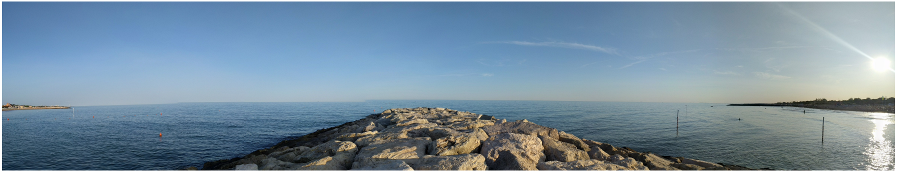
On truncation effects in Dyson-Schwinger equations



Markus Q. Huber

Institute of Physics, University of Graz

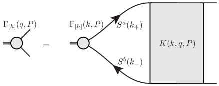
Functional Methods in Hadron and Nuclear Physics, Trento, Italy

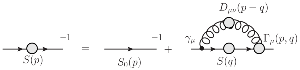
Aug. 23, 2017

FWF Der Wissenschaftsfonds. UNI GRAZ NAWI Graz Natural Sciences INSTITUT FÜR PHYSIK

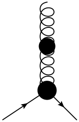
Where Yang-Mills theory matters

- Successful (functional) bottom-up approaches in QCD:
  Model the Yang-Mills part





  Example: Maris-Tandy-like interaction for quark-gluon interaction incl. gluon propagator


$$= i\, g\, T^a \gamma_\mu \mathcal{G}(k^2)$$

- Self-contained calculations in QCD necessarily contain Yang-Mills part.
- Glueballs

Overview

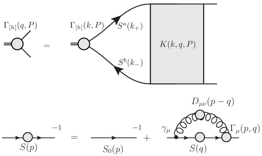
- Where the Yang-Mills part enters
- Some details on the framework
- Testing truncations:
  - Hierarchy of diagrams (testing in 3 dimensions)
  - Extensions of truncations in 4 dimensions:

    $\rightarrow$ Two-loop terms

    $\rightarrow$ Non-primitively divergent correlation functions

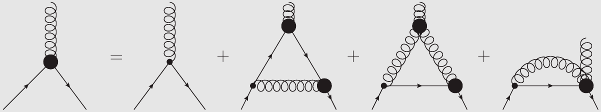
Where Yang-Mills theory enters

Widely used truncation: Rainbow + ladder + variant of Maris-Tandy interaction



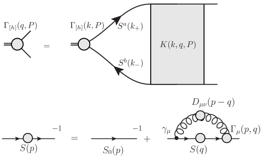
How to reduce model dependence

- Improve kernel K
- Use explicit gluon propagator + quark-gluon vertex



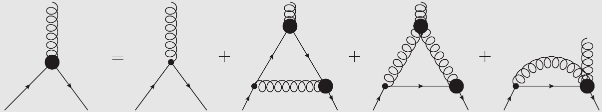
Where Yang-Mills theory enters

Widely used truncation: Rainbow + ladder + variant of Maris-Tandy interaction



How to reduce model dependence

- Improve kernel K
- Use explicit gluon propagator + quark-gluon vertex



$\longrightarrow$ We need full control over the gluonic sector.

- Gluon propagator
- Three-gluon vertex
- ...?

Landau gauge QCD

$$\mathcal{L} = \bar{q}(-\not{D} + m)q + \frac{1}{2}F^2 + \mathcal{L}_{gf} + \mathcal{L}_{gh}$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + i\,g\,[A_\mu, A_\nu]$$

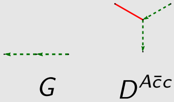
Landau gauge QCD

$$\mathcal{L} = \bar{q}(-\not{D} + m)q + \frac{1}{2}F^2 + \mathcal{L}_{gf} + \mathcal{L}_{gh}$$

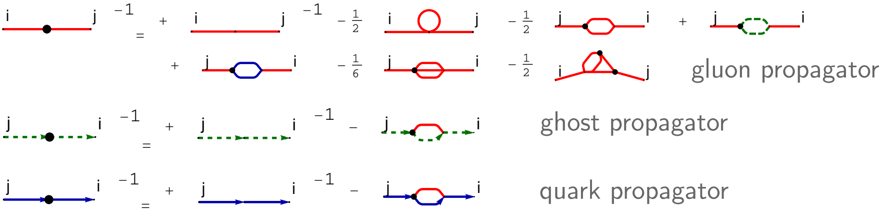
$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + i\,g\,[A_\mu, A_\nu]$$

Landau gauge

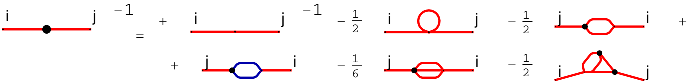
- simplest one for functional equations
- $\partial_\mu A_\mu = 0$: $\quad \mathcal{L}_{gf} = \frac{1}{2\xi}(\partial_\mu A_\mu)^2, \quad \xi \to 0$
- requires ghost fields: $\quad \mathcal{L}_{gh} = \bar{c}\,(-\Box + g\,A\,\times)\,c$



The tower of DSEs

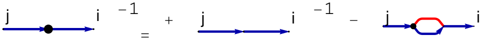


The tower of DSEs

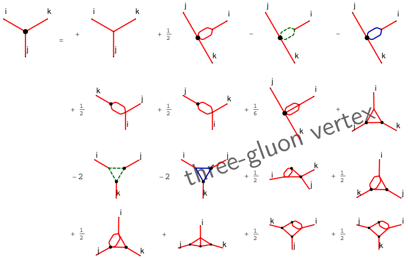


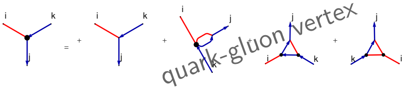
gluon propagator

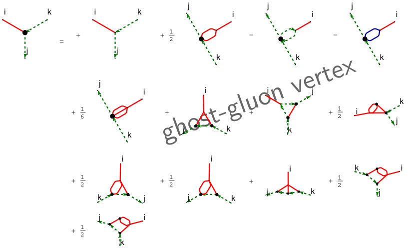
ghost propagator



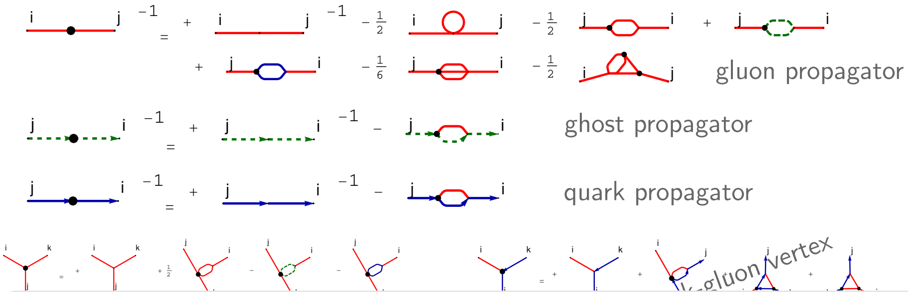
quark propagator





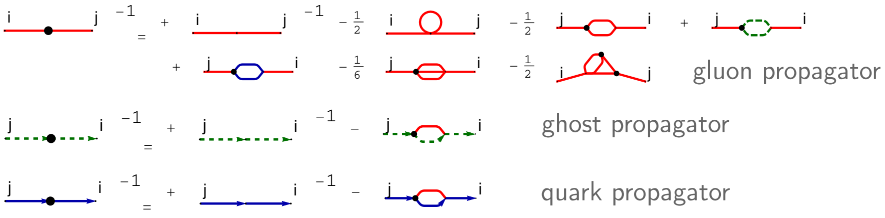


The tower of DSEs



Infinitely many equations. In QCD, every n-point function depends on $(n+1)$- and possibly $(n+2)$-point functions.

The tower of DSEs



Infinitely many equations. In QCD, every n-point function depends on $(n+1)$- and possibly $(n+2)$-point functions.

Is it possible to find and solve a truncation with all relevant contributions?

Questions about truncations

- Influence of higher correlation functions?
- Hierarchy of diagrams/correlation functions?
- Model dependence $\leftrightarrow$ Self-contained truncation?
- How to realize resummation?
- Equivalence between different functional methods?

$d = 3$ Yang-Mills theory as testing ground

Advantages:

- UV finite: no renormalization, no anomalous running
- Spurious divergences easier to handle
- UV behavior 'easier': $\propto \dfrac{g^2}{p}$ instead of resummed logarithm

$\rightarrow$ Many complications from $d = 4$ absent.
$\rightarrow$ Disentanglement of UV easier.

$\Rightarrow$ 'Cleaner' system $\rightarrow$ Focus on truncation effects.

$d = 3$ Yang-Mills theory as testing ground

Advantages:

- UV finite: no renormalization, no anomalous running
- Spurious divergences easier to handle
- UV behavior 'easier': $\propto \dfrac{g^2}{p}$ instead of resummed logarithm

$\rightarrow$ Many complications from $d = 4$ absent.
$\rightarrow$ Disentanglement of UV easier.
$\Rightarrow$ 'Cleaner' system $\rightarrow$ Focus on truncation effects.

Historically interesting because cheaper on the lattice $\rightarrow$ easier to reach the IR.

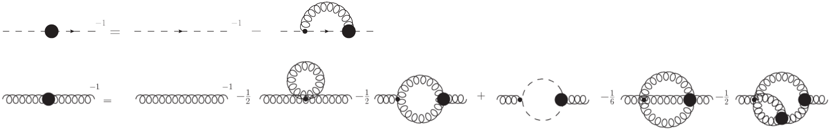
Numerically not cheaper for functional equations of 2- and 3-point functions.

Continuum results:

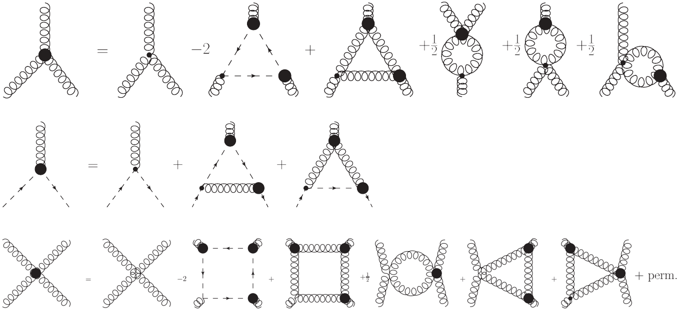
- Coupled propagator DSEs: [Maas, Wambach, Grüter, Alkofer '04]
- (R)GZ: [Dudal, Gracey, Sorella, Vandersickel, Verschelde '08]
- DSEs of PT-BFM: [Aguilar, Binosi, Papavassiliou '10]
- YM + mass term: [Tissier, Wschebor '10, '11]

Dyson-Schwinger equations: Truncation

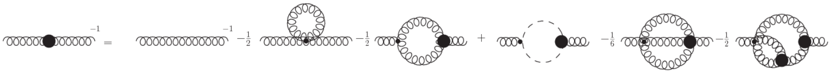
Untruncated propagators:

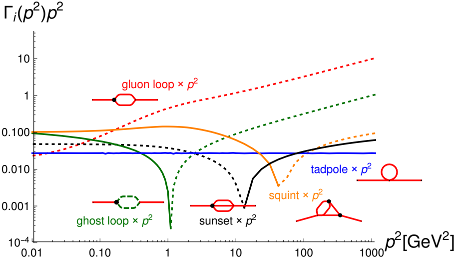


Truncated vertices:



Gluon propagator: Single diagrams



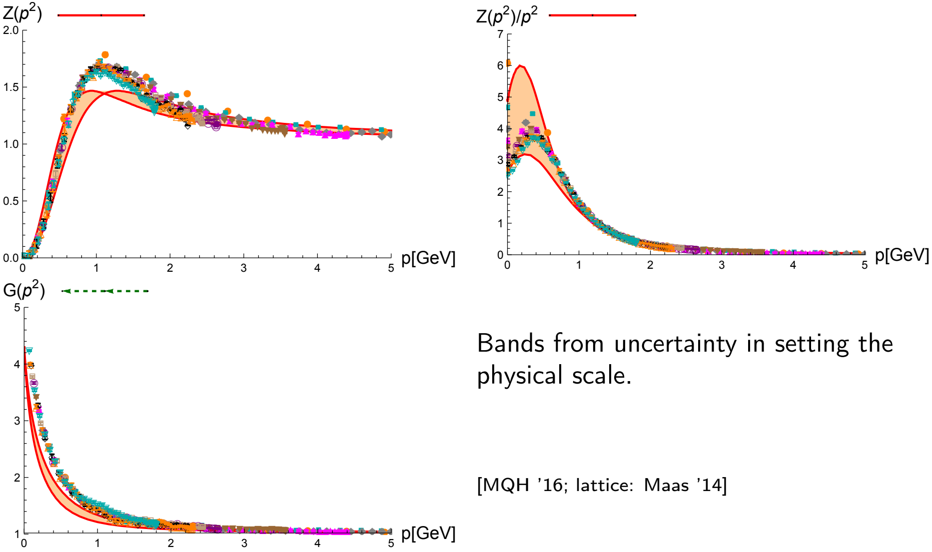


[MQH '16]

Clear hierarchies identified:

- UV: as expected perturbatively
- non-perturbative: squint important, sunset small
  (d=4: [Mader, Alkofer '13; Meyers, Swanson '14])

Results: Propagators

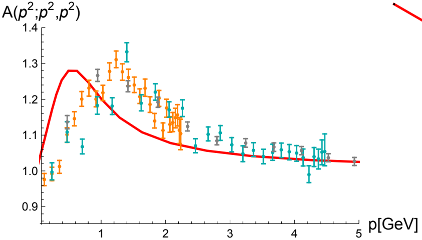


Bands from uncertainty in setting the physical scale.

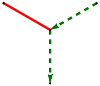
[MQH '16; lattice: Maas '14]

Comparison of three-point functions with lattice results

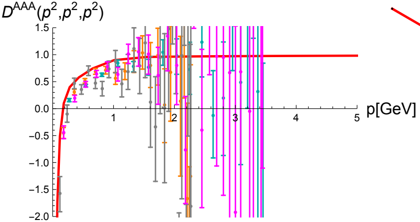
Dressings:



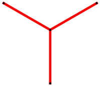
[MQH '16; lattice: Maas, unpublished]



- Deviation from tree-level in midmomentum 'small'
- Maximum position shifted.
- Bump height ok.



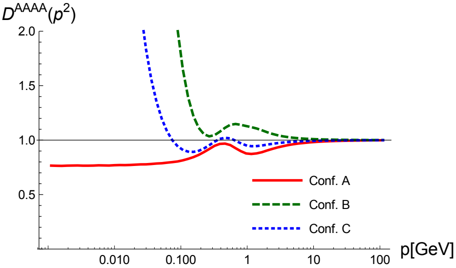
[MQH '16; lattice: Cucchieri, Maas, Mendes



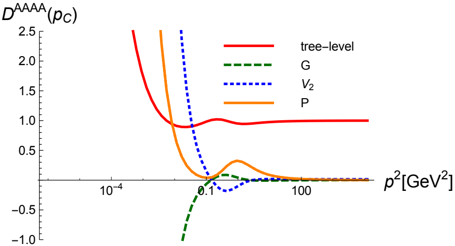
- Close to tree-level above 1 GeV
- Good agreement with lattice data.
- Linear IR divergence [Pelaez, Tissier, Wschebor '13; Aguilar et al. '14]

Four-gluon vertex

Different momentum configurations:



Different projections:

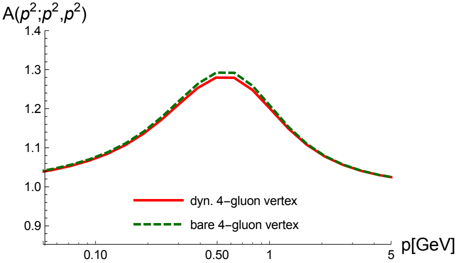


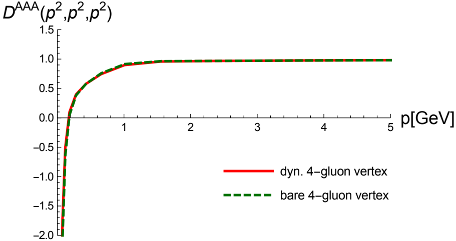
Four-gluon vertex:

- Close to tree-level down to 1 GeV

$\rightarrow$ Corrections small individually?

Influence of four-gluon vertex on three-point functions

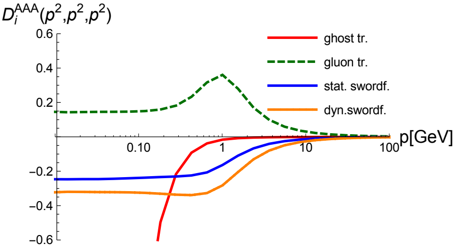




- Influence of four-gluon vertex small.

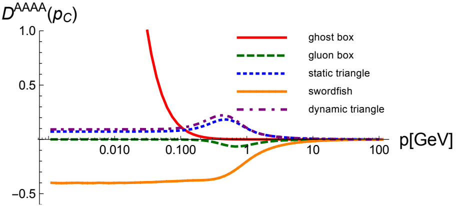
Cancellations in gluonic vertices

Three-gluon vertex:



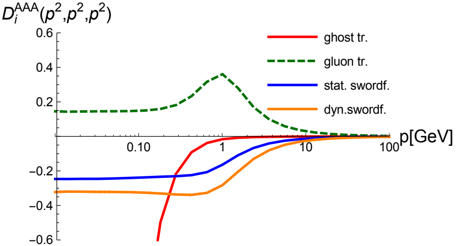
- Individual contributions large.
- Sum is small!

Four-gluon vertex:



Cancellations in gluonic vertices

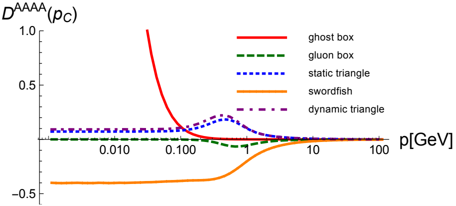
Three-gluon vertex:



- Individual contributions large.
- Sum is small!

$\Downarrow$

Four-gluon vertex:



Higher contributions:

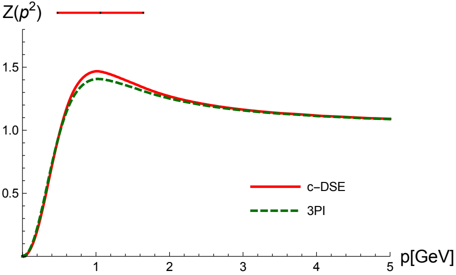
- Higher vertices close to 'tree-level'? $\rightarrow$ Small.
- If pattern changes (higher vertices large): cancellations required.

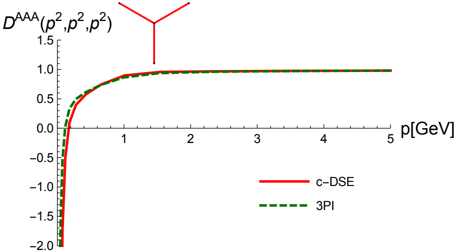
Solution from the 3PI effective action

Different set of functional equations:
equations of motion from 3PI effective action (at three-loop level)

Solution from the 3PI effective action

Different set of functional equations:
equations of motion from 3PI effective action (at three-loop level)





$\rightarrow$ Very similar results.

[MQH '16]

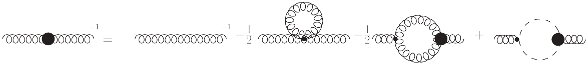
Summary about three dimensions

- Hierarchy of correlation functions and diagrams
- Cancellations
- Some degree of stability (but no complete list of checks done) when
  - varying *system* of equations.
  - varying *equations* of system.
- Discrepancies with lattice results:
  - Nonperturbative gauge fixing?
  - Incomplete tensor bases for some vertices?
  - Missing diagrams for vertices?
  - Lattice systematics?

UV behavior of the gluon propagator

Resummed one-loop order: anomalous dimension $\gamma = -13/22$

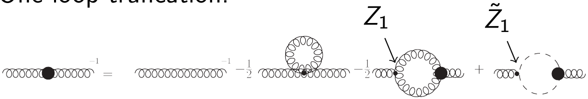
One-loop truncation:



UV behavior of the gluon propagator

Resummed one-loop order: anomalous dimension $\gamma = -13/22$

One-loop truncation:



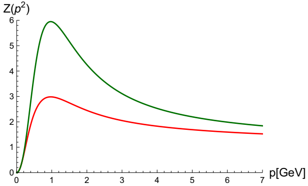
Self-consistent solution puts constraints on UV behavior of vertices [von Smekal, Hauck, Alkofer '97]:

- Ghost-gluon vertex: $\sim$ *const.* $\to \checkmark$
- Three-gluon vertex: $\propto (\log p)^{17/22}$
  Anomalous dimension $\gamma_{3g} = 17/44 \to$ ☹
  Solutions: $Z_1 \to Z_1(p^2) \leftrightarrow$ modified three-gluon vertex model [von Smekal, Hauck, Alkofer '97; Fischer, Alkofer '02]

Truncation artifact!

Resummed behavior

- Resolving the UV behavior within this truncation leads to an additional parameter dependence $\rightarrow$ part of the model
  Extreme example:



- Study for three-gluon vertex: [Eichmann, Williams, Alkofer, Vujinovic '14]
- However, correct UV behavior is required for self-consistency.

One-loop resummation

One-loop anomalous dimension

Origin in resummation of higher order diagrams.

$$\left(1+\frac{\alpha(s)11N_c}{12\pi}\ln\frac{p^2}{s}\right)^\gamma = 1 + c_1\, g^2 \ln p^2 + c_2\, g^4 \ln^2 p^2 + \mathcal{O}(g^6)$$

- $\mathcal{O}(g^2)$: One-loop diagrams
- $\mathcal{O}(g^4)$: Iterated one-loop diagrams, squint (*not* sunset)

Resummed behavior

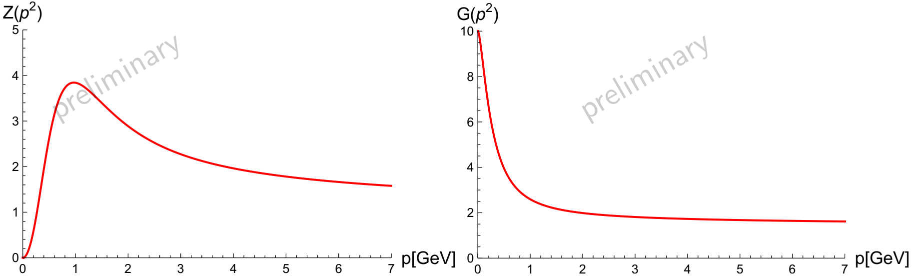
Minimal requirements to obtain one-loop resummed behavior:

- Squint diagram
- Proper models for three-point functions (with correct anom. dimensions)
- Correct renormalization

Resummed behavior

Minimal requirements to obtain one-loop resummed behavior:

- Squint diagram
- Proper models for three-point functions (with correct anom. dimensions)
- Correct renormalization



[propagator eqs. full, 3-point models, bare 4-gluon vertex]

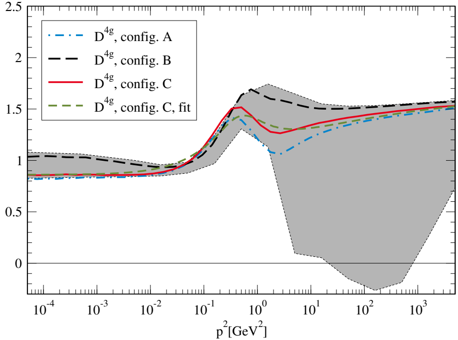
- Resummed behavior is recovered.

Four-gluon vertex

Full calculation with fixed input:

[Cyrol, MQH, von Smekal '14]

Computationally expensive!

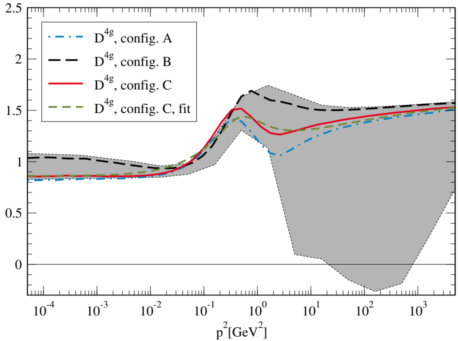


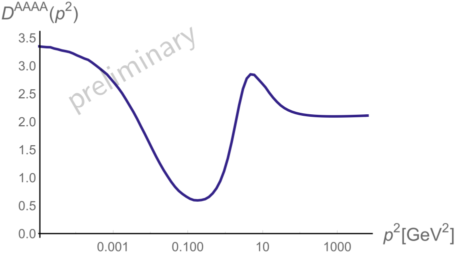
Four-gluon vertex

Full calculation with fixed input:

[Cyrol, MQH, von Smekal '14]

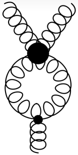
Computationally expensive!





For dynamic inclusion: Resort to a one-momentum approximation (symmetric point).

Effect of four-gluon vertex



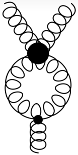
In three-gluon vertex DSE:

Important for convergence within current truncations in $d = 4$

[Blum, MQH, Mitter, von Smekal '14; Eichmann, Williams, Alkofer, Vujinovic '14]

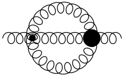
$\rightarrow$ Related to renormalization.

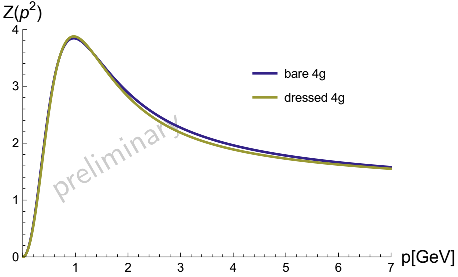
Effect of four-gluon vertex

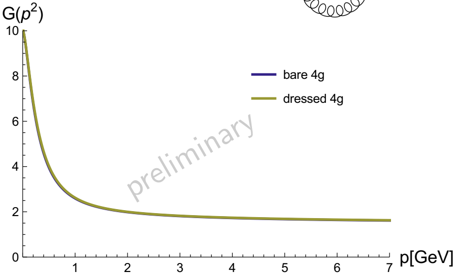


In three-gluon vertex DSE:
Important for convergence within current truncations in $d = 4$
[Blum, MQH, Mitter, von Smekal '14; Eichmann, Williams, Alkofer, Vujinovic '14]
$\rightarrow$ Related to renormalization.

In gluon propagator: Via sunset diagram, small contribution of tree-level dressing; model studies: [Mader, Alkofer '13; Meyers, Swanson '14]







[propagator eqs. full, 3-gluon model, ghost-gluon fixed, 4-gluon fixed]

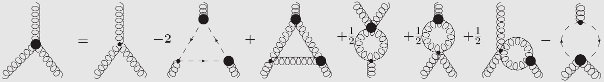
Extending truncations of three-point functions

Extend truncations of three-point functions:

Extending truncations of three-point functions

Extend truncations of three-point functions:

Three-gluon vertex



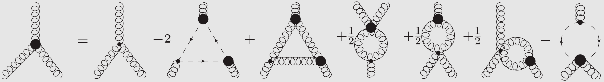
$\rightarrow$ One-loop complete equation.

$\Rightarrow$ Requires the two-ghost-two-gluon vertex.

Extending truncations of three-point functions

Extend truncations of three-point functions:

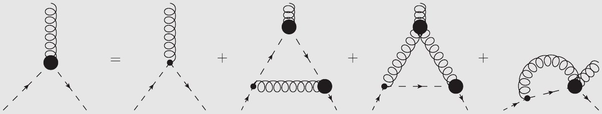
Three-gluon vertex



$\rightarrow$ One-loop complete equation.

$\Rightarrow$ Requires the two-ghost-two-gluon vertex.

Ghost-gluon vertex



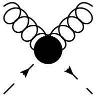
$\rightarrow$ Complete equation.

Four-ghost vertex:

In alternative ghost-gluon vertex DSE and in four-point functions.

The two-ghost-two-gluon vertex

Non-primitively divergent correlation function ($\rightarrow$ no guide from tree-level tensor) appearing in three-point and higher DSEs.



$$\Gamma^{AA\bar{c}c,abcd}_{\mu\nu}(p,q;r,s) = \mathbf{g^4} \sum_{k=1}^{\mathbf{25}} \rho^{k,abcd}_{\mu\nu} D^{AA\bar{c}c}_k(p,q;r,s) \qquad \text{with } \rho^{abcd}_{\mu\nu} = \tau^{abcd} \otimes \sigma_{\mu\nu}$$

Color basis: 8 tensors. Neglect symmetric d^{abc} $\rightarrow$ 5 tensors.

$$\tau^{abcd}_1 = -2f^{acd}f^{bde} + f^{abd}f^{cde}, \qquad \tau^{abcd}_2 = \delta^{ab}\delta^{cd}, \qquad \tau^{abcd}_3 = \delta^{ad}\delta^{bc} + \delta^{ac}\delta^{bd},$$

$$\tau^{abcd}_4 = -\delta^{ad}\delta^{bc} + \delta^{ac}\delta^{bd}, \qquad \tau^{abcd}_5 = f^{abe}f^{cde}.$$

Lorentz basis transverse wrt gluon legs $\rightarrow$ 5 tensors.

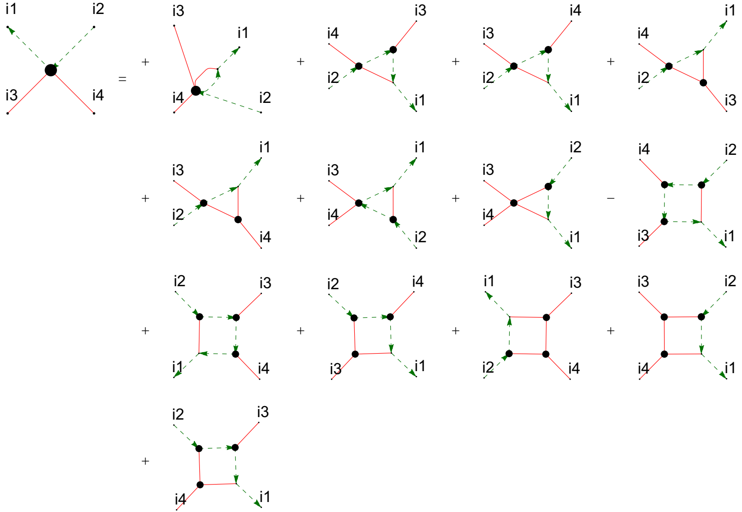
$$\sigma^1_{\mu\nu}(p,q;r,s) = t_{\mu\nu}(p,q), \qquad \sigma^2_{\mu\nu}(p,q;r,s) = t_{\mu\alpha}(p,p)t_{\alpha\nu}(r,q) + t_{\mu\alpha}(p,r)t_{\alpha\nu}(q,q),$$

$$\sigma^4_{\mu\nu}(p,q;r,s) = t_{\mu\alpha}(p,p)t_{\alpha\nu}(q,q), \qquad \sigma^3_{\mu\nu}(p,q;r,s) = t_{\mu\alpha}(p,p)t_{\alpha\nu}(r,q) - t_{\mu\alpha}(p,r)t_{\alpha\nu}(q,q),$$

$$\sigma^5_{\mu\nu}(p,q;r,s) = t_{\mu\alpha}(p,r)t_{\alpha\nu}(r,q), \qquad \text{with } t_{\mu\nu}(p,q) = g_{\mu\nu}\, p\cdot q - p_\mu q_\nu.$$

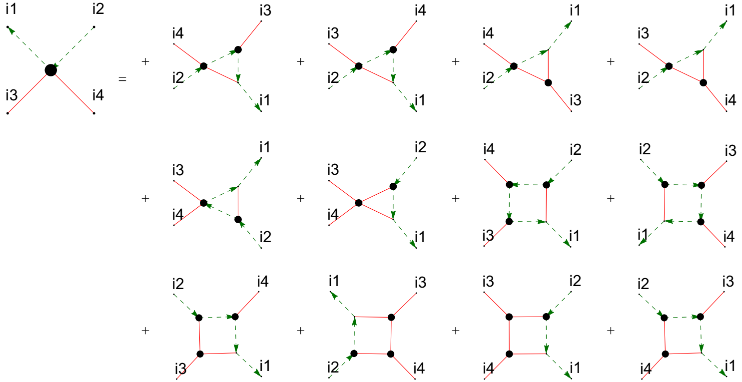
The two-ghost-two-gluon vertex DSE

2 DSEs, choose the one with the ghost leg attached to the bare vertex
$\rightarrow$ Truncation discards only one diagram.



The two-ghost-two-gluon vertex DSE

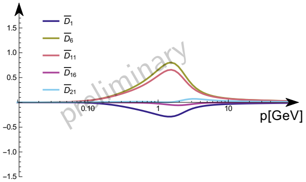
2 DSEs, choose the one with the ghost leg attached to the bare vertex
$\rightarrow$ Truncation discards only one diagram.

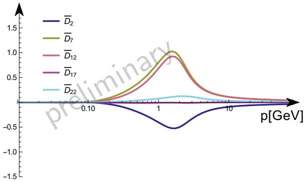


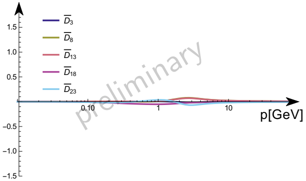
Results for the two-ghost-two-gluon vertex

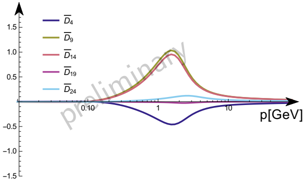
Kinematic approximation: one-momentum configuration

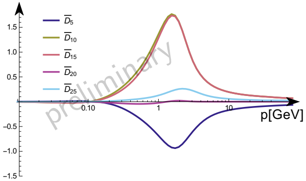
- Dimensionless dressing functions $\overline{D}_k$.
- Each plot one Lorentz tensor.







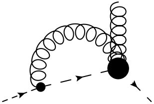


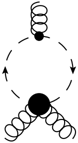


$\rightarrow$ Two classes of dressings: 13 very small, 12 not small

Influence of two-ghost-two-gluon vertex

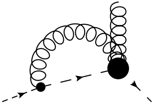
Coupled system of ghost-gluon, three-gluon and four-gluon vertices with and without two-ghost-two-gluon vertex in three-point functions:

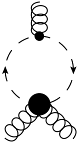


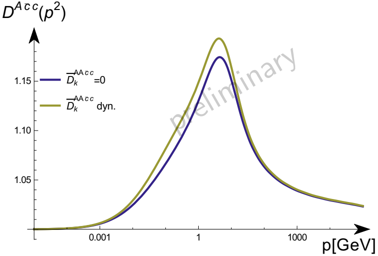


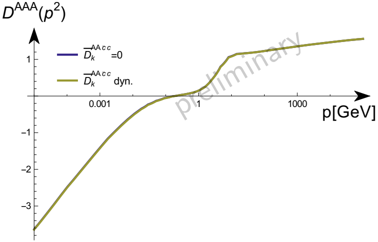
Influence of two-ghost-two-gluon vertex

Coupled system of ghost-gluon, three-gluon and four-gluon vertices with and without two-ghost-two-gluon vertex in three-point functions:





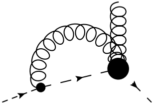


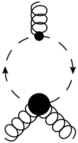


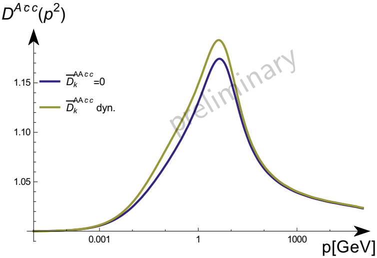
- Small influence on ghost-gluon vertex ($< 1.7\%$)
- Negligible influence on three-gluon vertex

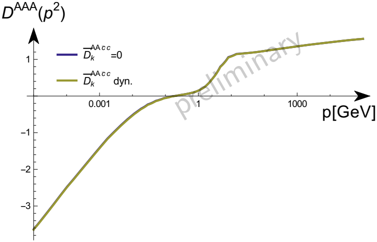
Influence of two-ghost-two-gluon vertex

Coupled system of ghost-gluon, three-gluon and four-gluon vertices with and without two-ghost-two-gluon vertex in three-point functions:









- Small influence on ghost-gluon vertex ($< 1.7\%$)
- Negligible influence on three-gluon vertex
  (only $\overline{D}_{k=16,\dots,25}$ contribute $\leftarrow$ the small ones)

Summary and conclusions

Based on

- tests in $d = 3$ including comparison with 3PI calculations
- analysis of one-loop resummation
- testing a non-primitively divergent correlation function

a non-perturbative hierarchy of correlations functions and diagrams can be identified.

Three- and four-gluon vertices:

1. Cancellations between diagrams
2. Negligible diagrams

Two-loop diagrams in propagators:

Required quantitatively and for self-consistency.

Summary and conclusions

Based on

- tests in $d = 3$ including comparison with 3PI calculations
- analysis of one-loop resummation
- testing a non-primitively divergent correlation function

a non-perturbative hierarchy of correlations functions and diagrams can be identified.

Three- and four-gluon vertices:

1. Cancellations between diagrams
2. Negligible diagrams

Two-loop diagrams in propagators:

Required quantitatively and for self-consistency.

Outlook:

- Two-ghost-two-gluon vertex impact on four-gluon vertex.
- Combine vertex and propagator calculations.

Summary and conclusions

Based on

- tests in $d = 3$ including comparison with 3PI calculations
- analysis of one-loop resummation
- testing a non-primitively divergent correlation function

a non-perturbative hierarchy of correlations functions and diagrams can be identified.

Three- and four-gluon vertices:

1. Cancellations between diagrams
2. Negligible diagrams

Two-loop diagrams in propagators:

Required quantitatively and for self-consistency.

Outlook:

- Two-ghost-two-gluon vertex impact on four-gluon vertex.
- Combine vertex and propagator calculations.

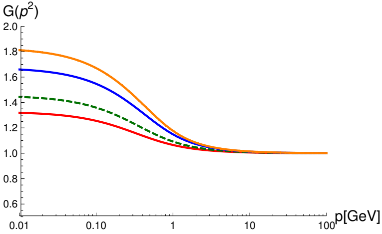
Thank you for your attention!

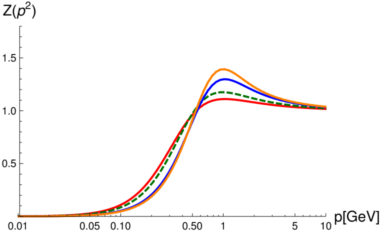
Family of solutions in three dimensions

Cf. FRG results: Bare mass parameter from modified STIs [Cyrol, Fister, Mitter, Pawlowski, Strodthoff '16].

DSEs: Enforce family of solutions by fixing the gluon propagator at $p^2 = 0$.

Simple toy system with bare vertices [MQH, 1606.02068]:





$\Rightarrow$ Possibility of family of solutions.

NB: Effect overestimated here since vertices are fixed.