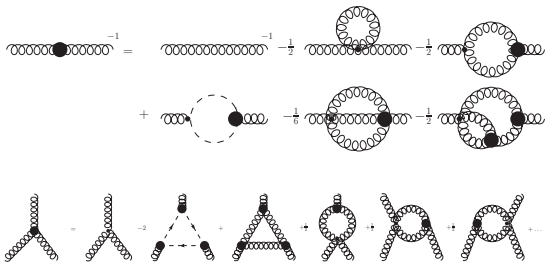
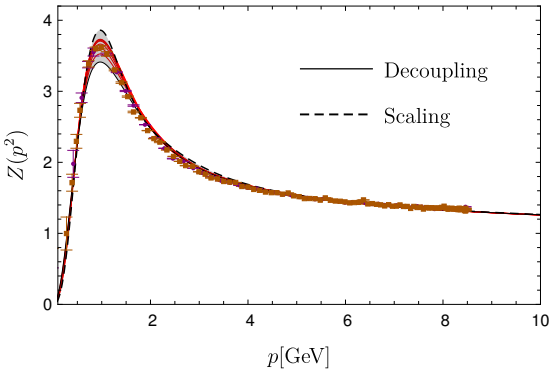
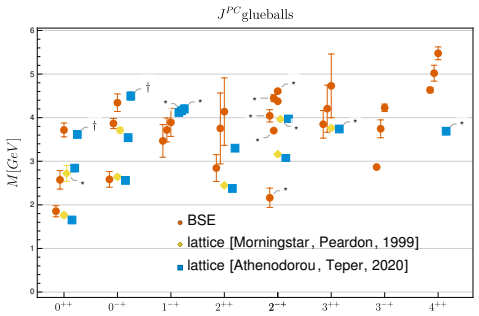
With functional methods from propagators and vertices to glueballs







Markus Q. Huber

Institute of Theoretical Physics, Giessen University

MQH, Phys.Rev.D 101, arXiv:2003.13703

MQH, Fischer, Sanchis-Alepuz, Eur.Phys.J.C 80, arXiv:2004.00415

FunQCD, Valencia, Spain

March 29, 2021

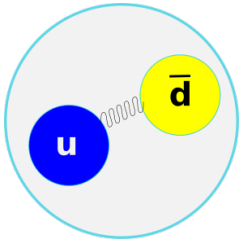
FWF Der Wissenschaftsfonds.

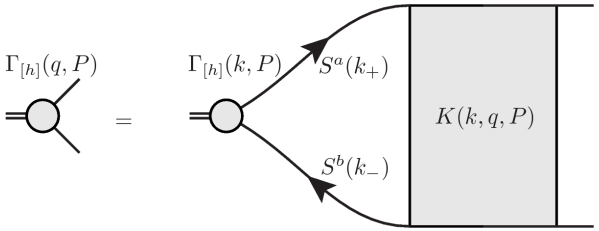
JUSTUS-LIEBIG-UNIVERSITÄT GIESSEN

DFG Deutsche Forschungsgemeinschaft

Hadrons from bound state equations

Example: Meson



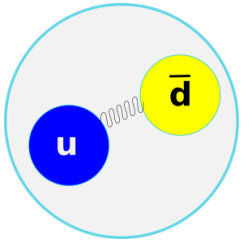


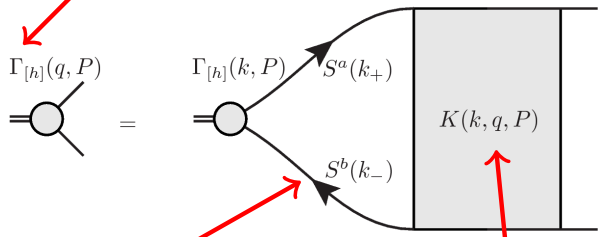
Integral equation: $\Gamma(q,P) = \int dk\, \Gamma(k,P)\, S(k_+)\, S(k_-)\, K(k,q,P)$

Hadrons from bound state equations

Bethe-Salpeter amplitude

Example: Meson





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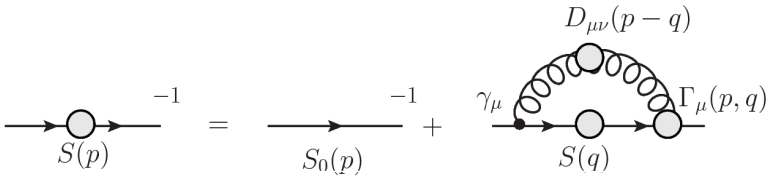
Ingredients:

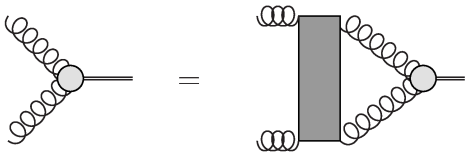
- Quark propagator S



Nonperturbative diagram: full momentum dependent dressings $\rightarrow$ numerical solution

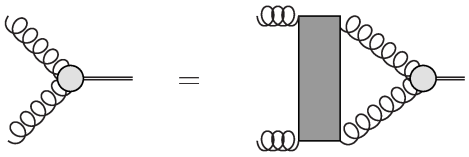
- Interaction kernel K
- Constrained by symmetries

Glueball BSE



Need  and  , solve for  . $\rightarrow$ Mass

Glueball BSE



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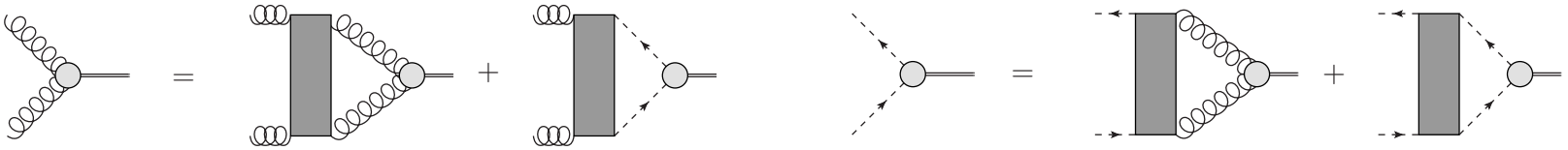
Not quite...

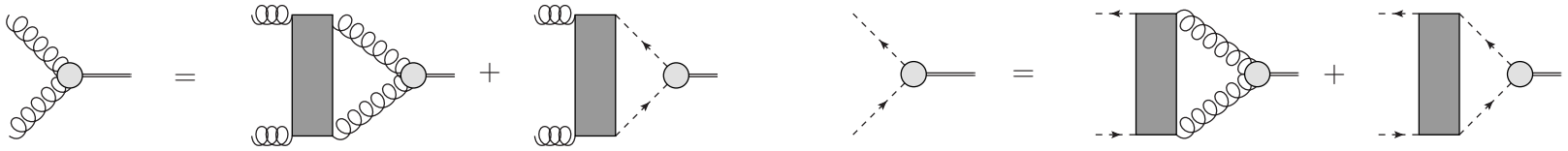
Glueball BSE



Gluons couple to ghosts → Include 'ghostball'-part.

(First step: no quarks → Yang-Mills theory)

Glueball BSE



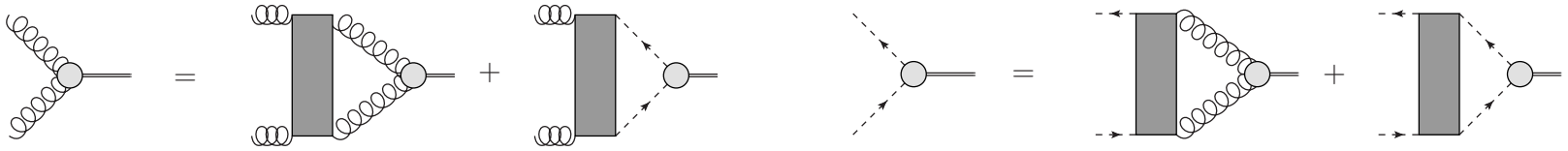
Gluons couple to ghosts → Include 'ghostball'-part. (First step: no quarks → Yang-Mills theory)

Need  ,  and 4×  , solve for  and  . → Mass

Construction of kernel

Consistency with input: Apply same construction principle.

Glueball BSE



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Need  ,  and 4×  , solve for  and  . → Mass

Construction of kernel

Consistency with input: Apply same construction principle.

Previous BSE calculations for glueballs:

- [Meyers, Swanson '13]
- [Sanchis-Alepuz, Fischer, Kellermann, von Smekal '15]
- [Souza et al. '20]
- [Kaptari, Kämpfer '20]

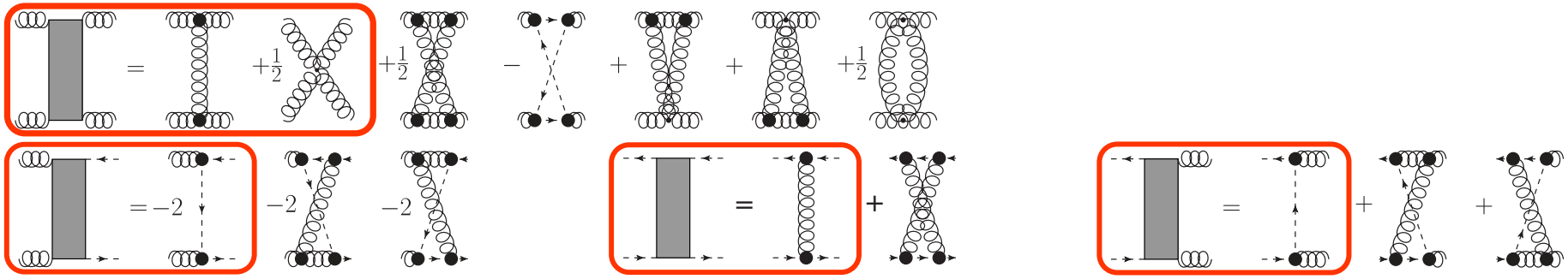
⇒ Input is important for quantitative predictive power!

[MQH, Fischer, Sanchis-Alepuz '20]

Kernel construction

From 3PI effective action truncated to three-loops:

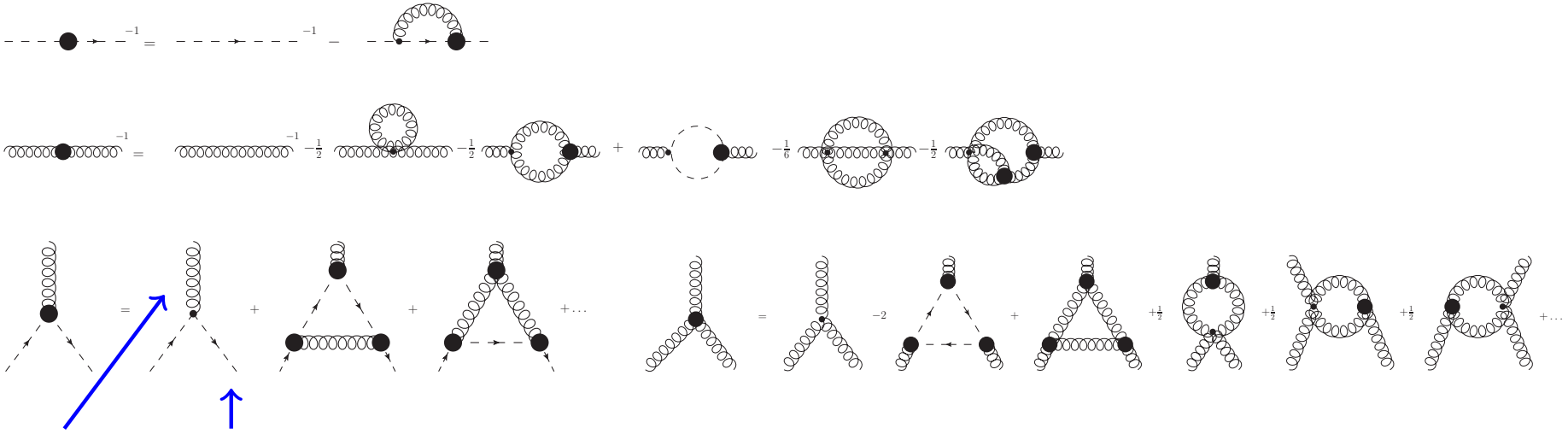
[Fukuda '87; McKay, Munczek '89; Sanchis-Alepuz, Williams '15; MQH, Fischer, Sanchis-Alepuz '20]



$\rightarrow$ Need  .

- Some diagrams vanish for certain quantum numbers.
- Full QCD: Same for quarks $\rightarrow$ Mixing with mesons.

Equations of motion from 3-loop 3PI effective action

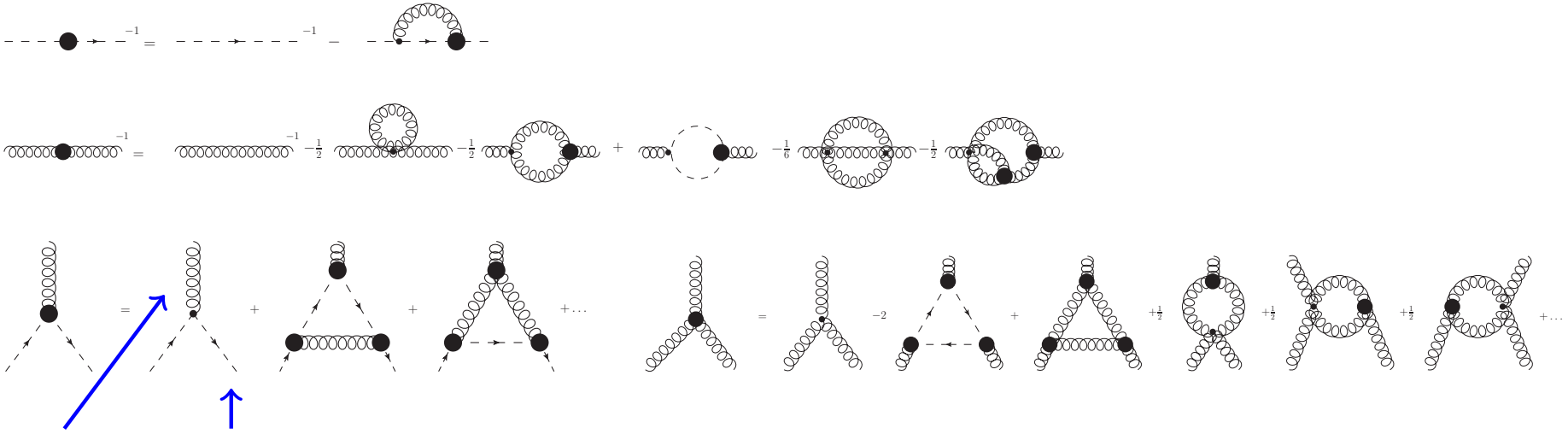


Gluon and ghost fields: Elementary fields of Yang-Mills theory in the Landau gauge

Self-contained system of equations with the scale as the only input.

Truncation?

Equations of motion from 3-loop 3PI effective action



Gluon and ghost fields: Elementary fields of Yang-Mills theory in the Landau gauge

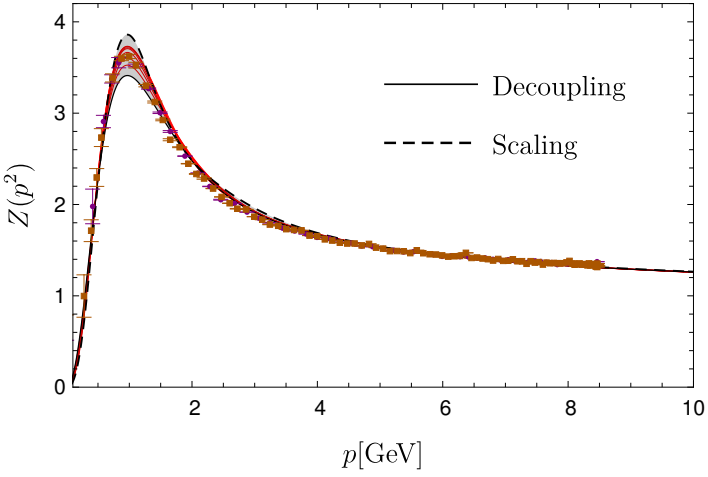
Self-contained system of equations with the scale as the only input.

Truncation→ 3-loop expansion of 3PI effective action [Berges '04]

- 4 coupled integral equations with full kinematic dependence.
- Sufficient numerical accuracy required for renormalization.
- One- and two-loop diagrams [Meyers, Swanson '14; MQH '17].

Landau gauge propagators

Gluon dressing function:



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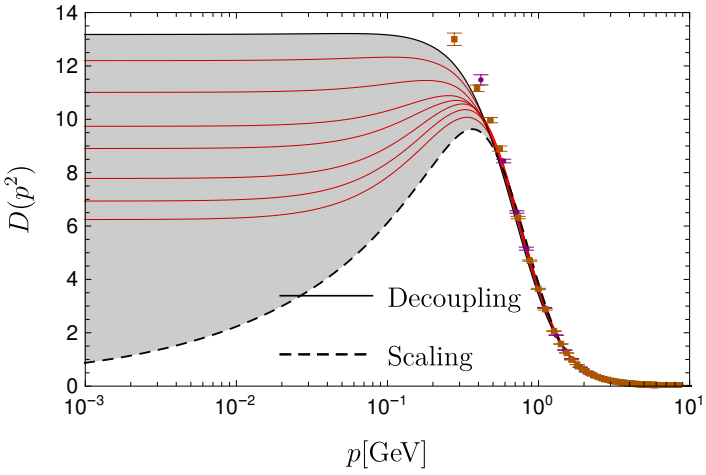
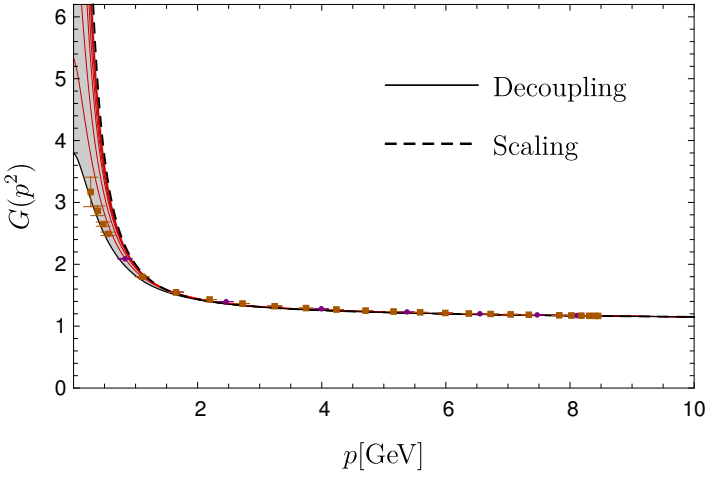


- Family of solutions: Nonperturbative completions of Landau gauge [Maas '10]
- Realized by condition on $G(0)$ [Fischer, Maas, Pawlowski '08; Alkofer, Huber, Schwenzer '08]

Ghost dressing function:

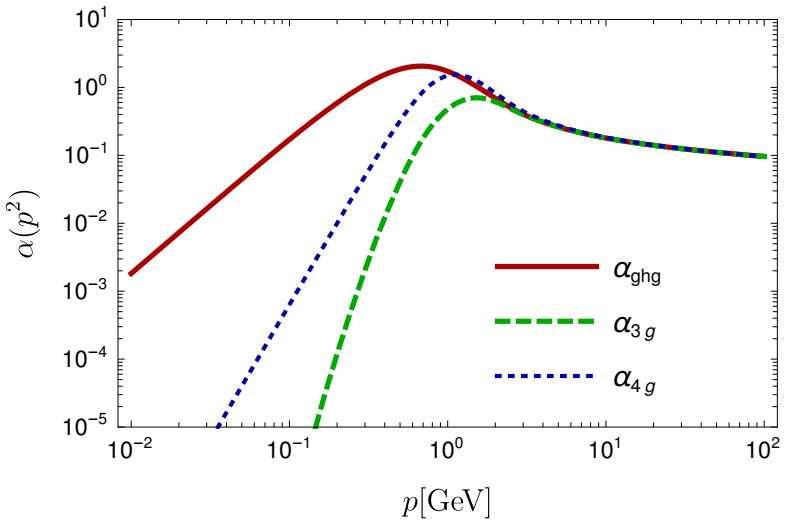


[Sternbeck '06; MQH '20]

Some properties of the Landau gauge solution

[MQH '20]

- Slavnov-Taylor identities (gauge invariance): Vertex couplings agree down to GeV regime



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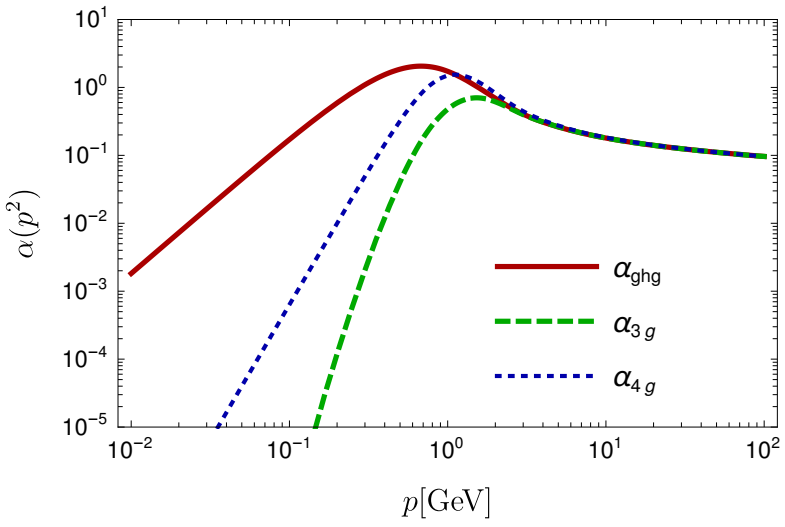
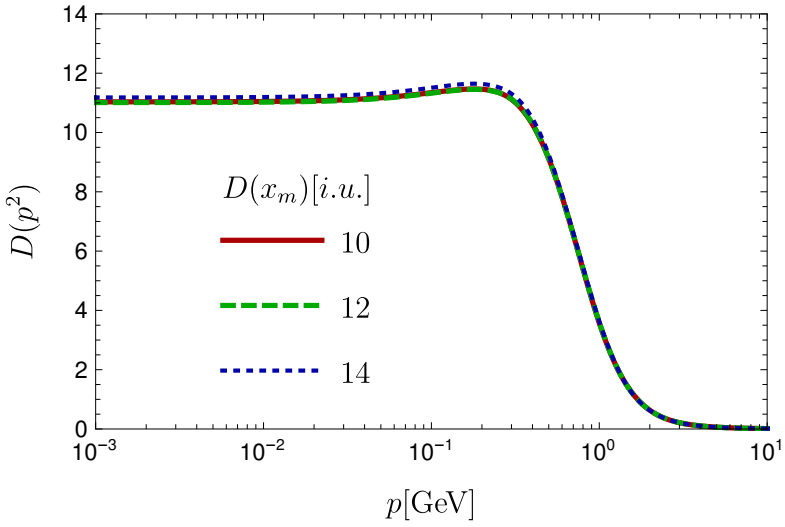


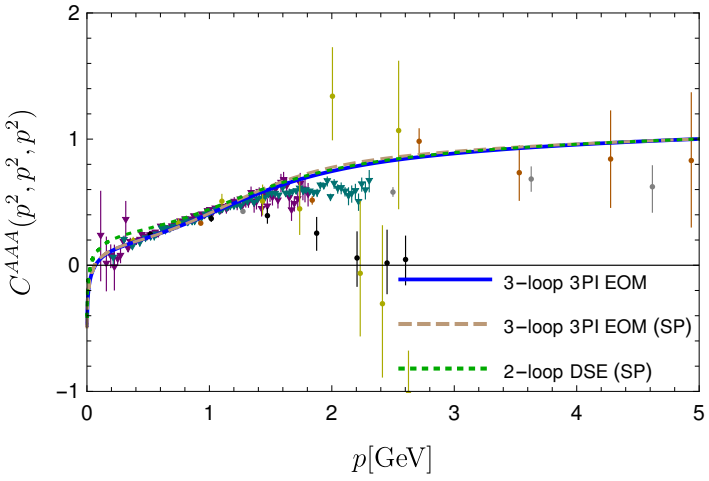
- Renormalization: First parameter-free subtraction of quadratic divergences
  $\Rightarrow$ One unique free parameter (family of solutions)



Concurrence of functional methods

Exemplified with three-gluon vertex.

3PI vs. 2-loop DSE:



[Cucchieri, Maas, Mendes '08; Sternbeck et al. '17; MQH '20]

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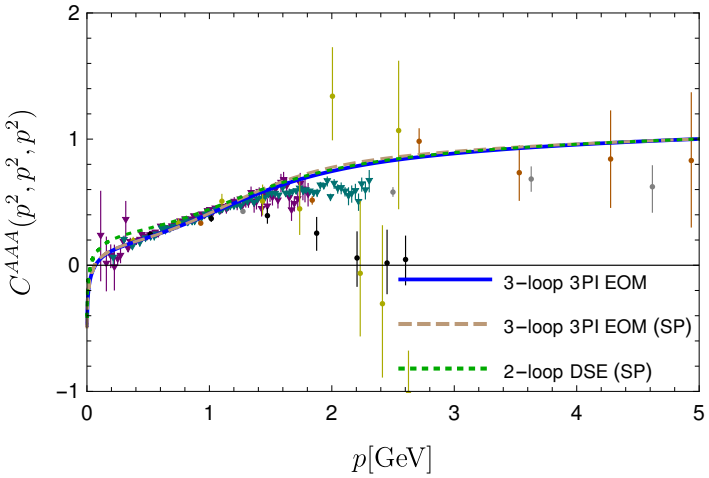

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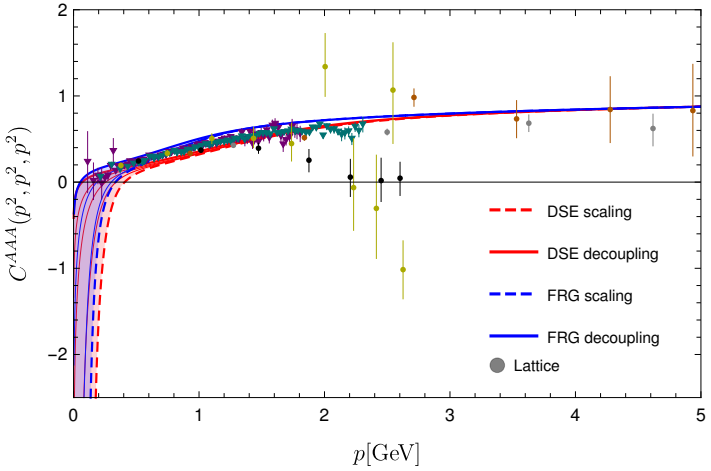
DSE vs. FRG:



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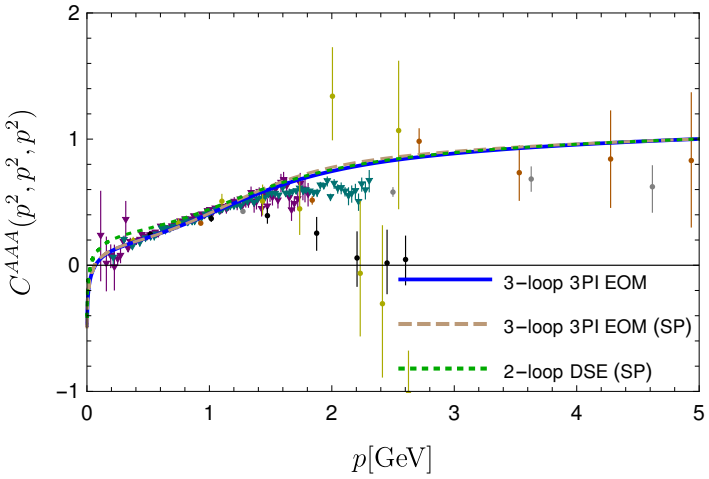

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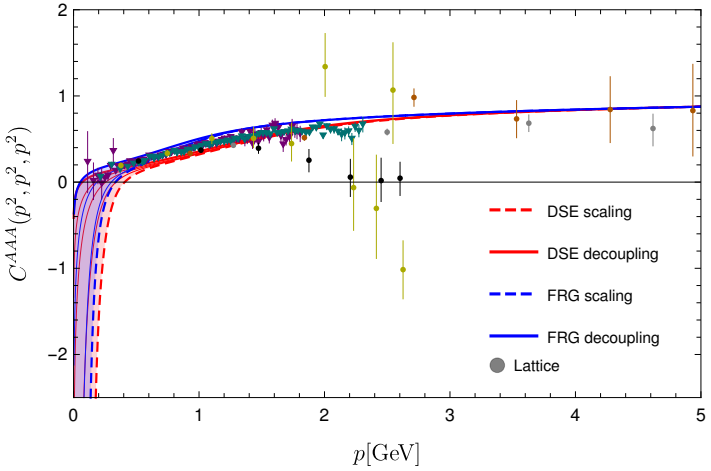
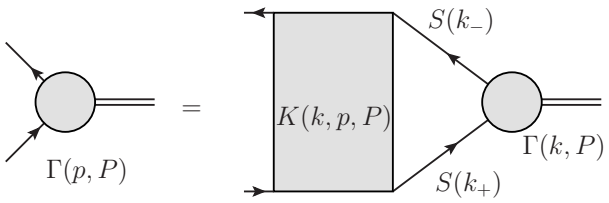


[Cucchieri, Maas, Mendes '08; Sternbeck et al. '17; Cyrol et al. '16; MQH '20]

Beyond this truncation

- Further dressings of three-gluon vertex [Eichmann, Williams, Alkofer, Vujinovic '14]
- Effects of four-point functions [MQH '16, MQH '17, Corell et al. '18, MQH '18]

Solving a BSE



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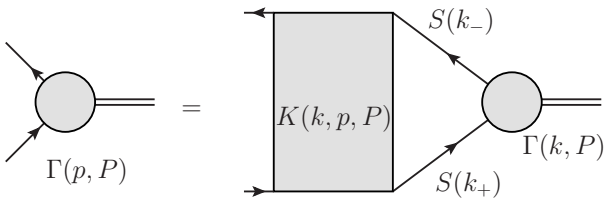

Consider the eigenvalue problem (Γ is the BSE amplitude)

$$\mathcal{K} \cdot \Gamma(P) = \boldsymbol{\lambda(P)}\, \Gamma(P).$$

$\lambda(P^2) = 1$ is a solution to the BSE $\Rightarrow$ Glueball mass $P^2 = -M^2$

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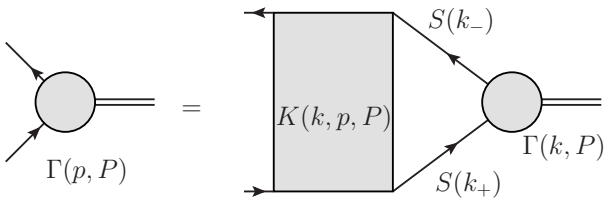

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Calculation requires quantities for

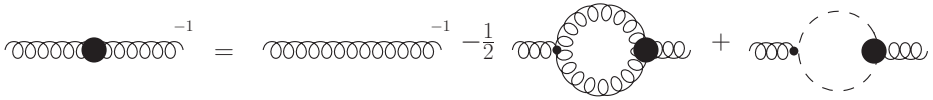
$$k_\pm^2 = P^2 + k^2 \pm 2\sqrt{P^2\, k^2}\cos\theta = -M^2 + k^2 \pm 2\, \boldsymbol{i}\, M\, \sqrt{k^2}\, \cos\theta.$$

$\Rightarrow$ Complex momentum arguments.

Landau gauge propagators in the complex plane

Technique to resp. analyticity (avoid branch cuts in integrand): Contour deformation

Simpler truncation:



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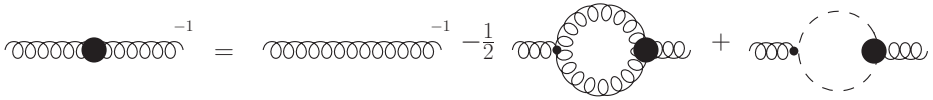
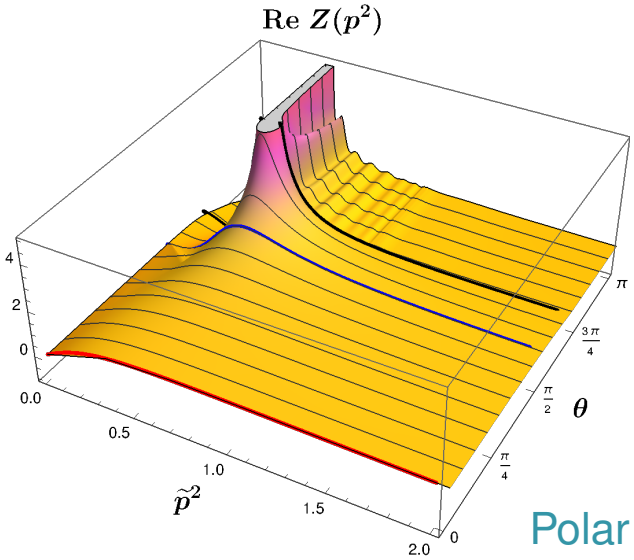
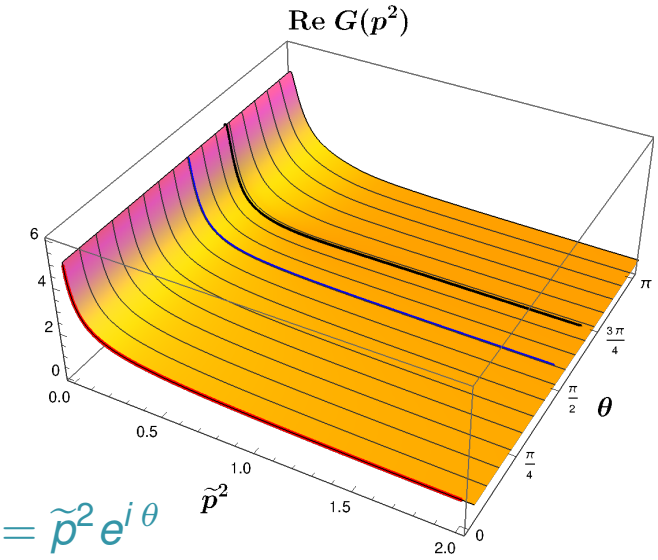






Polar coordinates: $p^2 = \widetilde{p}^2 e^{i\,\theta}$

- Current truncation leads to a pole-like structure in the gluon propagator.
- Analyticity up to 'pole' confirmed by various tests (Cauchy-Riemann, Schlessinger, reconstruction)
- No proof of existence of complex conjugate poles due to simple truncation.

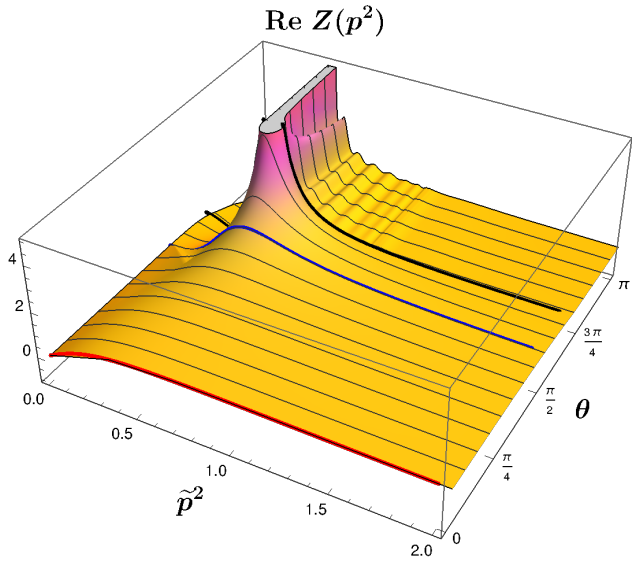
[Fischer, MQH '20]

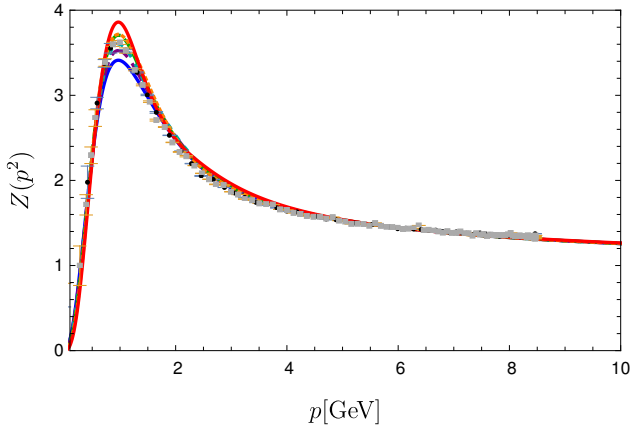
Input for glueballs

Low quality results in complex plane

vs.

Quantitative results for real momenta





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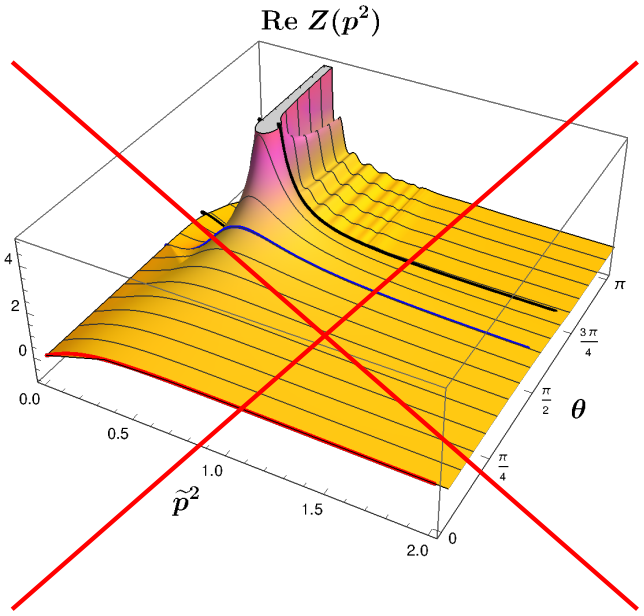

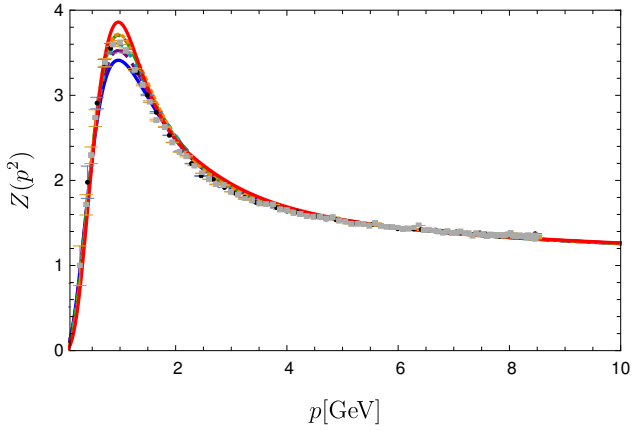

$\Rightarrow$ Solve eigenvalue problem for $P^2 > 0$ and extrapolate $\lambda(P^2)$ to glueball mass.

Extrapolation of $\lambda(P^2)$

Extrapolation method

- Extrapolation to time-like P^2 using Schlessinger's continued fraction method (proven superior to default Padé approximants) [Schlessinger '68]
- Average over extrapolations using subsets of points for error estimate

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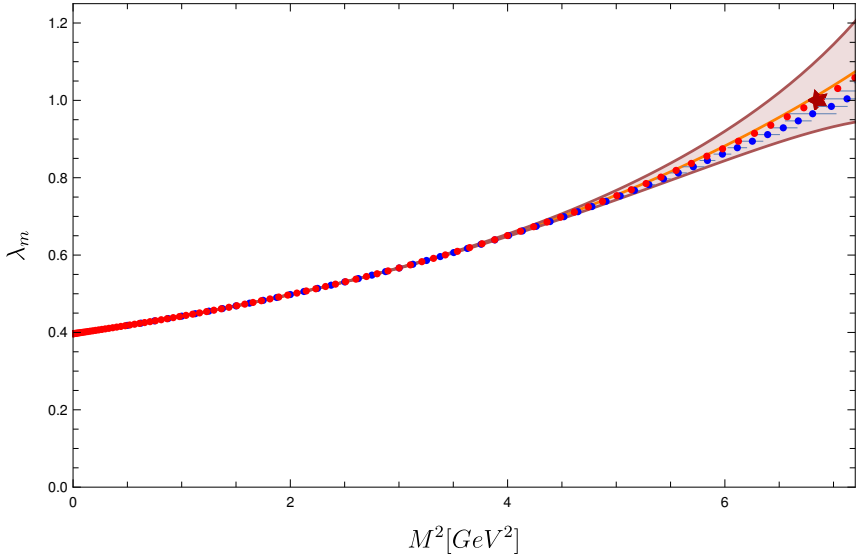
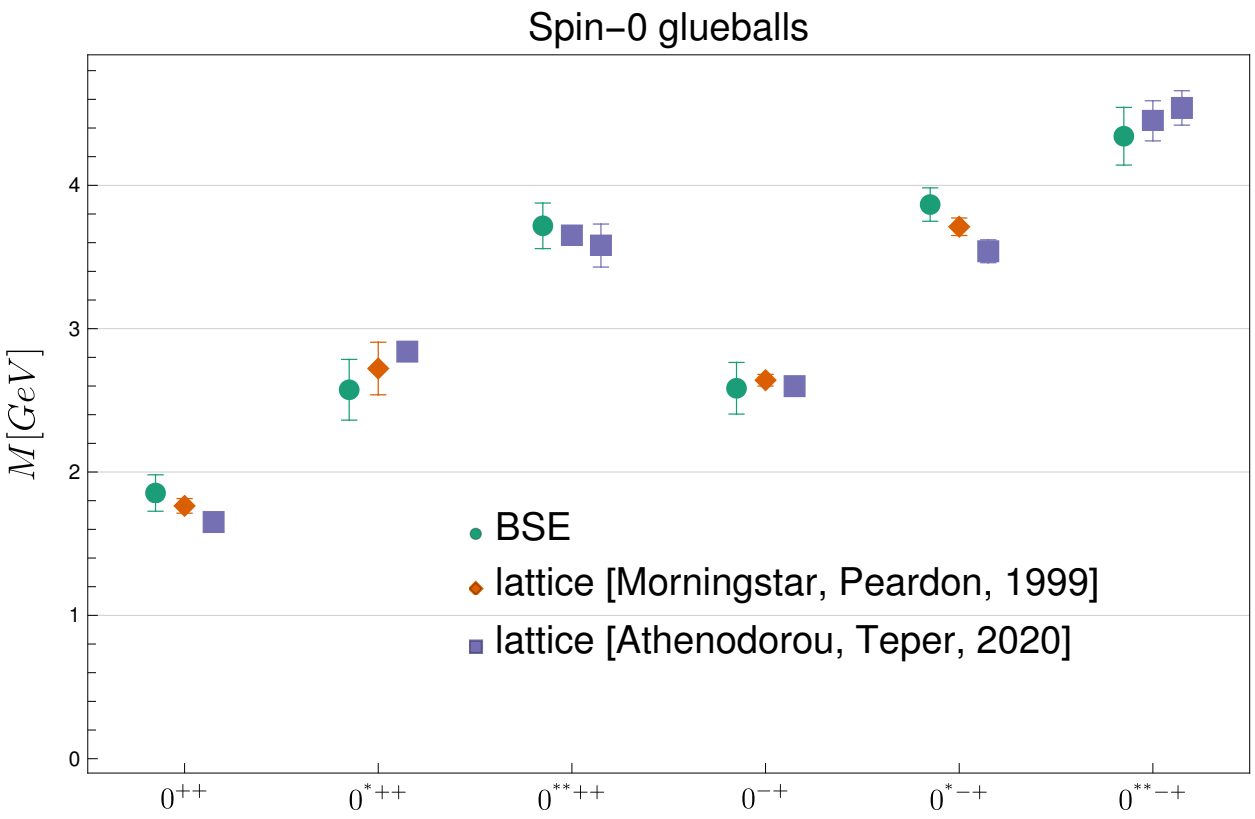
Test extrapolation for solvable system: Heavy meson



[MQH, Sanchis-Alepuz, Fischer '20]

Glueballs masses for $0^{\pm+}$



Lattice 0^{**++}: Conjectured based on irred. rep. of octahedral group

All results for $r_0 = 1/418(5)$ MeV.

[MQH, Fischer, Sanchis-Alepuz '20]

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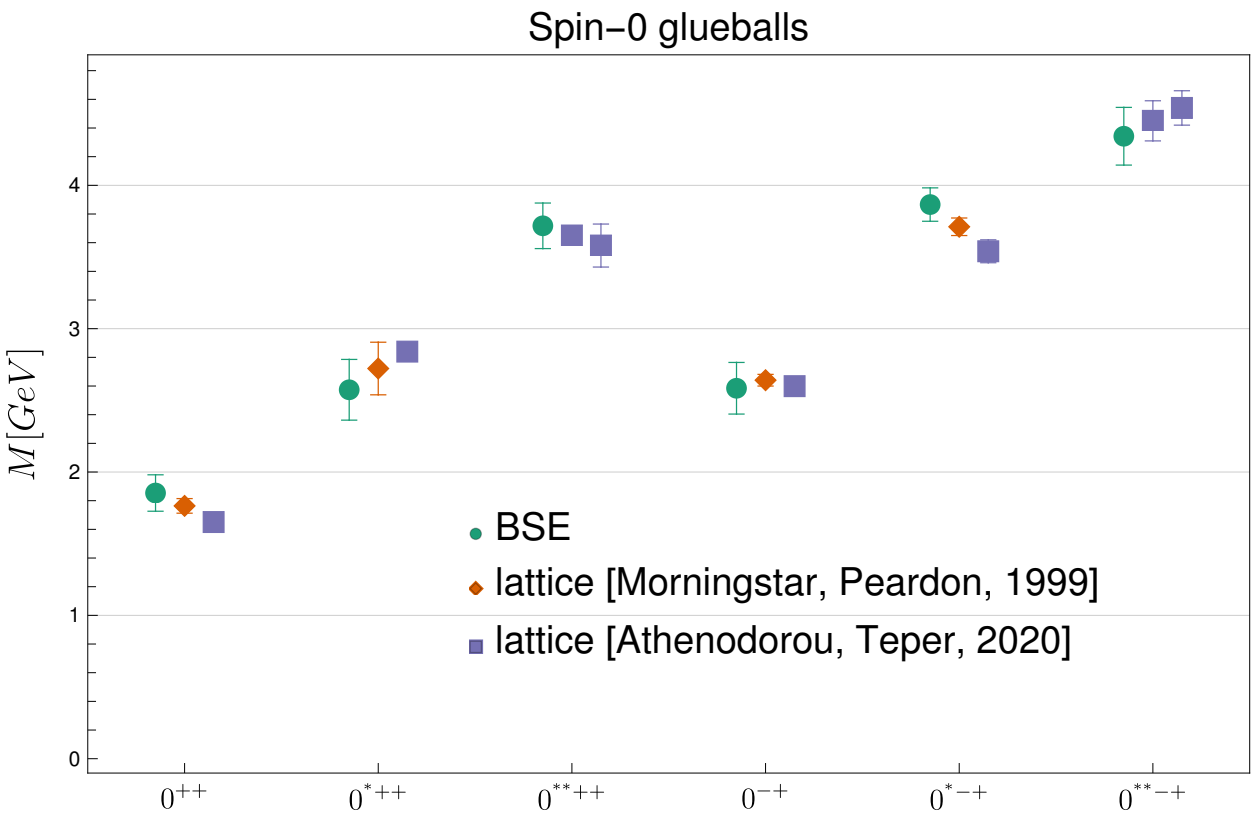

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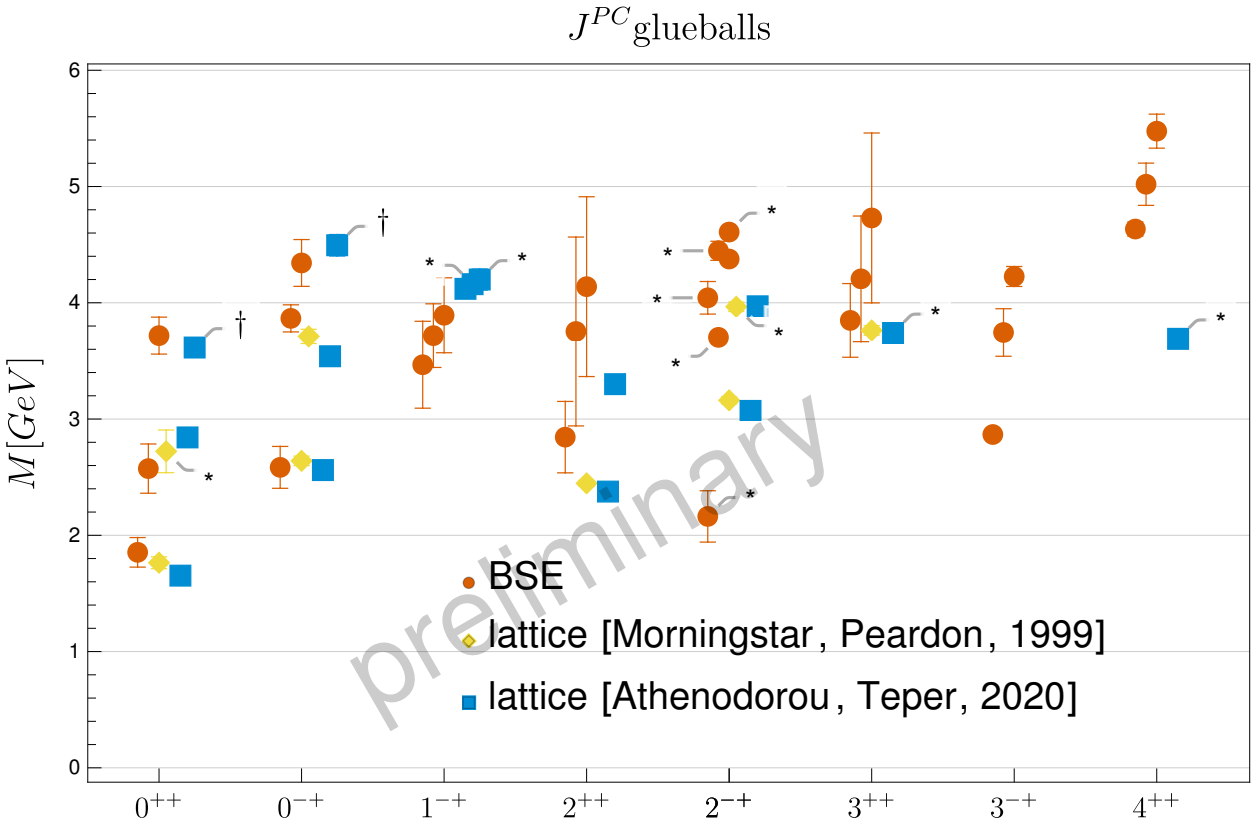
All results for $r_0 = 1/418(5)$ MeV.

[MQH, Fischer, Sanchis-Alepuz '20]

Under conjecture that choice of solution is a gauge choice: Explicit test of gauge independence!

Tested that results are independent of family of solutions.

Glueball masses for $J^{\pm+}$



Lattice:
*: identification with some uncertainty
†: conjecture based on irred. rep of octahedral group

[MQH, Fischer, Sanchis-Alepuz, in preparation]

Summary

Parameter-free determination of glueball masses from functional methods.

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  - Comparison with lattice results
  - Concurrence of different functional methods
- Connection to observables: Glueballs
- Systematic improvements (now) possible
- Direct access to analytic structure

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Parameter-free determination of glueball masses from functional methods.

- Quantitatively reliable correlation functions (Euclidean) from functional equations
  - Comparison with lattice results
  - Concurrence of different functional methods
- Connection to observables: Glueballs
- Systematic improvements (now) possible
- Direct access to analytic structure

Thank your for your attention.

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Hadron masses from correlation functions of color singlet operators.

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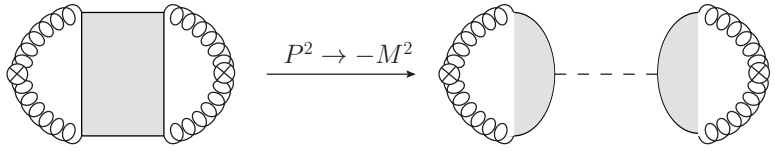
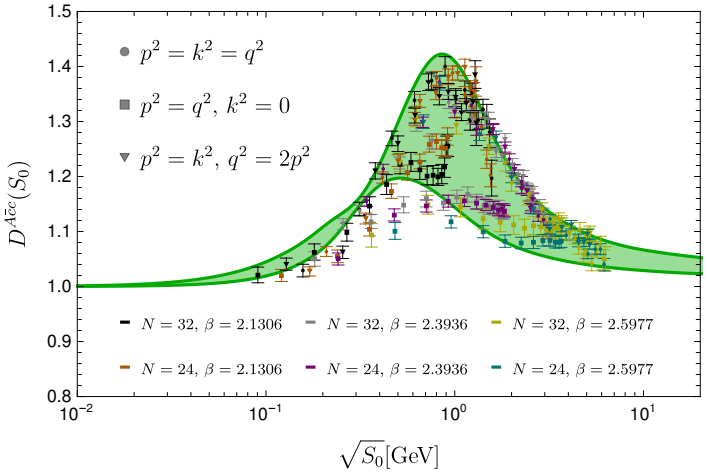
Put total momentum on-shell and consider individual 2-, 3- and 4-gluon contributions. $\rightarrow$ Each can have a pole at the glueball mass.

A^4-part of $D(x-y)$, total momentum on-shell:



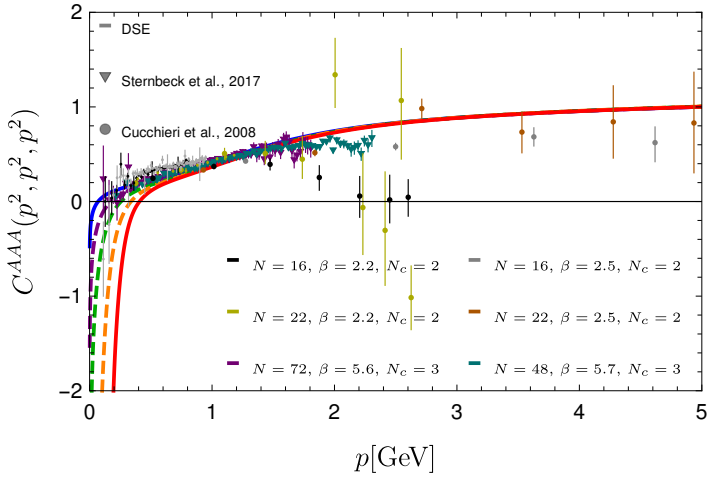
Landau gauge vertices

Ghost-gluon vertex:



[Maas '19; MQH '20]

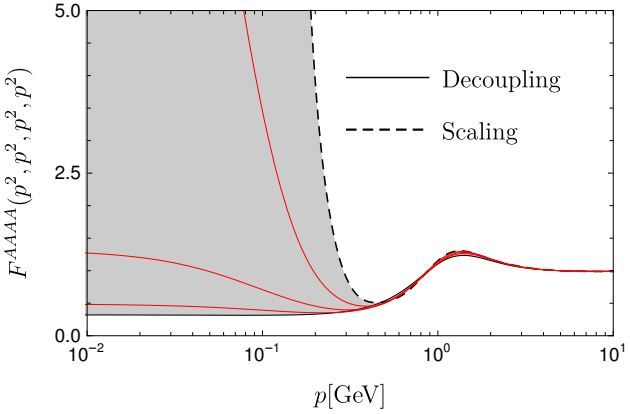
Three-gluon vertex:



[Cucchieri, Maas, Mendes '08; Sternbeck et al. '17; MQH '20]

- Nontrivial kinematic dependence of ghost-gluon vertex
- Simple kinematic dependence of three-gluon vertex
- Four-gluon vertex from solution

Four-gluon vertex:



[MQH '20]

Landau gauge propagators in the complex plane

Propagators for complex momenta

- Reconstruction from Euclidean results: mathematically ill-defined, bias in solution [talk by Oliveira]
- Direct calculation from functional methods possible, e.g., contour deformation or spectral DSEs [Horak, Pawlowski, Wink '20; → talk by Horak]

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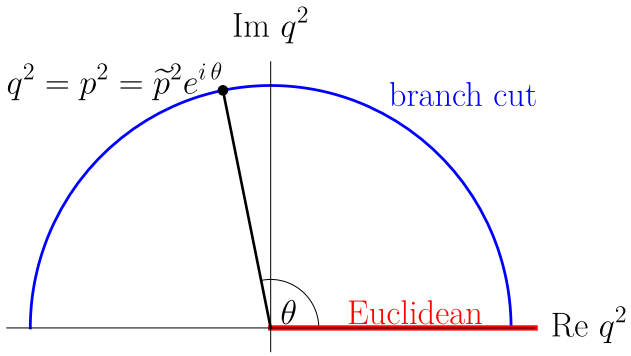
Contour deformation: Special technique to respect analyticity (avoid branch cuts in the integrand)

- QED3 [Maris '95 (QED)]
- Quark propagator [Alkofer, Fischer, Detmold, Maris '04]
- Self-consistent solution: Ray technique, YM propagators [Strauss, Fischer, Kellermann '12; Fischer, MQH '20]
- Glueball correlators [Windisch, Alkofer, Haase, Liebmann '13; Windisch, MQH, Alkofer '13]
- Meson decays [Weil, Eichmann, Fischer, Williams '17; Williams '18]
- Spectral functions at $T > 0$ [Pawlowski, Strodthoff, Wink '18]
- Quark-photon vertex [Miramontes, Sanchis-Alepuz '19]
- Scalar scattering amplitude [Eichmann, Duarte, Pena, Stadler '19]

Landau gauge propagators in the complex plane



Appearance of branch cuts for complex momenta forbids integration directly to cutoff.

Deformation of integration contour necessary [Maris '95]. Recent resurgence: [Alkofer et al. '04; Windisch, MQH, Alkofer, '13; Williams '19; Miramontes, Sanchis-Alepuz '19; Eichmann et al. '19], ...

Ray technique for self-consistent solution of a DSE: [Strauss, Fischer, Kellermann; Fischer, MQH '20].

Extrapolation of $\lambda(P^2)$ for glueballs

Higher eigenvalues: Excited states.

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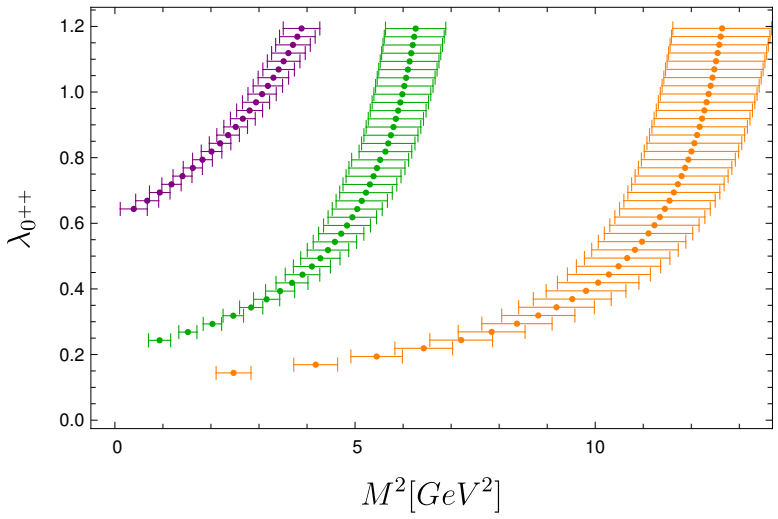

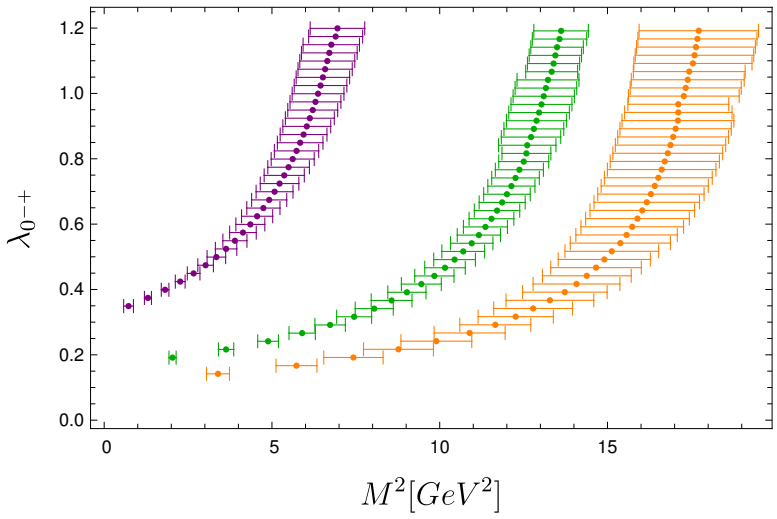

Physical solutions for $\lambda(P^2) = 1$.