# Infrared Behavior of Yang-Mills Green Functions in the Maximally Abelian Gauge 

Reinhard Alkofer, Markus Q. Huber, Kai Schwenzer Instituu für Physik, Fachbereich Theoretische Physik

## Infrared scaling solutions

The Green functions of Yang-Mills theory have been intensively studied during the last few years using functional approaches such as Dyson-Schwinger equations (DSEs) and Functional Renormalization Group equations (RGEs). Landau gauge (LG) is the best studied gauge using these techniques, and its IR scaling solution [1, 2], i.e. all dressing functions are characterized by power laws with so-called infrared exponents (IRE), can explain gluon [3] as well as quark confinement [2]. A possible strategy to learn about gauge-independent aspects is to investigate also other gauges. We present here a method to assess the IR behavior of gauges based on the information obtained in LG and show the results for the maximally Abelian gauge.

## The maximally Abelian gauge (MAG)

For the MAG we decompose the gauge fields as

$$
A_{\mu}=A_{\mu}^{i} T^{i}+B_{\mu}^{a} T^{a}
$$

where $T^{i}$ are the diagonal generators of the group $S U(N)$ and $T^{a}$ the off-diagonal ones. Consequently the $A^{i}$ field is called the diagonal field and $B^{a}$ the off-diagonal one. The notion of this gauge goes back to the idea of Abelian dominance in the IR [4], which means that the diagonal field components dominate. This can be achieved by fixing the gauge such that the norm of the off-diagonal components is minimized with respect to gauge transformations. Also the ghost fields $c$ split. Usually the remaining gauge freedom of the diagonal fields is fixed to the LG condition.
A further complication in MAG is the necessity of additional terms including a quartic ghost interaction to guarantee renormalizability. One simplification is the fact that the diagonal ghosts decouple from the system. In total the number of interactions is considerably larger than that of LG and depends on the gauge group. The table below shows the possible interactions for $\operatorname{SU}(2)$ and the additional ones for $\operatorname{SU}(3)$.

> |  | Three-Point | Four-Point |
| :--- | :--- | :--- |
| SU(2) | ABB, Acc | AABB, AAcc, BBBB, cccc, BBcc |
| SU(3) | $+\mathrm{Bcc}, \mathrm{BBB}$ | $+\mathrm{ABcc}, \mathrm{ABBB}$ |

The right-hand sides of the DSEs for the diagonal gluon (red), the off-diagonal gluon (magenta) and the off-diagonal ghost (green) propagators are:


## General approach

Assuming power laws for all dressing functions one can write down the IRE $\delta_{v}$ of an arbitrary diagram $\boldsymbol{v}$. We denote the set of fields by $X_{i}$, which is $\{A, B, c\}$ for MAG. Using topological relations this expression can be rewritten such that it does no longer depend on the number of internal propagators but on the number of legs $m^{X_{i}}$, the number of dressed and bare vertices $n_{d}^{X_{1} \ldots X_{k}}$ and $n_{b}^{X_{1} \ldots X_{k}}$, respectively, and the IREs $\delta_{X_{i}}$ of propagators and $\delta_{X_{1} \ldots X_{k}}$ of vertices. $k_{X_{i}}^{X_{1} \ldots X_{k}}$ denotes the number of times the field $X_{i}$ appears in the vertex $X_{1} \ldots X_{k}$. The resulting expression is valid for generic diagrams, i.e.
including those of DSEs and RGEs:

$$
\begin{aligned}
\delta_{v}= & -\frac{1}{2} \sum_{i} m^{X_{i}} \delta_{X_{i}}+\sum_{\text {vertices }, k \geq 3} n_{b}^{X_{1} \ldots X_{k}} \sum_{i} \frac{1}{2} k_{X_{i}}^{X_{1} \ldots X_{k}} \delta_{X_{i}}+ \\
& +\sum_{\text {vertices, } k \geq 3} n_{d}^{X_{1} \ldots X_{k}}\left(\delta_{X_{1} \ldots X_{k}}+\frac{1}{2} \sum_{i} k_{X_{i}}^{X_{1} \ldots X_{k}} \delta_{X_{i}}\right)
\end{aligned}
$$

For the remaining analysis important input comes from constraints given by the DSEs and RGEs in the IR. For example one can immediately tell that the IRE of off-diagonal fields is non-negative, which is due to the following graphs:

Constraints from the RGEs are stronger, since the DSEs always contain one bare vertex. For example using the following diagrams and generalizations for other vertex functions one can conclude that the coefficients of $n_{b}^{X_{1} \ldots X_{k}}$ and $n_{b}^{X_{1} \ldots X_{k}}$ are non-negative:


The same conclusion can be drawn from the assumption of a non-divergent skeleton expansion, an argument employed in ref. [5] for LG. Therefore the maximally IR divergent solution does not depend on any kind of vertex and is given by

$$
\delta_{v}=-\frac{1}{2} \sum_{i} m^{X_{i}} \delta_{X_{i}} .
$$

All constraints together restrict the system sufficiently to determine the leading diagrams of every DSE.

## General results of the IR analysis:

- Self-interacting fields have a non-negative IRE.
- The maximally IR divergent solution depends solely on the numbers of external legs $m_{X_{i}}$.
- The IREs of individual diagrams in the maximally IR divergent solution are determined by their bare vertices, i.e. all diagrams of an RGE have the same IRE.
- A negative IRE for a propagator is only possible if the tree-level term is subtracted in the renormalization process, i.e. by imposing appropriate boundary conditions.


## IR scaling solution for the MAG

The solution for the propagator IREs is unique:

$$
0 \geq \delta_{A}=-\delta_{B}=-\delta_{c}
$$

The leading diagrams in the propagator DSEs are the sunsets with four-point interactions containing two diagonal gluons and possibly the corresponding squint diagrams. Whether those scale equally as the sunsets is still to be determined and depends on the IR behavior of three-point and higher n-point functions. For the former there exists besides the maximally IR divergent solution a second one. The qualitative solution for the propagators is in accordance with the Abelian dominance hypothesis.

All DSEs and diagrams of this poster were produced using DoDSE [6].
[1] R. Alkofer, C. S. Fischer and F. J. Llanes-Estrada, Phys. Lett. B $\mathbf{6 1 1}$ (2005) 279
[2] R. Alkofer, C. S. Fischer, F. J. Llanes-Estrada and K. Schwenzer, Annals Phys. (in print)
[3] L. von Smekal, A. Hauck and R. Alkofer, Annals Phys. 267 (1998) 1
[4] Z. F. Ezawa and A. Iwazaki, Phys. Rev. D 25 (1982) 2681
[5] R. Alkofer, M. Q. Huber and K. Schwenzer, arXiv:0801.2762 [hep-th]
[6] R. Alkofer, M. Q. Huber and K. Schwenzer, arXiv:0808.2939 [hep-th]

