Dyson-Schwinger studies of Yang-Mills vertices at zero and non-vanishing temperatures

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Excited QCD 2015, Tatranska Lomnica

March 13, 2015

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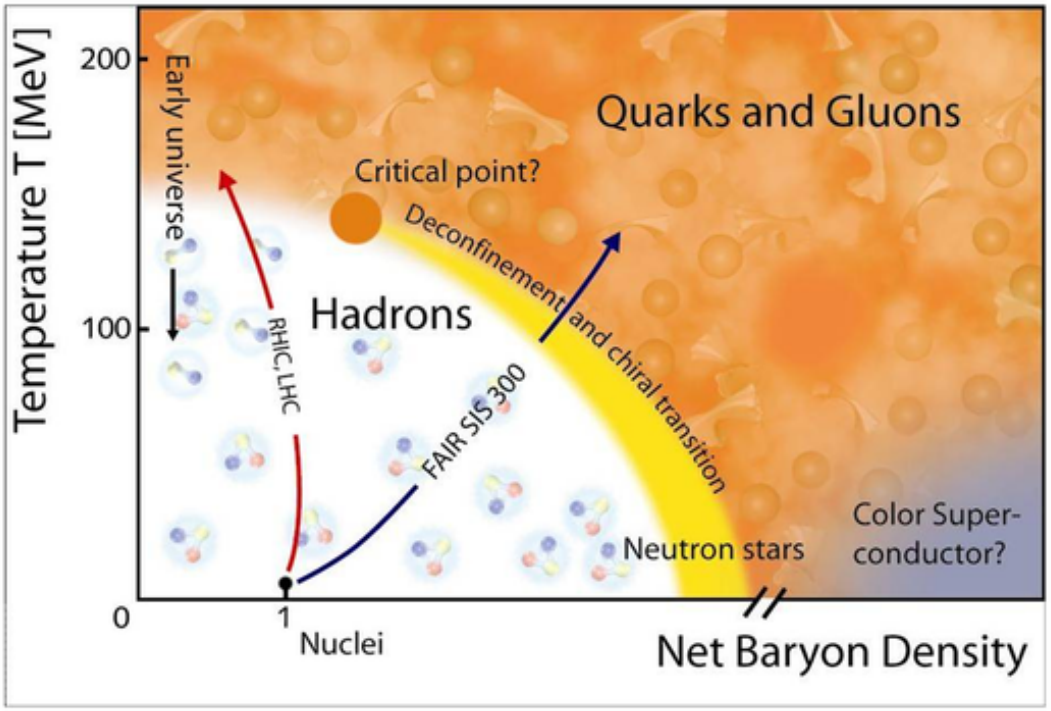
Phases of QCD

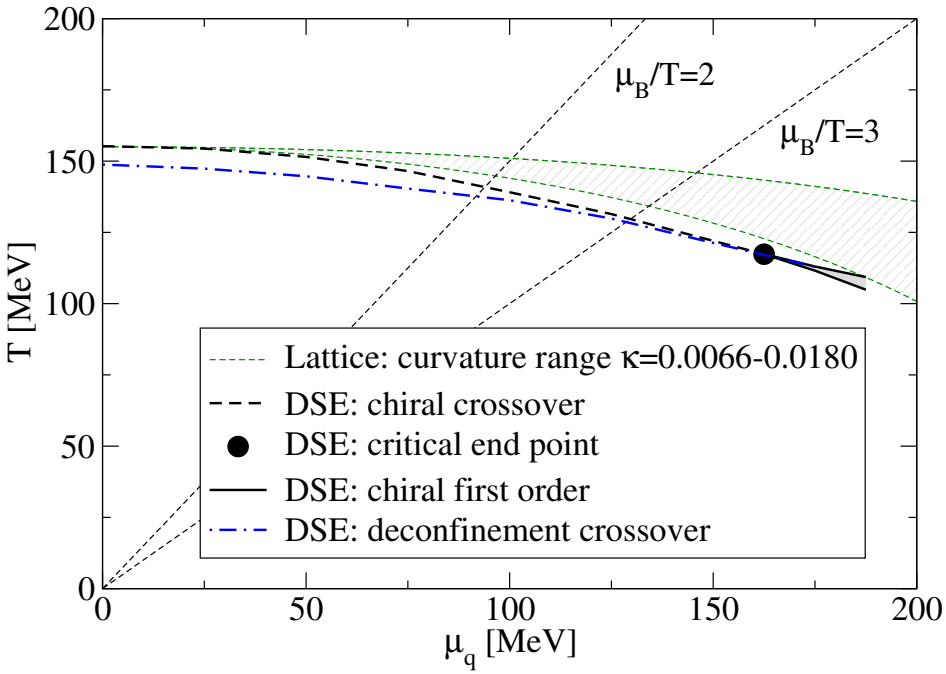


Challenges at non-zero density:

- Lattice: complex phase problem → complex Langevin (talk by B. Jäger), Lefschetz thimble, dual variables, ...
- Functional framework: truncations

Phases of QCD from Dyson-Schwinger equations

Phase diagram for 2+1 quark flavors:



[Fischer, Lücker, Welzbacher '14]

Similar conclusions from functional renormalization group, e.g., [Herbst, Pawlowski, Schaefer '13]

Currently dependence on models (fixed at $\mu = 0$).

Quantitative results

- Understanding of vacuum improved during the last few years.
- Push truncations to a similar level at $T, \mu > 0$ to reduce/eliminate model dependence.

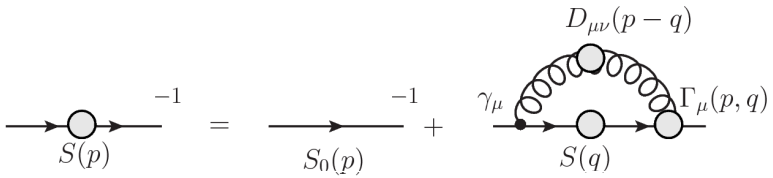
Outline

1. Functional equations
2. $T = 0$: Recent developments and modern truncations
3. $T > 0$: Three-point functions

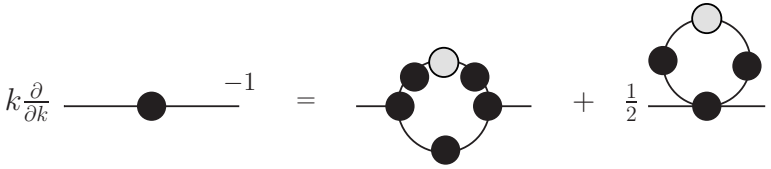
The family of functional equations

Coupled integro-differential/integral equations.

- Dyson-Schwinger equations: eqs. of motion for correlation functions



- Functional renormalization group: flow equations, RG scale k, regulator



- N-PI effective action

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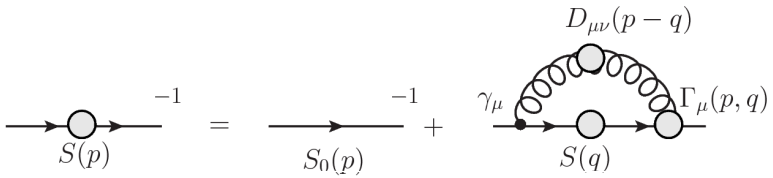

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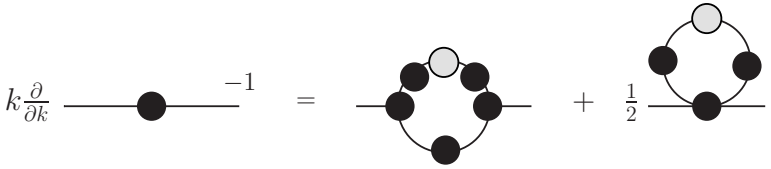

- N-PI effective action

Non-perturbative in the sense:

- Exact equations.
- No small coupling required.

In reality they cannot be solved exactly (with a few exceptions).

From Green functions to 'observables'

Functional equations are expressed in terms of propagators and vertices/ Green functions/correlation functions/n-point functions $\Gamma_{i_1 \dots i_n}$.

The effective action is the generating functional of 1PI Green functions.

$$\Gamma[\Phi] = \sum_{n=0}^{\infty} \frac{1}{n!} \Phi_1 \dots \Phi_n \Gamma_{i_1 \dots i_n}$$

$\longleftrightarrow$

The set of **all** Green functions describes the theory completely.

$$\Gamma_{ij} = \frac{\delta^2 \Gamma[\Phi]}{\delta \Phi_i \delta \Phi_j},$$

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Green functions $\rightarrow$ 'observables'?

Examples:

- Bound state equations $\rightarrow$ masses and properties of hadrons ($\rightarrow$ talks of G. Eichmann, V. Sauli)
- Analytic properties of Green functions $\rightarrow$ confinement
- Phases and transitions: (Pseudo-)Order parameters

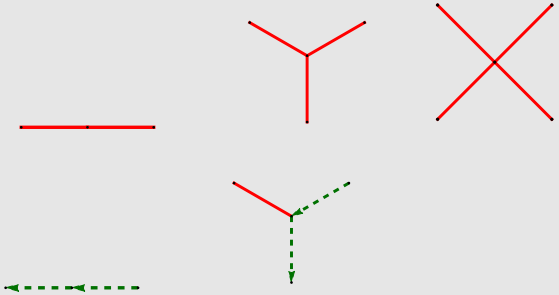
Landau gauge Yang-Mills theory

Gluonic sector of quantum chromodynamics: Yang-Mills theory

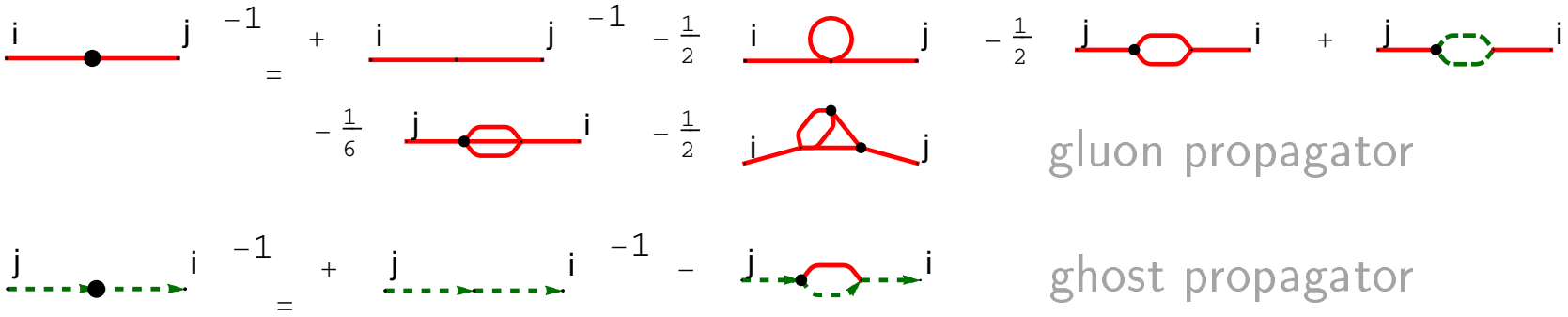
$$\mathcal{L} = \frac{1}{2} F^2 + \mathcal{L}_{gf} + \mathcal{L}_{gh}$$
$$F_{\mu\nu} = \partial_\mu \boldsymbol{A}_\nu - \partial_\nu \boldsymbol{A}_\mu + i\, g\, [\boldsymbol{A}_\mu, \boldsymbol{A}_\nu]$$

Landau gauge

- simplest one for functional equations
- $\partial_\mu \boldsymbol{A}_\mu = 0$: $\quad \mathcal{L}_{gf} = \frac{1}{2\xi} (\partial_\mu \boldsymbol{A}_\mu)^2, \quad \xi \to 0$
- requires ghost fields: $\quad \mathcal{L}_{gh} = \bar{\boldsymbol{c}} \left(-\Box + g\, \boldsymbol{A} \times \right) \boldsymbol{c}$



The tower of DSEs



gluon propagator

ghost propagator

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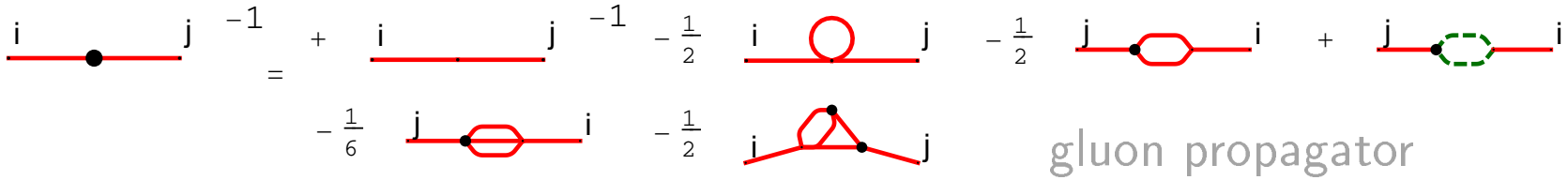

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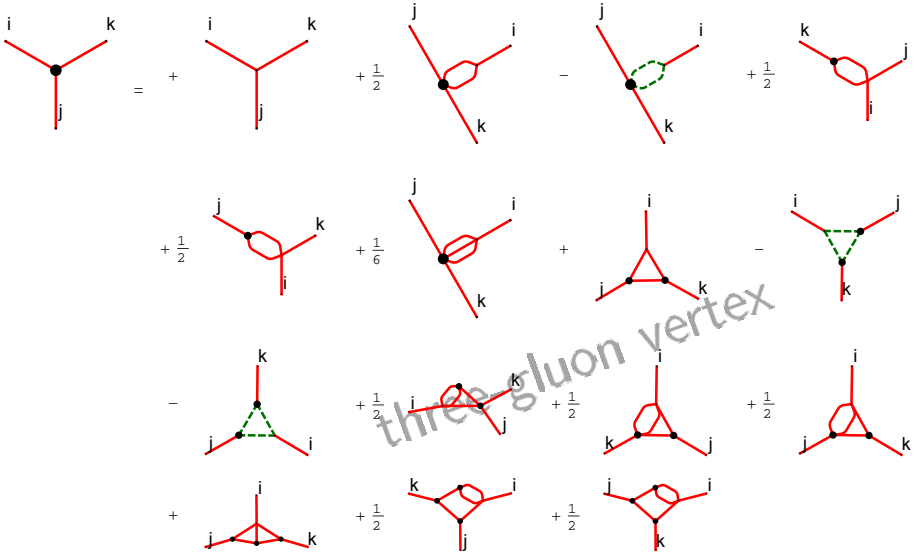
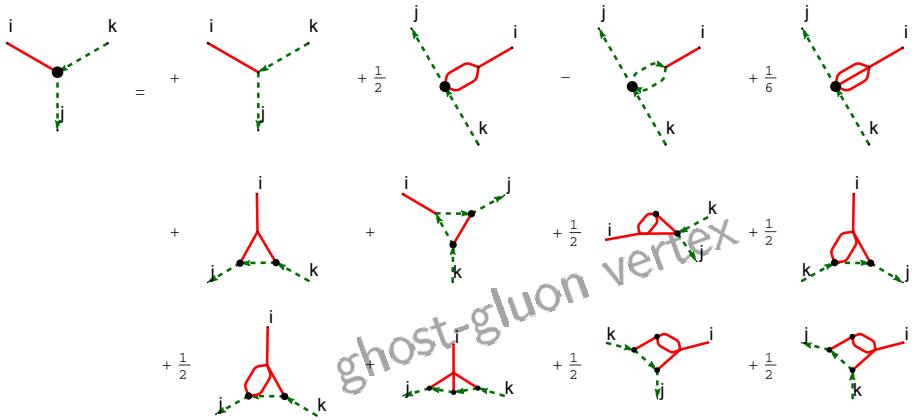
Self-consistence!





Infinitely many equations. In QCD, every n-point function depends on $(n+1)$- and possibly $(n+2)$-point functions.

Dealing with the equations

DoFun [Braun, MQH '11, http://tinyurl.com/dofun2]:

- *Mathematica* package for deriving functional equations
- Automatization!

CrasyDSE [Huber, Mitter '11, http://tinyurl.com/crasydse]:

- Automatic creation of kernels for *C++*
- *C++* framework for DSEs

→ Allows to treat large systems in a modular way.

Taming the equations

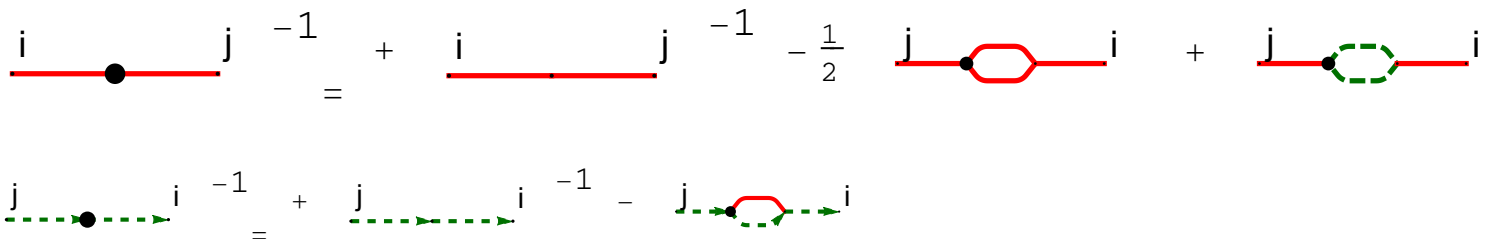
Keep most important parts!

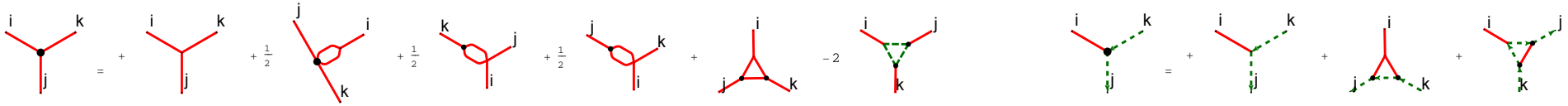
- Drop quantities
- Model quantities

Taming the equations

Keep most important parts! The art...

- Drop quantities
- Model quantities





Most important parts

- UV leading (perturbation theory)
- IR leading (analytic, lattice)

A note on truncations

No small expansion parameter. $\rightarrow$ Systematic expansion?

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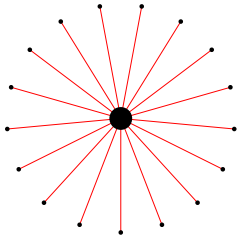
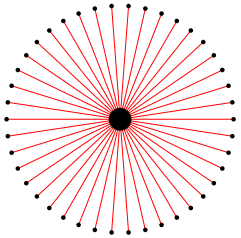
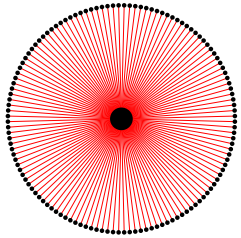
- Is nature so mean that the 17-, 42- and 128-point functions are important?







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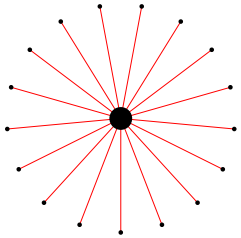
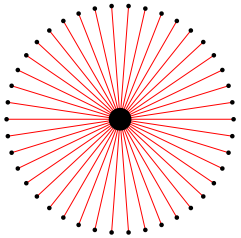
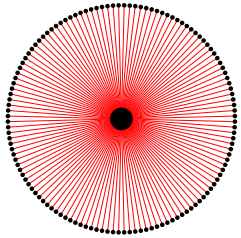

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  Lattice has a finite number of points but can describe physics quantitatively.
  ⇒ There must be an N so that the n-point function with $n > N$ is no longer important.

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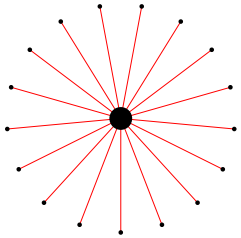
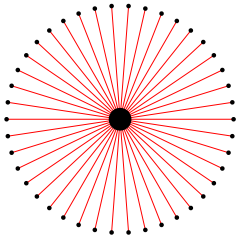
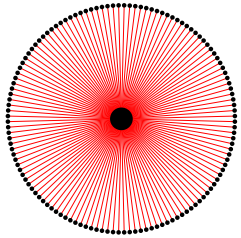

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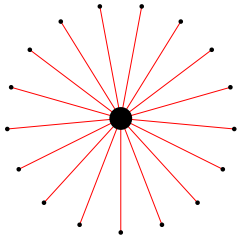
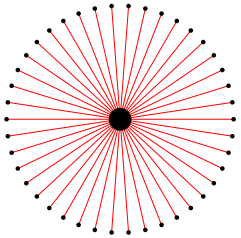
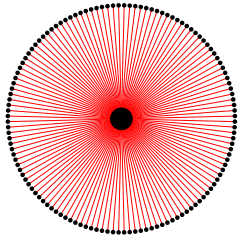

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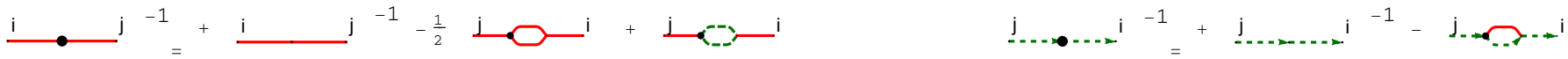
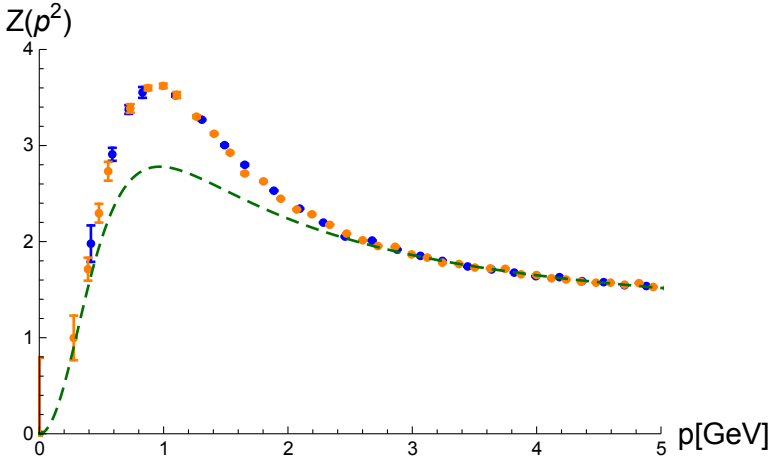
→ Let's try numerically. . .

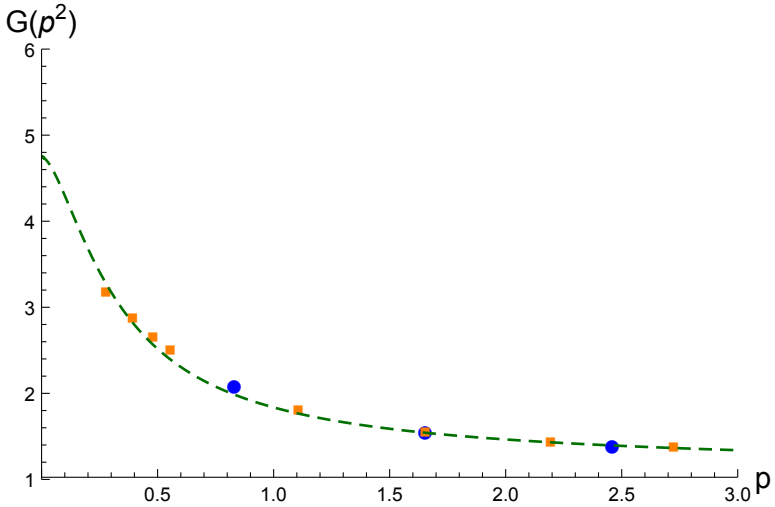
Effect of truncations: Propagators



Long-time standard truncation (1997 - 2012)

- No four-gluon vertex
- Ghost-gluon vertex: bare
- Three-gluon vertex: model





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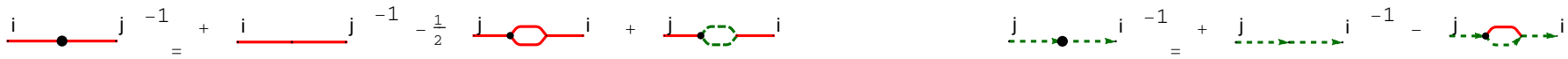

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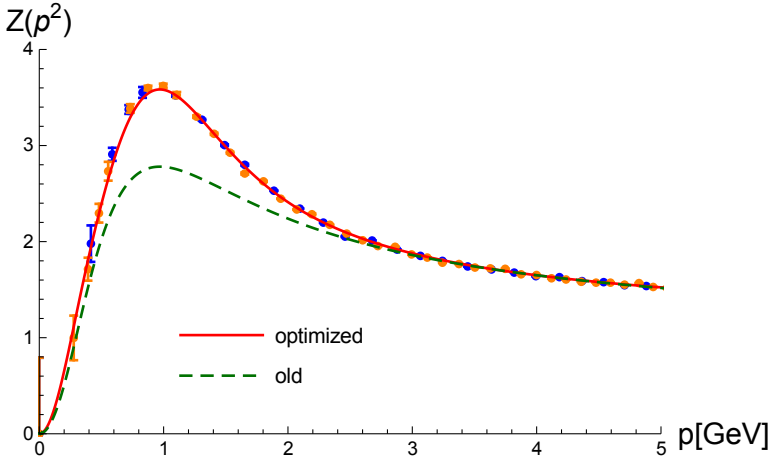

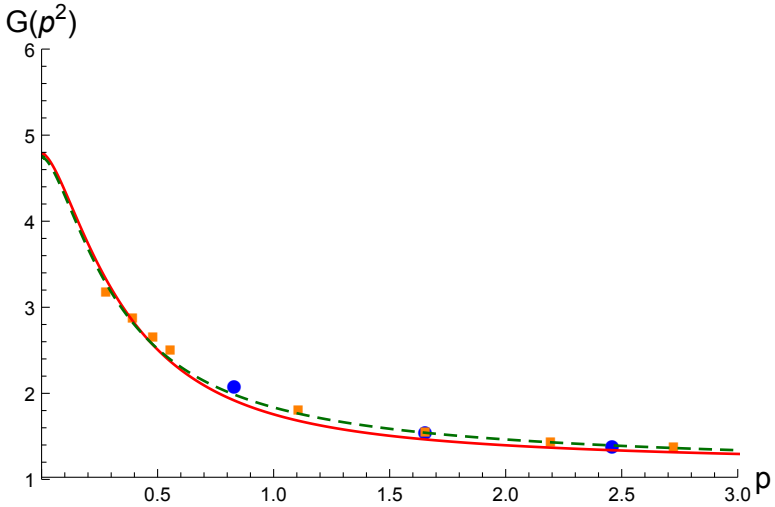

[MQH, von Smekal '12; lattice; Sternbeck '06]

→ Role of three-gluon vertex?

→ Use as input in other calculations.

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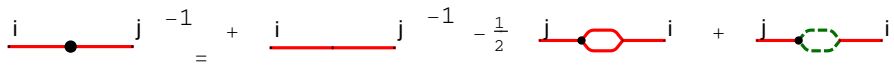

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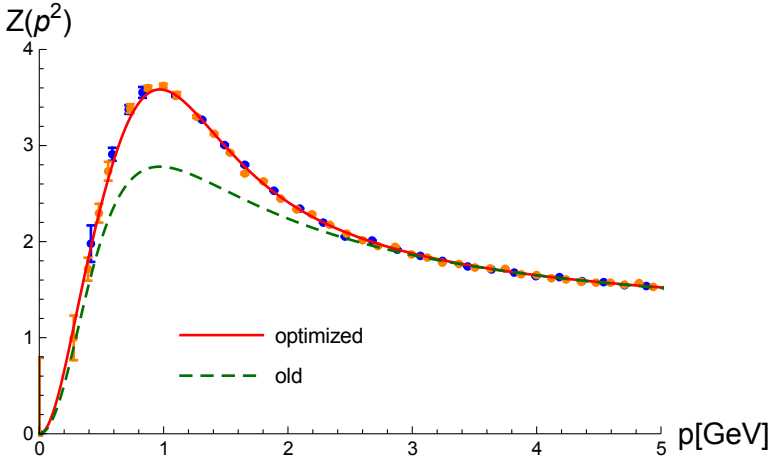
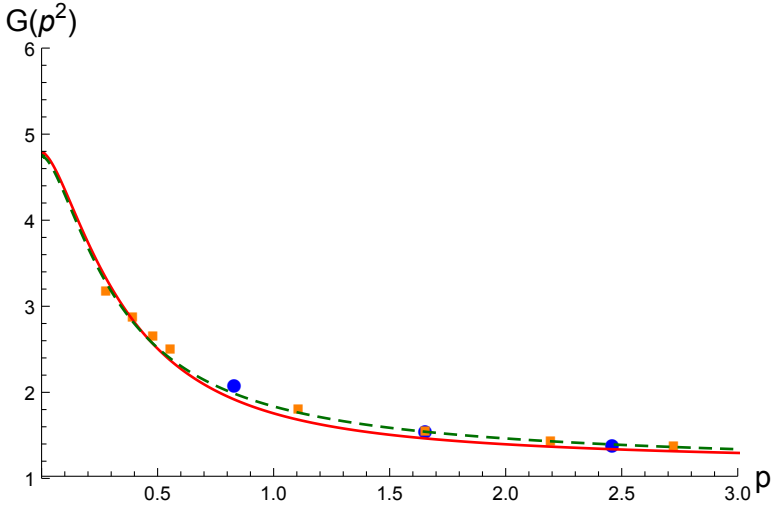
Missing strength in mid-momentum regime:
→ two-loop diagrams
[Blum, MQH, Mitter, von Smekal '14]





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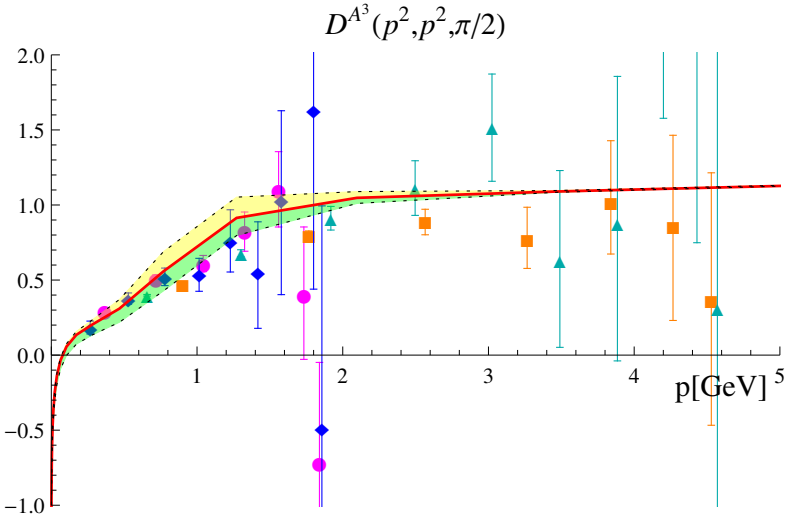
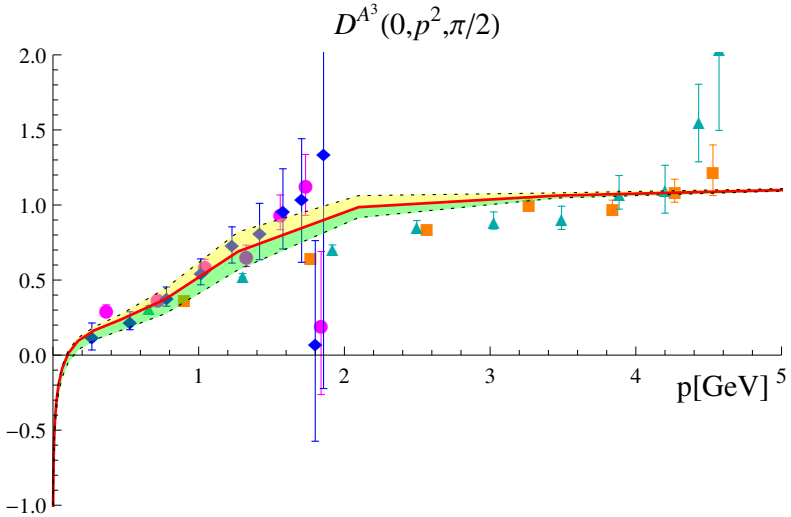
The three-gluon vertex

→ See talk of A. Blum.





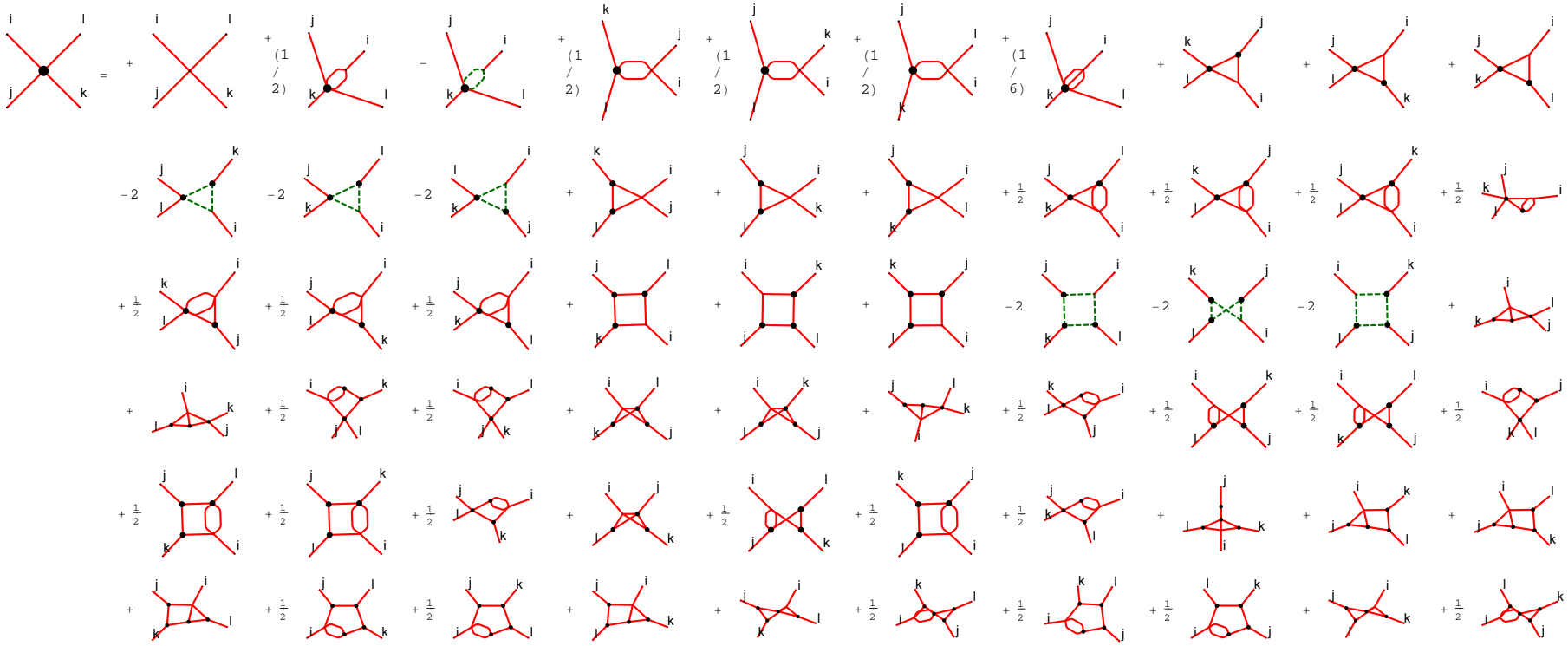
[Blum, MQH, Mitter, von Smekal '14; lattice: Cucchieri, Maas, Mendes '08]

→ Truncation reliable. Neglected terms, incl. two-loop diagrams, suppressed.

See also results by [Eichmann, Williams, Alkofer, Vujinovic '14], esp. other dressings, and [Peláez, Tissier, Wschebor '13].

Four-gluon vertex

- 20 one-loop, 39 two-loop diagrams



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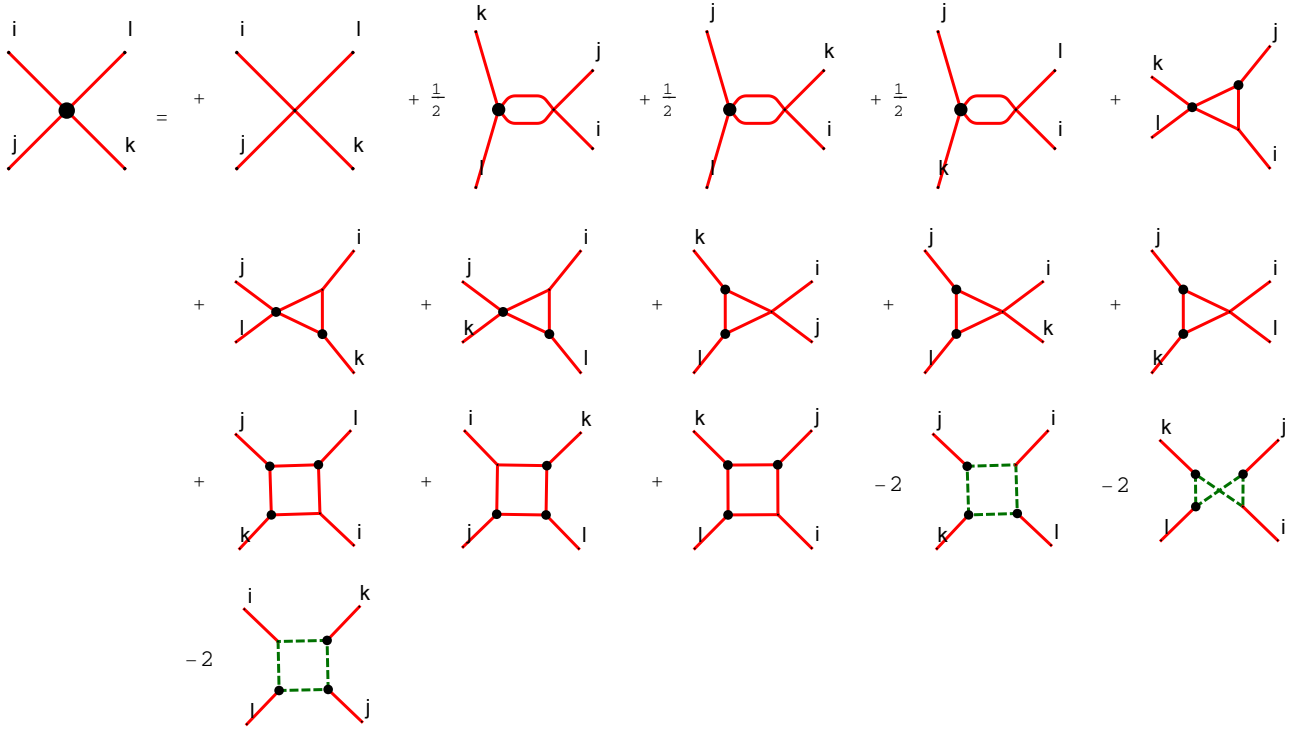

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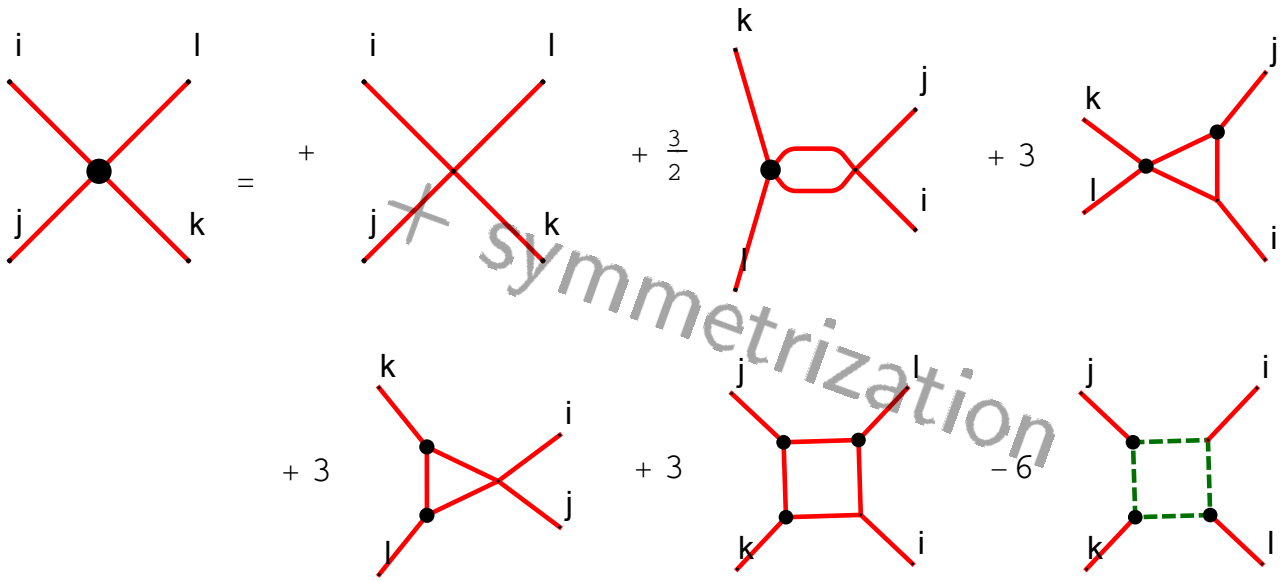

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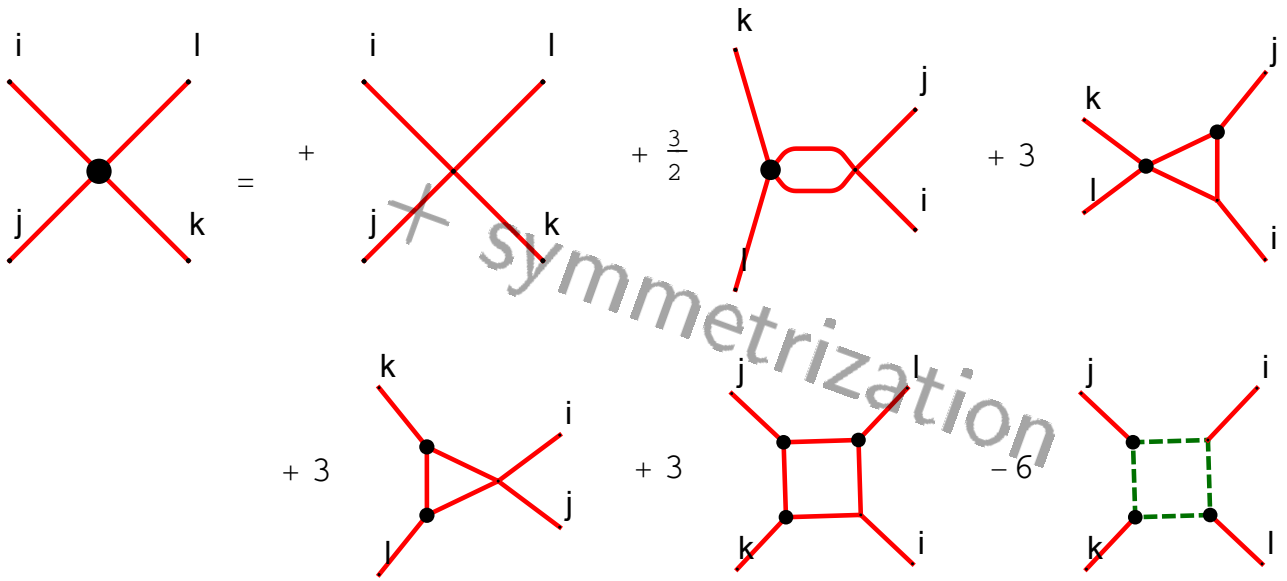

No model dependence! → 'Truncation closes.'

Four-gluon vertex: Results

	ghost and gluon propagators	four-gluon vertex
ext. variables:	1	6
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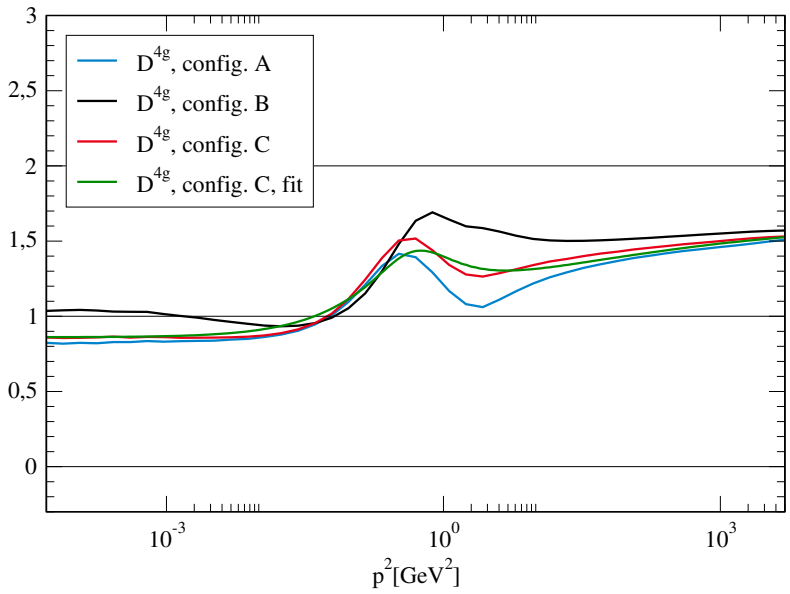

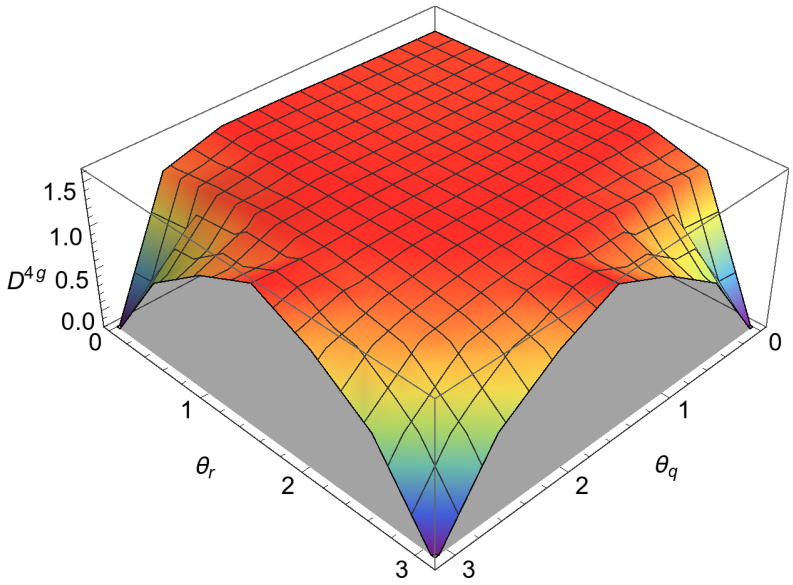

[Cyrol, MQH, von Smekal '14]

2-parameter fit:

$$D^{4g,\,\mathrm{dec}}_{\mathrm{model}}(p,\,q,\,r,\,s) = \left(a \tanh\left(b/\bar{p}^2\right) + 1\right) D^{4g}_{\mathrm{RG}}(p,\,q,\,r,\,s)$$

Beyond Landau gauge: Coulomb gauge

- Variational approach, trial ansatz for vacuum wave functional
  - Gaussian: qualitative picture (IR regime, perimeter law for 't Hooft loop, dual superconductor picture of confinement, deconfinement transition, ...) [Reinhardt et al. '04-'13]
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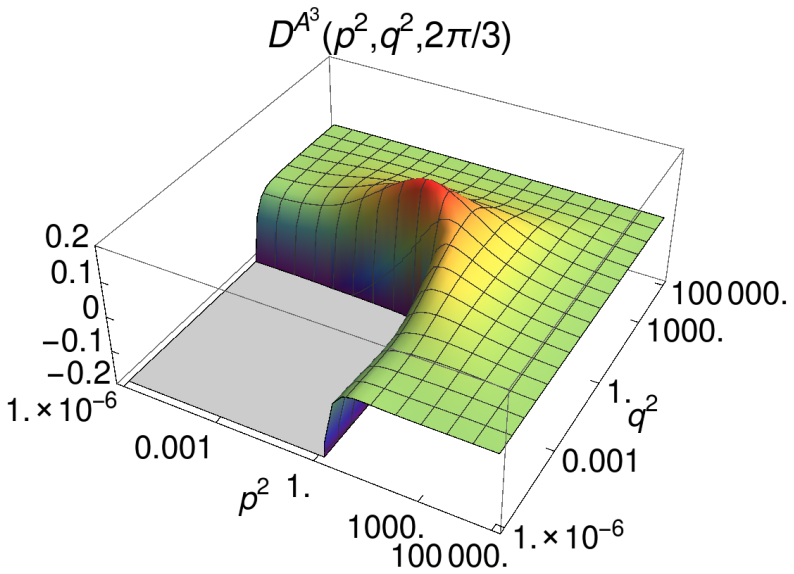


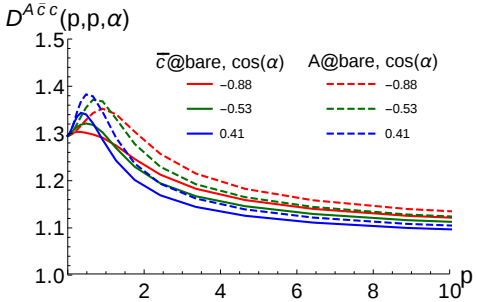
[MQH, Campagnari, Reinhardt '14]

Three-gluon vertex:

- Zero crossing
- IR divergent like p^{-3}

Ghost-gluon vertex:

- Different truncations quite similar



Beyond Landau gauge: Linear covariant gauges

- Gaussian distribution $e^{-\frac{1}{2\xi}(\partial A)^2}$ around Landau gauge in path integral
- Test of gauge (in)dependence of observables possible.
- Well-known Landau gauge is endpoint: $\xi = 0$
- Special choices convenient perturbatively, e.g., Feynman gauge $\xi = 1$; non-perturbatively no advantage

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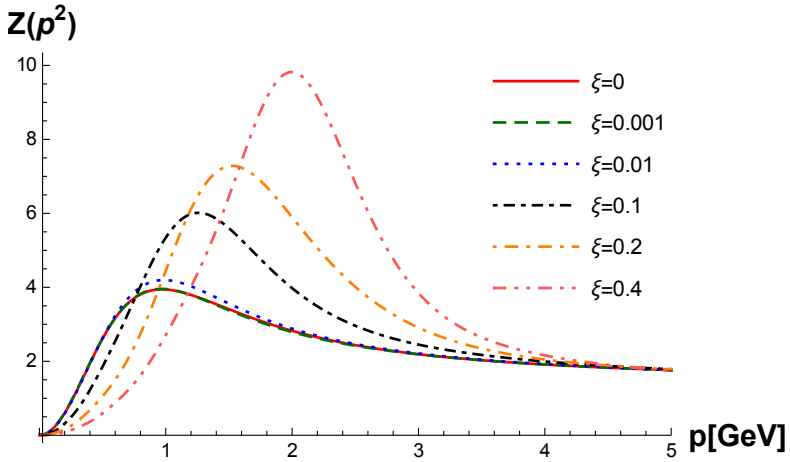
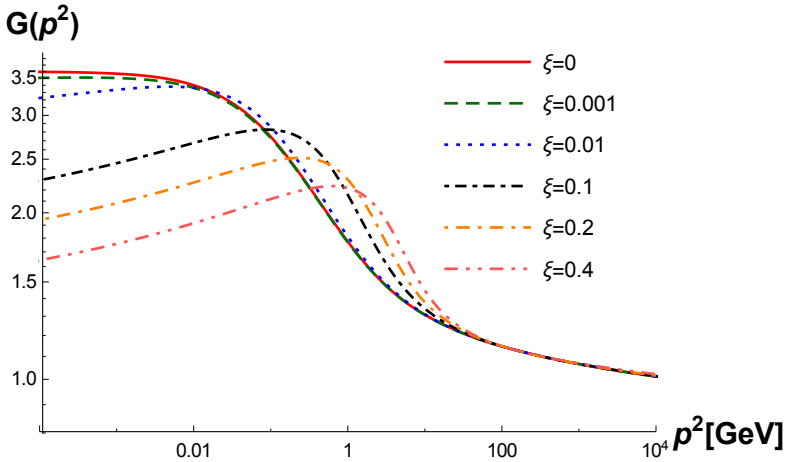




[MQH '15]

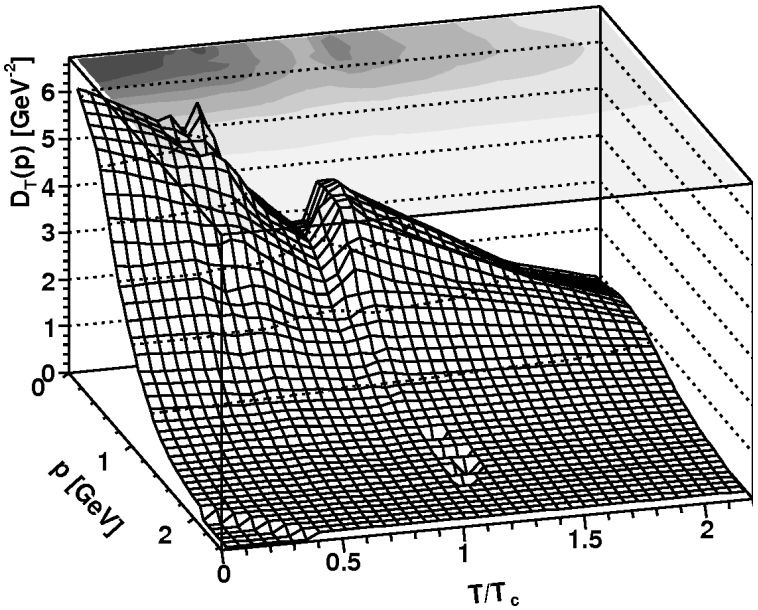
- Ghost vanishes logarithmically in IR, see also [Aguilar, Binosi, Papavassiliou '15].
- Gluon propagator IR finite.

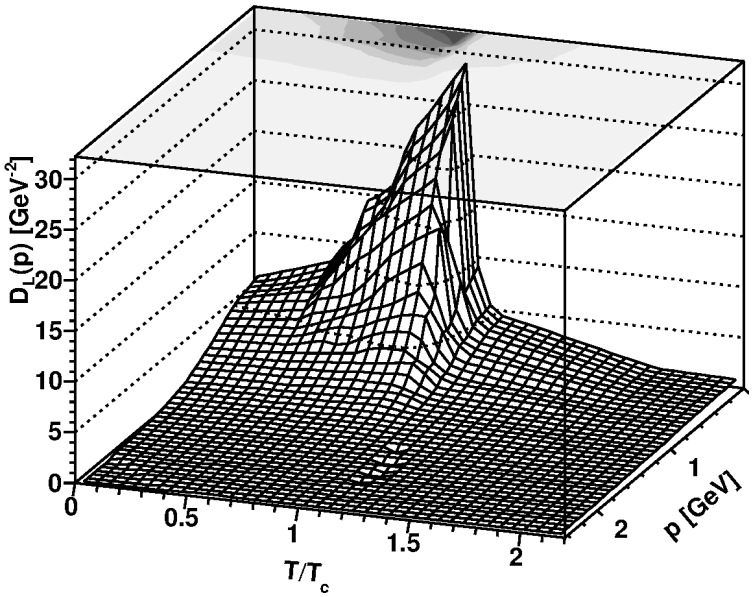
$$T > 0$$

Gluon propagators

Chromomagnetic and -electric gluons from the lattice [Fischer, Maas, Müller '10]:

Lattice data:





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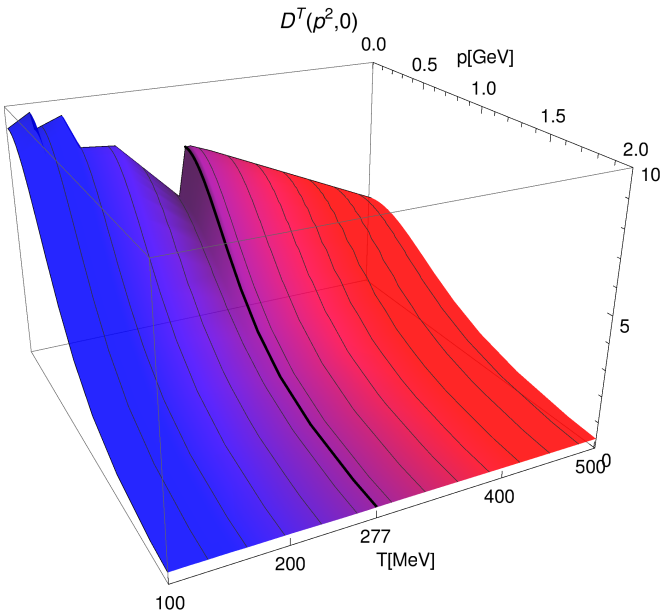
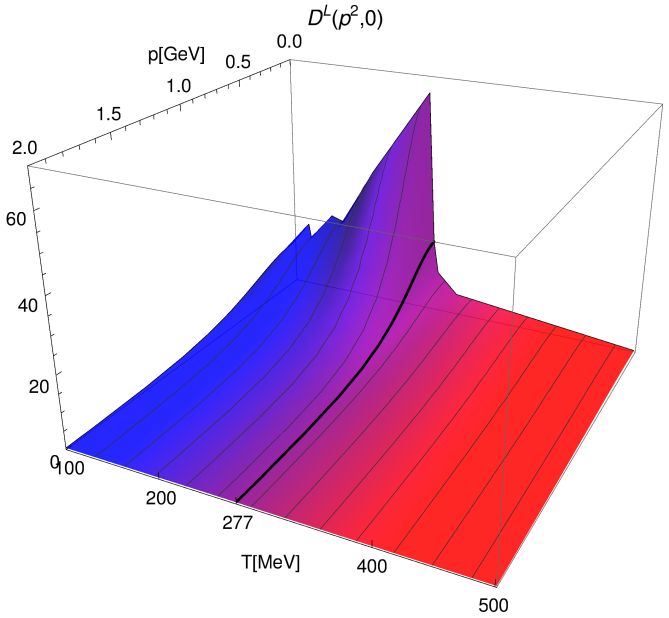


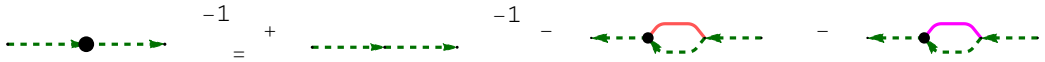


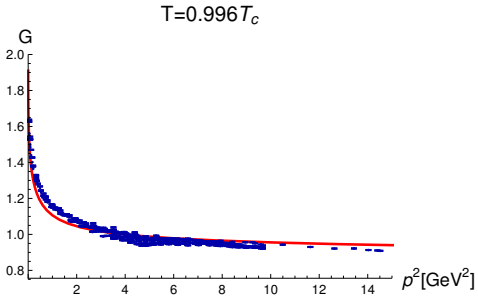
Input for DSEs to calculate quantities difficult for lattice

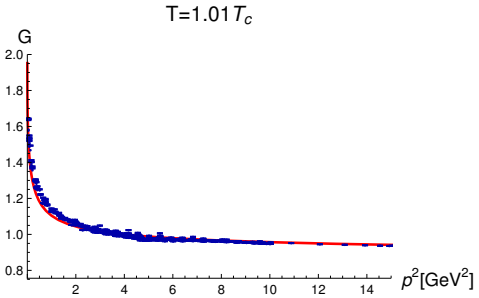
- Gluon DSE inconveniently difficult: spurious divergences, two-loop diagrams
- No truncation effects for input!
- Lattice artifacts? $\rightarrow$ [Cucchieri, Mendes '11]

Simple test: Ghost propagator

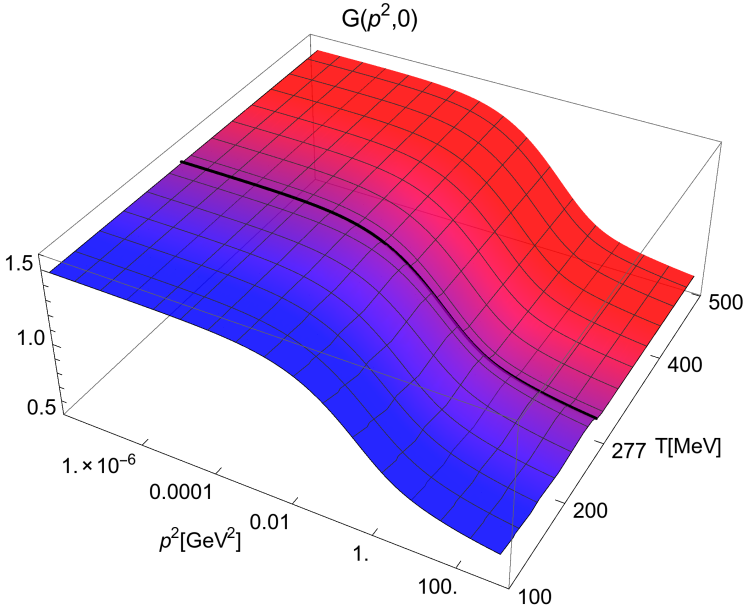
Ghost dressing $G(p^2)$ from DSE [MQH, von Smekal '13]:







[Fischer, Maas, Müller '10]



Propagators from other sources:

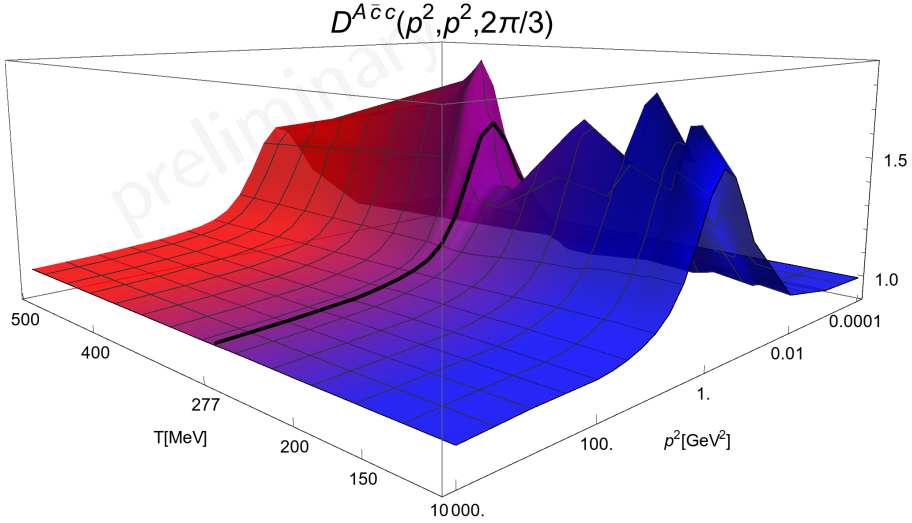
Lattice: [Fischer, Maas, Müller '10, Cucchieri, Mendes '11, Silva, Oliveira, Bicudo, Cardoso '13]

FRG: [Fister, Pawlowski '11]

Massive Yang-Mills: [Reinosa, Serreau, Tissier, Wschebor '13]

Three-point warm-up: Ghost-gluon vertex

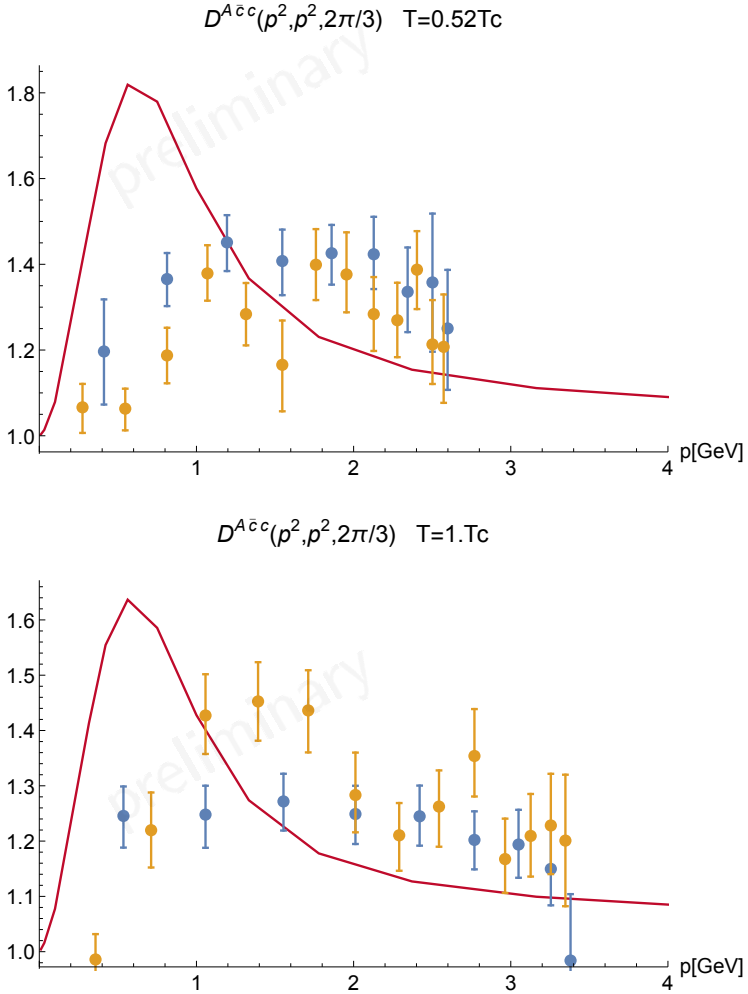
DSE calculation: self-consistent solution of truncated DSE, zeroth Matsubara frequency only

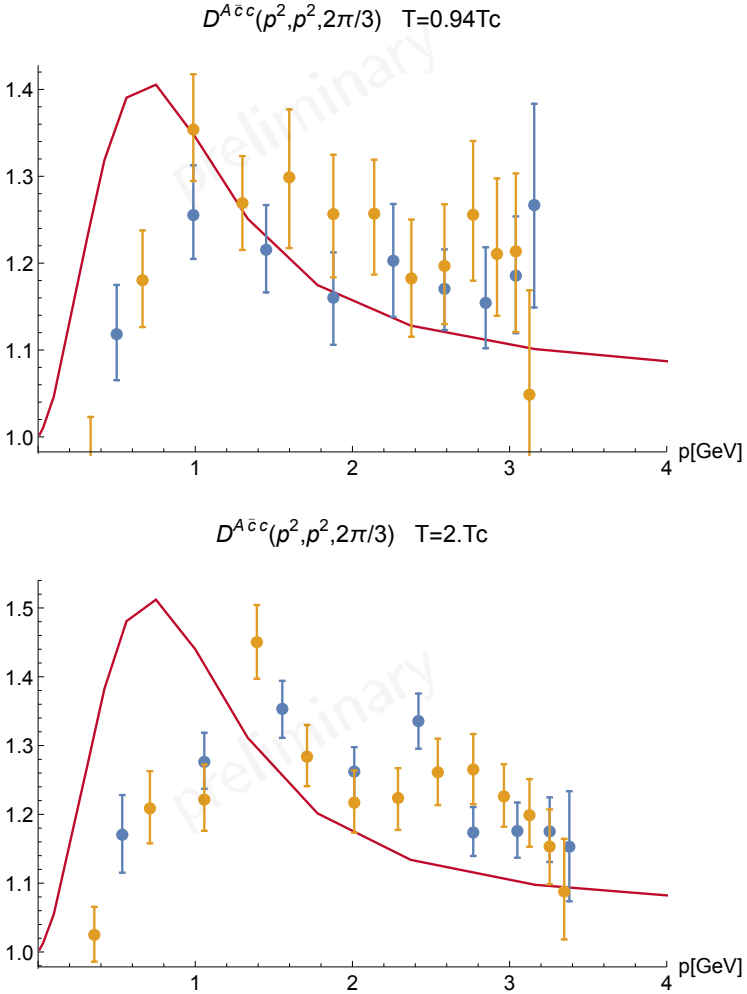


- Vertices quite expensive on lattice.
- Full momentum dependence from functional equations.

Vertex from FRG: [Fister, Pawlowski '11]

Ghost-gluon vertex: Continuum and lattice





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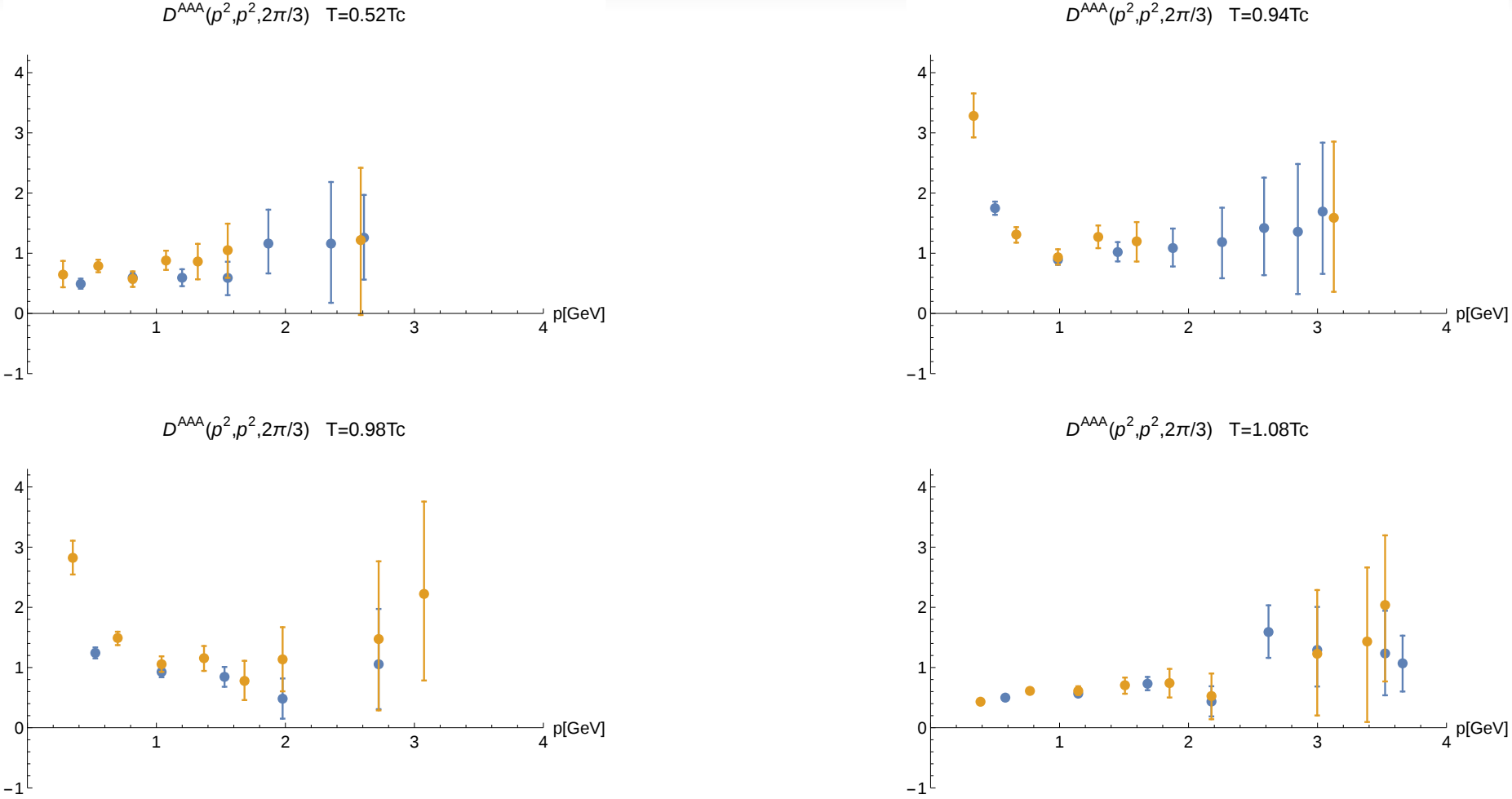

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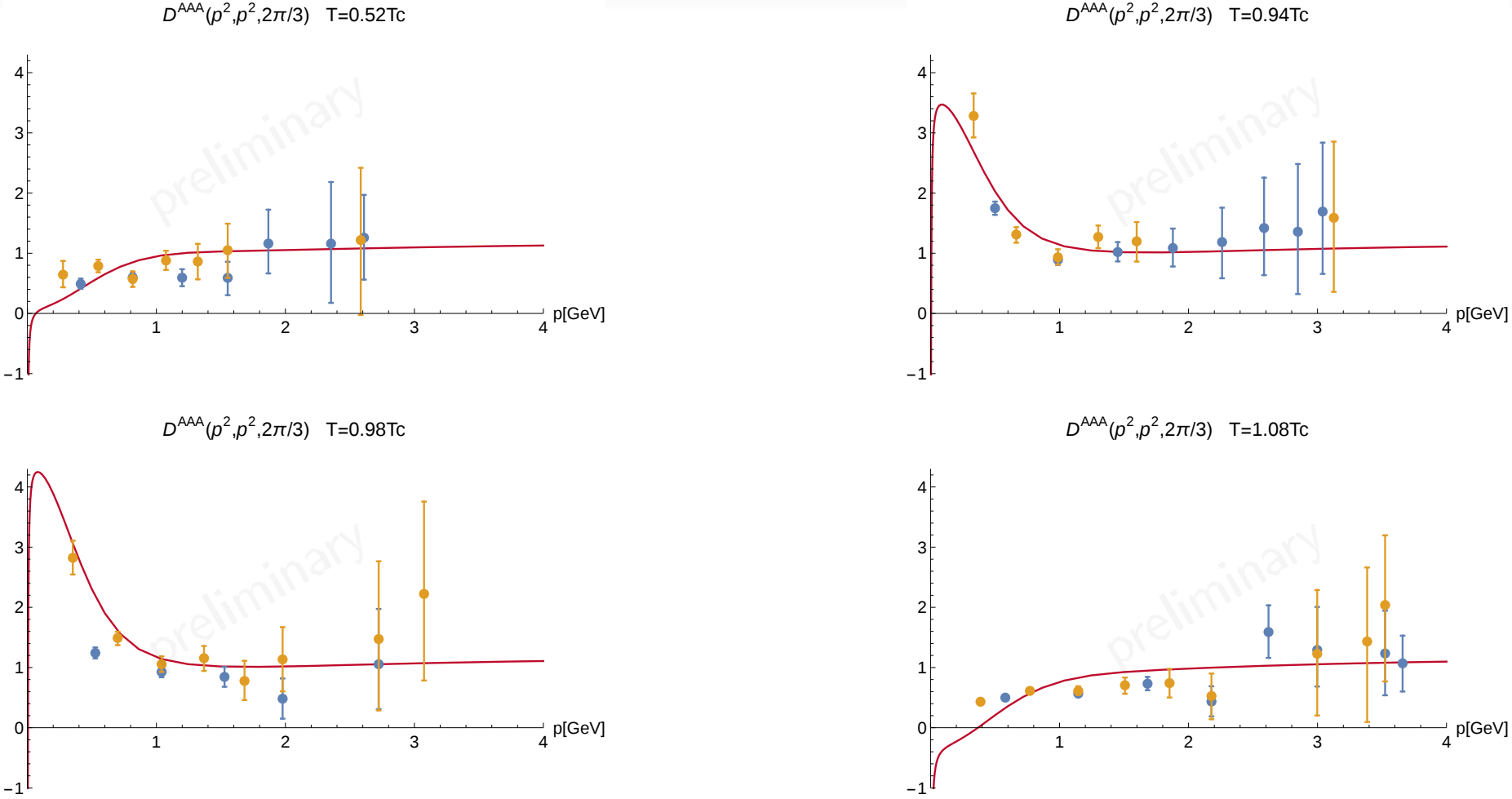

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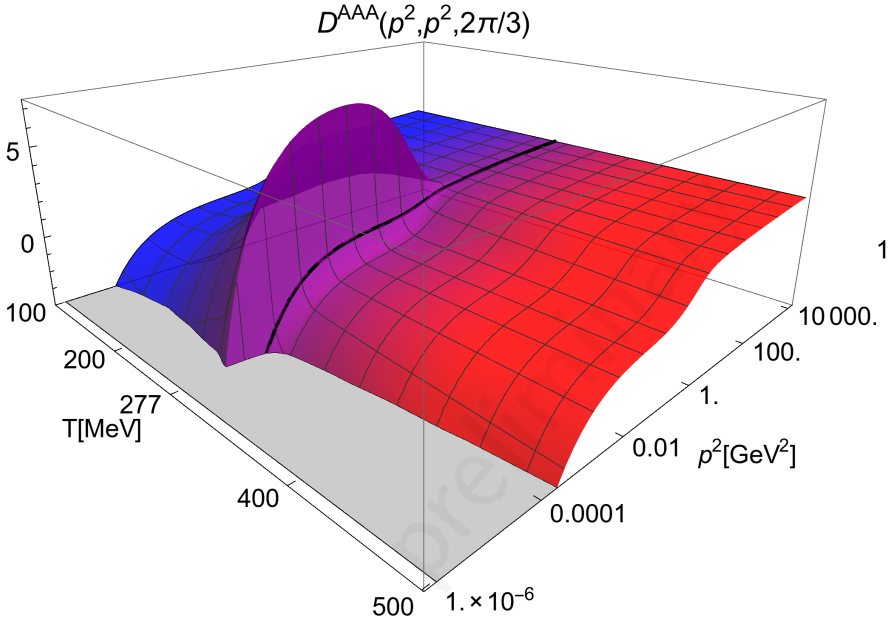
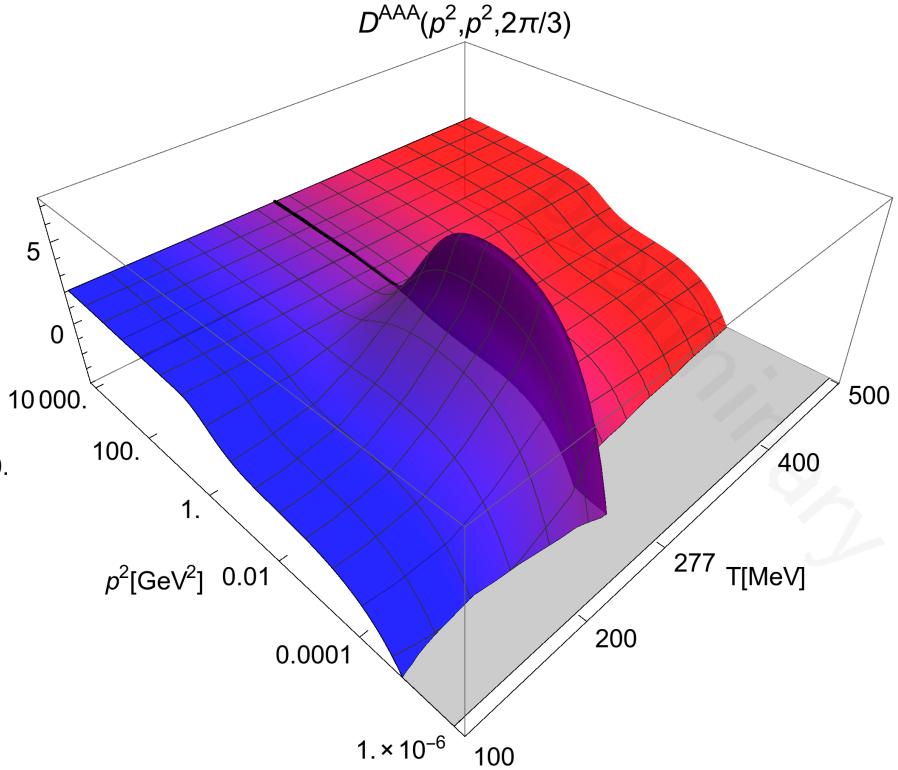
Lattice volume artifacts inherited from lattice gluon propagators in functional calculation!

Three-gluon vertex

DSE calculation: semi-perturbative approximation (first iteration only)





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Intermediately: model dependence

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