely many equations. In QCD, every n -point function depends on $(n+1)$ - and possibly $(n+2)$ -point functions.

$$\text{ghost-gluon vertex} = \text{tree} + \text{ghost loop} + \text{quark loop} + \text{quark-gluon vertex}$$

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The tower of DSEs

$$\begin{aligned}
 \text{quark propagator} &= \text{tree} + \text{self-energy} + \text{polarization} + \dots \\
 \text{gluon propagator} &= \text{tree} + \text{self-energy} + \text{polarization} + \dots \\
 \text{ghost propagator} &= \text{tree} + \text{self-energy} + \dots
 \end{aligned}$$

gluon propagator

ghost propagator

quark propagator

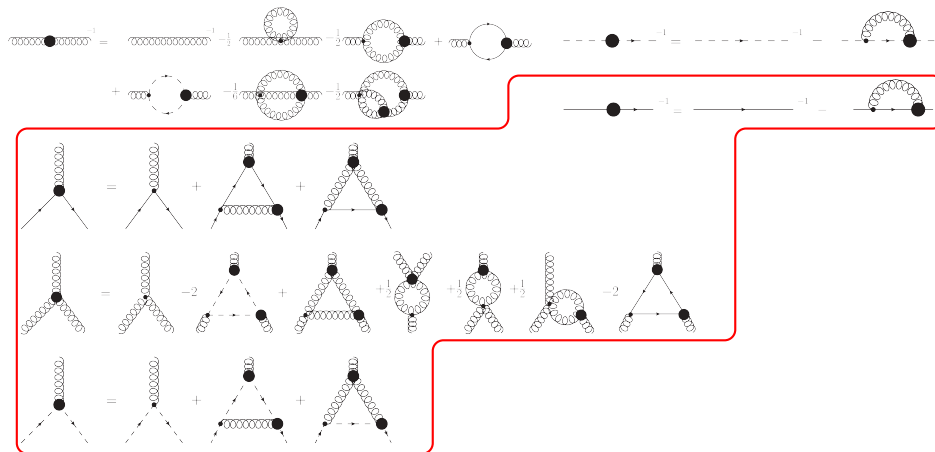
Infinitely many equations. In QCD, every n -point function depends on $(n+1)$ - and possibly $(n+2)$ -point functions.

Is it possible to find and solve a truncation with all **relevant contributions**?

three-gluon vertex

ghost-gluon vertex

Dyson-Schwinger equations: Truncations



quark propagator + 3-point functions: [Williams, Fischer, Heupel '15]

Dyson-Schwinger equations: Truncations

$$\text{Gluon Propagator}^{-1} = \text{Tree-level}^{-1} + \text{Gluon Loop} + \text{Ghost Loop} + \dots$$

$$\text{Ghost Propagator}^{-1} = \text{Tree-level}^{-1} + \text{Gluon Loop}$$

$$\text{Ghost-Gluon Vertex}^{-1} = \text{Tree-level}^{-1} + \text{Gluon Loop}$$

$$\text{3-point Gluon Vertex} = \text{Tree-level} + \text{Gluon Loop} + \text{Ghost Loop} + \dots$$

$$\text{4-point Gluon Vertex} = \text{Tree-level} + \text{Gluon Loop} + \text{Ghost Loop} + \dots$$

$$\text{Ghost-Gluon-Gluon Vertex} = \text{Tree-level} + \text{Gluon Loop} + \text{Ghost Loop} + \dots$$

$$\text{4-point Ghost-Gluon Vertex} = \text{Tree-level} + \text{Gluon Loop} + \text{Ghost Loop} + \dots + \text{perm.}$$

Yang-Mills propagators, 3-point functions + 4-gluon vertex in $d = 3$: [Huber '16]

Yang-Mills theory

- Truncations: From qualitative to quantitative level. \rightarrow Testing required.
- Glueballs!

Yang-Mills theory

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Solving the gluon propagator DSE:

$$\text{Full Gluon Propagator} = \text{Tree Level} - \frac{1}{2} \text{Gluon Loop} + \text{Ghost Loop} - \frac{1}{6} \text{Three-Vertex Loop} - \frac{1}{2} \text{Four-Vertex Loop}$$

- Requires ghost propagator and three- and four-point functions
- Some hidden obstacles: correct UV behavior, spurious divergences

$d = 3$ Yang-Mills theory as testing ground

Advantages:

- UV finite: no renormalization, no anomalous running
- Spurious divergences easier to handle
- UV behavior 'easier'

⇒ Many complications from $d = 4$ absent. → Focus on truncation effects.

$d = 3$ Yang-Mills theory as testing ground

Advantages:

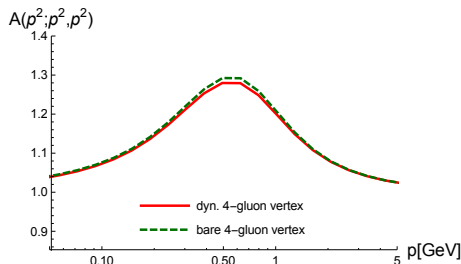
- UV finite: no renormalization, no anomalous running
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Historically interesting because cheaper on the lattice → easier to reach the IR.

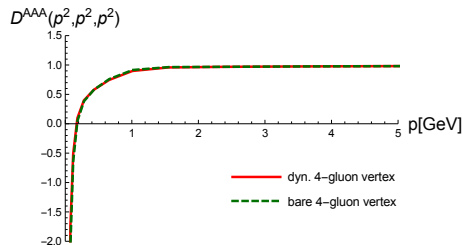
However: Numerically not cheaper for functional equations of 2- and 3-point functions.

Three-point functions



Ghost-gluon vertex:

- Deviation from tree-level in midmomentum 'small'

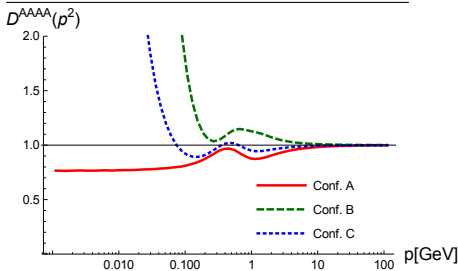


Three-gluon vertex:

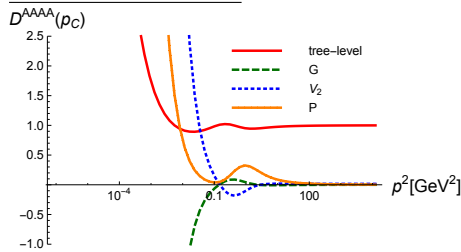
- Close to tree-level above 1 GeV
 - Non-tree-level dressings negligible
- [d=4: Eichmann, Williams, Alkofer, Vujanovic '14]

Four-gluon vertex

Different momentum configurations:



Different projections:



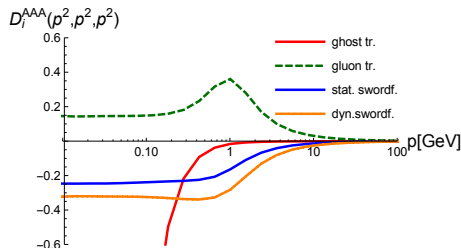
Four-gluon vertex:

- Close to tree-level down to 1 GeV

→ Corrections small individually?

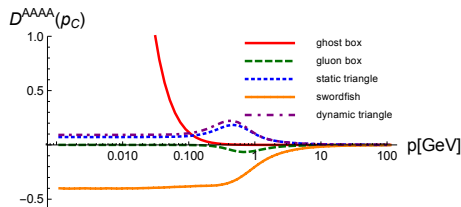
Cancellations in gluonic vertices

Three-gluon vertex:



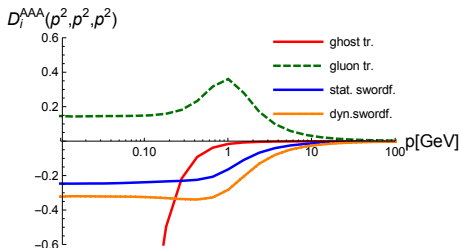
- Individual contributions large.
- **Sum is small!**

Four-gluon vertex:



Cancellations in gluonic vertices

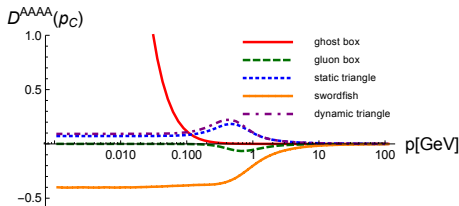
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Four-gluon vertex:



Higher contributions:

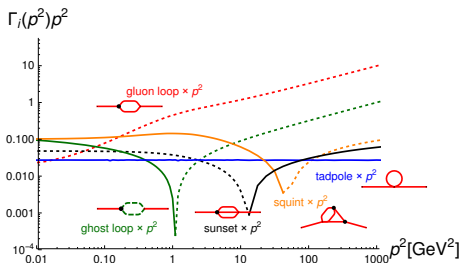
- Higher vertices close to 'tree-level'?
→ Small.
- If pattern changes (higher vertices large): cancellations required.

Gluon propagator: Single diagrams

$$\begin{aligned}
 & \text{Diagram 1}^{-1} = + \text{Diagram 2}^{-1} - \frac{1}{2} \text{Diagram 3} - \frac{1}{2} \text{Diagram 4} \\
 & \quad - \frac{1}{6} \text{Diagram 5} - \frac{1}{2} \text{Diagram 6}
 \end{aligned}$$

The diagrams represent various loop corrections to the gluon propagator:

- Diagram 1: Bare propagator (red line with a black dot).
- Diagram 2: Gluon loop (red line with a red loop).
- Diagram 3: Ghost loop (red line with a green dashed loop).
- Diagram 4: Sunset (red line with two red loops).
- Diagram 5: Squint (red line with a red loop and a red triangle).
- Diagram 6: Tadpole (red line with a red loop and a red triangle).



- Squint important in midmomentum regime.
- Sunset contribution small.

Solution from the 3PI effective action

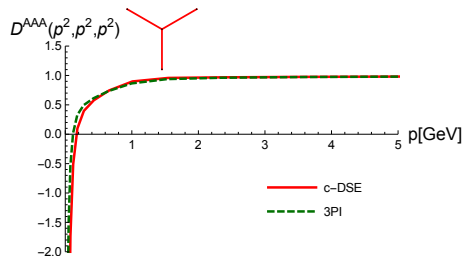
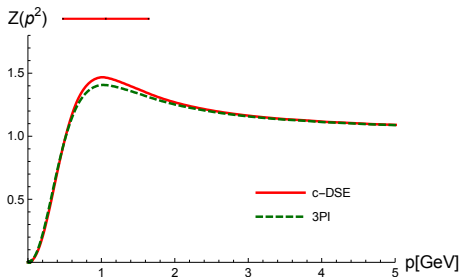
Different set of functional equations:

equations of motion from 3PI effective action (at three-loop level)

Solution from the 3PI effective action

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equations of motion from 3PI effective action (at three-loop level)



→ Very similar results.

[MQH '16]

UV behavior of the gluon propagator

Resummed **one-loop** order: anomalous dimension $\gamma = -13/22$

One-loop truncation:

$$\text{Gluon line with black dot}^{-1} = \text{Gluon line with black dot}^{-1-\frac{1}{2}} - \frac{1}{2} \text{Gluon line with black dot}^{-\frac{1}{2}} \text{Gluon loop} - \frac{1}{2} \text{Gluon line with black dot}^{-\frac{1}{2}} \text{Ghost loop} + \text{Gluon line with black dot}^{-\frac{1}{2}} \text{Ghost loop with dashed line}$$

UV behavior of the gluon propagator

Resummed **one-loop** order: anomalous dimension $\gamma = -13/22$

One-loop truncation:

$$\text{Gluon line with black dot}^{-1} = \text{Gluon line with self-energy loop}^{-1} - \frac{1}{2} \text{Gluon line with ghost loop}^{-1} + \dots$$

Self-consistent solution puts constraints on UV behavior of vertices [von Smekal, Hauck, Alkofer '97]:

- Ghost-gluon vertex: $\sim \text{const.} \rightarrow \checkmark$
- Three-gluon vertex: $\propto (\log p)^{17/22}$
Anomalous dimension $\gamma_{3g} = 17/44 \rightarrow \odot$

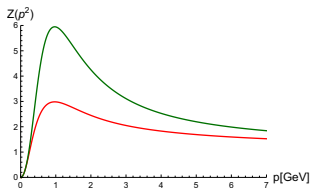
Solutions: $Z_1 \rightarrow Z_1(p^2) \leftrightarrow$ modified three-gluon vertex model [von Smekal, Hauck, Alkofer '97; Fischer, Alkofer '02]

Truncation artifact!

Resummed behavior

- Resolving the UV behavior within this truncation leads to an additional parameter dependence \rightarrow part of the **model**

Extreme example:

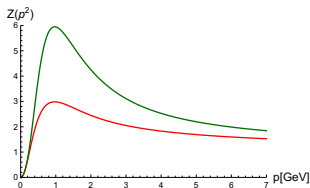


- Study for three-gluon vertex: [Eichmann, Williams, Alkofer, Vujanovic '14]
- However, **correct UV behavior** is minimal demand on truncation and also required for **self-consistency**.

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One-loop anomalous dimension

Origin in resummation of higher order diagrams.

- Some are included in truncation.
- Some are missing, e.g., squint diagram.
- Sunset does not contribute at $O(g^4)$.

Resummed behavior

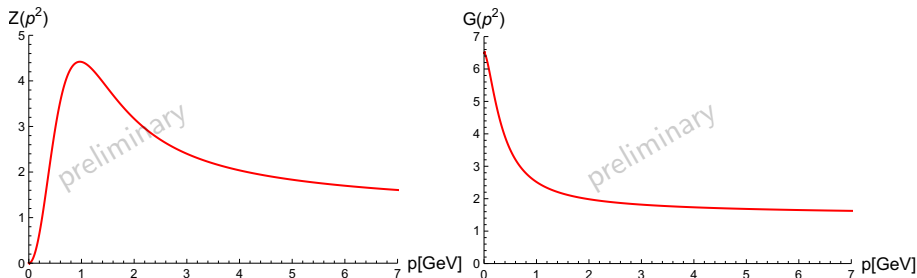
Minimal requirements to obtain one-loop resummed behavior:

- Squint diagram
- Proper models for three-point functions (with correct anom. dimensions)

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[propagator eqs. full, 3-point models, bare 4-gluon vertex]

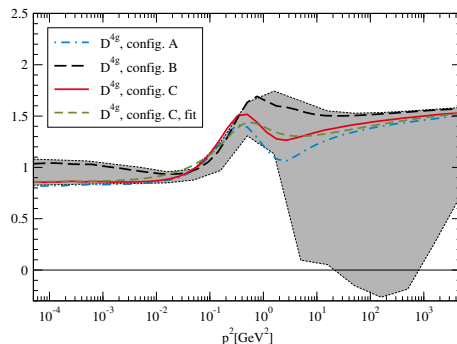
- Resummed behavior is recovered.
- Coupling: Consistent with results for $N_f = 0$ from lattice: [Sternbeck, Maltman, Müller-Preussker, on Smekal '12]

Four-gluon vertex

Full calculation with fixed input:

[Cyrol, MQH, von Smekal '14]

Computationally expensive!

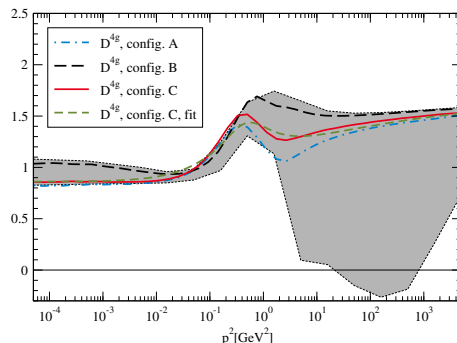
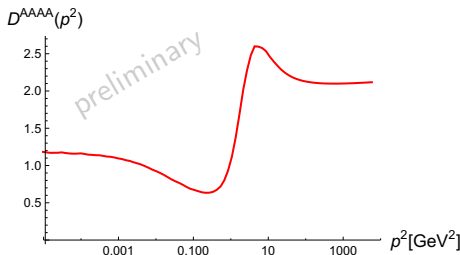


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For dynamic inclusion: Resort to a
one-momentum approximation
 (symmetric point).

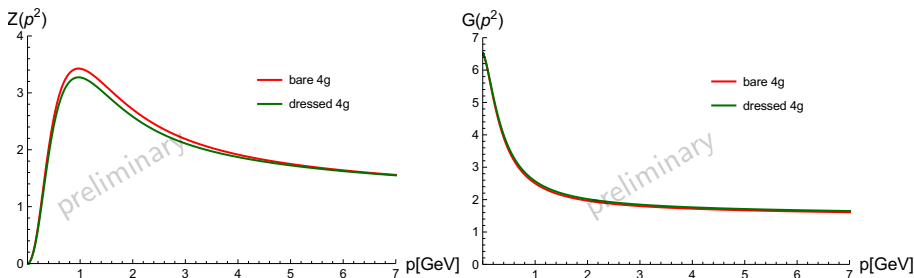
Effect of four-gluon vertex

Three-gluon vertex: Important for convergence within current truncations in $d = 4$ [Blum, MQH, Mitter, von Smekal '14; Eichmann, Williams, Alkofer, Vujanovic '14]

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In gluon propagator: Via sunset diagram, small contribution of tree-level dressing



[propagator eqs. full, 3-gluon model, ghost-gluon dyn., 4-gluon fixed]

Summary and conclusions

Test truncation effects in $d = 3$, where spurious divergences and RG resummation are understood [Huber '16]:

- Truncation **stable under** all tested **variations**:
 - comparison with 3PI
 - changing the four-gluon vertex
 - different DSEs for the ghost-gluon vertex
- **Hierarchy of diagrams** identified.

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Extension to $d = 4$:

- Renormalization: Anomalous running
- Two-loop diagrams: Required quantitatively and for self-consistency
- Ghost-gluon vertex, four-gluon vertex included

Outlook:

- Include three-gluon vertex dynamically
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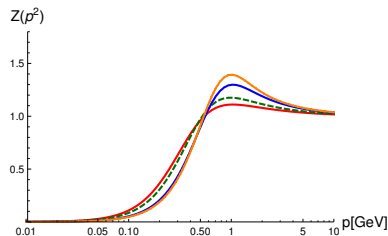
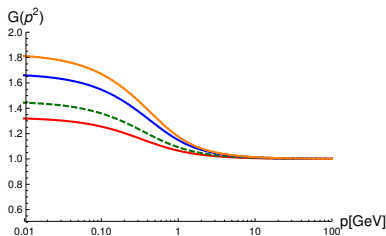
Thank you for your attention!

Family of solutions

Cf. FRG results: Bare mass parameter from modified STIs [Cyrol, Fister, Mitter, Pawłowski, Strodthoff '16].

DSEs: Enforce family of solutions by fixing the gluon propagator at $p^2 = 0$.

Simple toy system with bare vertices [MQH, 1606.02068]:



⇒ Possibility of family of solutions.

NB: Effect overestimated here since vertices are fixed.