Nonperturbative correlation functions from Dyson-Schwinger equations

Markus Q. Huber

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Institute of Theoretical Physics, Giessen University

Bound states in QCD and beyond III

St. Goar, Germany

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FWF Der Wissenschaftsfonds.

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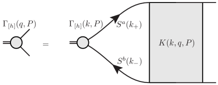
DFG Deutsche Forschungsgemeinschaft

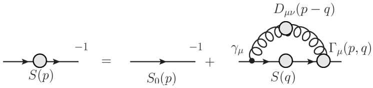
Hadronic bound states

Bound state equations: E.g., meson



Ingredients:

- Interaction kernel K
- Quark propagator S



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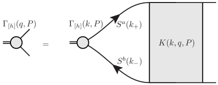

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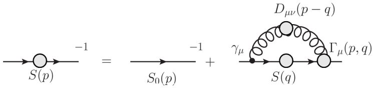

Approaches:

- Phenomenological (bottom-up): Model interactions
- From first principles (top-down): Piecing together the elementary pieces

Bottom-up vs. top-down

Bottom-up:

- Modeling to describe certain quantities, symmetries as guiding principles
- Example: Rainbow-ladder truncation with Maris-Tandy interaction:

1 function, 2 parameters
$$\mathcal{G}(k^2)$$

$\rightarrow$ Good description of, e.g., pseudoscalars

Top-down:

- 9 dressings for gluon propagator and quark-gluon vertex:
  $D(k^2)$, $\Gamma_i^{A\bar{q}q}(p,q,r)$, $i = 1, \ldots, 8$
  $\rightarrow$ Technically complex
- Maximal flexibility $\leftrightarrow$ consistency not easy to achieve
- Parameters of QCD only

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More parameters?

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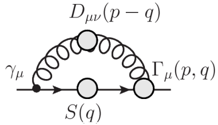
In the past, mostly bottom-up (hadron pheno) and top-down with bottom-up admixtures.

The elementary pieces



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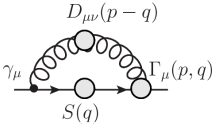

Bottom-up:

Effective interaction via $g^2 D_{\mu\nu}(p)\Gamma_\mu(p,q) \to Z_2 \widetilde{Z}_3 D^{(0)}_{\mu\nu}(p)\gamma_\mu \mathcal{G}((p+q)^2)$

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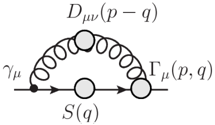

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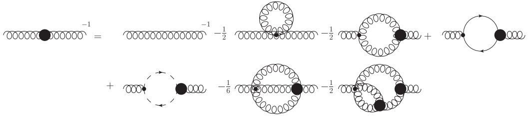
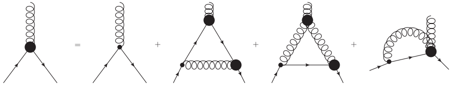
Top-down:

Gluon propagator $D_{\mu\nu}(p^2)$:



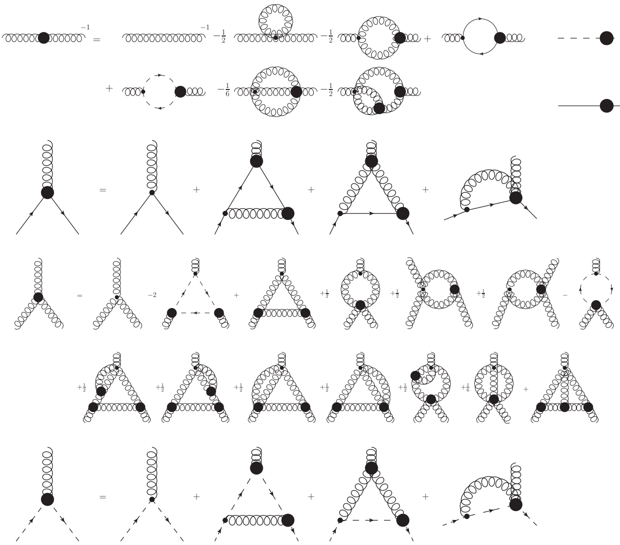
Quark-gluon vertex $\Gamma_\mu(p,q)$:

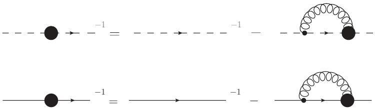


$\to$ Couple to infinity of equations.

$\to$ Gluonic part is crucial. $\Rightarrow$ Understand pure QCD (Yang-Mills theory) first.

Dyson-Schwinger equations





Infinitely large coupled system of integral equations.

Truncations

- Influence of higher correlation functions?

  qualitative? quantitative? negligible?

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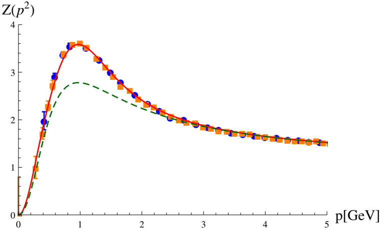
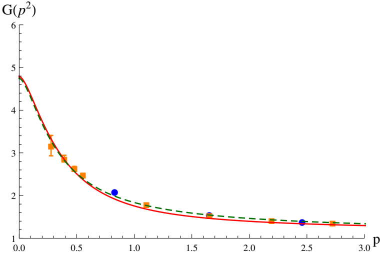
- How to realize resummation?
  higher loop contributions?
- Systematics and tests?
  comparison to other methods, self-tests?

Example of a bottom-up calculation

Propagators and ghost-gluon vertex with three-gluon vertex model:
One-loop truncation of gluon propagator with an optimized effective model (contains zero crossing) [MQH, von Smekal '13; lattice: Sternbeck '06]:





Good quantitative agreement for ghost and gluon dressings.

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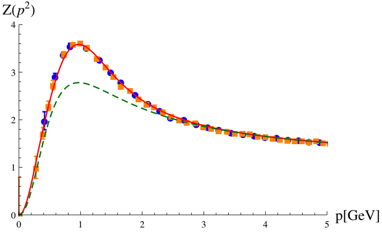

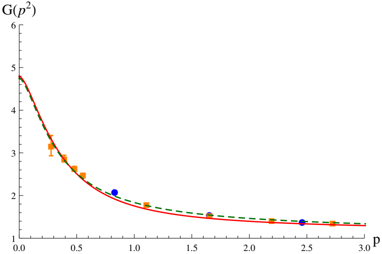

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QCD is only this:

$$\boxed{\mathcal{L} = -\frac{1}{2}\mathrm{Tr}\left(F_{\mu\nu}F^{\mu\nu}\right) + \sum_j \bar{\varphi}_j[i\gamma^\mu D_\mu - m_j]\varphi_j}$$

WOBEI $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + ig[A_\mu, A_\nu]$

UND $D_\mu = \partial_\mu + igA_\mu$

Can we do with only that?

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Error estimation difficult!

Extending truncations

Various ways to extend truncations.

Extension of the previous one:

- Vertex tensors beyond tree-level tensor
- Include neglected diagrams
- Include neglected correlation functions

Extensions also test the previous truncations!

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Three-gluon vertex DSE

[Cucchieri, Maas, Mendes '08; Alkofer, MQH, Schwenzer '09; Pelaez, Tissier, Wschebor '13; Aguilar et al. '13; Blum, et al. '14; Eichmann, Alkofer, Vujinovic '14; Cyrol et al. '16; Williams, Fischer, Heupel '16; Sternbeck '16; Athenodorou et al. 16; Duarte et al. '16; Boucaud et al. '17]

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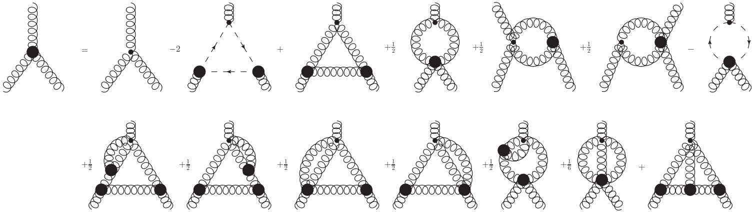

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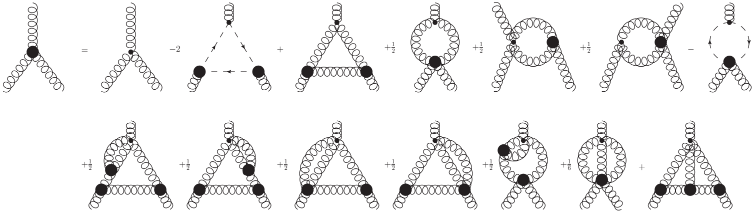
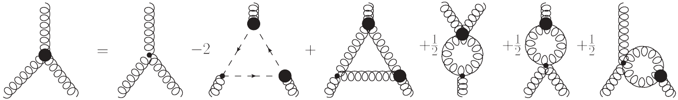


Perturbative one-loop truncation [Blum, MQH, Mitter von Smekal '14; Eichmann, Alkofer, Vujinovic '14; Williams, Fischer, Heupel '16]:



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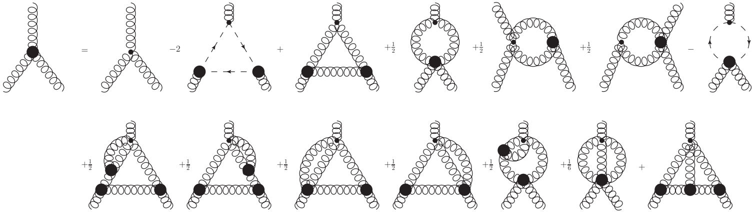
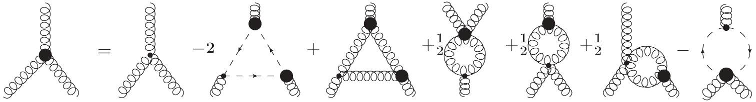
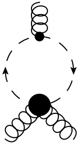


Non-perturbative one-loop truncation [MQH '17]:



Influence of two-ghost-two-gluon vertex



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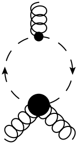
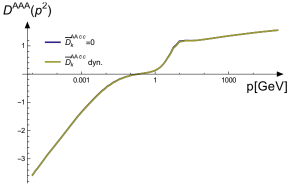


Coupled system of ghost-gluon, three-gluon and four-gluon vertices with and without two-ghost-two-gluon vertex [MQH '17]:



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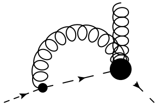
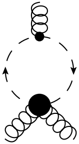
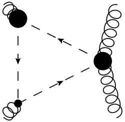

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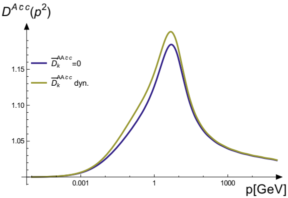
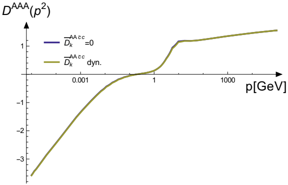
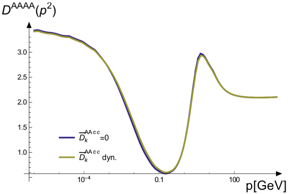
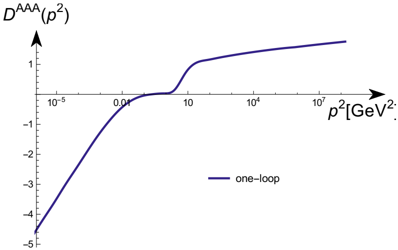






- Color structure: only small dressings in the three-gluon DSE $\rightarrow$ no change.
- Small influence on ghost-gluon vertex ($< 1.7\%$)

Three-gluon vertex results



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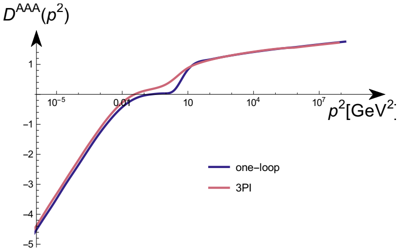

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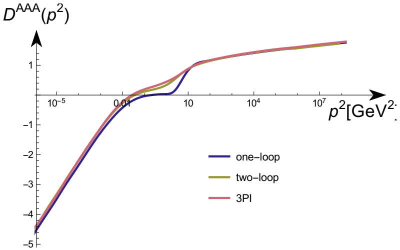

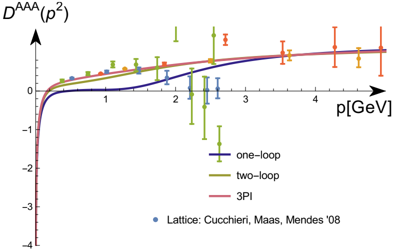

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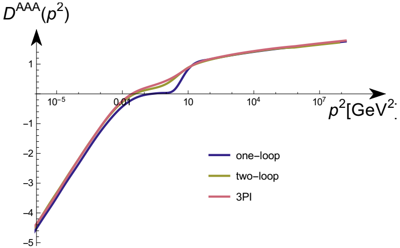
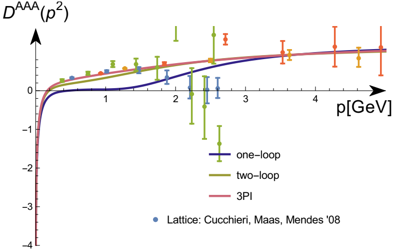
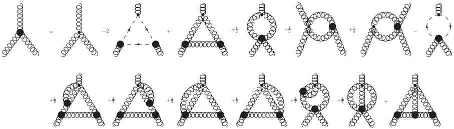




- Two-loop truncation: All diagrams except the one with a five-point function.
- One-momentum configuration approximation.



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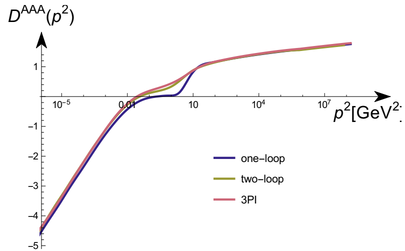
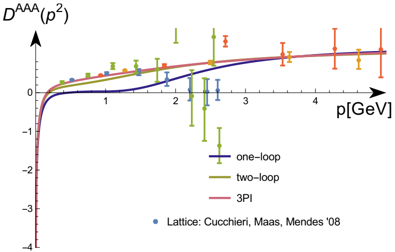
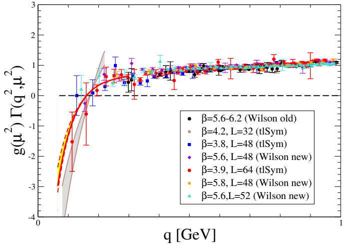




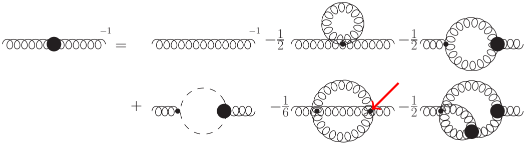
- Difference between two-loop DSE and 3PI smaller than lattice error.
- Zero crossing in agreement with other approaches, e.g., [Pelaez et al. '13; Aguilar et al. '13; Cyrol et al. '15; Athenodorou et al. '16; Duarte et al. '16; Sternbeck et al. '17]

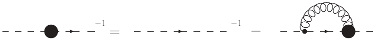


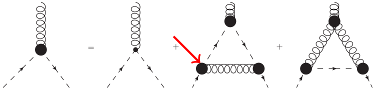
[Athenodorou et al. '16, '18]

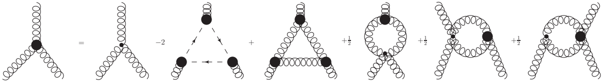
3PI system of primitively divergent correlation functions

Three-loop expansion of 3PI effective action [Berges '04]:



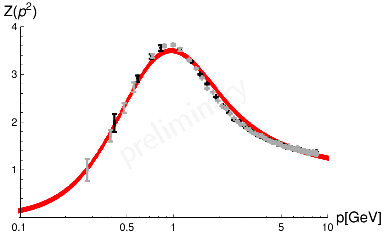






Results for fully coupled 3PI system

[lattice: Sternbeck '06]



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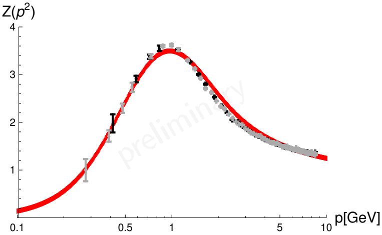
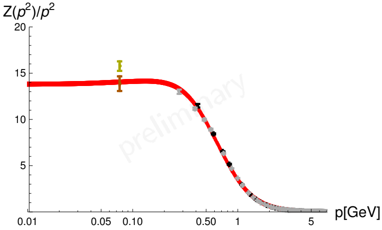
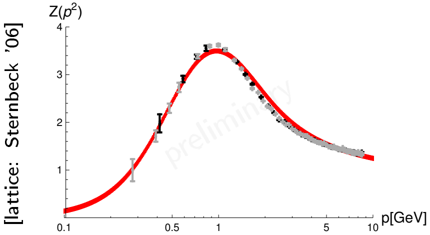
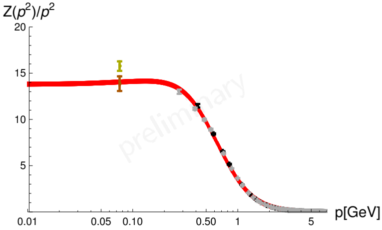
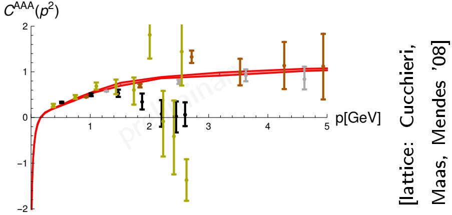




Results for fully coupled 3PI system







- Details of renormalization crucial!
- Other details also important.
- Very small angle dependence of three-gluon vertex.
- Slight bending down of gluon propagator in IR.

Open checks

- Effects of larger tensor bases, in particular of the three-gluon vertex
- Renormalization

What tests can be done?

Couplings

Couplings can be defined from every vertex, e.g., [Allés et al. '96; Alkofer et al., '05; Eichmann et al. '14]:

$$\alpha_{\text{ghg}}(p^2) = \alpha(\mu^2)\left(D^{A\bar{c}c}(p^2)\right)^2 G^2(p^2)Z(p^2),$$
$$\alpha_{3\text{g}}(p^2) = \alpha(\mu^2)\left(C^{AAA}(p^2)\right)^2 Z^3(p^2),$$
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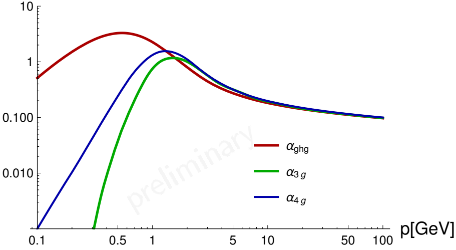

Ghost-gluon vs. other couplings: Further checks required.

Renormalization with a hard UV cutoff

Introduces quadratic divergences.
Note: Appears already perturbatively!

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The breaking of gauge covariance by the UV regularization leads to spurious (quadratic) divergences.

Due to the anomalous running, this behaves as (at one-loop resummed level)

$$\frac{\Lambda_{\mathrm{QCD}}^2}{p^2}(-1)^{2\delta}\Gamma(1+2\delta,-\ln(\Lambda^2/\Lambda_{\mathrm{QCD}}^2))$$

Many ways to deal with them, e.g., Brown-Pennington projector, modifications of integrands/vertices, fitting, seagull identities, ... [MQH, von Smekal '14].

Examples for renormalization with a hard UV cutoff

Here: Second renormalization condition

Breaking of gauge covariance $\rightarrow$ mass counter term [Collins '84]

Renormalization condition: $D(0) = c$ [Meyers, Swanson '14]

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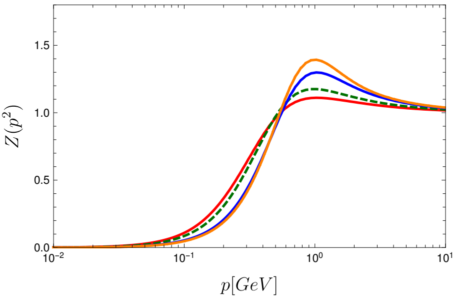
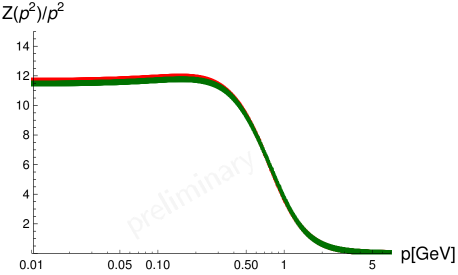
Extreme example: One-loop truncation with bare vertices in three dimensions [MQH '16].



Better example: Full system with one-momentum configuration approximation.



Results for fully coupled 3PI system revisited

Vary the renormalization condition $D(0)$:

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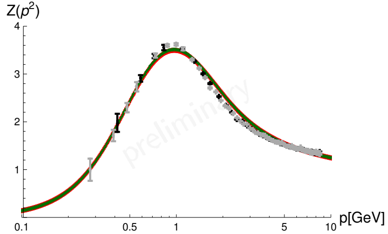

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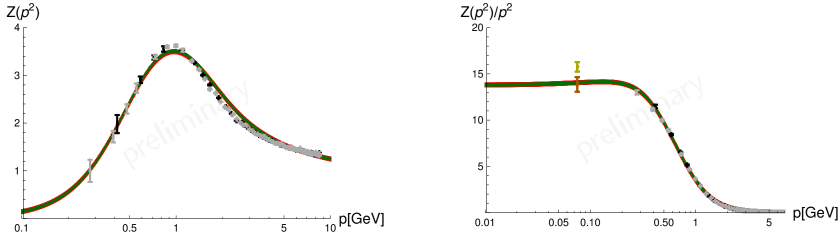

→ Two solutions on top of each other. $D(0)$ is not a parameter anymore?
→ If so: Provides a self-test of a truncation.

Summary and conclusions

Towards a systematic understanding of truncations of functional equations to access their full potential.

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- Non-classical tensors in gluonic vertices
- Add quarks
- Finite temperature
- Bound states
- Finite density
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Thank you for your attention!

Four-gluon vertex

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Organize via S_4 permutation group [Eichmann, Fischer, Heupel '15] and restrict to singlet and doublet. $\rightarrow$ Three variables.

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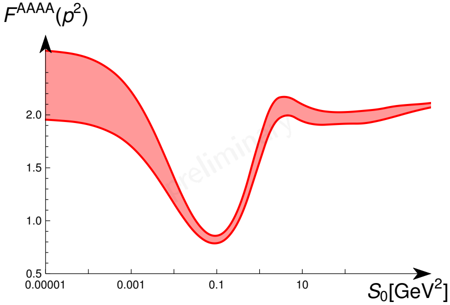

Deviations from leading perturbative behavior small, weak angle dependence.

The two-ghost-two-gluon vertex

Non-primitively divergent correlation function $\rightarrow$ No guide from tree-level tensor. $\rightarrow$ Use full basis.

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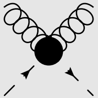


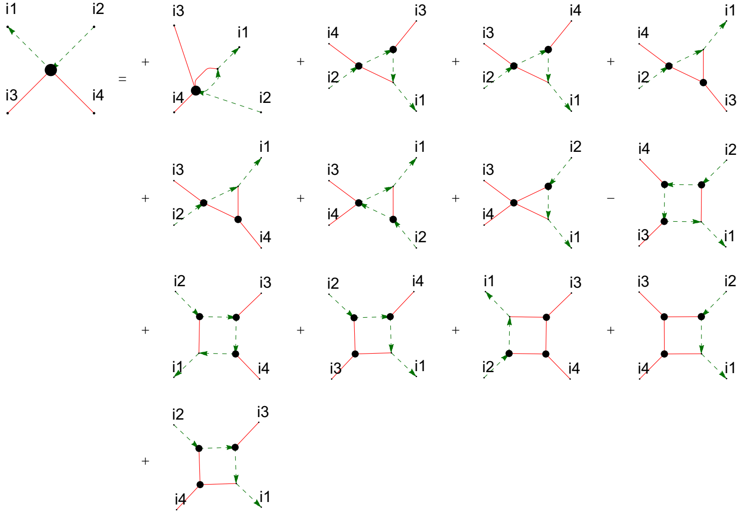
$$\Gamma^{AA\bar{c}c,abcd}_{\mu\nu}(p,q;r,s) = \boldsymbol{g}^{4} \sum_{k=1}^{\mathbf{40}} \rho^{k,abcd}_{\mu\nu} D^{AA\bar{c}c}_{k(i,j)}(p,q;r,s)$$

with

$$\rho^{k,abcd}_{\mu\nu} = \sigma^{abcd}_{i} \tau^{j}_{\mu\nu}, \qquad k = k(i,j) = 5(i-1)+j$$

The two-ghost-two-gluon vertex DSE

2 DSEs, choose the one with the ghost leg attached to the bare vertex
$\rightarrow$ Truncation discards only one diagram.



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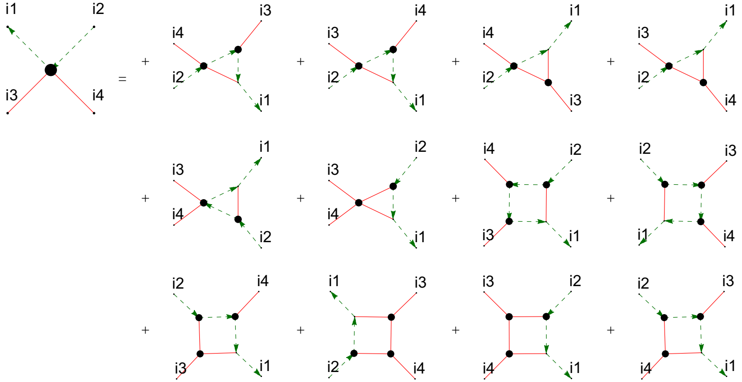
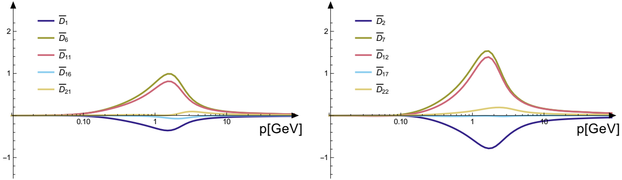
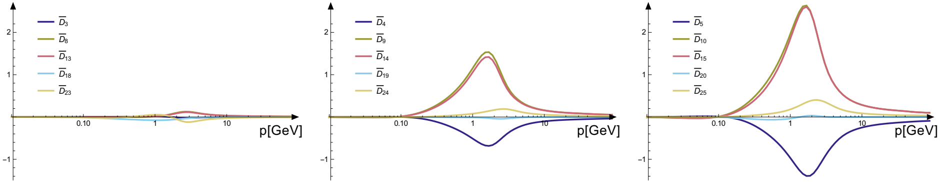


Results for the two-ghost-two-gluon vertex

Kinematic approximation: one-momentum configuration

- Dimensionless dressing functions $\overline{D}_k$.
- Each plot one Lorentz tensor.





$\rightarrow$ Two classes of dressings: 13 very small, 12 not small

$\rightarrow$ No nonzero solution for $\{\sigma_6, \sigma_7, \sigma_8\}$ found.

[MQH '17]

Results for the four-ghost vertex

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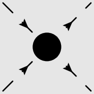

$$\Gamma^{\bar{c}\bar{c}cc,abcd}(p,q,r,s) = g^4 \sum_{k=1}^{8} \sigma^{k,abcd} E_k^{\bar{c}\bar{c}cc}(p,q,r,s).$$

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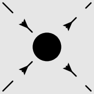

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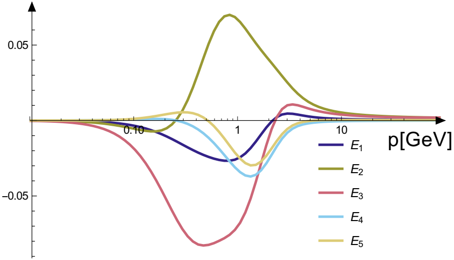

→ All dressings very small.

[MQH '17]

A lesson from three dimensions?

Three-dimensional Yang-Mills theory is finite.

$\rightarrow$ No renormalization

$\rightarrow$ Leading pertubative contributions $\propto g^2/p$

$\Rightarrow$ Testbed for functional calculations.

Various methods employed:

- Lattice: [Bornyakov, Cucchieri, Maas, Mendes, Mitrjushkin, Rogalyov, ...]
- Coupled propagator DSEs: [Maas, Wambach, Grüter, Alkofer '04]
- (R)GZ: [Dudal, Gracey, Sorella, Vandersickel, Verschelde '08]
- DSEs of PT-BFM: [Aguilar, Binosi, Papavassiliou '10]
- YM + mass term: [Tissier, Wschebor '10, '11]
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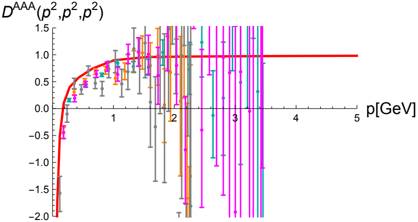
Study effect of individual diagrams...

Cancellations in three-gluon vertex



[MQH '16; lattice: Cucchieri, Maas, Mendes '08]

- Close to tree-level above 1 GeV
- Good agreement with lattice data.
- Linear IR divergence [Pelaez, Tissier, Wschebor '13; Aguilar et al. '13]
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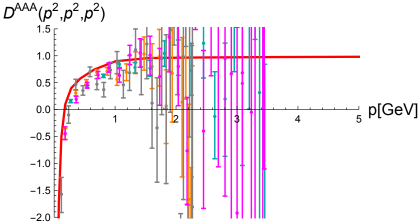

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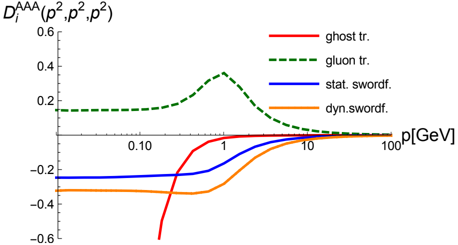

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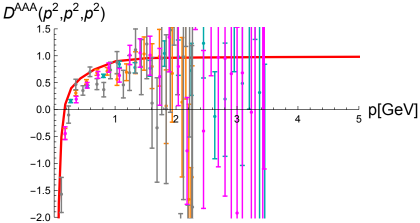

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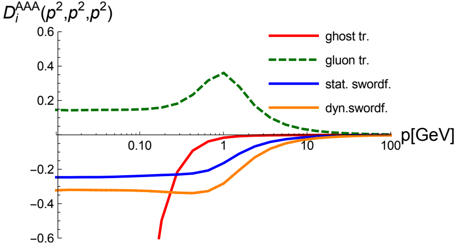

[MQH '16]

- Individual contributions large.
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$\rightarrow$ In four dimensions similar qualitative effects, but renormalization complicates things.

$\rightarrow$ Do not try with fewer diagrams!